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The NLO+NLL Top Total Cross-Section

Matteo Cacciari
LPTHE - Paris 6,7 and CNRS

Outline

- Previous work and state of the art
- NLO+NLL results
- General considerations on theoretical uncertainties

Timeline (I)

[NB: certainly approximate/incomplete]

1988	1992	1994-1996	1997-1998
NLO calculation	MNR exclusive MC	LL threshold resummation	NLL threshold resummation
Nason, Dawson, Ellis Beenakker et al.	Mangano, Nason, Ridolfi	Laenen, Smith, van Neerven Berger, Contopanagos Catani, Mangano, Nason, Trentadue (In N, minimal prescription)	Kidonakis, Sterman Bonciani, Catani, Mangano, Nason

Timeline (II)

Kidonakis, Laenen, Moch, Vogt (ln β)

MC, Frixione, Mangano, Nason, Ridolfi (NLO+NLL)

Kidonakis, Vogt (NNLL, ...)

Moch, Uwer ($NNLO_{approx}$)

MC, Frixione, Mangano, Nason, Ridolfi

Kidonakis, Vogt (NNLL, ...)

Czakon, Mitov

(+'perfect' NLL)

- 2001: approx NNLL resummation
- 2003: Tevatron phenomenology
- LHC phenomenology
- 2008: Fully analytical NLO calculation

The next challenge is NNLO

Many ingredients have recently become available:

M. Czakon, A. Mitov and S. Moch, Nucl. Phys. B798 (2008) 210, arXiv:0707.4139 [hep-ph],

M. Czakon, A. Mitov and S. Moch, Phys. Lett. B651 (2007) 147, arXiv:0705.1975 [hep-ph],

M. Czakon, Phys. Lett. B664 (2008) 307, arXiv:0803.1400 [hep-ph],

J.G. Korner, Z. Merebashvili and M. Rogal, Phys. Rev. D77 (2008) 094011, arXiv:0802.0106 [hep-ph]

B. Kniehl et al., Phys. Rev. D78 (2008) 094013, arXiv:0809.3980 [hep-ph],

R. Bonciani et al., JHEP 07 (2008) 129, arXiv:0806.2301 [hep-ph],

C. Anastasiou and S.M. Aybat, (2008), arXiv:0809.1355 [hep-ph],

. . . .

However, a full calculation will still be elusive for some time

Soft massive anom. dim. at two loops

Factorization: $|M\rangle = \prod J \times S|H\rangle$

$$\mathrm{P}\exp\left[-\int_0^{\mu}\frac{d\tilde{\mu}}{\tilde{\mu}}\mathbf{\Gamma}_{S_{\mathrm{f}}}\left(\beta_i\cdot\beta_j,\bar{\alpha_s}\left(\tilde{\mu},\varepsilon\right)\right)\right]$$

For massless partons $\mathbf{\Gamma}^{(2)}_{S_{\mathrm{f}}}(\beta_i) = \frac{K}{2}\mathbf{\Gamma}^{(1)}_{S_{\mathrm{f}}}(\beta_i)$

What happens with massive quark lines?

Apparently, not a simple proportionality [Mitov, Sterman,sung]

Jury still out on NNLL heavy quark threshold resummation?

NLO+NLL

$$\hat{\sigma}^{(res)}_{ij,N}(m^2,\alpha_s(\mu^2),\mu^2) = \frac{\alpha_s^2(\mu^2)}{m^2} \sum_{\mathbf{I}=1,8} f^{(corr)}_{ij,\mathbf{I},N}(\alpha_s(\mu^2),\mu^2/m^2)\ \Delta_{ij,\mathbf{I},N+1}\left(\alpha_s(\mu^2),\frac{\mu^2}{m^2}\right)$$

$$\left(f^{(0)}_{ij,\mathbf{I},N} + 4\pi\alpha_s(\mu^2) f^{(1),\mathrm{Coul}}_{ij,\mathbf{I},N}\right)\left[1+\frac{\alpha_s(\mu^2)}{\pi}C_{ij}(\mu^2/m^2)\right]$$

$$\begin{aligned}
C_{q\bar{q}}(\frac{\mu^2}{m^2}) &= \overline{C}_2(\frac{\mu^2}{m^2}) + 2C_F\left[\ln^2 2 + (\gamma_E - 2)\ln\frac{\mu^2}{4m^2} + \frac{\pi^2}{2} + \gamma_E(\gamma_E - 4)\right] \\
&+ (8C_F + C_A)(\gamma_E - 2 - \ln 2)\ , \\
C_{gg}(\frac{\mu^2}{m^2}) &= \overline{C}_3(\frac{\mu^2}{m^2}) + 2C_A\left[\ln^2 2 + (\gamma_E - 2)\ln\frac{\mu^2}{4m^2} + \frac{\pi^2}{2} + \gamma_E(\gamma_E - 4)\right] \\
&+ \frac{C_A(9N_c^2 - 20)}{N_c^2 - 2}(\gamma_E - 2 - \ln 2)\ .
\end{aligned}$$

[Exact analytical results by Czakon, Mitov]

The constants produce subleading logs. Are they the correct ones?

This expression is then matched to the full NLO calculation to yield NLO+NLL:

$$\sigma^{(res)}_N(m^2) = \sum_{ij=q\bar{q},gg} F_{i,N+1}(\mu^2)\ F_{j,N+1}(\mu^2)\left[\hat{\sigma}^{(res)}_{ij,N}(m^2,\alpha_s(\mu^2),\mu^2) - \left(\hat{\sigma}^{(res)}_{ij,N}(m^2,\alpha_s(\mu^2),\mu^2)\right)_{\alpha_s^3}\right] + \sigma^{(NLO)}_N(m^2)$$

Sudakov Factor

$$\Delta_{ij,\mathbf{I},N}\left(\alpha_s(\mu^2),\mu^2/m^2\right) = \exp\left\{\ln N\, g^{(1)}_{ij}(b_0\,\alpha_s(\mu^2)\ln N) + g^{(2)}_{ij,\mathbf{I}}(b_0\,\alpha_s(\mu^2)\ln N,\mu^2/m^2) + \alpha_s\, g^{(3)}_{ij,I}\right\}$$

LL NLL NNLL

(Moch, Uwer)

$$\ln\Delta_{q\bar{q},\mathbf{1},N} = \int_0^1 dz\,\frac{z^{N-1}-1}{1-z}\int_{\mu^2}^{4m^2(1-z)^2}\frac{dq_\perp^2}{q_\perp^2}\,2A_q\left(\alpha_s(q_\perp^2)\right) + \mathcal{O}(\alpha_s(\alpha_s\ln N)^k)\,,$$

$$\ln\Delta_{q\bar{q},\mathbf{8},N} = \int_0^1 dz\,\frac{z^{N-1}-1}{1-z}\left\{\int_{\mu^2}^{4m^2(1-z)^2}\frac{dq_\perp^2}{q_\perp^2}\,2A_q\left(\alpha_s(q_\perp^2)\right) + D_{Q\bar{Q}}\left(\alpha_s\left(4m^2(1-z)^2\right)\right)\right\} + \mathcal{O}(\alpha_s(\alpha_s\ln N)^k)\,,$$

$$\ln\Delta_{gg,\mathbf{1},N} = \int_0^1 dz\,\frac{z^{N-1}-1}{1-z}\int_{\mu^2}^{4m^2(1-z)^2}\frac{dq_\perp^2}{q_\perp^2}\,2A_g\left(\alpha_s(q_\perp^2)\right) + \mathcal{O}(\alpha_s(\alpha_s\ln N)^k)\,,$$

$$\ln\Delta_{gg,\mathbf{8},N} = \int_0^1 dz\,\frac{z^{N-1}-1}{1-z}\left\{\int_{\mu^2}^{4m^2(1-z)^2}\frac{dq_\perp^2}{q_\perp^2}\,2A_g\left(\alpha_s(q_\perp^2)\right) + D_{Q\bar{Q}}\left(\alpha_s\left(4m^2(1-z)^2\right)\right)\right\} + \mathcal{O}(\alpha_s(\alpha_s\ln N)^k)\,,$$

A_q, A_g, D_{QQ} known for NLL, see next talks for NNLL

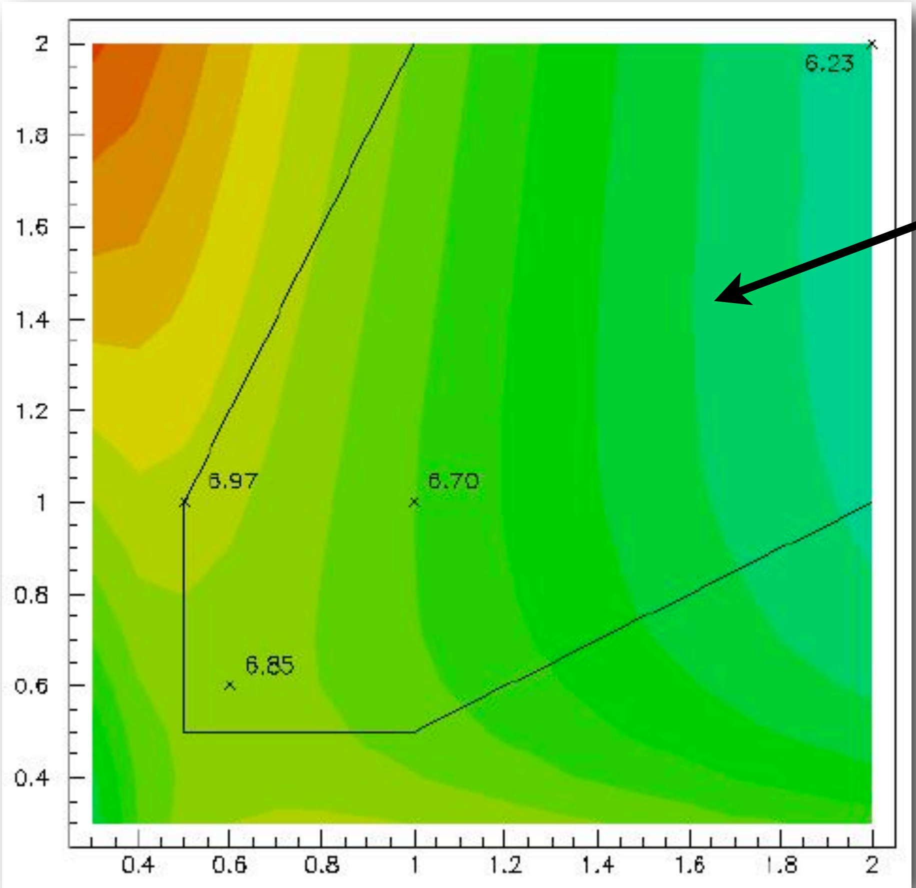
Very little residual theoretical uncertainty (~3-4%) observed. More in Sven's talk

Two components: scales and subleading terms

Scales: vary them independently, in region

$0.5 < \mu_{R,F}/m < 2$ && $0.5 < \mu_R/\mu_F < 2$

NLO+NLL top x-sect @ Tevatron, m = 175 GeV



"Fiducial" region

Order **±5%** uncertainty along the diagonal, a little more considering independent scale variations

Independent variation not critical in this case, but crucial elsewhere (e.g. total bottom x-sect at LHC goes from 4% to 40% uncertainty)

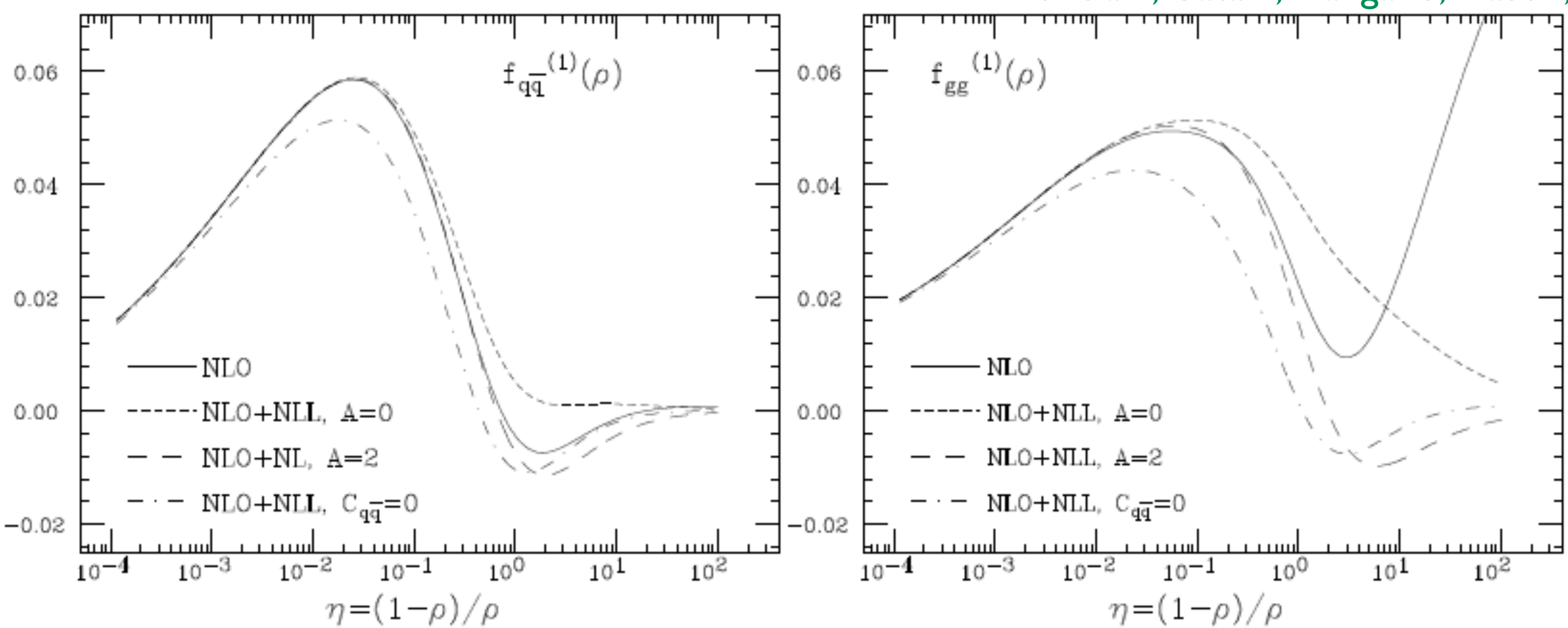
Theoretical error

Subleading logs: we do not control subleading terms beyond NLL

Try vary them by replacing $C_{ij} \to C_{ij} \left(1 - \dfrac{A}{N + A - 1}\right)$

and playing with the value of A (default: A=2)

Bonciani, Catani, Mangano, Nason,



Including the C_{ij} generally improves the approximation, but different values of A yield visually equivalent descriptions of full NLO

Tevatron

$$\sigma_{t\bar{t}}^{\rm NLO+NLL}(\text{Tev}, m_t = 171\ \text{GeV}, \text{CTEQ6.5}) = 7.61\ ^{+0.30(3.9\%)}_{-0.53(6.9\%)}\ (\text{scales})\ ^{+0.53(7\%)}_{-0.36(4.8\%)}\ (\text{PDFs})\ \ \text{pb}$$

$$\sigma_{t\bar{t}}^{\rm NLO}(\text{Tev}, m_t = 171\ \text{GeV}, \text{CTEQ6.5}) = 7.35\ ^{+0.38(5.1\%)}_{-0.80(10.9\%)}\ (\text{scales})\ ^{+0.49(6.6\%)}_{-0.34(4.6\%)}\ (\text{PDFs})\ \ \text{pb}$$

Theoretical error
NLO+NLL < NLO

LHC (14 TeV)

$$\sigma_{t\bar{t}}^{\rm NLO+NLL}(\text{LHC}, m_t = 171\ \text{GeV}, \text{CTEQ6.5}) = 908\ ^{+82(9.0\%)}_{-85(9.3\%)}\ (\text{scales})\ ^{+30(3.3\%)}_{-29(3.2\%)}\ (\text{PDFs})\ \ \text{pb}$$

$$\sigma_{t\bar{t}}^{\rm NLO}(\text{LHC}, m_t = 171\ \text{GeV}, \text{CTEQ6.5}) = 875\ ^{+102(11.6\%)}_{-100(11.5\%)}\ (\text{scales})\ ^{+30(3.4\%)}_{-29(3.3\%)}\ (\text{PDFs})\ \ \text{pb}$$

Theoretical error
NLO+NLL ≈ NLO

At LHC 10 TeV the improvement is intermediate/similar to LHC 14 TeV: , 11.5% → 9%

NLO+NLL: effect of subleading terms

$$\sigma_{t\bar{t}}^{\mathrm{NLO+NLL(A=0)}}(\mathrm{LHC}, m_t = 171\ \mathrm{GeV}, \mathrm{CTEQ6.5}) = 945\ ^{+95(10\%)}_{-85(9.0\%)}\ (\mathrm{scales})\ \ \mathrm{pb} \qquad [\mu_{\mathrm{F}} \neq \mu_{\mathrm{R}}]$$

$$\sigma_{t\bar{t}}^{\mathrm{NLO+NLL(A=0)}}(\mathrm{LHC}, m_t = 171\ \mathrm{GeV}, \mathrm{CTEQ6.5}) = 945\ ^{+19(2\%)}_{-7(0.7\%)}\ (\mathrm{scales})\ \ \mathrm{pb} \qquad [\mu_{\mathrm{F}} = \mu_{\mathrm{R}}]$$

to be compared with the default result (A=2):

$$\sigma_{t\bar{t}}^{\mathrm{NLO+NLL}}(\mathrm{LHC}, m_t = 171\ \mathrm{GeV}, \mathrm{CTEQ6.5}) = 908\ ^{+82(9.0\%)}_{-85(9.3\%)}\ (\mathrm{scales})\ ^{+30(3.3\%)}_{-29(3.2\%)}\ (\mathrm{PDFs})\ \ \mathrm{pb}$$

$$\sigma_{t\bar{t}}^{\mathrm{NLO+NLL}}(\mathrm{LHC}, m_t = 171\ \mathrm{GeV}, \mathrm{CTEQ6.5}) = 908\ ^{+61(6.7\%)}_{-48(5.3\%)}\ (\mathrm{scales})\ \ \mathrm{pb} \qquad [\mu_F = \mu_R]$$

A=0 yields a very little scale uncertainty, unless independent scale variations are used

Theoretical uncertainty

We perform enormously complicated calculations, but still have no agreed upon (never mind rigorous) procedure to estimate their uncertainty

While a fully objective way of determining a theoretical uncertainty probably does not exist, we should at least find a way to standardize the procedure and facilitate the communication and comparison of its results

One vague (very vague) proposal:

a bayesian framework, were a set of subjective priors (which we may even agree upon) is processed in a rigorous way into a confidence level for a cross section interval, using the available knowledge along the way

We may then be able to simple exchange or communicate the priors, and the rest will be automatic and well defined

Conclusions

NLO+NLL predictions for LHC (10 TeV)

$$\sigma_{t\bar{t}}^{\rm NLO+NLL}({\rm LHC\ 10\ TeV}, m_t = 171\ {\rm GeV}, {\rm CTEQ6.5}) = 414\ {}^{+36(8.7\%)}_{-38(9.2\%)}\ ({\rm scales})\ {}^{+20(4.8\%)}_{-18(4.3\%)}\ ({\rm PDFs})\ \ {\rm pb}$$

$$\sigma_{t\bar{t}}^{\rm NLO+NLL}({\rm LHC\ 10\ TeV}, m_t = 171\ {\rm GeV}, {\rm MRST2006nnlo}) = 446\ {}^{+40(9.0\%)}_{-41(9.2\%)}\ ({\rm scales})\ {}^{+8(1.8\%)}_{-8(1.8\%)}\ ({\rm PDFs})\ \ {\rm pb}$$

- PDF results/uncertainties not wonderfully compatible. Upgrade to more recent PDF sets and addition of NNPDF easily doable, though I would not expect drastic changes
- How phenomenologically relevant is an O(<5%) knowledge of the top total cross section?
- Top production a very interesting process, containing most of the features that make calculations hard (mass, colour,…).It could become a benchmark to test our ability to improve it and to estimate the uncertainty of our predictions