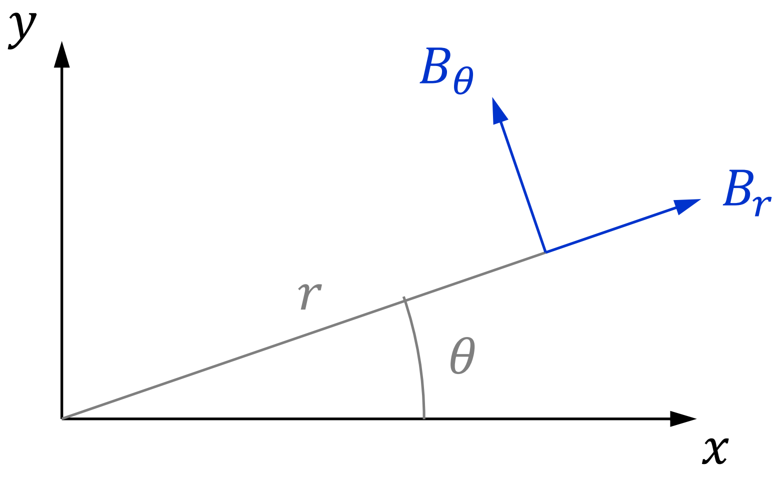
In synchrotrons / transfer lines the B field as seen from the beam is usually expressed as a series of multipoles

$$B_r = \sum_{n=1}^{\infty} \left(\frac{r}{R}\right)^{n-1} [B_n \sin(n\theta) + A_n \cos(n\theta)]$$

$$B_\theta = \sum_{n=1}^{\infty} \left(\frac{r}{R}\right)^{n-1} [B_n \cos(n\theta) - A_n \sin(n\theta)]$$




$$B_y(z) + iB_x(z) = \sum_{n=1}^{\infty} (B_n + iA_n) \left(\frac{z}{R}\right)^{n-1}$$

$$z = x + iy = re^{i\theta}$$

## Usually, for optics calculation, the field or multipole component is given, together with the (magnetic) length; these are a few definitions from MAD-X

<u>Dipole</u>

bend angle $\alpha$ [rad] & length L [m]

$k_0$ [1/m] & length L [m] obsolete

$k_0 = B / (B\rho)$ $B = B_1$

<u>Quadrupole</u>

quadrupole coefficient $k_1$ [1/m$^2$] $\times$ length L [m]

$k_1 = (dB_y/dx) / (B\rho)$

$G = dB_y/dx = B_2/R$

<u>Sextupole</u>

sextupole coefficient $k_2$ [1/m$^3$] $\times$ length L [m]

$k_2 = (d^2B_y/dx^2) / (B\rho)$

$(d^2B_y/dx^2)/2! = B_3/R^2$

```
BEAM, PARTICLE=ELECTRON,PC=3.0;
DEGREE:=PI/180.0;
QF: QUADRUPOLE,L=0.5,K1=0.2;
QD: QUADRUPOLE,L=1.0,K1=-0.2;
B: SBEND,L=1.0,ANGLE=15.0*DEGREE;
```

$(B\rho) = 10^9/c*PC$ = 10^9/299792485*3.0 = 10.01 Tm

dipole (SBEND)

$B = |ANGLE|/L*(B\rho) = (15*pi/180)/1.0*10.01 = 2.62$ T

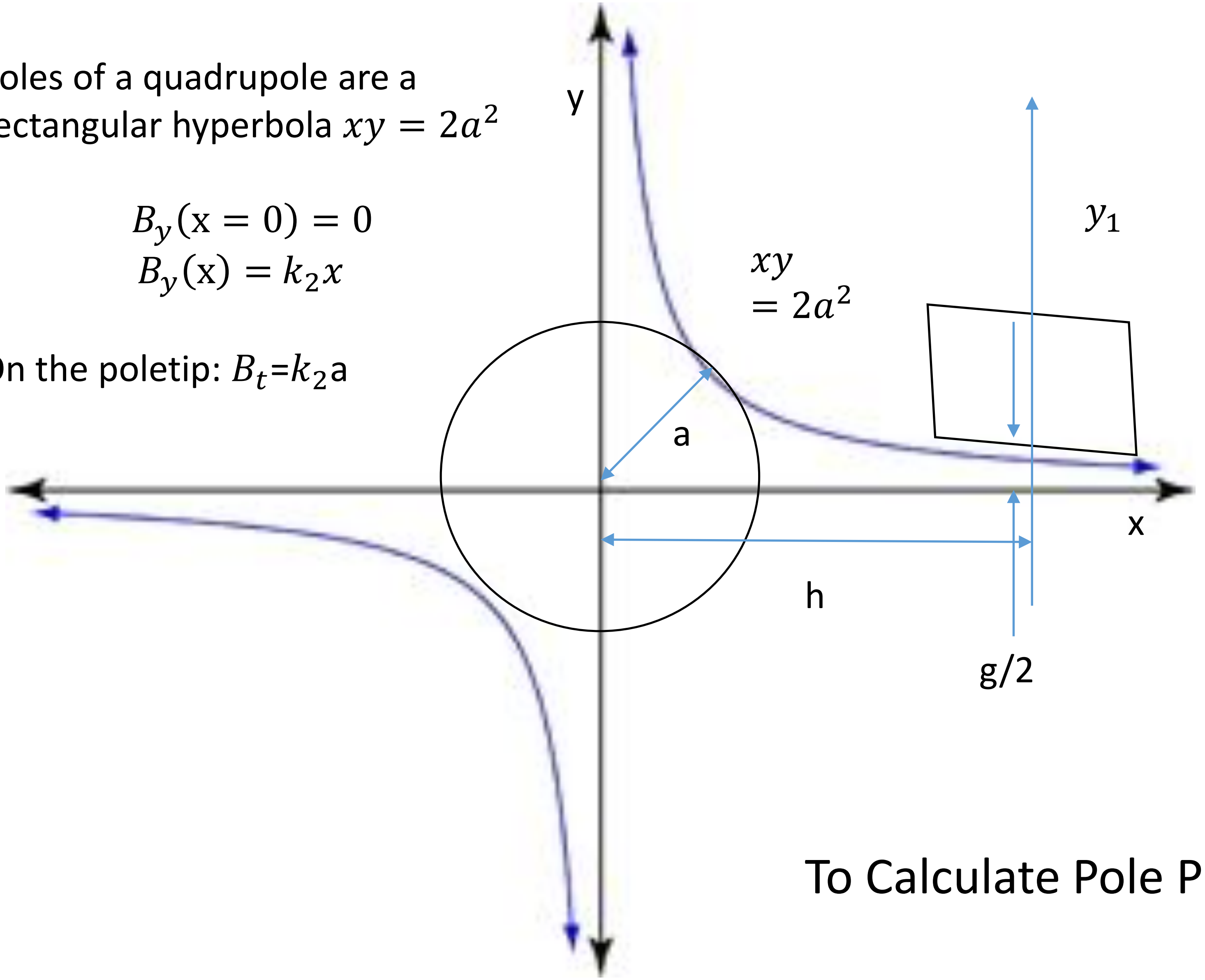
quadrupole

$G = |K1|*(B\rho) = 0.2*10.01 = 2.00$ T/m

Poles of a quadrupole are a rectangular hyperbola $xy = 2a^2$

$$B_y(\mathrm{x} = 0) = 0$$
$$B_y(\mathrm{x}) = k_2 x$$

On the poletip: $B_t$=$k_2$a




Combined function nagnet is like a quadrupole shifted by distance, h

In new coords $(x, y_1)$ the poles have equation (x+h) $y_1 = 2a^2$

$B_0$=$k_2 h$, $B_z'$=$k_2$

$h = B_0/B_z'$, $h\frac{g}{2} = 2a^2$

Given $B_0$ and $B_z'$, solve above equns to find:

*h and a*

*Then profile is* $y_1$= $2a^2$/(x+h)

# To Calculate Pole Profile of CF Magnet

# Definition of field error coefficients

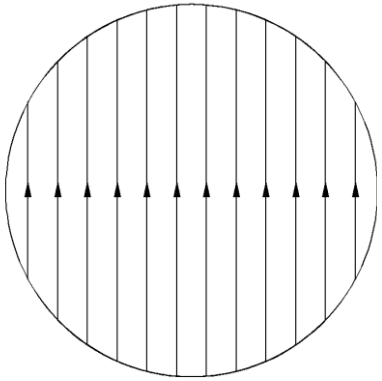
$$B_{y,id}(x) = B_1$$

$$B_y(x) = B_1 + \frac{B_1}{10000}\left[b_2\left(\frac{x}{R}\right) + b_3\left(\frac{x}{R}\right)^2 + b_4\left(\frac{x}{R}\right)^3 + \cdots\right]$$

$$\frac{\Delta B}{B}(x) = \frac{1}{10000}\left[b_2\left(\frac{x}{R}\right) + b_3\left(\frac{x}{R}\right)^2 + b_4\left(\frac{x}{R}\right)^3 + \cdots\right]$$

The harmonic decomposition is very handy to describe the field quality, that is, deviations of the actual B vs. the ideal one

(normal) dipole



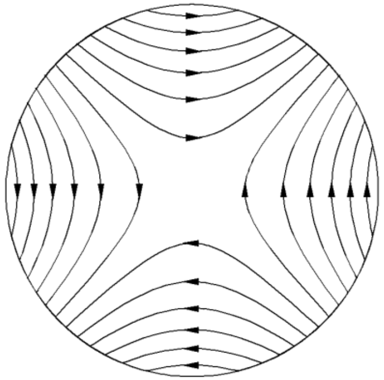
$$\vec{B}_{id}(x,y) = B_1\vec{j}$$

$$B_y(z) + iB_x(z) = \\ = B_1 + \frac{B_1}{10000}\left[ia_1 + (b_2 + ia_2)\left(\frac{z}{R}\right) + (b_3 + ia_3)\left(\frac{z}{R}\right)^2 + (b_4 + ia_4)\left(\frac{z}{R}\right)^3 + \cdots\right]$$

$$b_2 = 10000\frac{B_2}{B_1} \quad b_3 = 10000\frac{B_3}{B_1} \quad a_1 = 10000\frac{A_1}{B_1} \quad a_2 = 10000\frac{A_2}{B_1} \quad \ldots$$

# The same expression can be written for a quadrupole

(normal) quadrupole



$$\vec{B}_{id}(x,y) = B_2[x\vec{j} + y\vec{i}]\frac{1}{R}$$

$$B_y(z) + iB_x(z) =$$
$$= B_2\frac{z}{R} + \frac{B_2}{10000}\left[ia_2\left(\frac{z}{R}\right) + (b_3 + ia_3)\left(\frac{z}{R}\right)^2 + (b_4 + ia_4)\left(\frac{z}{R}\right)^3 + \cdots\right]$$

$$b_3 = 10000\frac{B_3}{B_2} \quad b_4 = 10000\frac{B_4}{B_2} \quad a_2 = 10000\frac{A_2}{B_2} \quad \ldots$$