Resummation and top-pair production

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Work with:

Michael Czakon
arXiv:0812.0353
arXiv:0811.4119
in progress …

George Sterman and Ilmo Sung
arXiv:0903.3241
in progress …

The partonic cross-section

The observed cross-section is an integral over the product of:

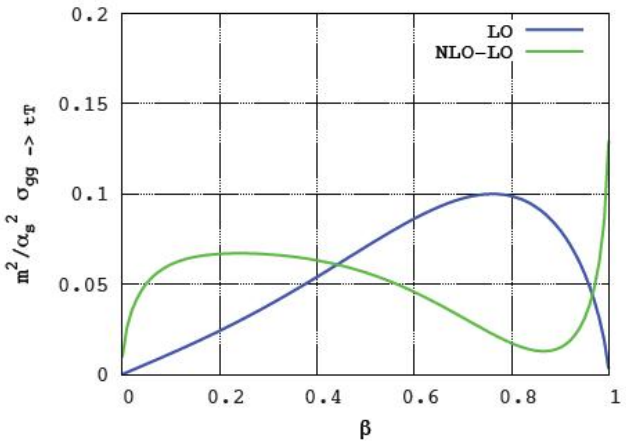
- Partonic cross-section,
- Partonic flux.

$$\sigma(s_{\text{had}}) = \sum_{ij} \int_0^{\beta_{\max}} d\beta\, \Phi(\beta)\, \hat{\sigma}_{\text{part}}(\beta)$$

$$\rho = \frac{4m_t^2}{s}$$

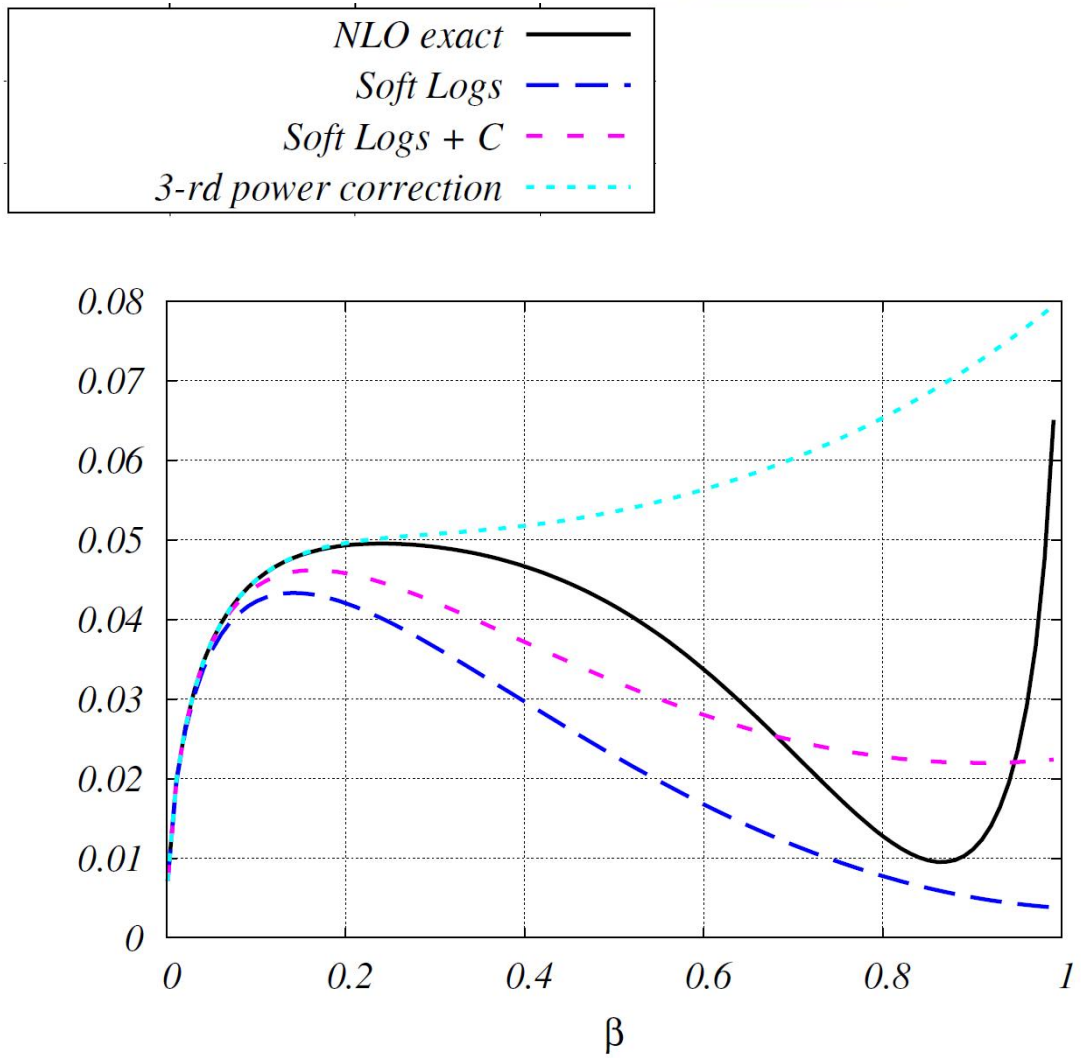
$$\beta = \sqrt{1-\rho}$$

$$\eta = \frac{s}{4m_t^2} - 1$$

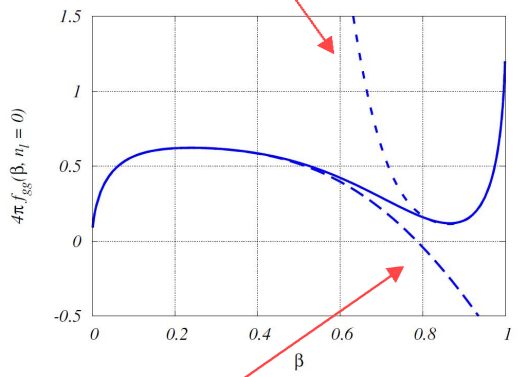


Large NLO corrections!

The partonic cross-section: power expansion



Massless expansion (6 powers)

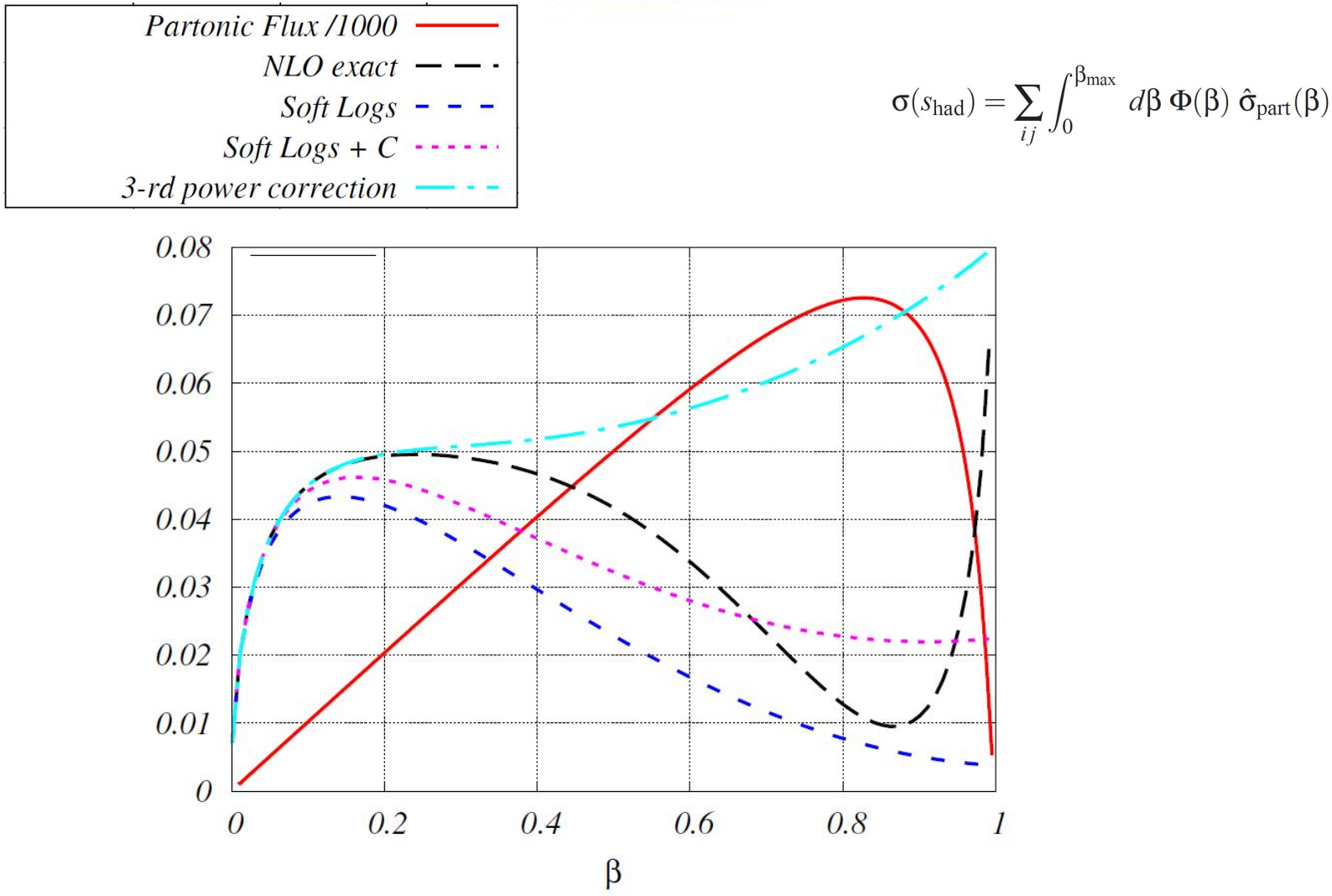


Threshold expansion (6 powers)

Czakon, AM '08

The sub-leading powers are large (and divergent)!

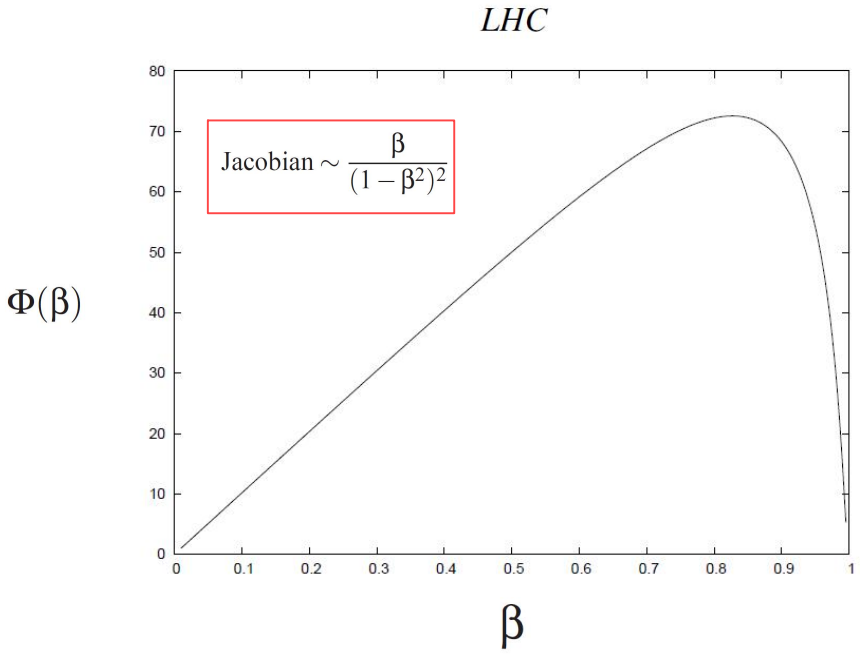
Does the flux change things?

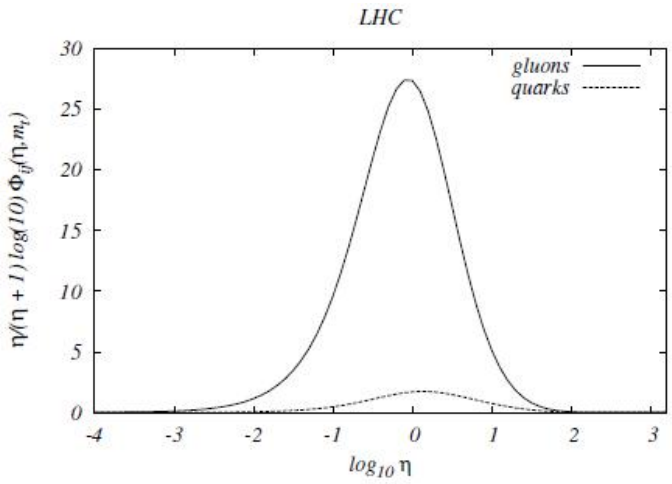


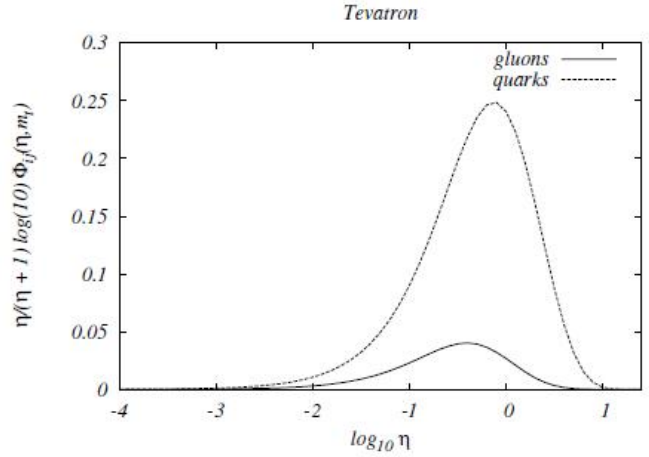
$$\sigma(s_{\rm had}) = \sum_{ij} \int_0^{\beta_{\rm max}} d\beta\, \Phi(\beta)\, \hat{\sigma}_{\rm part}(\beta)$$

The flux does not predominantly sample the threshold region!

The partonic flux







What do we learn from the known NLO results?

- The soft approximation is not a good approximation to FO
- The partonic flux does not help much either ☺
- Yet, at hadronic level, the soft approximation is only few % away from the exact NLO.

This requires the hard matching constant!
Constant itself not predicted by the resummation;
comes from FO calculation.

Let's say we try to use it for guessing NNLO;

how does the soft resummation work?

How is the threshold resummation done?

The resummation of soft gluons is driven mostly by kinematics:

Sterman '87
Catani, Trentadue '89

- Only soft emissions possible due to phase space suppression (hence kinematics)
- That's all there is for almost all "standard" processes: Higgs, Drell-Yan, DIS, e^+e^-

Key: the number of hard colored partons < 4

In top pair production (hadron colliders) new feature arises:

Color correlations due to soft exchanges (n>=4)

Non-trivial color algebra in this case.

How is the threshold resummation done?

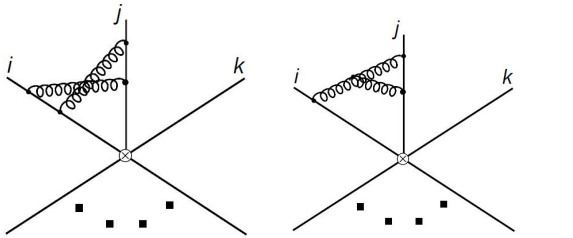
For a general n-colored parton amplitude we have:

$$M_I = J \cdot S_{IJ} \cdot H_J$$

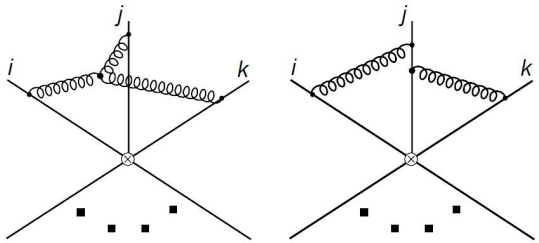
Soft function:

- contains all soft color correlations,
- matrix (in color space) for > 3 partons

Two-particle correlations:
(easy ☺ - massive formfactor)



Three-particle correlations:
(very hard – massive box)



They are never small no matter how soft the gluons!

How is the threshold resummation done?

The soft function (and its anomalous dimension) control the single poles of n-loop amplitudes; appear in resummations at hadron colliders.

In the massive case it is know explicitly at 1 loop

Kidonakis, Sterman '97
Catani, Dittmaier, Trocsanyi '01

First 2-loop results appeared very recently:

A.M., Sterman, Sung '09
Becher, Neubert '09
N. Kidonakis '09

The results are not totally explicit (yet),
but few very important properties were established:

- The matrix diagonalizes at 2-loops close to partonic threshold,
- Its structure is very different from the massless case.

How is the threshold resummation done?

How is all that related to top resummation?

The relation $M_I = J \cdot S_{IJ} \cdot H_J$

Hard matching coefficients

Implies that, close to threshold, one has:

$$M_I \sim e^{\Gamma_{IJ}} \cdot H_J = e^{\Gamma_1} H_1 + e^{\Gamma_8} H_8$$

Singlet-octed eigenvalues

Implication: for top-pair we need two independent Sudakov exponents.

$$\sigma_{ij}^{\mathrm{TOT}}(N) = \sigma_{ij,\mathbf{1}}(N) + \sigma_{ij,\mathbf{8}}(N)$$

$$\sigma_{ij,\mathbf{I}}(N) = \sigma_{ij,\mathbf{I}}^{\mathrm{Born}}(N)\, \sigma_{ij,\mathbf{I}}^{\mathrm{H}}\, \Delta_{ij,\mathbf{I}}(N)$$

Sudakov exponent

i,j=gg,qq

N-Mellin moment: $f(N) = \int_0^1 \rho^{N-1} f(\rho) d\rho$.

Get the cross-section

The NLO hard matching coefficients were calculated only very recently:

$$\sigma^{\mathrm{H}}_{q\bar{q},\mathbf{1}} = O(\alpha_s^2),$$

$$\sigma^{\mathrm{H}}_{q\bar{q},\mathbf{8}} = 1+\frac{\alpha_s}{\pi}\left[C_F\left(-8+\frac{2\pi^2}{3}+3\log 2\right)+C_A\left(\frac{59}{9}-\frac{\pi^2}{4}-3\log 2\right)\right.$$
$$\left. + n_l\left(-\frac{5}{9}+\frac{2\log 2}{3}\right)-\frac{8}{9}+\log\left(\frac{\mu^2}{m^2}\right)\left(-\frac{3}{2}C_F+\frac{11}{6}C_A-\frac{n_l+1}{3}\right)\right]+O(\alpha_s^2)$$

M. Czakon, A.M. '08

$$\sigma^{\mathrm{H}}_{gg,\mathbf{1}} = 1+\frac{\alpha_s}{\pi}\left[C_F\left(-5+\frac{\pi^2}{4}\right)+C_A\left(1+\frac{5\pi^2}{12}\right)-\frac{1}{3}\log\left(\frac{\mu^2}{m^2}\right)\right]+O(\alpha_s^2)$$

$$\sigma^{\mathrm{H}}_{gg,\mathbf{8}} = 1+\frac{\alpha_s}{\pi}\left[C_F\left(-5+\frac{\pi^2}{4}\right)+C_A\left(3+\frac{7\pi^2}{24}\right)-\frac{1}{3}\log\left(\frac{\mu^2}{m^2}\right)\right]+O(\alpha_s^2)$$

Surprisingly simple expressions!

Agree with results from quarkonium production:

Hagiwara, Sumino, Yokoya '08
Kuhn, Mirkes '93
Petrelli, Cacciari, Greco, Maltoni, Mangano '97

Numerical findings

At that point we have everything we need at NLO.

By putting it together we find unexpected numerical effects:

Threshold expansion $\beta \to 0$: (i.e. $4m^2 \to s$)

$$f_{gg}^{(1)}(\beta) = \frac{1}{4\pi^2} f_{gg}^{(0)}(\beta) \left(\left(C_F - \frac{(N^2-4)\,C_A}{2(N^2-2)} \right) \frac{\pi^2}{2\beta} + 2C_A \log^2\left(8\beta^2\right) - \frac{(9N^2-20)C_A}{N^2-2} \log\left(8\beta^2\right) \right.$$
$$+ C_A \left(\frac{21N^2-50}{N^2-2} - \frac{(17N^2-40)\,\pi^2}{24\,(N^2-2)} + \frac{(N^2-4)\log 2}{N^2-2} - 2\log^2 2 \right)$$
$$\left. + C_F \left(-5 + \frac{\pi^2}{4} \right) + o(\beta) \right) . \qquad (27)$$

Extraction of the constant in the threshold limit:

$$C_A \left(\frac{21N^2-50}{N^2-2} - \frac{(17N^2-40)\,\pi^2}{24\,(N^2-2)} + \frac{(N^2-4)\log 2}{N^2-2} - 2\log^2 2 \right) + C_F \left(-5 + \frac{\pi^2}{4} \right) \qquad \frac{768\pi}{7} a_0^{gg} \simeq 37.25 .$$
$$= \frac{1111}{21} - \frac{283\pi^2}{168} + \frac{15\log 2}{7} - 6\log^2 2 \simeq 34.88 ,$$

Used in earlier literature

Interested lesson in numerics: the earlier numeric cross-section is better than 1% but its derivative (close to threshold) is 7% off.

Numerical findings

From resummation, the following 2 loop logs can be predicted:

$$\sigma_{gg}(\beta) = \sigma_{gg}^{\text{Born}}(\beta) + \frac{\alpha_s}{4\pi}\sigma_{gg}^{(1)} + \left(\frac{\alpha_s}{4\pi}\right)^2 \sigma_{gg}^{(2)} + O(\alpha_s^3)$$

$$\sigma_{gg}^{(2)} = \sigma_{gg}^{\text{Born}}(\beta)\left(4608\log^4\beta + 1894.9\log^3\beta - 3.4811\log^2\beta + O(\log\beta)\right)$$

Moch Uwer '08

It turns out the coefficient of $\ln^2(\beta)$ is of the form: $-14306.9505 + 384C_3$

where: $C_3 = 37.23$ Old numeric value

$C_3 = 34.88$ New analytic value

Therefore the coefficient of $\ln^2(\beta)$ becomes -912.35

Note: the reason is pure numerics!

i.e. a change by a factor of 260 !

Get the cross-section

How we put all this to work?

- Match fixed order and resummed results:

$$\sigma_{RESUM} = \sigma_{NLO} + \sigma_{SUDAKOV} - \sigma_{OVERLAP}$$

Known at NLO, not at NNLO

- σ_{NLO} is known exactly,
- $\sigma_{SUDAKOV}$: anomalous dimensions and matching coefficients needed.

Known at NLO M. Czakon, A.M. '08

Numerical Findings

$$\sigma_{gg}^{H\,(\mathrm{BCMN})} = 1+\frac{\alpha_s}{\pi}\,14.39+\mathcal{O}(\alpha_s^2),$$

$$\sigma_{gg}^{H\,(\mathrm{BCMN})}|_{C_3\ \mathrm{exact}} = 1+\frac{\alpha_s}{\pi}\,12.04+\mathcal{O}(\alpha_s^2),$$

$$\sigma_{gg,\mathbf{1}}^{H} = 1+\frac{\alpha_s}{\pi}\,9.16+\mathcal{O}(\alpha_s^2),$$

$$\sigma_{gg,\mathbf{8}}^{H} = 1+\frac{\alpha_s}{\pi}\,13.19+\mathcal{O}(\alpha_s^2),$$

C_3 numerics: -5%,
color singlet channel: -12%,
color octet channel: -3%,

Their implications :

- ✓ Formally these effects are beyond NLL; yet significant numerically
- ✓ Must be taken into account beyond NLL !

Top quark pair: "the numbers"

The central values (LHC):

- FO NLO / FO LO: 50%
- NLL / FO NLO: 4%
- New NLO effects / FO NLO: 1-1.5% Czakon, AM
- Beyond NLL effects / FO NLO: 0.8% Moch, Uwer

Current theory error estimate (NLO/NLL): ~ 10%

Uncertainty =/= just scale variation !!!

Important: No genuine NNLO term is known (could easily give 5%) !

Top quark pair: pdf's

Comparison of central values for:

- m_{top}=172.4 GeV
- μ=m
- correct exact hard matching coefficients
- Coulombic effects not elaborated upon.

$\alpha_s(M_Z)$:

CTEQ 6.6: 0.118
MRST 2006 nnlo: 0.119
MSTW 2008 nnlo: 0.117
MSTW 2008 nlo: 0.120

MRST 2006 nnlo

NLO = 890 pb
NLO+NLL = 918 pb

5.4 %

CTEQ 6.6

NLO = 844 pb
NLO+NLL = 871 pb

MSTW 2008 nnlo

NLO = 857 pb
NLO+NLL = 885 pb

1.6 %

MSTW 2008 nlo

NLO = 906 pb
NLO+NLL = 935 pb

7.3 %

Czakon, AM in progress

Conclusions

Only now we have complete NLO/NLL. M. Czakon, A.M. (2008)
This is the most complete (and consistent) description of top-pair we have at the moment.

- Soft gluon approximation is not really good at parton level
- Partonic flux does not predominantly sample the threshold region (LHC)

- Yet, when integrated to hadron level SG approx produces good results!

Question: The uncertainty/how to = ? (it is close to 10%, not 3%)

- Now we have good understanding of the "A-constant":

Bonciani, Catani, Mangano, Nason '98

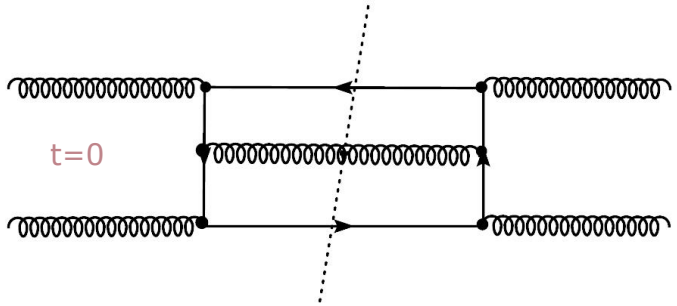
Sub-leading powers are non-negligible; in fact are equally important!

- The new set MSTW 2008 NNLO is (much) closer to CTEQ6.6 (for top-pair)

- Two-loop anomalous dimensions will appear. That is the best that can be done before the NNLO calculation is ready!

Top quark: How complicated is the NLO?

Here are few sample diagrams at NLO:



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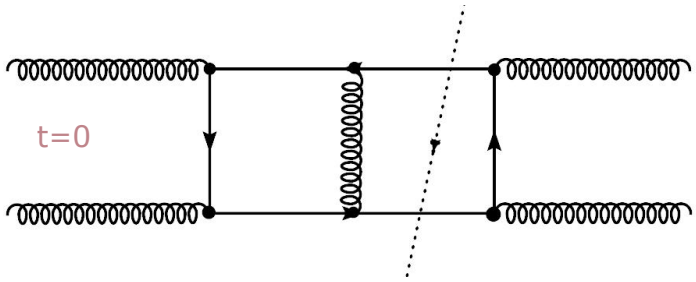
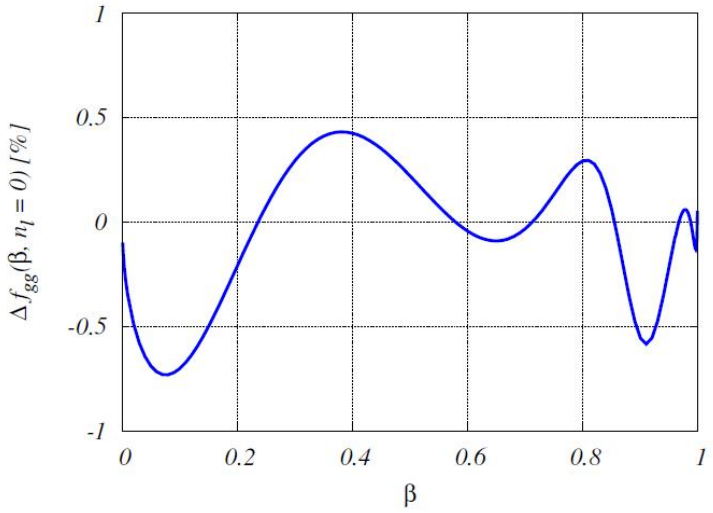


+ crossed

- Note: these are 2 loop (cut) boxes with masses.
  Not studied before.

Main details of the exact NLO calculation

- For 20 years σ_{TOT} was known as a numerically derived fit
- Newly calculated analytical results (new techniques):
  - The whole problem is mapped into 37 masters (real+virtual)
  - We find that the cross-section develops new unphysical singularities!
  - Appearance of elliptic functions,
  - We confirm the high numerical accuracy of the earlier FO results (< 1%)

Czakon, AM '08