

1) The energy lost per turn for a single electron is given by:

$$U_0 (\text{keV}) = \frac{e^2 \gamma^2}{3\epsilon_0 \rho} = 88.5 \cdot \frac{E^4 (\text{GeV})}{\rho (\text{m})}$$

electron energy
 ← bend radius
 for 0.84×10^3 fill factor

The FCC energy range is $45 \rightarrow 175 \text{ GeV}$.

The corresponding energy lost per turn is:

$$\frac{88.5 \times 45^4}{11 \times 10^3} = 3.299 \times 10^4 \text{ keV} \quad [45 \text{ GeV}]$$

$$= \underline{\underline{0.03299 \text{ GeV}}}$$

$$\frac{88.5 \times 175^4}{11 \times 10^3} = 7.546 \times 10^6 \text{ keV} \quad [175 \text{ GeV}]$$

$$= \underline{\underline{7.55 \text{ GeV}}}$$

- This confirms the data in the parameters table

$$\frac{88.5 \times 120^4}{11 \times 10^3} = 1.6683 \times 10^3 \text{ keV}$$

$$= \underline{\underline{1.668 \text{ GeV}}}$$

This loss per turn per electron can be converted into total power loss.

- Power = $\frac{\text{Energy}}{\text{Time}}$ - if we have E (per electron per turn) then for N electrons in a beam: $E_{\text{beam per turn}} = N E_{\text{elect per turn}}$

- Then a current can be defined as the rate of flow of charge:

- per beam there are N electrons of charge e , so total charge Ne .

- They circuit it time T

$$I_b = \frac{Ne}{T}$$

Therefore, the power $P[\text{W}] = U_0[\text{eV}] \cdot I_b[\text{A}]$

$$P_{45} = 0.03299 \times 10^9 [\text{eV}] \times 1.450 [\text{A}] \\ = \underline{47.84 \text{ MW}}$$

$$P_{175} = 1.668 \times 10^9 [\text{eV}] \times 30 \times 10^{-3} [\text{A}] = 50.04 \times 10^6 \text{ W} \\ = \underline{50.04 \text{ MW}}$$

Again, this confirms the data in the table

Also, All values for P_{beam} are 50 MW-
because of the limit set on synch ~~radiation~~^{radiation}
Power.

- The rf system must compensate for
the power loss. Therefore, per turn, the
rf must provide:

50 MW of power to each
beam

- Then converting that power to energy

$$\begin{aligned} E &= 50 \text{ MW} \times T_{\text{rev}} = 50 \text{ MW} \times 3.33 \times 10^{-4} \text{ [s]} \\ &= 1.665 \times 10^4 \text{ J} \\ &= \underline{16.65 \text{ kJ}} \end{aligned}$$

This is assuming an even distribution.

In the end, the synchrotron radiation is "collimated"
in a specific direction for $\beta=1$ so maybe
the heating on the outer side will be
greater.

-you could change this to temp using
specific heat.

2) Any angle greater than the critical angle
has a small contribution. The radiation
peaks at $\approx 0.3 \theta_c$.

$$\theta_c = \frac{1}{\gamma} \left(\frac{W_c}{W} \right)^{1/3} \quad W_c = \frac{3}{2} \frac{c}{\rho} \gamma^3$$

$$W_c = \frac{3}{2} \times \frac{3 \times 10^8}{11 \times 10^3} \left(\frac{46.5}{0.6005} \right)^3 = 1.798 \times 10^6 \text{ Hz} = \underline{\underline{3.08 \times 10^{19}}}$$

~~$= 1.798 \text{ MHz}$~~

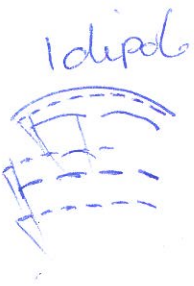
$$f_c = \frac{W_c}{2\pi} = 4.9019 \times 10^{18} \text{ Hz}, \quad \lambda_c = \frac{3 \times 10^8}{4.9019 \times 10^{18}} = \underline{\underline{6.12 \times 10^{-11} \text{ m}}}$$

3) The dipole filling factor is the factor of the lattice cell that is filled with dipoles.

$$\text{Filling factor} = \frac{L_{\text{dipoles}}}{L_{\text{cell}}}$$

There is a trade off between the requirement of low beta and the requirements of less synchrotron radius. Ideally a fill factor of 1 would reduce the magnet field strength. However, there need to be insertion devices and diagnostics.

$$L \uparrow \theta \uparrow, N \uparrow L \downarrow \theta \downarrow$$



~~filling factor \propto $L_{\text{dipole}}, N_{\text{dipoles}}, \theta$~~

~~well you could argue for a fill factor of 1 there is 1 dipole and the more dipoles, the lower the fill factor~~

fill factor \propto L, θ, N

Also, But the more ~~the~~ magnets, the less consistent the field will be!

4) $E_{min} = \frac{C_j \theta^3 \gamma^2}{J_x \sqrt{15} \cdot 12}$ if $N \uparrow$ then $\theta \downarrow$ $\theta = \frac{2\pi}{N}$
 $\Rightarrow E_{min} \propto \frac{1}{N^3}$ $\theta^3 = \frac{(2\pi)^3}{N^3}$
 100 Feds \circledast

$E_{min} = (2\pi)^3 C_j \gamma^2$
 $J_x N^3$
 $C_j = 3.84 \times 10^{-10} \text{ m}$
 1- β Damping \circledast

$\gamma = \frac{175 \times 10^9}{0.5 \times 10^6}$

$\Rightarrow E_{min} = (2\pi)^3 \times 3.84 \times 10^{-10} \text{ mm} \cdot \left(\frac{175 \times 10^9}{0.5 \times 10^6}\right)^2$

$12 \cdot \sqrt{15} N^3$

~~$= \frac{11.668 \times 100}{12 \sqrt{15} N^3} \text{ [mm]} \frac{25106 \text{ [mm]}}{N^3}$~~

The target emittance at 175 GeV is 2nm. So, as the fill factor increases with increasing N, which in turn decreases the synchrotron radiation, the largest N possible should be chosen, as to minimise the emittance; bearing in mind that the fill factor $\neq 1$ and there needs to be space for diagnostic and experiments.

for $E_x = 2 \text{ nm}$ ~~2.324 = N~~ $N = 1800$ This seems to low?
 for the proposed fill factor
 $N = \frac{12 \times 6.4 \times 0.84 \times 10^3}{10 \text{ m}} = 6854$
 $E_{min} = 36 \text{ pm}$ at 175 GeV

just for the minimum horizontal E_x
 $N = 2324$, but we have not counted
for the vertical E_y . ~~If the same~~
~~formula applies:~~ $N_{ey} =$ for the N and a
length of $10m$, the fill factor is quite low
at ~ 0.37

