

Top quark production at Tevatron and LHC

— *precision issues* —

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– CERN Theory Institute, *TOP09 : Top quark physics - from the Tevatron to the LHC*, Geneva, May 29, 2009 –

In a nutshell

What this talk will not tell you:

- We have the complete NNLO QCD result for top quark hadro-production.

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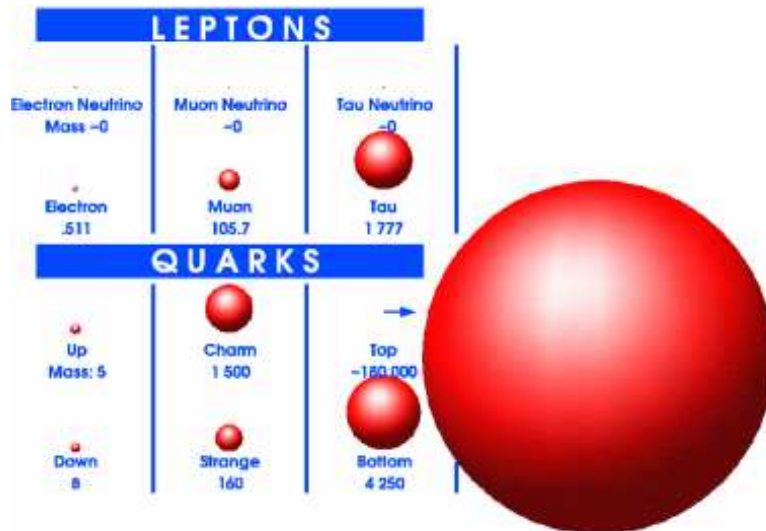
- We have the complete NNLO QCD result for top quark hadro-production.
- Top quark production at Tevatron/LHC is only due to threshold logarithms.

What this talk will tell you:

- Important parts of the NNLO QCD result are known – and they contribute in the entire phase-space.
- We have presently the best phenomenological prediction.

Plan

- Some new results on the heaviest elementary particle



- Report based on recent work done in collaboration with
 - P. Uwer on [arXiv:0804.1476](https://arxiv.org/abs/0804.1476) and on [arXiv:0807.2794](https://arxiv.org/abs/0807.2794)
 - Y. Kiyo, J.H. Kühn, M. Steinhauser and P. Uwer on [arXiv:0812.0919](https://arxiv.org/abs/0812.0919)
 - U. Langenfeld on [arXiv:0901.0802](https://arxiv.org/abs/0901.0802)
 - U. Langenfeld and P. Uwer to appear

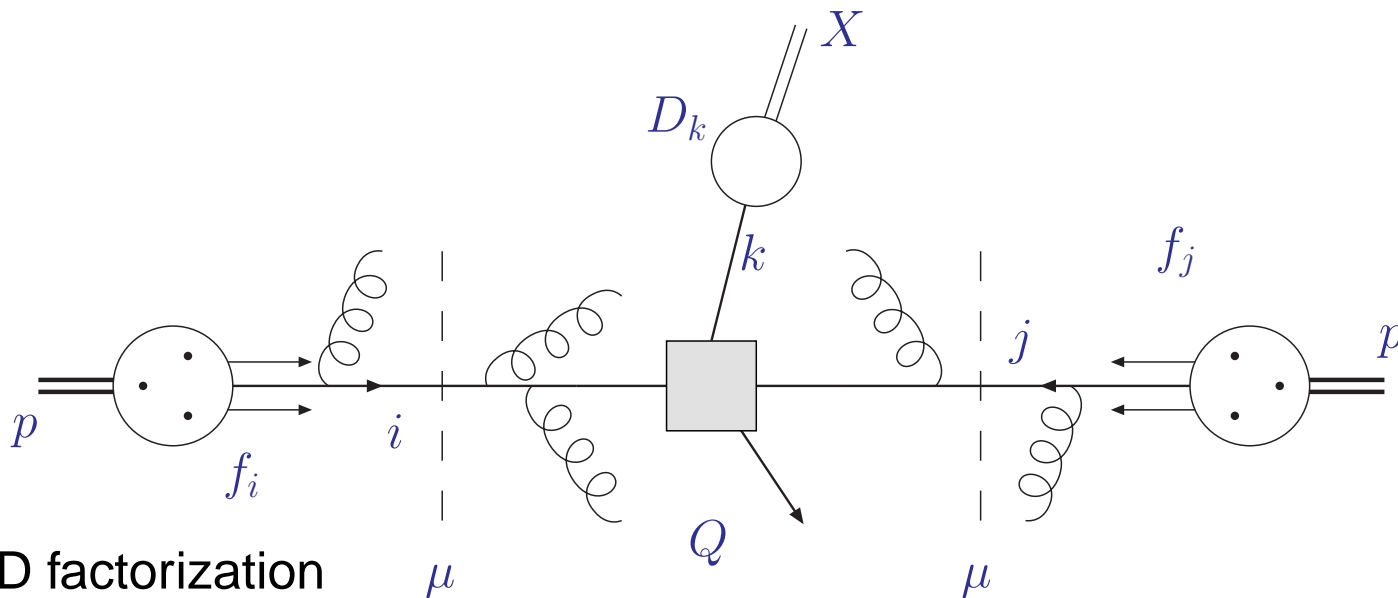
Proton colliders

- Tevatron: energy frontier at $\sqrt{S} = 1.96\text{TeV}$
top quark discovery
- LHC: in commissioning phase
Higgs boson search at highest energies: $\sqrt{S} = 14\text{TeV}$



Perturbative QCD at colliders

- Hard hadron-hadron scattering
 - constituent partons from each incoming hadron interact at short



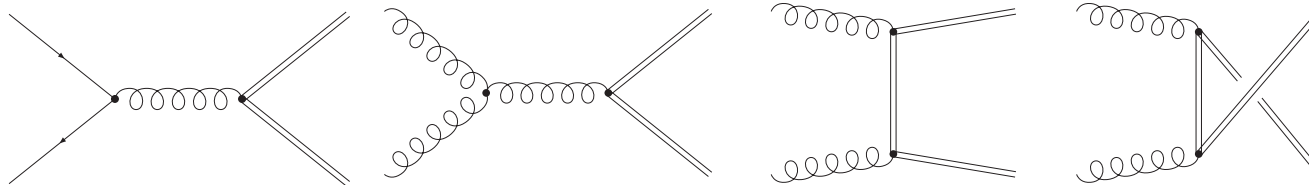
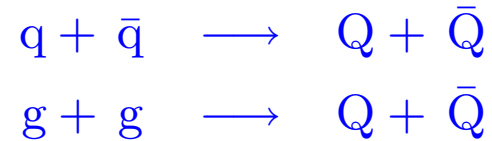
- QCD factorization
 - separate sensitivity to dynamics from different scales

$$\sigma_{pp \rightarrow X} = \sum_{ijk} f_i(\mu^2) \otimes f_j(\mu^2) \otimes \hat{\sigma}_{ij \rightarrow k}(\alpha_s(\mu^2), Q^2, \mu^2) \otimes D_{k \rightarrow X}(\mu^2)$$

- factorization scale μ , subprocess cross section $\hat{\sigma}_{ij \rightarrow k}$ for parton types i, j and hadronic final state X

Top quark production

- Leading order Feynman diagrams



- NLO in QCD [Nason, Dawson, Ellis '88](#); [Beenakker, Smith, van Neerven '89](#); [Mangano, Nason, Ridolfi '92](#); [Bernreuther, Brandenburg, Si, Uwer '04](#); [Mitov, Czakon '08](#); ...
 - accurate to $\mathcal{O}(15\%)$ at LHC
- Much activity towards higher orders in QCD
 - one-loop squared terms (NLO \times NLO) [Anastasiou, Mert Aybat '08](#); [Kniehl, Merebashvili, Körner, Rogal '08](#)
 - analytic two-loop fermionic corrections for $q\bar{q} \rightarrow t\bar{t}$ [Bonciani, Ferroglia, Gehrmann, Maitre, Studerus '08](#)
 - numerical result for two-loop virtual $q\bar{q} \rightarrow t\bar{t}$ [Czakon '08](#)

Strategy

- First steps towards higher orders in QCD: explore limits

Strategy

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Small mass limit

- Study of massive QCD amplitudes in limit $m \rightarrow 0$
 - look at soft and collinear limits
 - exploit relation of massive to massless amplitudes
- Two-loop virtual corrections to $q\bar{q} \rightarrow t\bar{t}$ and $gg \rightarrow t\bar{t}$ in small-mass limit $m^2 \ll s, t, u$ S.M., Czakon, Mitov '07

Threshold resummation

- Partonic threshold $s \simeq 4m^2$
 - Sudakov-type logarithms $\ln \beta$ with velocity of heavy quark $\beta = \sqrt{1 - 4m^2/s}$
 - long history of resummation Kidonakis, Sterman '97; Bonciani, Catani, Mangano, Nason '98; Kidonakis, Laenen, S.M., Vogt '01; ...

Threshold resummation

- Threshold at $s \simeq 4m_t^2$
 - parton cross section exhibit Sudakov-type logarithms $\ln(\beta)$ with velocity of heavy quark $\beta = \sqrt{1 - 4m_t^2/s}$ at n^{th} -order
- All order resummation of large logarithms $\alpha_s^n \ln^{2n}(\beta) \longleftrightarrow \alpha_s^n \ln^{2n}(N)$
 - resummation in Mellin space (renormalization group equation)
- Resummed cross section in Mellin space

$$\frac{\hat{\sigma}_{ij,I}^N(m^2)}{\hat{\sigma}_{ij,I}^{(0),N}(m^2)} = g_{ij,I}^0(m^2) \cdot \exp\left(G_{ij,I}^{N+1}(m^2)\right) + \mathcal{O}(N^{-1} \ln^n N)$$

- exponent in singlet-octet color basis decomposition $I = 1, 8$

$$G_{q\bar{q}/gg,I}^N = G_{\text{DY}/\text{Higgs}}^N + \delta_{I,8} G_{Q\bar{Q}}^N$$

- Renormalization group equations for functions $G_{\text{DY}/\text{Higgs}}^N$ and $G_{Q\bar{Q}}^N$
 - well-known exponentiation from factorization in soft/collinear limit

The radiative factors

- Production of color singlet final state from parton-parton scattering described by $G_{\text{DY/Higgs}}^N$

$$G_{\text{DY/Higgs}}^N =$$

$$\int_0^1 dz \frac{z^{N-1} - 1}{1-z} \int_{\mu_f^2}^{4m^2(1-z)^2} \frac{dq^2}{q^2} 2 A_i(\alpha_s(q^2)) + D_i(\alpha_s(4m^2[1-z]^2))$$

- well known anomalous dimensions A_i (collinear gluon emission) and D_i (process dependent gluon emission at large angles)
Vogt '00; Catani, Grazzini, de Florian, Nason '03; S.M., Vermaseren, Vogt '05

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- $G_{Q\bar{Q}}^N$ accounts for gluon emission from octet final state

$$G_{Q\bar{Q}}^N = \int_0^1 dz \frac{z^{N-1} - 1}{1-z} D_{Q\bar{Q}}(\alpha_s([1-z]^2 4m^2))$$

- anomalous dimension $D_{Q\bar{Q}}$ (cf. pole of form factor for massive quarks)

$$D_{Q\bar{Q}}^{(1)} = -A_g^{(1)}, \quad D_{Q\bar{Q}}^{(2)} = -A_g^{(2)} = -A_g^{(1)} K$$

$$D_{Q\bar{Q}}^{(2)} \text{ consistent with } \text{Mitov, Sterman, Sung '09; Becher, Neubert '09}$$

Accuracy under control

- Control over logarithms $\ln(N)$ with $\lambda = \beta_0 \alpha_s \ln(N)$ to N^k LL accuracy

$$G_{ij, I}^N = \ln N \cdot g_{ij}^1(\lambda) + g_{ij, I}^2(\lambda) + \alpha_s g_{ij, I}^3(\lambda) + \dots$$

- $g^1(\lambda)$: LL

Laenen, Smith, v.Neerven '92; Berger, Contopanagos '95; Catani, Mangano, Nason, Trentadue '96

- $g^2(\lambda)$: NLL

Bonciani, Catani, Mangano, Nason '98; Kidonakis, Laenen, S.M., Vogt '01

- $g^3(\lambda)$: NNLL

S.M., Uwer '08

- Resummed G^N predicts fixed orders in perturbation theory
 - generating functional for towers of large logarithms

New results

- NNLO cross section for heavy-quark hadro-production near threshold (all powers of $\ln \beta$ and Coulomb corrections) S.M., Uwer '08; Langenfeld, S.M., Uwer to appear
 - e.g. gg -fusion for $n_f = 5$ light flavors at $\mu = m_t$

$$\hat{\sigma}_{gg \rightarrow t\bar{t}}^{(1)} = \hat{\sigma}_{gg \rightarrow t\bar{t}}^{(0)} \left\{ 96 \ln^2 \beta - 9.5165 \ln \beta + 35.322 + 5.1698 \frac{1}{\beta} \right\}$$

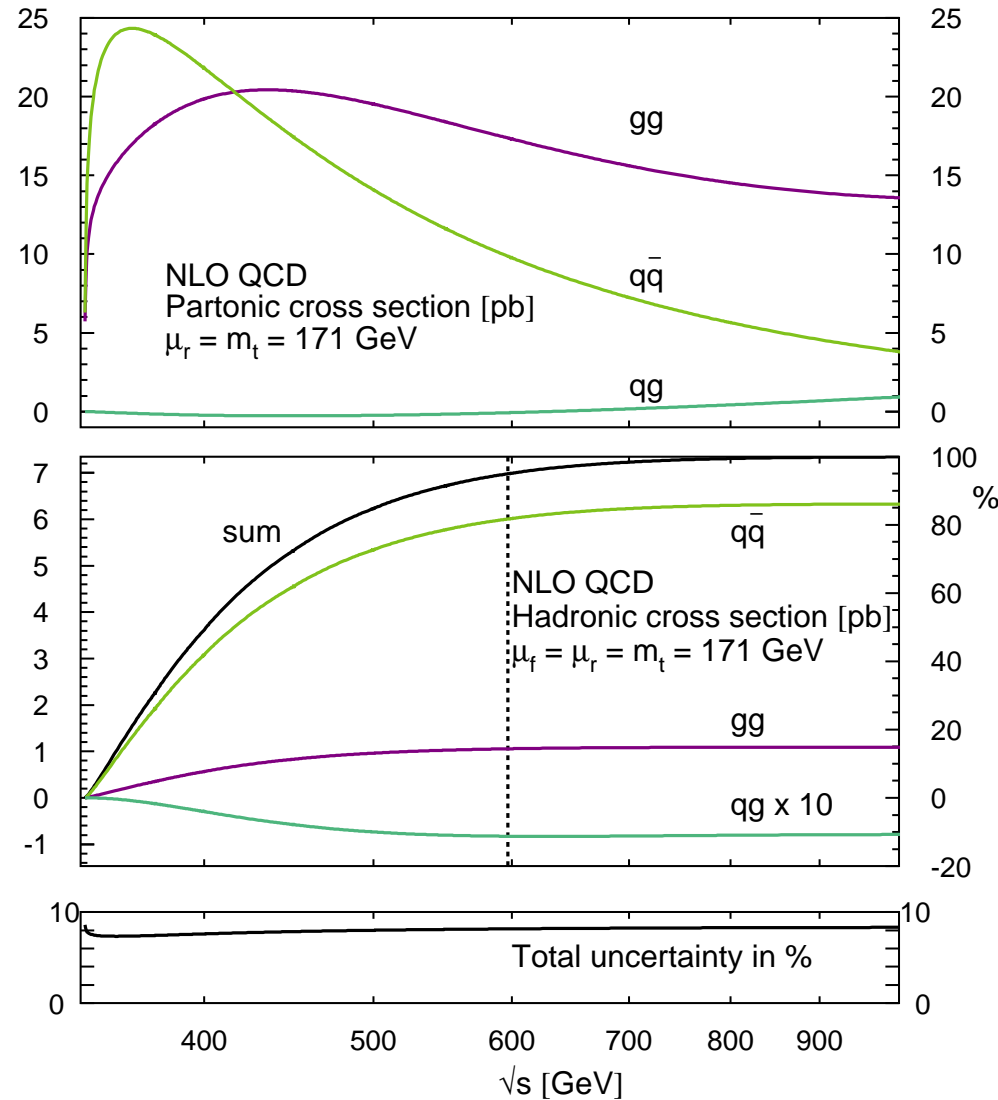
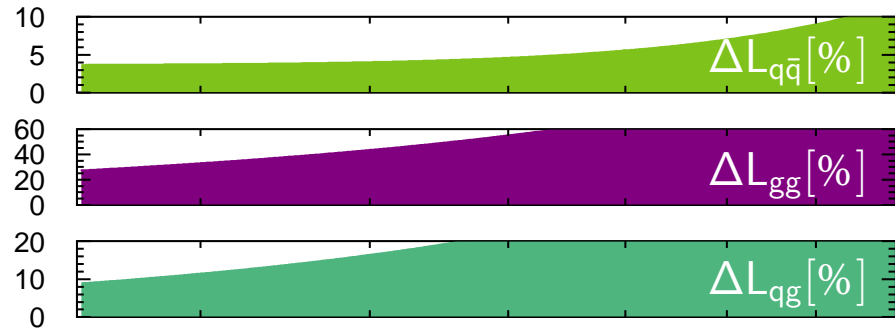
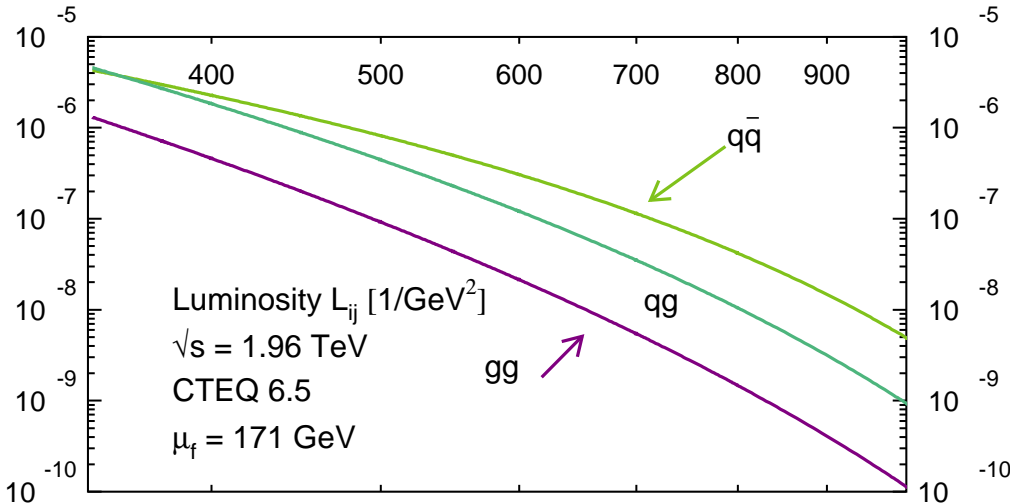
$$\hat{\sigma}_{gg \rightarrow t\bar{t}}^{(2)} = \hat{\sigma}_{gg \rightarrow t\bar{t}}^{(0)} \left\{ 4608 \ln^4 \beta - 1894.9 \ln^3 \beta + \left(-912.35 + 496.30 \frac{1}{\beta} \right) \ln^2 \beta \right. \\ \left. + \left(3031.1 + 321.14 \frac{1}{\beta} \right) \ln \beta + 68.547 \frac{1}{\beta^2} - 196.93 \frac{1}{\beta} + C_{gg}^{(2)} \right\}$$

Upshot

- Best approximation to complete NNLO
- Similar results for new massive colored particles (4th generation quarks, squarks, gluinos, ...)
S.M., Uwer '08; S.M., Langenfeld '08

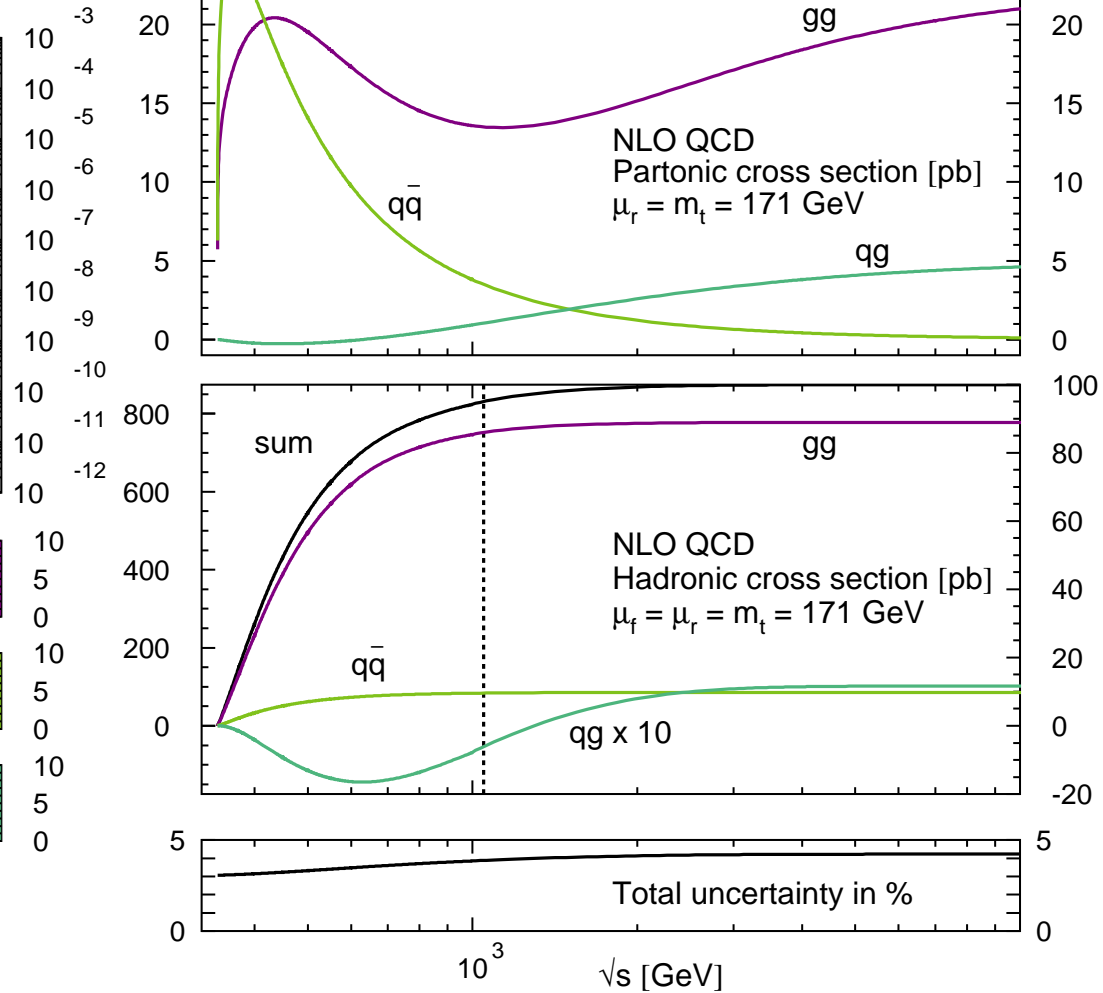
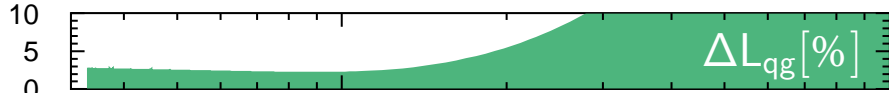
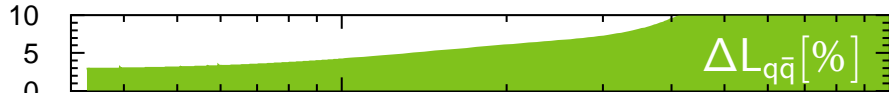
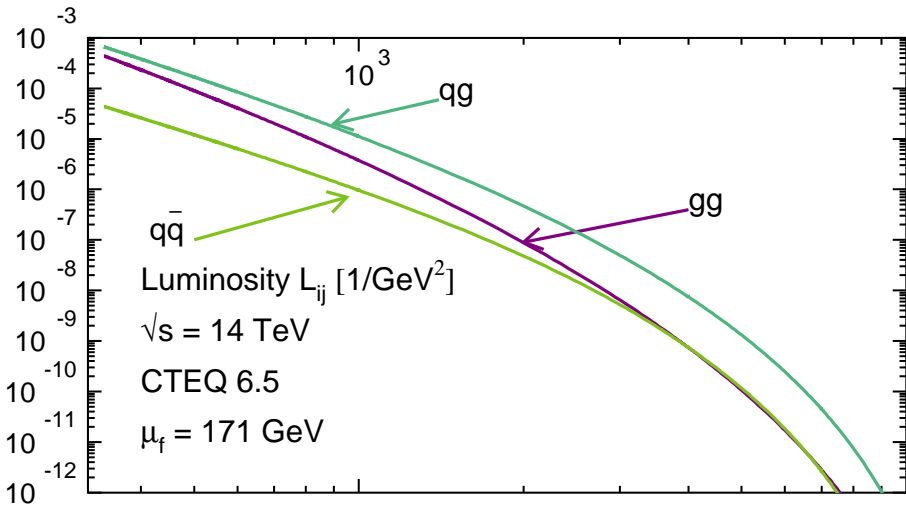
Total cross section at Tevatron

$$\sigma_{pp \rightarrow t\bar{t}} = \sum_{ij} f_i \otimes f_j \otimes \hat{\sigma}_{ij \rightarrow t\bar{t}}$$



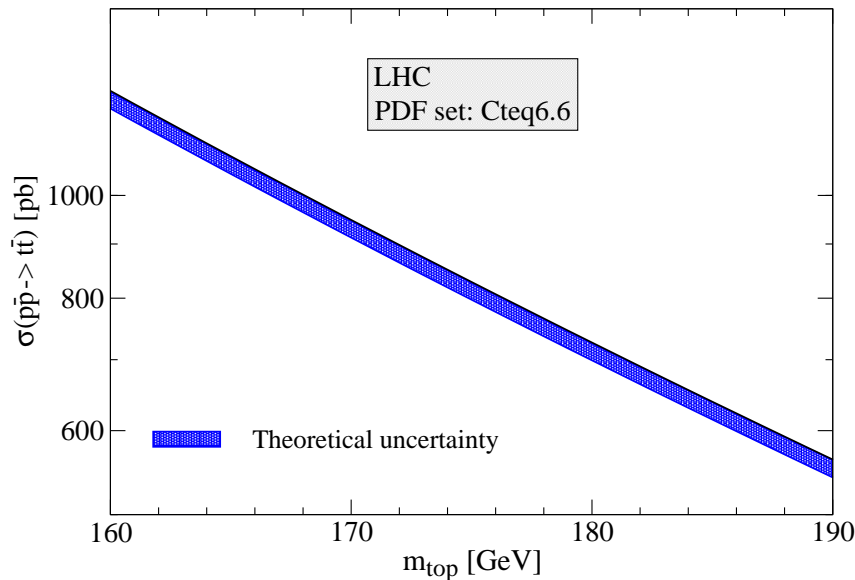
Total cross section at LHC

$$\sigma_{pp \rightarrow t\bar{t}} = \sum_{ij} f_i \otimes f_j \otimes \hat{\sigma}_{ij \rightarrow t\bar{t}}$$

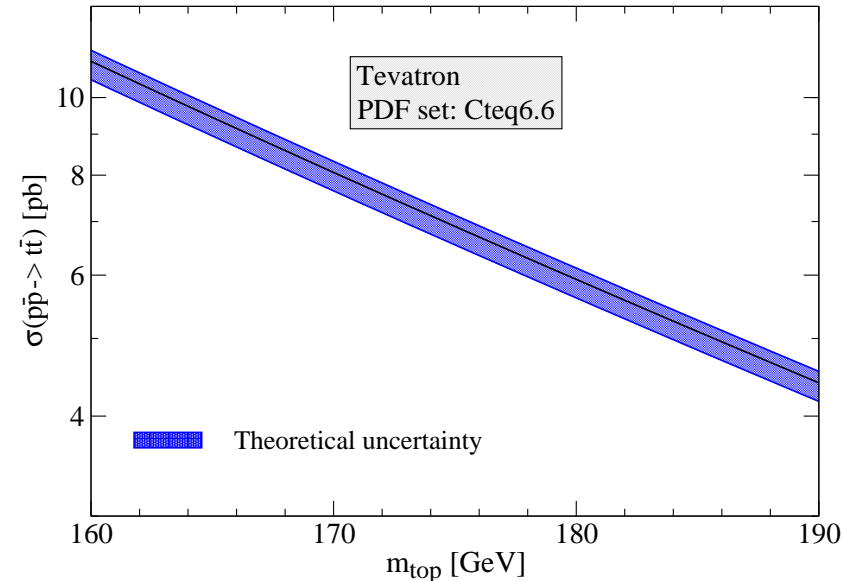


Mass dependence

- Parametrize mass dependence with a fit around $x = (m_t/\text{GeV} - 173)$
$$\sigma(\mu) = a + bx + cx^2 + dx^3 + ex^4 + fx^5 + gx^6$$
- fit precise to per mille accuracy in range $150 \text{ GeV} \leq m_t \leq 220 \text{ GeV}$
- various scale and PDF choices: $\mu = m_t/2, m_t, 2m_t$, CTEQ6.6, MSTW2008, ...



LHC



Tevatron

Recent theory activities

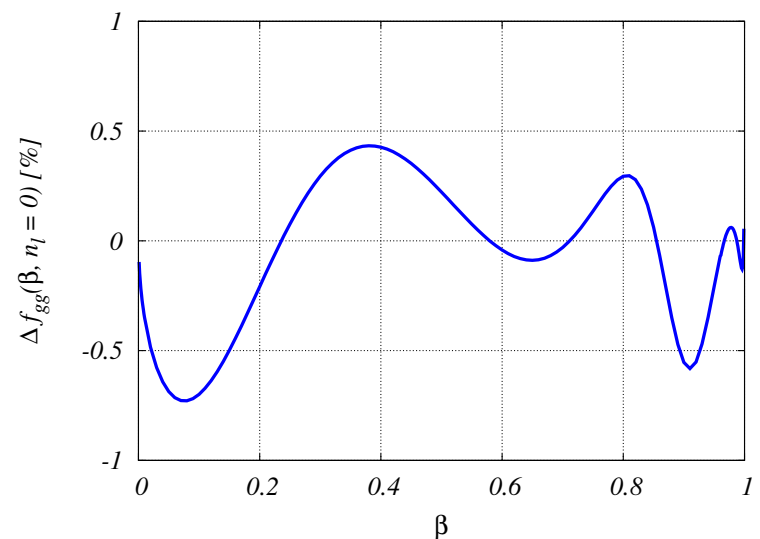
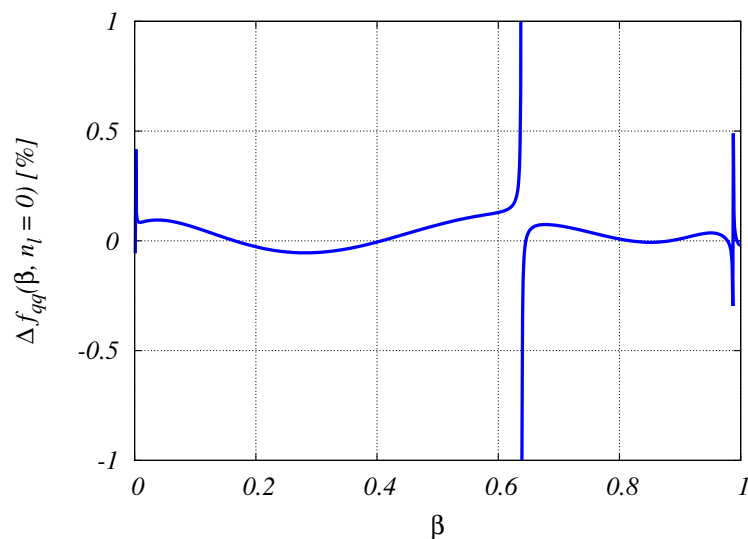
- Updates of cross section predictions based on resummation
Cacciari, Frixione, Mangano, Nason, Ridolfi '08; Kidonakis, Vogt '08
- Correlation of cross section at NLO with gluon PDFs
Nadolsky, Lai, Cao, Huston, Pumplin, Stump, Tung, Yuan '08;

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Progress at NLO

- Complete analytic calculation at NLO Mitov, Czakon '08
 - check quality of fit of NDE Nason, Dawson, Ellis '88 (fit OK to $\mathcal{O}(1\%)$)



Theory improvements

- Improved matching at NLO
 - consistent color-singlet and color-octet contributions at NLO
Petrelli, Cacciari, Greco, Maltoni, Mangano '97; Hagiwara, Sumino, Yokoya '08
- Subleading logarithms $\beta^2 \ln(\beta) \longleftrightarrow \ln(N)/N$
 - effect modeled in N -space (scheme $A = 2$)
Bonciani, Catani, Mangano, Nason '98
 $\hat{\sigma}_{\text{NLL}}^{\text{res}} \times (1 - A/(N + A - 1))$
 - recent progress in all-order exponentiation of power-suppressed terms (test cases F_L and Drell-Yan)
Laenen, Magnea, Stavenga '08; Moch, Vogt '09; Grunberg, Ravindran '09
- First step: contribution of qg -channel
 - leading term near threshold $\sim \beta^3 \ln^3 \beta$ (power suppressed by β^2)
 - include gq -channel at two loops because of large gq -parton luminosity at LHC

Scale dependence (I)

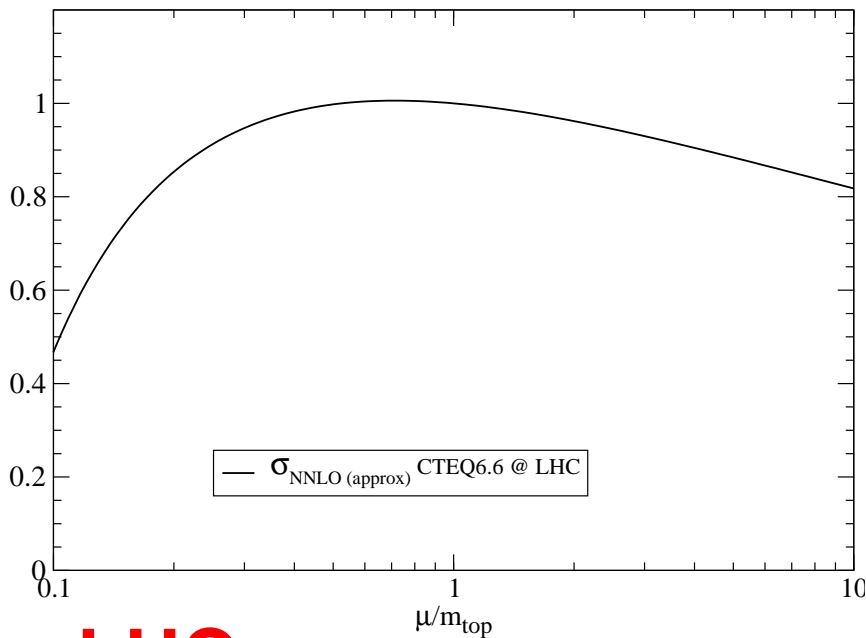
- Variation of renormalization and factorization scale $\mu_R \neq \mu_F$
- Renormalization group methods predict all terms $L = \ln(\mu^2/m_t^2)$

$$\begin{aligned}\sigma_{t\bar{t}} &= \sigma^{(0)} + \alpha_s(\mu) \left\{ \sigma^{(1)} + L \sigma_L^{(1)}(\sigma^{(0)}, \beta_0, P_0) \right\} \\ &+ \alpha_s^2(\mu) \left\{ \sigma^{(2)} + L \sigma_L^{(2)}(\sigma^{(0)}, \sigma^{(1)}, \beta_0, \beta_1, P_0, P_1) + L^2 \sigma_{L^2}^{(2)}(\sigma^{(0)}, \beta_0, P_0) \right\}\end{aligned}$$

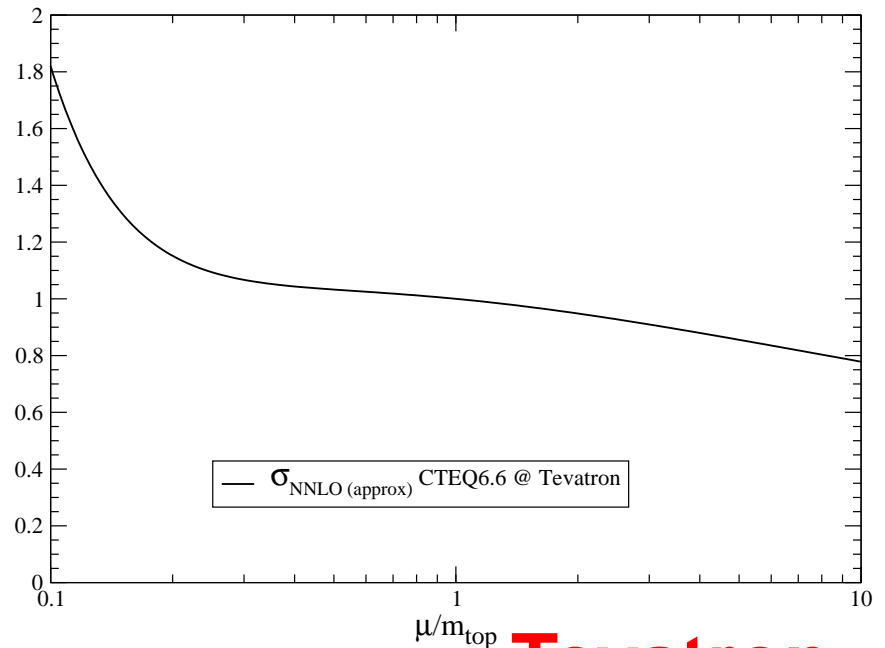
- relax $\mu = \mu_R = \mu_F$ in study of theoretical uncertainty
- allow for independent variation $\mu_R \neq \mu_F$

Scale dependence (II)

- Theoretical uncertainty from variation of scales $\mu = \mu_R = \mu_F$
 - plot with PDF set CTEQ6.6 (but largely independent on PDFs)
 - mass $m_t = 173 \text{ GeV}$ (from $m_t = 173.1 \pm 1.3 \text{ GeV}$ Tevatron winter '09)
 - very stable predictions in range $\mu \in [m_t/2, 2m_t]$
 - $-3\% \leq \Delta\sigma \leq +0.5\%$ at LHC
 - $-4\% \leq \Delta\sigma \leq +3\%$ at Tevatron



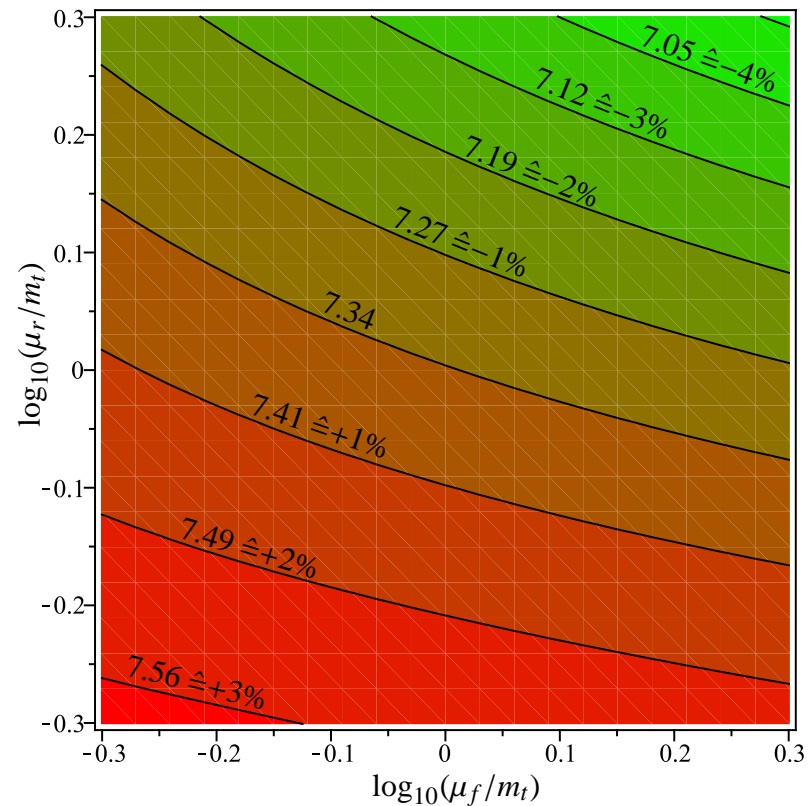
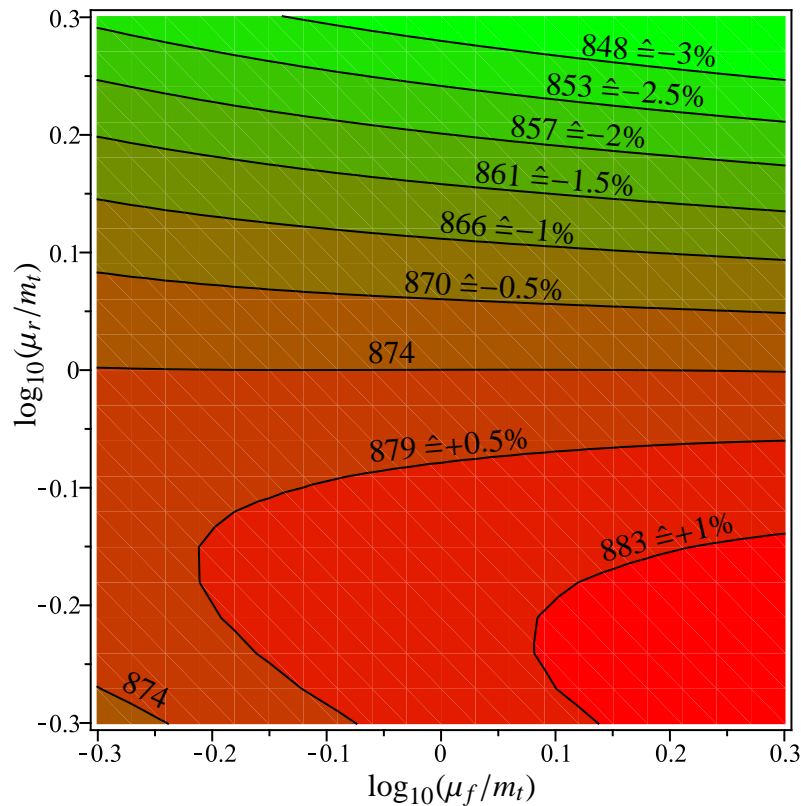
LHC



Tevatron

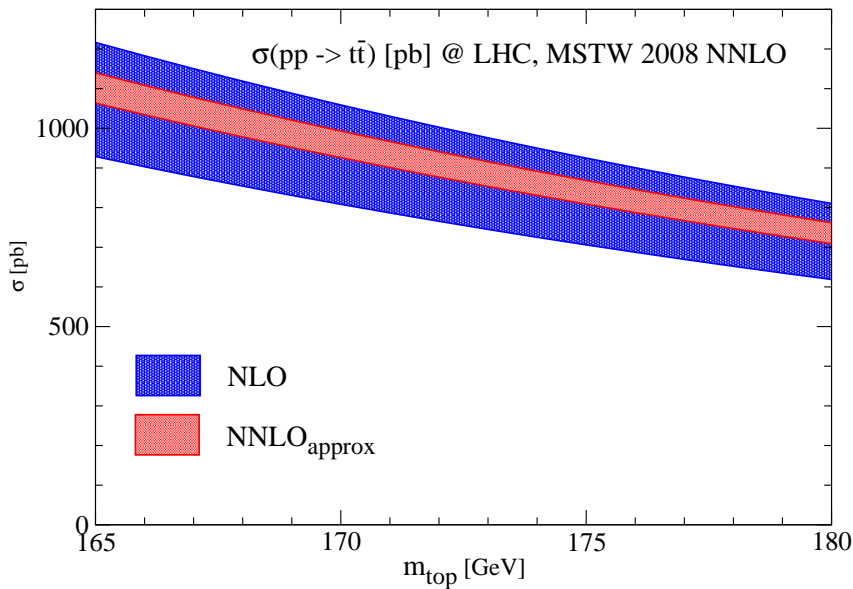
Scale dependence (III)

- Contour lines of total cross section for $\mu_R \neq \mu_F$
 - independent variation of renormalization and factorization scale
 - range corresponds to $\mu_R, \mu_F \in [m_t/2, 2m_t]$
 - plot with PDF set CTEQ6.6 (but largely independent on PDFs)

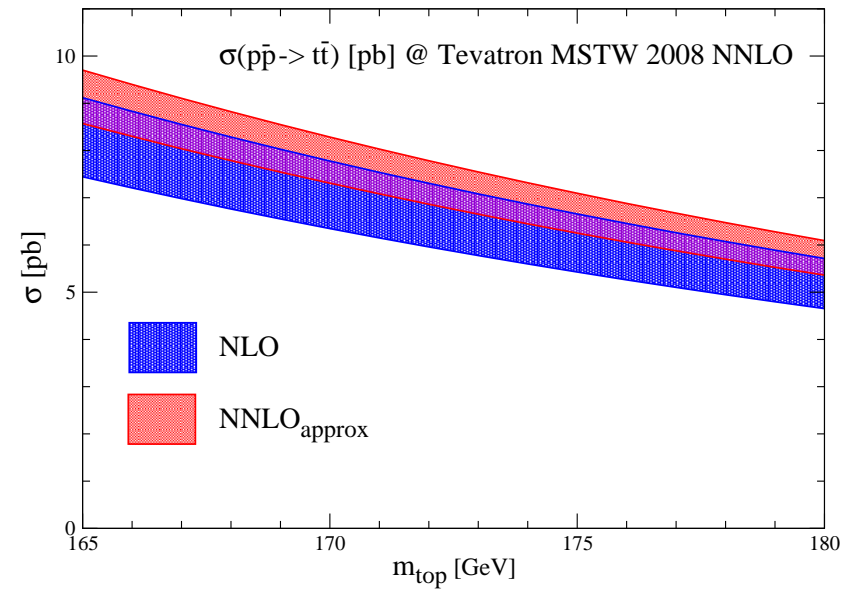


Total theory uncertainty

- NLO (with MSTW2008 PDF set)
 - scale uncertainty $\mathcal{O}(10\%) \oplus$ PDF uncertainty $\mathcal{O}(5\%)$
- NNLO_{approx} (with MSTW2008 PDF set)
 - scale uncertainty $\mathcal{O}(3\%) \oplus$ PDF uncertainty $\mathcal{O}(2\%)$
- Theory at NNLO matches anticipated experimental precision $\mathcal{O}(10\%)$



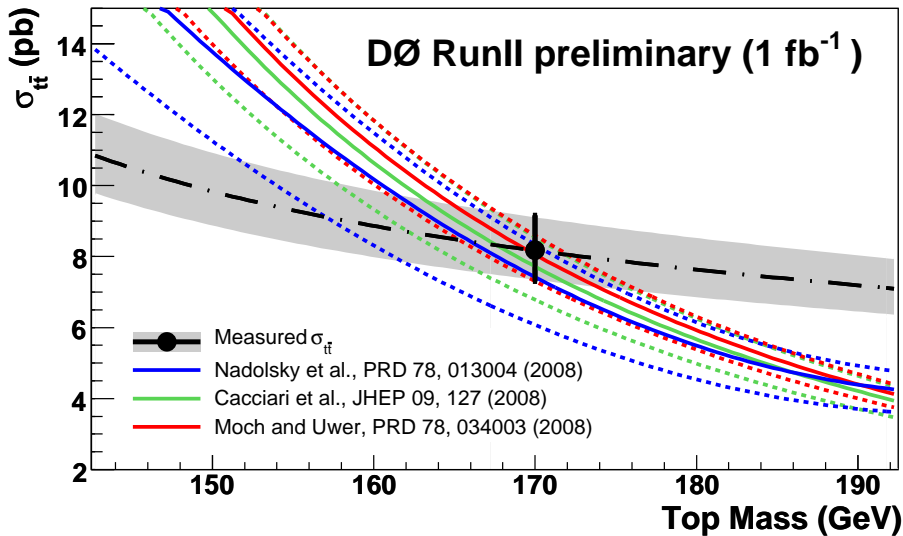
LHC



Tevatron

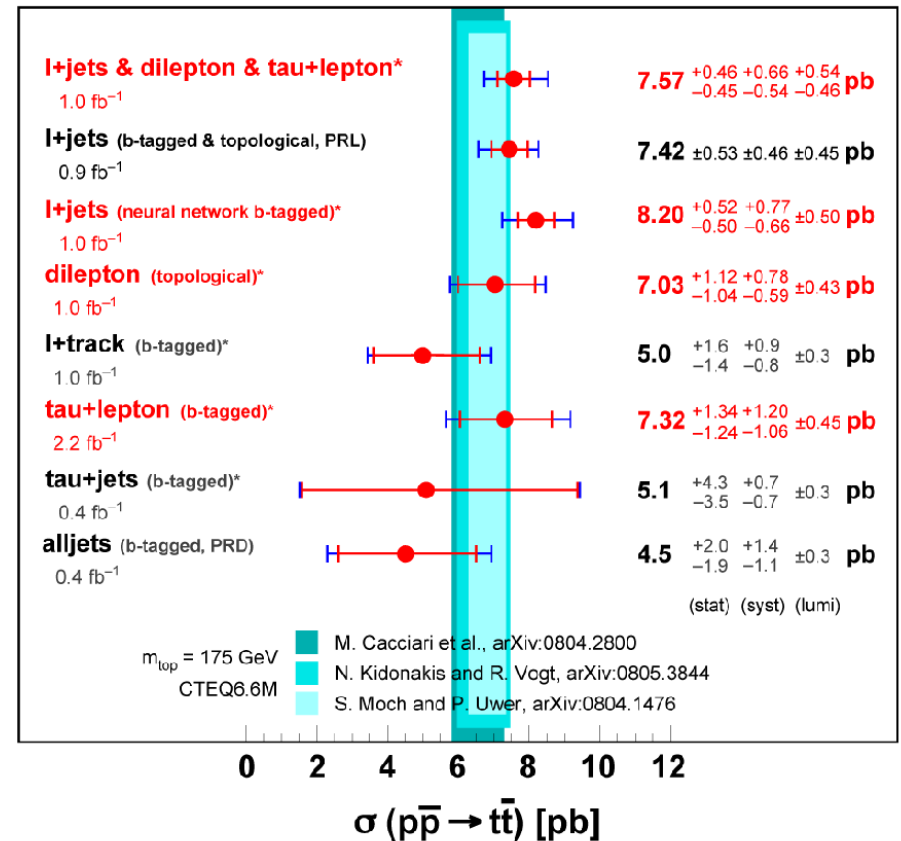
Tevatron analyses

- Total cross section and different channels of Tevatron analyses (theory uncertainty band from scale variation)
- NNLO allows for precision determinations of m_t from total cross section (slope $d\sigma/dm_t$)



DØ Run II * = preliminary

April 2009



Quality control

- Total cross section with $m_t = 173 \text{ GeV}$ at scale $\mu = \mu_R = \mu_F$
 - numbers from [arXiv:0804.1476](#), [arXiv:0807.2794](#)

	CTEQ6.6	MSTW08	CTEQ6.6	MSTW08
$\sigma_{\text{LO}}[\text{pb}]$	553		5.47	
$\sigma_{\text{NLO}}[\text{pb}]$	831		6.79	
$\sigma_{\text{NNLO}}[\text{pb}]$	872		7.34	

LHC **Tevatron**

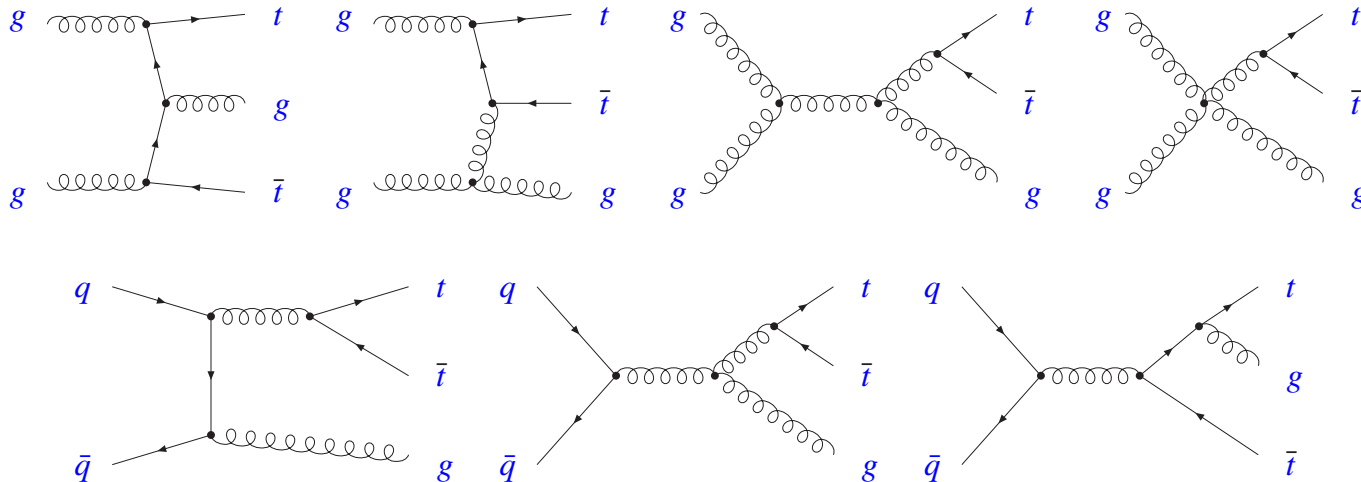
Quality control

- Total cross section with $m_t = 173 \text{ GeV}$ at scale $\mu = \mu_R = \mu_F$
 - numbers from this study

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	LHC		Tevatron	

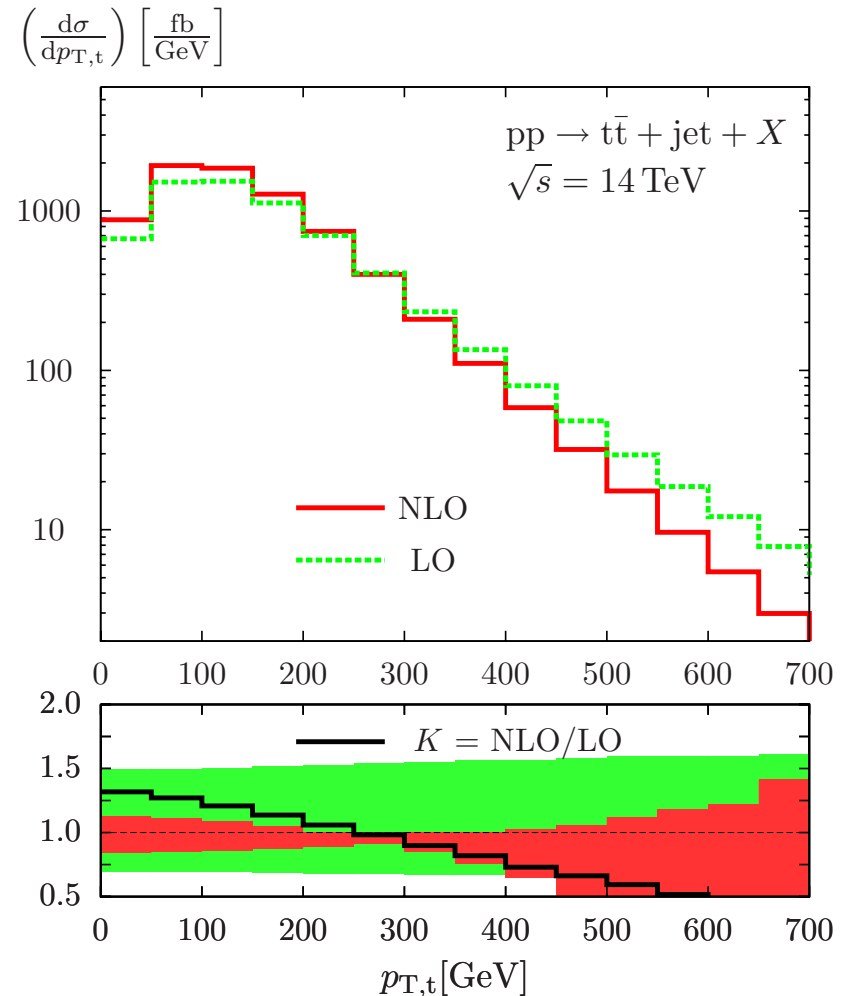
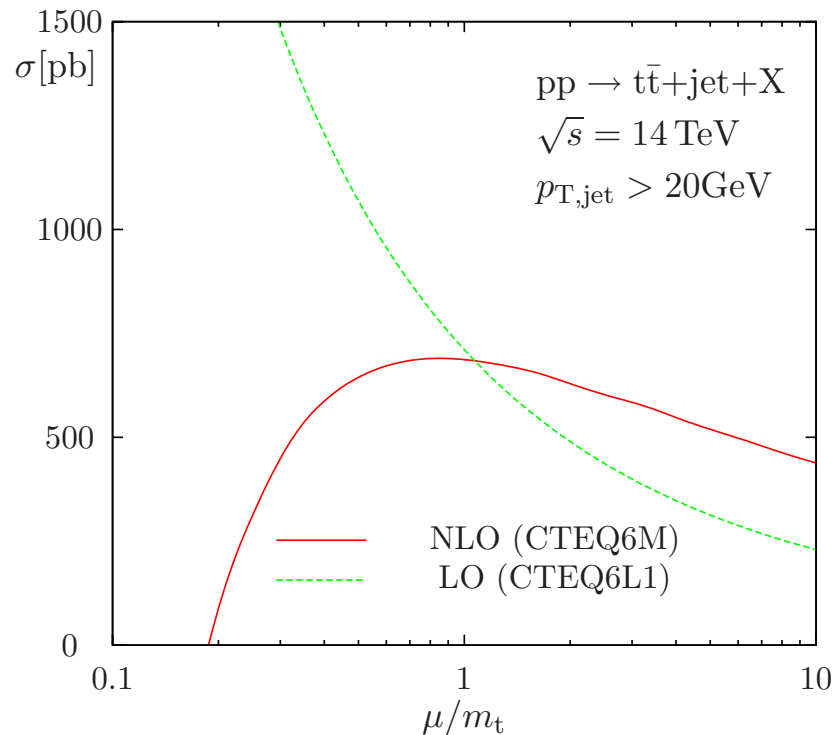
$t\bar{t} + \text{jet production (I)}$

- LHC: large rates for production of $t\bar{t}$ -pairs with additional jets
 - sample Feynman diagrams for $t\bar{t} + \text{jet}$ production at LO



- NLO corrections to $t\bar{t} + \text{jet}$ production are part of NNLO corrections for inclusive $t\bar{t}$ production
 - at scale $\mu_R = \mu_F = m_t$ corrections are almost zero
 - threshold resummation captures dominant contributions

$t\bar{t}$ + jet production (II)



- Impressive state-of-the-art NLO QCD result [Dittmaier, Uwer, Weinzierl '07-'08](#)
 - much improved scale dependence of total rate
 - transverse-momentum distributions of top-quark $p_{T,t}$ along with K-factor and scale variation $m_t/2 \leq \mu \leq 2m_t$

$t\bar{t}$ + jet production (III)

$p_{T,\text{jet,cut}}$ [GeV]	$\sigma_{t\bar{t}\text{jet}}$ [pb]	
	LO	NLO
20	$1.583(2)^{+0.96}_{-0.55}$	$1.791(1)^{+0.16}_{-0.31}$
30	$0.984(1)^{+0.60}_{-0.34}$	$1.1194(8)^{+0.11}_{-0.20}$
40	$0.6632(8)^{+0.41}_{-0.23}$	$0.7504(5)^{+0.072}_{-0.14}$
50	$0.4670(6)^{+0.29}_{-0.17}$	$0.5244(4)^{+0.049}_{-0.096}$

Tevatron

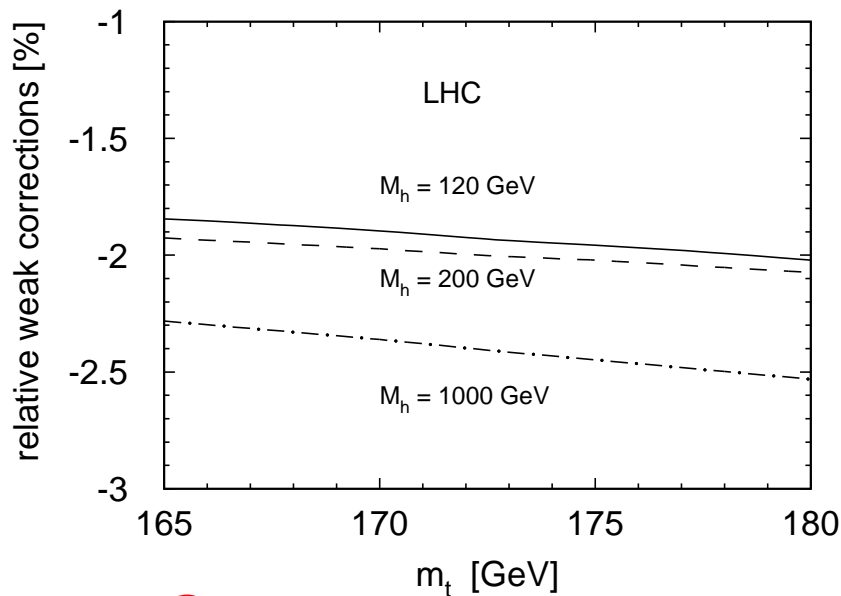
$p_{T,\text{jet,cut}}$ [GeV]	$\sigma_{t\bar{t}\text{jet}}$ [pb]	
	LO	NLO
20	$710.8(8)^{+358}_{-221}$	$692(3)3^{-40}_{-62}$
50	$326.6(4)^{+168}_{-103}$	$376.2(6)^{+17}_{-48}$
100	$146.7(2)^{+77}_{-47}$	$175.0(2)^{+10}_{-24}$
200	$46.67(6)^{+26}_{-15}$	$52.81(8)^{+0.8}_{-6.7}$

LHC

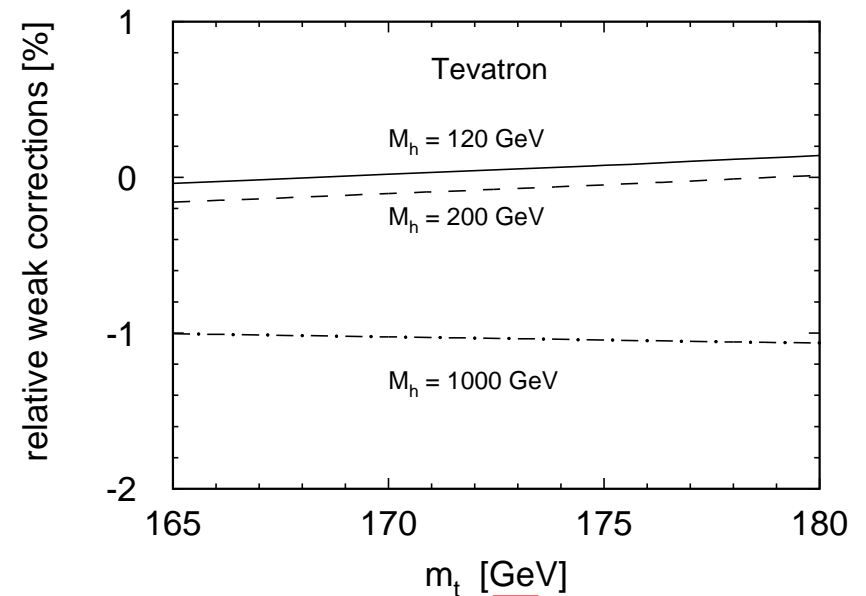
- Cross section $\sigma_{t\bar{t}\text{jet}}$ for different values of $p_{T,\text{jet,cut}}$ for $\mu = \mu_R = \mu_F = \{m_t/2, m_t, 2m_t\}$ with PDF sets CTEQ6L1, CTEQ6M
 Dittmaier, Uwer, Weinzierl '07-'08

Electroweak corrections

- Electroweak corrections (ratio of σ_{EW}/σ_{LO})
Bernreuther, Fückler '05; Kühn, Uwer, Scharf '06
- Effect depends on Higgs mass
(choices $m_H = 120\text{GeV}$, $m_H = 200\text{GeV}$, $m_H = 1000\text{GeV}$)



LHC

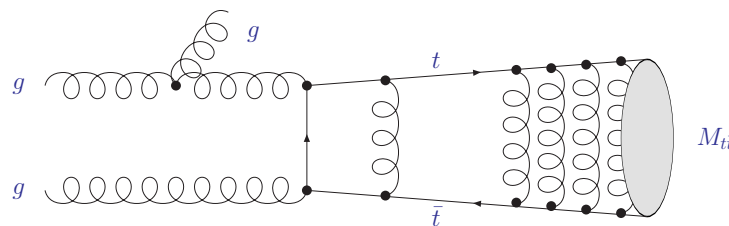


Tevatron

- Tevatron: vanishing contribution for light Higgs
- LHC: $\mathcal{O}(2\%)$ with respect to σ_{LO}
negative contribution to total cross section $\Delta\sigma_{EW} \simeq \mathcal{O}(10 - 15) \text{ pb}$

Coulomb corrections

- Heavy quark production very close to threshold
 - resummation of Coulomb corrections $\sim 1/\beta$ to all orders (non-relativistic QCD)
 - NRQCD factorization Bodwin, Braaten, Lepage '95
- Much work (theory and phenomenology) for ILC
 - fixed center-of-mass energy S allows threshold scan at $\sqrt{S} \sim 2m_t$
 - dominant color-singlet production $\rightarrow t\bar{t} \left({}^3S_1^{[1]} \right)$
- Effects on top-mass measurement at LHC Hagiwara, Sumino, Yokoya '08
- Detailed study in NRQCD assembling existing knowledge at NLO/NLL Kiyoyama, Kühn, S.M., Steinhauser, Uwer '08
 - complete NLO NRQCD result Petrelli, Cacciari, Greco, Maltoni, Mangano '97 (corrections by Hagiwara, Sumino, Yokoya '08)
 - NLL resummation Cacciari '99

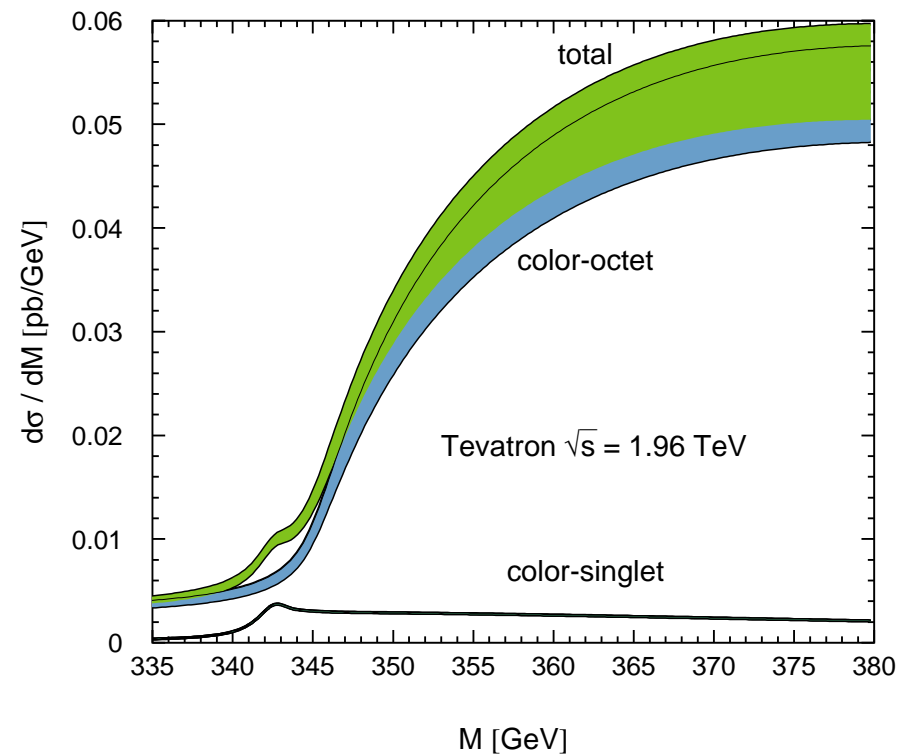
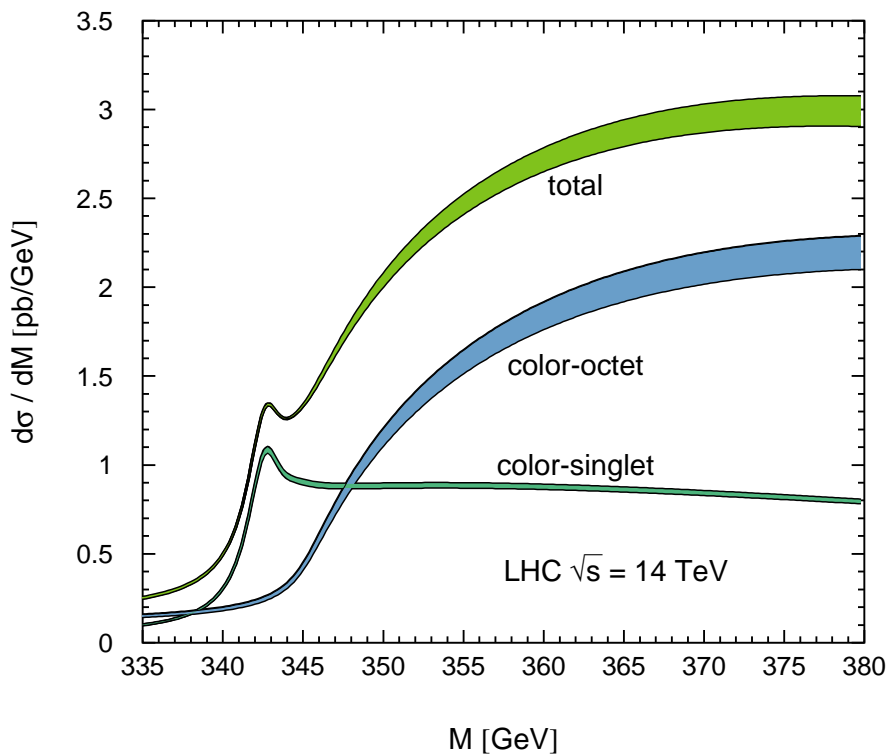


Coulomb corrections

- Recall master equation
$$\sigma_{pp \rightarrow t\bar{t}} = \sum_{ij} f_i \otimes f_j \otimes \hat{\sigma}_{ij \rightarrow t\bar{t}}$$
- Convolution with PDFs $f_i \otimes f_j$
 - top-quark pairs produced as color-singlets and color-octets
 $\rightarrow t\bar{t} \left({}^{2s+1}S_J^{[1,8]} \right)$
 - threshold at $M_{t\bar{t}} \sim 2m_t$ with $M_{t\bar{t}} = (p_t + p_{\bar{t}})^2$
- NRQCD factorization of partonic cross section into
$$\hat{\sigma}_{ij \rightarrow t\bar{t}} = F_{ij \rightarrow T} \otimes G(M_{t\bar{t}})$$
 - free $t\bar{t}$ production rate F
 - evolution factor into “boundstate” (Green’s function) G
- Differential kinematics
$$\frac{d\hat{\sigma}_{ij \rightarrow t\bar{t}}}{dM_{t\bar{t}}^2} = F_{ij \rightarrow T} \times \Im G^{[1,8]}(M_{t\bar{t}})$$
 - factorization of soft-collinear dynamics (real emission radiation)
 - matching at NLO and NLL resummation

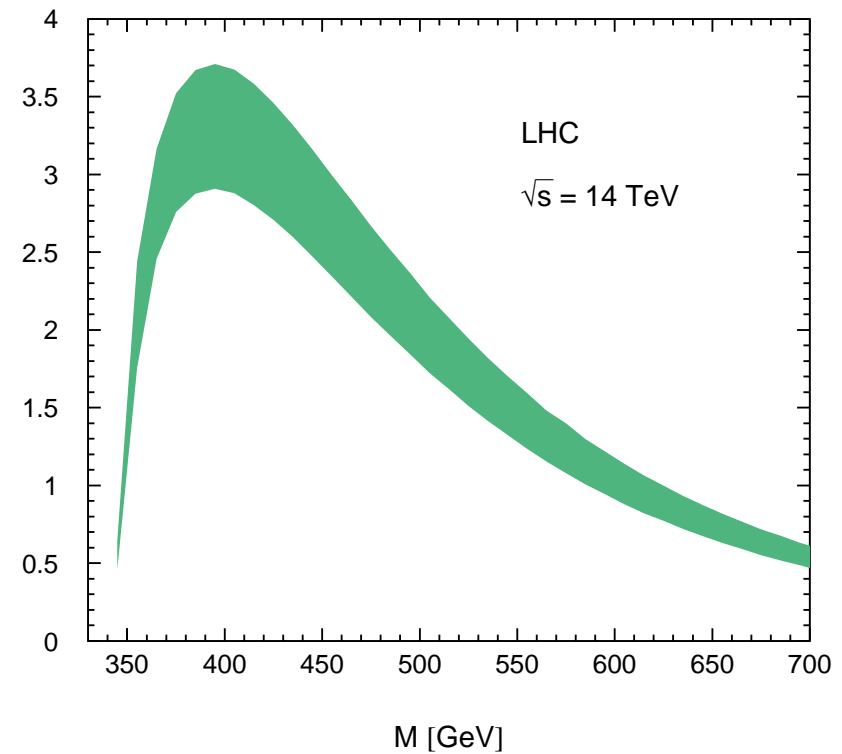
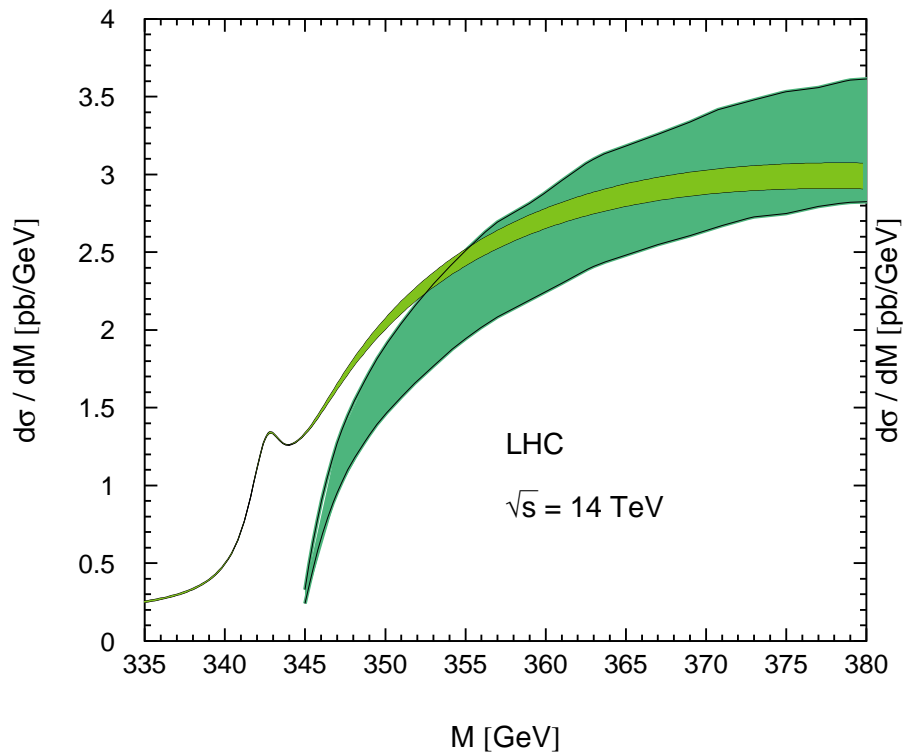
Invariant mass distribution

- $d\sigma/dM_{t\bar{t}}$ at LHC driven by large gluon luminosity
 - $gg \rightarrow t\bar{t} \left({}^1S_0^{[1]} \right)$ dominates
- $d\sigma/dM_{t\bar{t}}$ at Tevatron with small bound state effects
 - $q\bar{q}$ -channel large with only color-octet configurations only



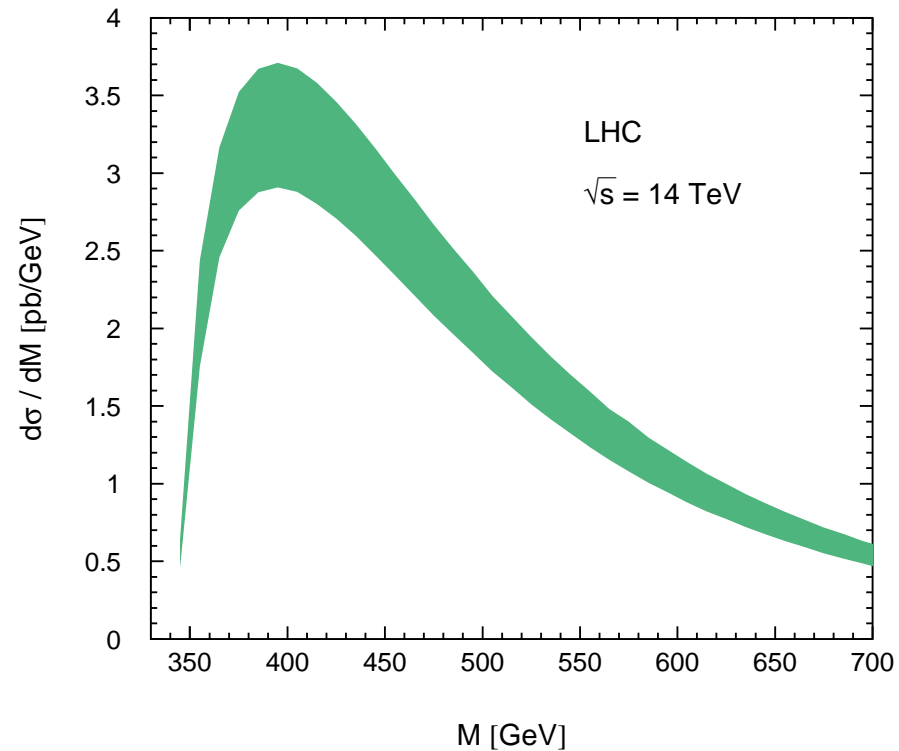
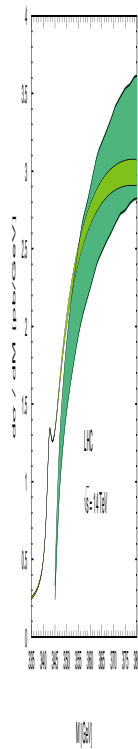
Matching to fixed order

- $d\sigma/dM_{t\bar{t}}$ with at LHC
 - compare NLL resummed result in NRQCD with (plain vanilla) NLO (use HVQMNR [Mangano, Nason, Ridolfi '92](#))
 - consistency check OK



Invariant mass distribution

- Resolution of bound state effects in $d\sigma/dM_{t\bar{t}}$ at LHC difficult (requires rather fine binning)
 - uncertainty of total cross section $\Delta\sigma \simeq \mathcal{O}(10)$ pb
 - extrapolation of $M_{t\bar{t}}$ -distribution affected by $gg \rightarrow t\bar{t} \left({}^1S_0^{[1]} \right)$



Summary

- Top quark theory
 - improved understanding of theory and application of new concepts
 - resummation important for Tevatron and LHC phenomenology
- Total cross section
 - NNLO_{approx} prediction with exact scale dependence $\mu_R \neq \mu_F$ ($\ln(\mu_R/m)$, $\ln(\mu_F/m)$ -terms)
 - cross check on systematics with NLO correction to $t\bar{t} + \text{jet}$
 - investigation of theoretical uncertainty (scale)
 - $\Delta\sigma \sim \mathcal{O}(\pm 3)\%$ at LHC
 - $\Delta\sigma \sim \mathcal{O}(\pm 4)\%$ at Tevatron
- Other corrections
 - electroweak corrections
 - bound state effects for $t\bar{t}$ -system
 - ...