

Largely based on:  
Lillie, Shu, TT JHEP 0804, 087 (2008)  
Kumar, TT, Vega-Morales, arXiv:0901.3808

# Fun with a Composite Top Quark

Tim M.P. Tait



CERN top  
May 26 2009



# Outline



- Why a composite top?
- KK Gauge Bosons.
- Four top signals at the LHC.
- Outlook and future directions.





# Why a composite top?



- Well, why not?
- This talk is intended to be more about exploring interesting signals than building models to address specific short-comings of the SM.
- But I would like to do a reasonable job of exploring the possibility that the right-handed top is composite.

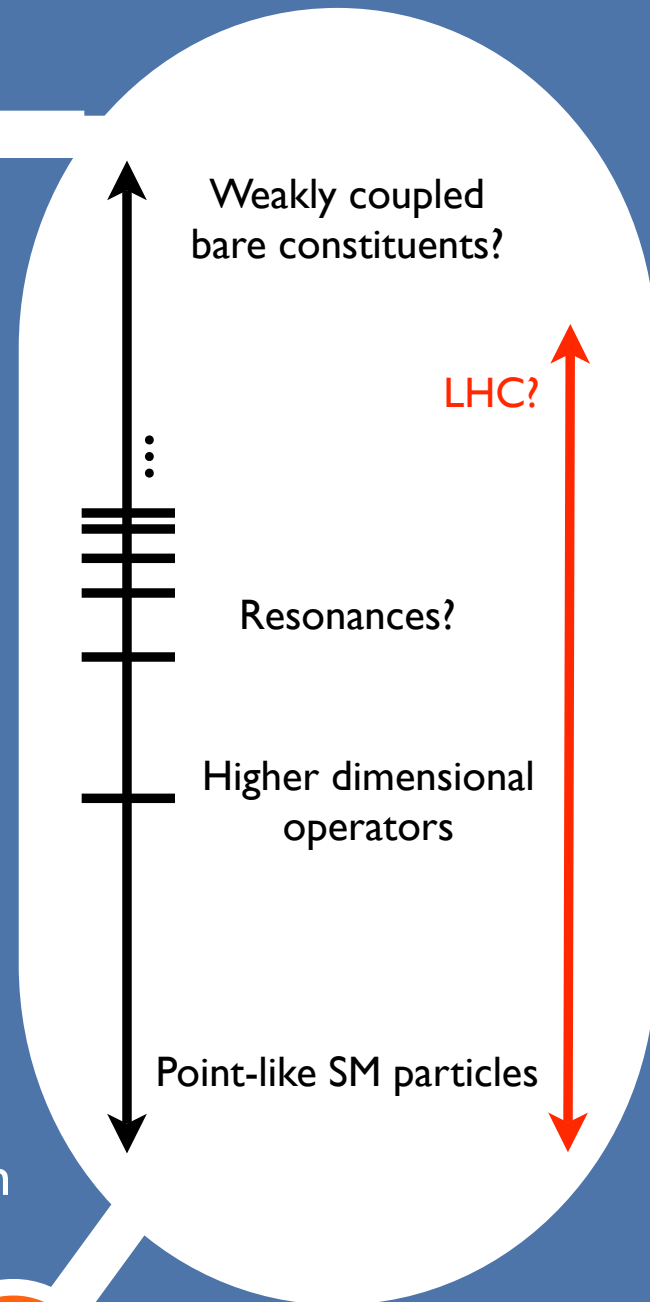
LH + RH cases: Pomarol, Serra PRD78, 074026 (2008)

- The right-handed top is perhaps the most weakly bound sector of the SM by current experimental data.
- How low can we get away with and not be ruled out?
- What would a very low scale of compositeness imply for the LHC?



# Seeing Constituents?

- We know the LHC can discover higher dimensional operators up to large  $\Lambda$ .
- We also have examples of composite theories (RS, Technicolor) for which it can also discover at least the first layer of the resonances. If quarks are composite, some of the resonances should be colored, which helps.
- Standard RS is not a good example to imagine going further than mapping resonances, because its approximate scale invariance implies the resonance description works up to  $\sim$  its UV cut-off which we usually take near  $M_{Pl}$ .



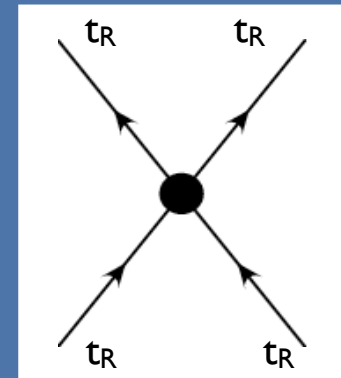
# Effective Field Theory

Eichten, Lane, Peskin PRL50, 811 (1983)

- As usual when talking about compositeness (especially without a specific UV theory in mind), I resort to effective field theory.
- The operator with four right-handed tops has a unique Lorentz structure, and several options for color structures.

$$\frac{g^2}{\Lambda^2} [\bar{t}^i \gamma^\mu P_R t_j] [\bar{t}^k \gamma_\mu P_R t_l]$$

Georgi, Kaplan, Morin, Schenk PRD51, 3888 (1995)



- Two interesting color structures are a pair of singlets and octets.

$$\delta_i^j \delta_k^l \quad (T^a)_i^j (T^a)_k^l$$



# The 4- $t_R$ Operator



- Even independently of any other interest, it is interesting to ask what are the bounds on this family of 4-top operators.
- Insertions of this operator into precision EW observables turn out to result in corrections of order the errors on  $S$  and  $T$  extracted from data, so not very strong constraints.
- The best bound comes from top pair production at the Tevatron.

Georgi, Kaplan, Morin, Schenk PRD51, 3888 (1995)



# Tevatron Bounds

- We'd like to use Tevatron data about top to find out how strong the bounds are - this will tell us whether the LHC has some hope to see constituents, or will have to be content to look for resonances.
- The 4-top operator is difficult to bound at the Tevatron. The natural thing to look for is four top production, but at Tevatron energies that process is negligible. What we need is a contribution to top pair production.
- So we look at operators which modify the top coupling to quarks & gluons:

$$\frac{g_1 g_s}{\Lambda^2} H [\bar{t} \sigma^{\mu\nu} T^a t] G_{\mu\nu}^a \quad \frac{g_2 g_s}{\Lambda^2} [\bar{t} \gamma^\mu D^\nu P_R t] G_{\mu\nu}^a \quad \frac{g_3 g_s}{\Lambda^2} [\bar{t} \gamma^\mu T^a P_R t] \sum_q [\bar{q} \gamma^\mu T^a q]$$

- In the compositeness picture, these operators represent hard gluons which probe the internal structure of the top, seeing the motion of the colored constituents inside it.

Buchmuller, Wyler NPB 268, 621 (1986)  
Atwood, Kagan, Rizzo PRD52, 6264 (1995)  
Hill, Parke PRD49, 4454 (1994)

# Top Pairs

- Naive dimensional analysis suggests the sizes for these operators at scale  $\Lambda$ :

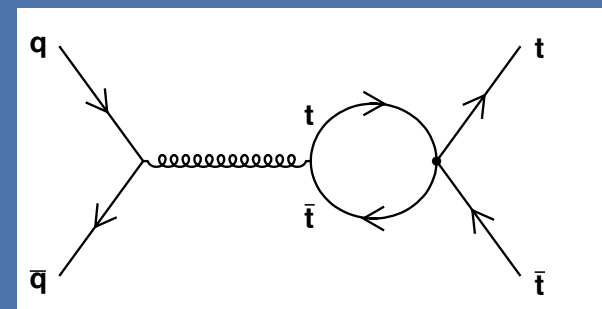
$$g_1(\Lambda) \sim \frac{1}{g} \quad g_2(\Lambda), g_3(\Lambda) \sim 1$$

- The second operator is induced from the 4-top operator through the RGEs.
- The third operator is related through the equations of motion to the second operator plus its Hermitean adjoint. Thus, we can set  $g_3=0$  at the cost of shifting the real part of  $g_2$ .
- We allow for complex  $g_1$  and  $g_2$  in our analysis of top pair production.

$$\frac{g_1 g_s}{\Lambda^2} H [\bar{t} \sigma^{\mu\nu} T^a t] G_{\mu\nu}^a$$

$$\frac{g_2 g_s}{\Lambda^2} [\bar{t} \gamma^\mu D^\nu P_{Rt}] G_{\mu\nu}^a$$

$$\frac{g_3 g_s}{\Lambda^2} [\bar{t} \gamma^\mu T^a P_{Rt}] \sum_q [\bar{q} \gamma^\mu T^a q]$$



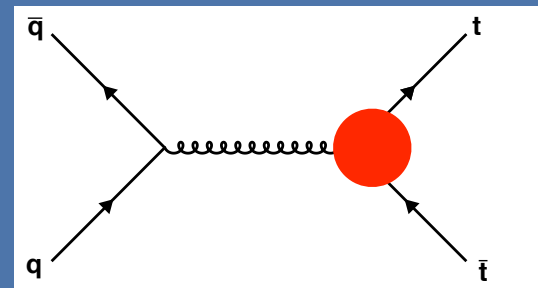
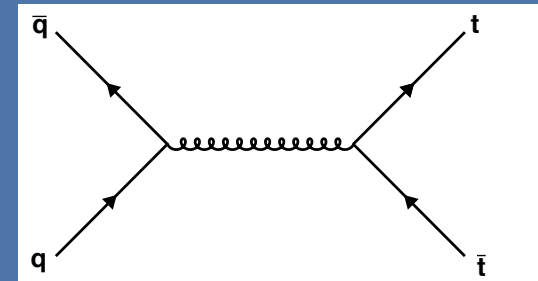


# Top Pairs

- We neglect the gluon fusion contribution, which is a bit less than 15% or so at the Tevatron.
- The dominant correction arises from the new physics amplitudes interfering with the Standard Model.
- We can write the partonic cross section to order  $1/\Lambda^2$  as a correction to the SM prediction:

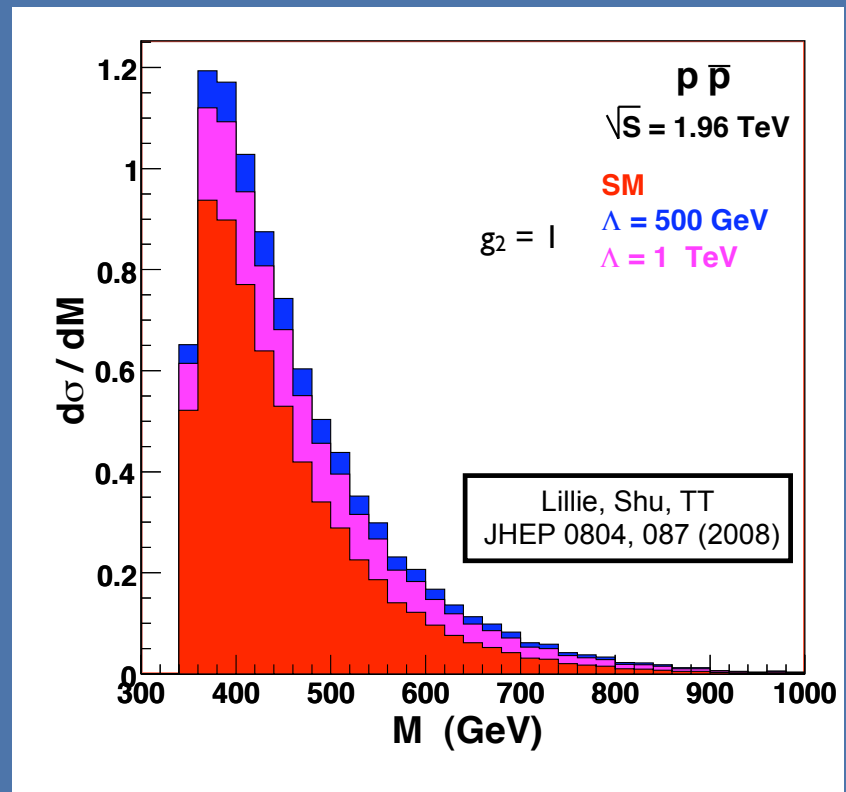
$$\hat{\sigma} = \sigma_{\hat{SM}} \left( 1 + \text{Re} \frac{g_1(16vm s^2) + g_2(4m^2 s^2 + s^3 + s(s + 2t - 2m^2)^2)}{2\Lambda^2(2m^4 + s^2 - 4m^2 t + 2st + 2t^2)} \right) + O\left(\frac{1}{\Lambda^4}\right)$$

- The relative correction grows with energy compared to the SM, and modifies invariant mass and production angle distributions.



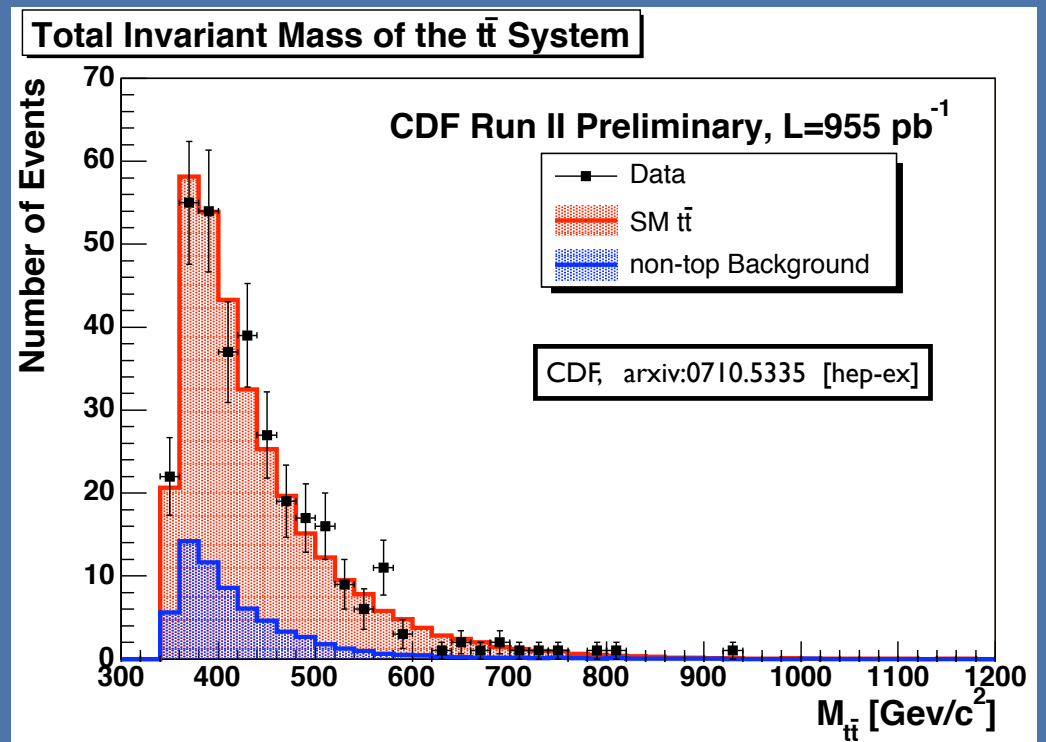
# Invariant Mass Distribution

- An obvious way to get a bound is to study the invariant mass of top pairs. The four top operator causes it to fall off less quickly with  $M$  than the SM prediction (or causes a deficit).
- The distribution shown is LO, and includes the (modified)  $qq$  initial state and (unmodified)  $gg$  initial state. The SM rate was generated at the parton level with MadEvent, and then the new physics was added by hand.



$$M_{t\bar{t}}$$

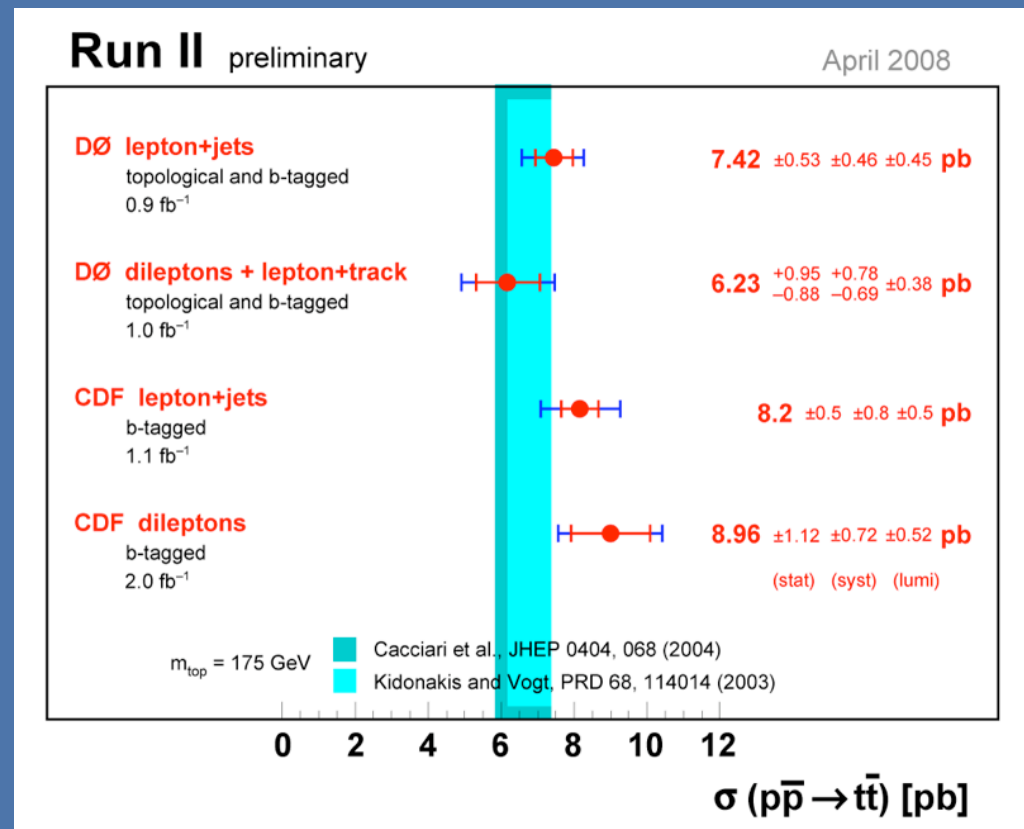
- CDF and D0 have results for top pairs binned in the invariant mass.
- It's not in a form that is immediately useful for a theorist, because it includes efficiencies and some non-top backgrounds.
- However, clearly there is good agreement between the theory expectation and the data.



This analysis puts a bound on narrow resonances decaying to top pairs.

# Total Cross Section

- Since the invariant mass distribution is difficult to extract, I can at least ask that the impact on the total cross section be within the experimental errors.
- Both CDF and D0 have consistent measurements, slightly on the high side of the best theory estimates (but consistent within error bars).



# Bounds

- Taking the most precise measurements:

$$\sigma_{exp} = 7.0 \pm 0.3 \pm 0.4 \pm 0.4 \text{ pb}$$

(statistical)                      (systematic)                      (luminosity)

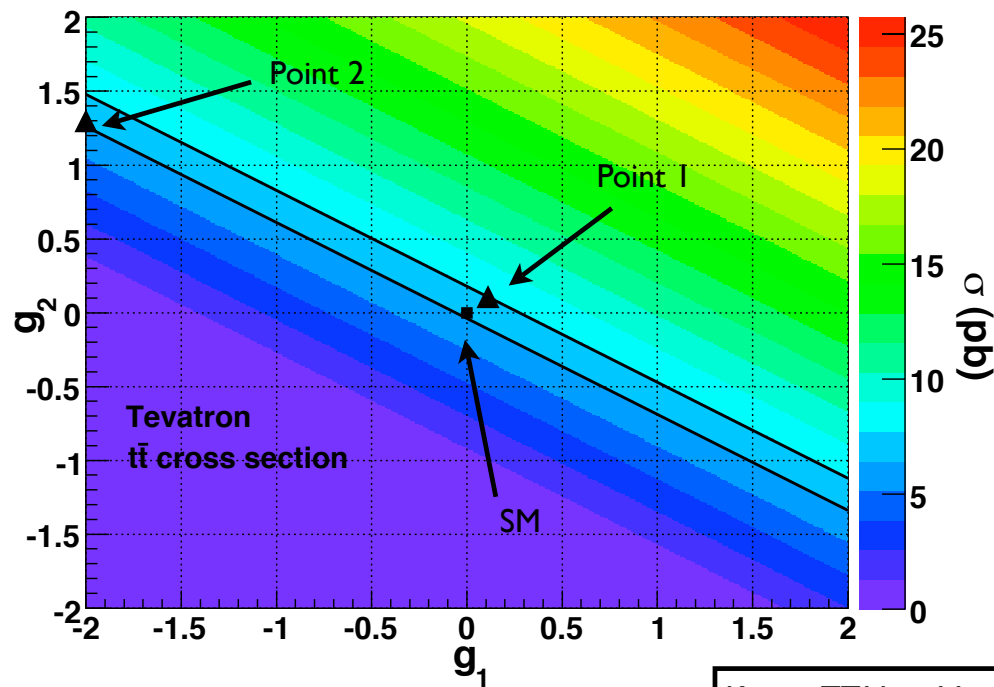
- Compared with the theory prediction:

$$\sigma_{SM} = 6.6 \pm 0.8 \text{ pb}$$

Cacciari, Frixione, Mangano, Nason, Ridolfi  
JHEP0809, 127 (2008)

- We fix  $\Lambda = 500 \text{ GeV}$  (for now) and compute the rate for different values of  $g_1$  and  $g_2$ .

# Re $g_1$ - Re $g_2$ Plane



$\Lambda = 500$  GeV

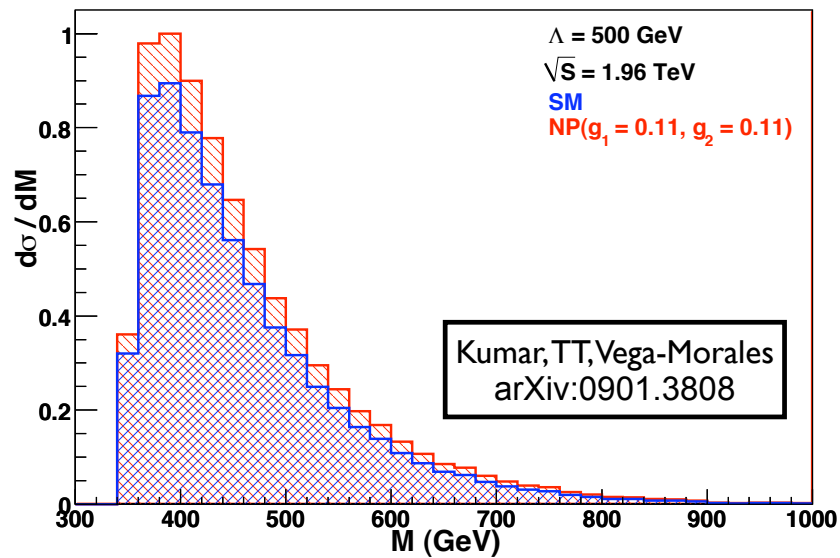
Kumar, TT, Vega-Morales  
arXiv:0901.3808

A swath of the  $g_1$ - $g_2$  plane is consistent with measurements.

In particular, if  $g_1$  and  $g_2$  take opposite signs, the effects of the two operators may partially cancel in the net cross section.

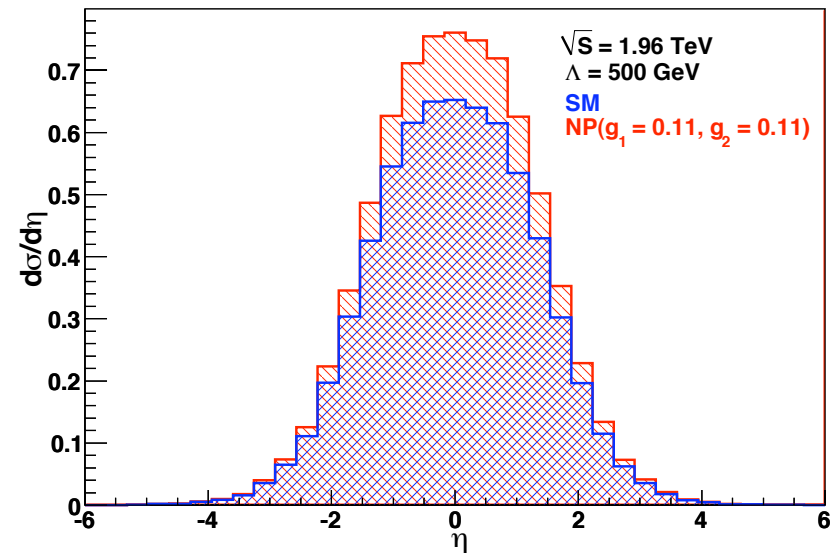
To illustrate the importance of distributions in going further, we consider two points consistent within one  $\sigma$  for the cross section.

# Point I

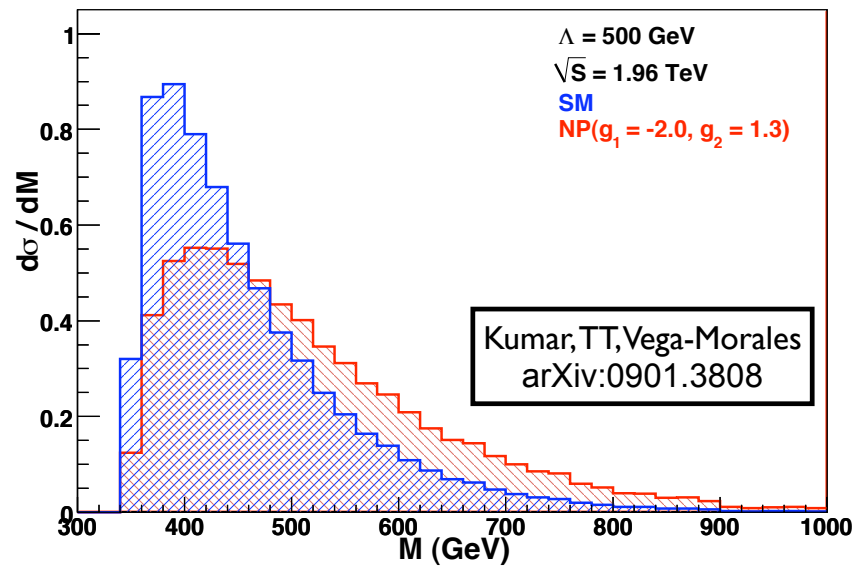


The top invariant mass distribution is shifted to slightly higher energies. The top rapidity distribution is a little more central.

Point I has no cancellations in the rate. The effects on distributions are modest.



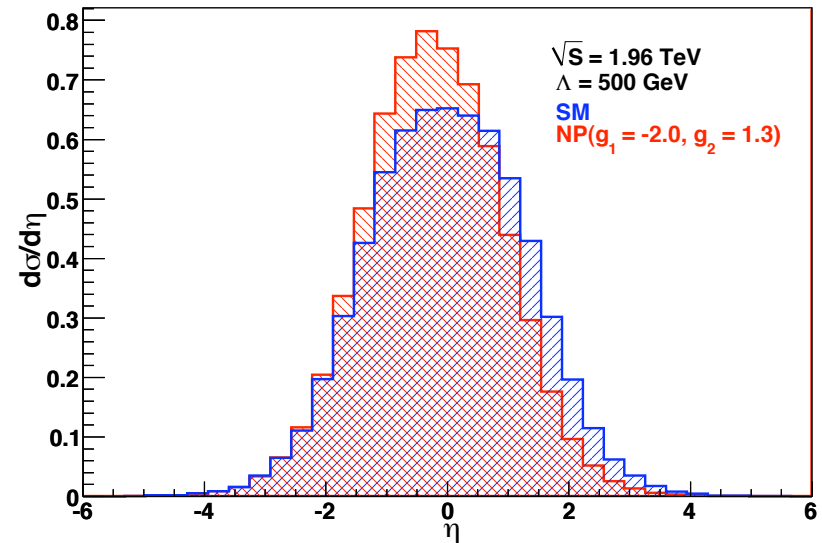
# Point 2



The top invariant mass distribution is very shifted. The top rapidity distribution is noticeably asymmetric.

James Wells talked about the asymmetry last week...

Point 2 has modest cancellations in the rate. The effects on distributions are pronounced.





# Tevatron Conclusion



- We saw that order one values of the  $g$ 's and  $\Lambda$  of about 500 GeV are (barely) consistent with the rate of top pair production.
- One can do better with distributions, but it is beyond the scope of what a theorist can easily do with the data available.
- Nonetheless, it is clear that compositeness scales of several hundred GeV are allowed by Tevatron data.
- This implies the possibility of large effects, including potentially the ability to see constituents at LHC energies.
- In the remaining time, I will explore some possible LHC signals of a composite top.

# Mapping to Resonances

● If the top is composite, some of the constituents should be colored. So a color octet vector meson is a typical element of the “higher resonances”.

● Even so, mapping the constraint on the operator to the properties of the vector is still model dependent...

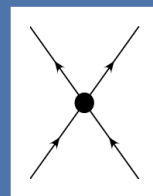
○ How many resonances?

○ How strongly coupled are they?

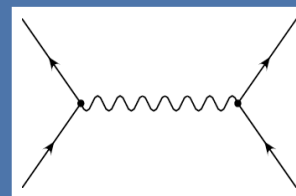
○ Is a single resonance a good description at all?

● Perhaps we need a momentum-dependent form-factor  $f(p^2)$ ?

● To go forward, I'll assume moderately strong coupling and that the bound is dominated by a single vector boson.



$$\sim \frac{g^2}{M_V^2} ?$$



I'll consider both color octet and color singlet vector resonances.



# RS “KK” Gluon



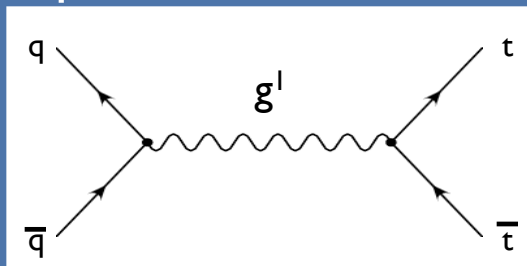
- A natural starting point at the LHC is the RS KK gluon. It is colored, and so has a relatively large production cross section.
- Another possibility, not explored here, would be some kind of higher resonance of top itself. (Sort of like a top-seesaw quark)
- Is the KK gluon typical of the kind of resonance we have in mind?
  - Bounds on its mass are relaxed, because we have avoided the worst of the precision constraints on standard RS.
  - It couples strongly to the  $t_R$ ,  $g \sim 4 g_s$ .
  - It has moderate coupling to the light quarks,  $g \sim g_s / 5$ , probably a little stronger than what I had in mind.



# Coupling to Quarks



- The main point where the KK gluon may not match to a more generic picture for a composite top is in how it couples to the light quarks.
- RS has substantial couplings to light quarks, and the main production mechanism is single production in the s-channel.



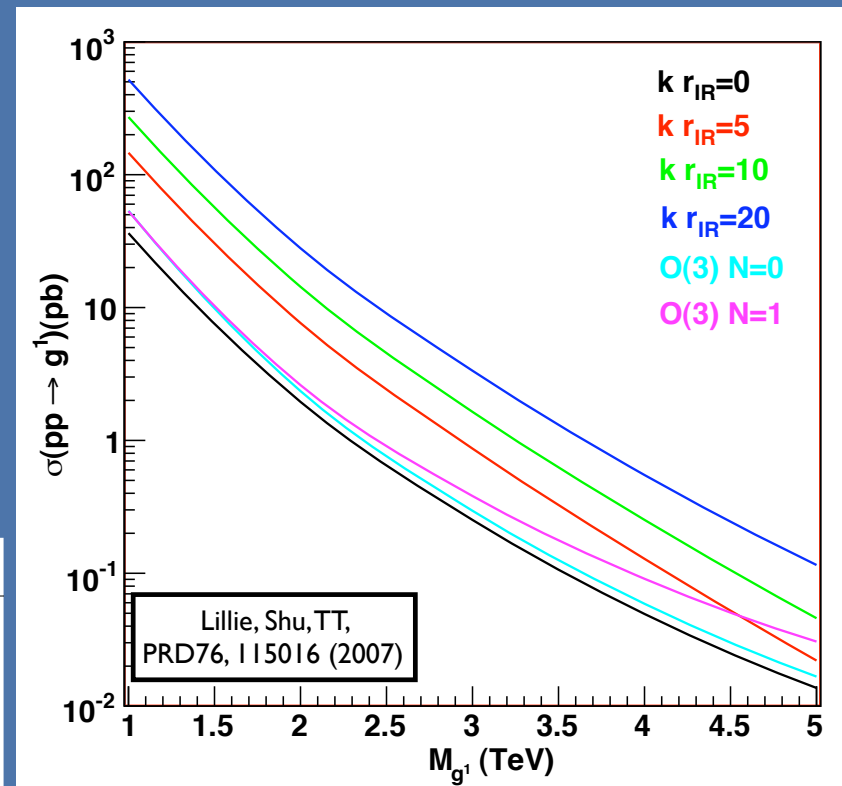
- Even in RS, there are parameters one can invoke to adjust the effective theory, and modify the composite model we think lies behind it. For example, we can include IR-brane kinetic terms for the KK gluon, which diminish its coupling to IR brane fields.

# KK $g$ Cross Sections



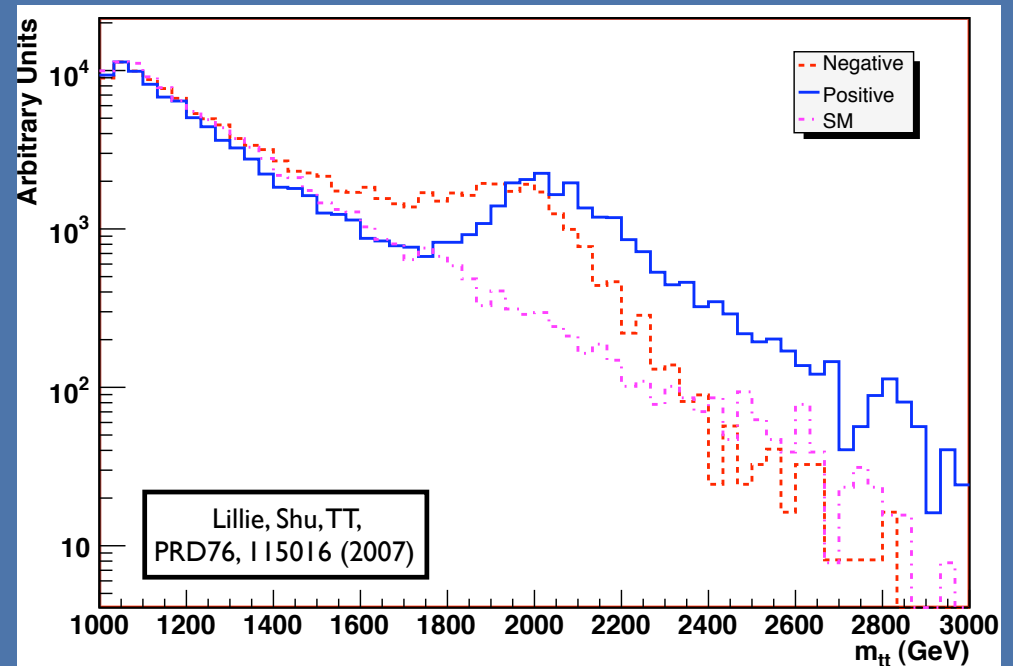
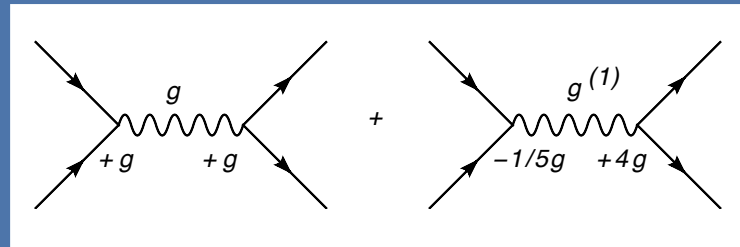
- Variations of RS can produce a variety of different couplings to the light quarks and to top.
- The cross section and branching ratios depend sensitively on the couplings, and thus reflect the underlying the parameters.

Model	top quarks	bottom quarks	light quarks	custodial partners	$\Gamma_{g^1}/M_{g^1}$
Basic RS	92.6%	5.7%	1.7%		0.14
$\kappa r_{IR} = 5$	2.6%	13.2%	84.2%		0.11
$\kappa r_{IR} = 20$	7.8%	15.1%	77.1%		0.05
$O(3), N = 0$	48.8%	49.0%	2.0%		0.11
$O(3), N = 1$	14.6%	14.6%	0.6%	70.2%	0.40



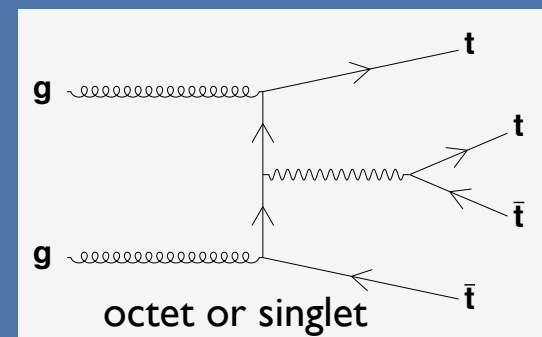
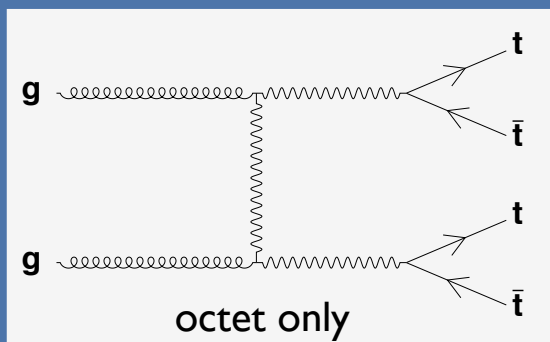
# Width and Interference

- In RS, as can be expected in any composite model, the KK gluons are strongly coupled, and have relatively large widths.
- The width may be directly measurable even with large LHC jet energy resolutions.
- Interference with the continuum  $t\bar{t}$  background tells us about the relative sign of the couplings.



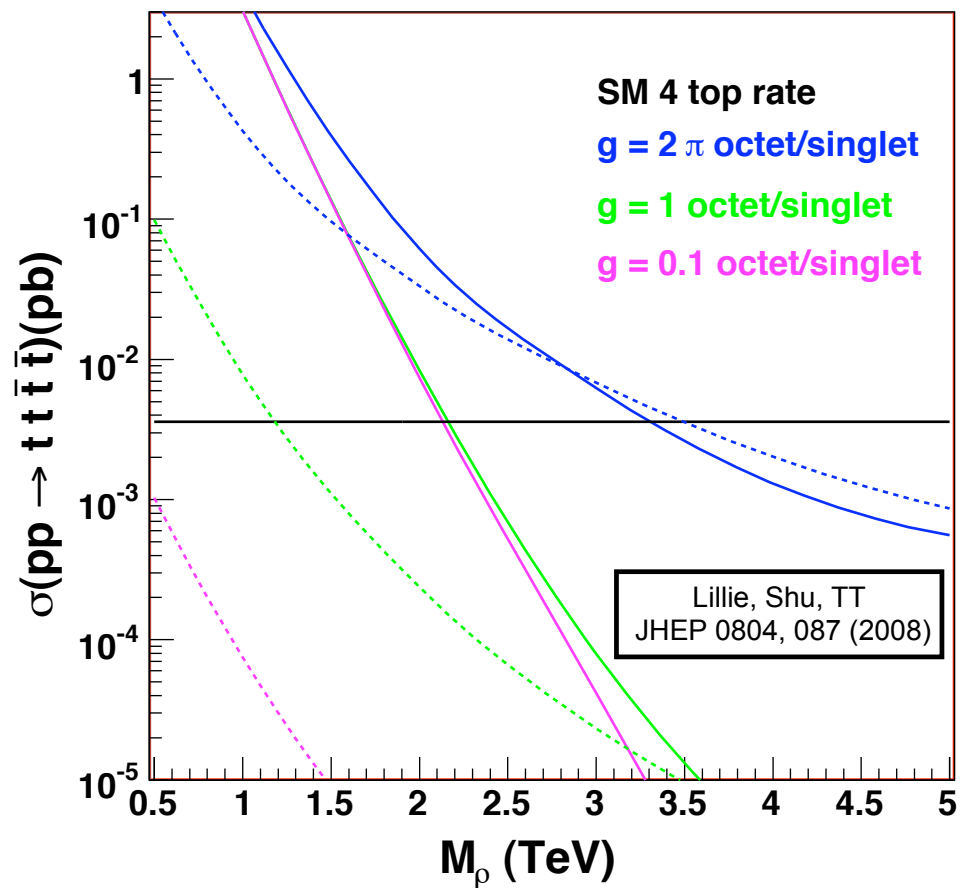
# Four Tops at the LHC

- Studies of the KK gluon are encouraging, but they rely strongly on the fact that traditional RS has a substantial coupling of the octet to light quarks.
- A more generic signature has color octet (and/or singlet) vector particles which couple strongly to top quarks, and perhaps negligibly weakly to light quarks. Can we do the case where the coupling to light quarks is too small to use as a production mechanism?
- A color octet vector can be pair-produced purely by QCD. A color singlet needs to be “radiated” from a top quark.



# Four Top Cross Sections

- Our resonances decay practically 100% of the time into top quarks, leading to a four top signal.
- The cross sections for octets and singlets show a very different dependence on the coupling of the resonance to top quarks.
- The SM four top rate is very small : a few fb.





# Four tops?





- So the question is: can we actually reconstruct four tops at the LHC?
- A recent study concluded we can, but used a jet mass technique which is probably very sensitive to underlying event and mis-measurement.  

Gerbush, Khoo, Phalen, Pierce, Tucker-Smith arXiv:0710.3133 [hep-ph]
- We went with a more conservative approach, and required two like-sign leptons (either electron or muon) together with 2 or more hard jets.
- After showing we can extract the signal from the background, we can ask additional questions to show it looks “4 top-like”.



# Backgrounds



- The backgrounds we simulate as part of the hard process are:
    - $W^\pm W^\pm + 2$  jets .
    - $W^\pm Z + 2$  jets.
    - $W^\pm + b\bar{b} +$  jet with a semi-leptonic b decay. single top!
    - $W^\pm + 3$  jets with a jet faking a lepton.
    - $W^+W^- + 2$  jets ( $t\bar{t}$ ) with a charge mis-identified.
- 
- 





# Simulation



- We simulate the hard processes using MadEvent.
- We run the events through PYTHIA to decay the tops and Ws, and to shower and hadronize the partons.
- We use PGS with the default LHC detector simulation to estimate the detection efficiency, reconstruct jets, etc.

Our point is not to do a fully realistic study, but to do a reasonable “back of the envelope” demonstration that the signal is feasible.

- The exception is the W + 3 jets background, which we cut at the parton level and apply a mistag rate of  $10^{-4}$ , after which it is small (but not negligible).
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- 



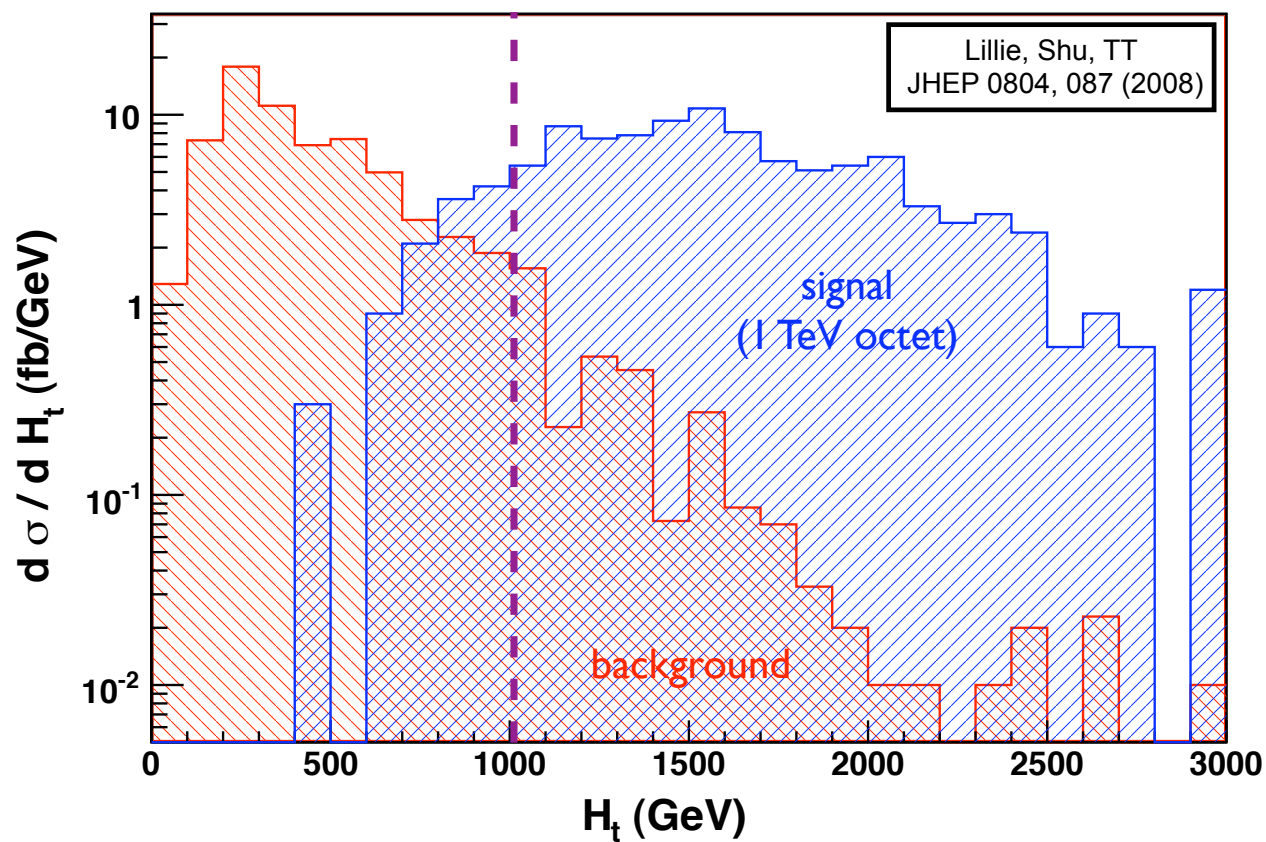
# Cuts



- We require two same-sign leptons, either electrons or muons with  $p_T > 30$  GeV,  $|y| < 2.5$ .
  - This should be good enough to trigger ATLAS.
- Two jets with  $p_T > 20$  GeV,  $|y| < 2.5$ .
- To help with the semi-leptonic b-decays, we impose a jet isolation cut around both leptons of  $\Delta R > 0.2$ .
- To get high energy events which have the possibility to correspond to 4 tops, we require  $H_t > 1$  TeV.



# $H_t$ Distribution



# Backgrounds

● After cuts, we are left with:

○ WW + jets: 1.1 fb (+/-: 0.8 fb / 0.3 fb)

○ WZ + jets: 1.5 fb (+/-: 1.1 fb / 0.4 fb)

○ Wbb + jet: 0.8 fb (+/-: 0.6 fb / 0.2 fb)

○ W + 3 jets: 0.6 fb (about equally + and -)

○ tt: 3.16 fb (Well simulated!?)

● The signal (for  $M \sim 1$  TeV,  $g \sim 2\pi$ , color octet) is about 112 fb.

(Efficiency of about 3% - mostly from the W BRs)



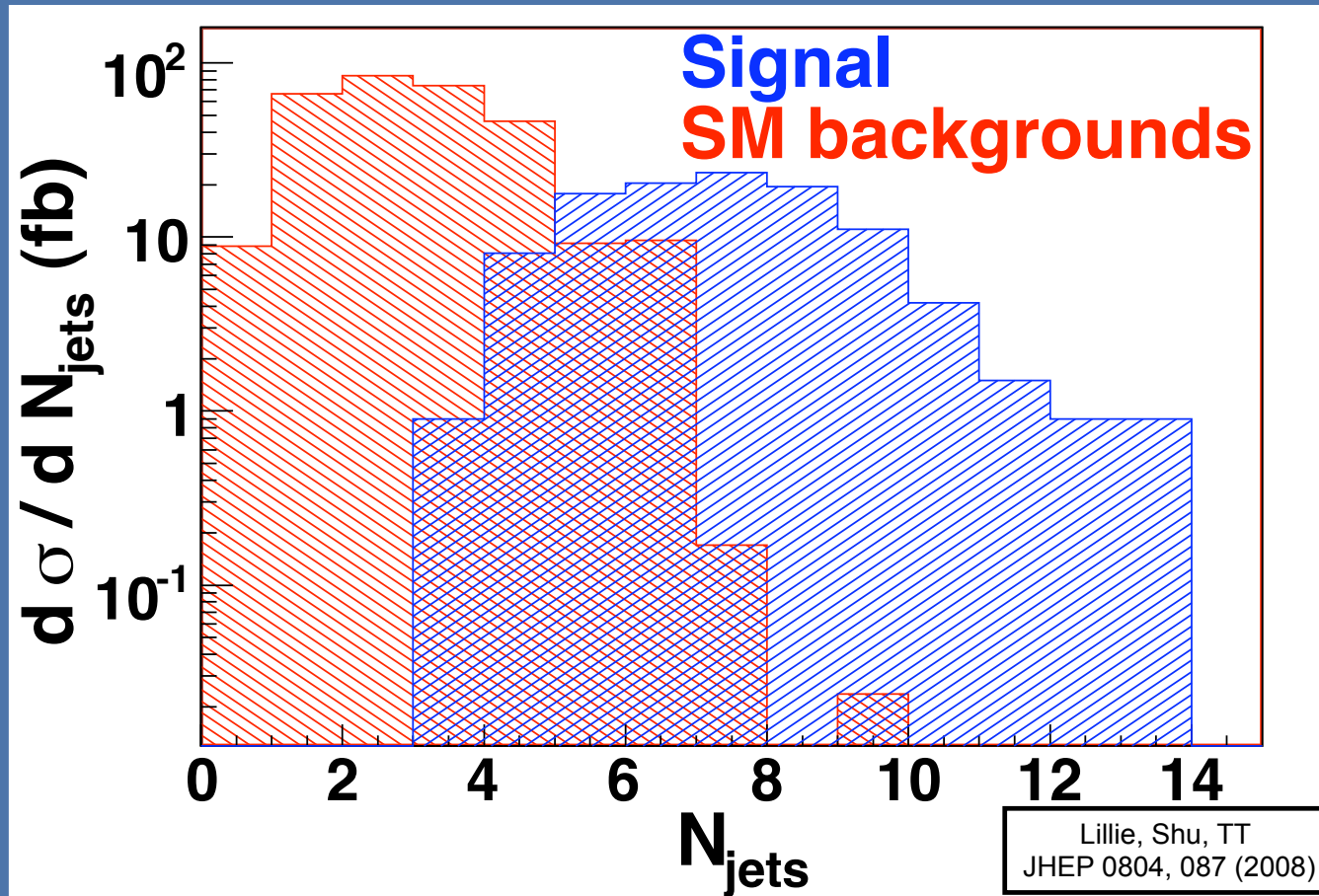
# Signal



- At this point we would ideally start reconstructing tops and Ws.
  - But the combinatorics seem to be flooding us.
- So I'll settle for a few observations that the signal looks more 4-top-like than not:
  - Four tops produces equal ++ and -- lepton pairs in our signal sample. Electroweak production of charged states will not.
  - There are b-tagged jets from the top decays.
  - In general, there is a lot of jet activity.



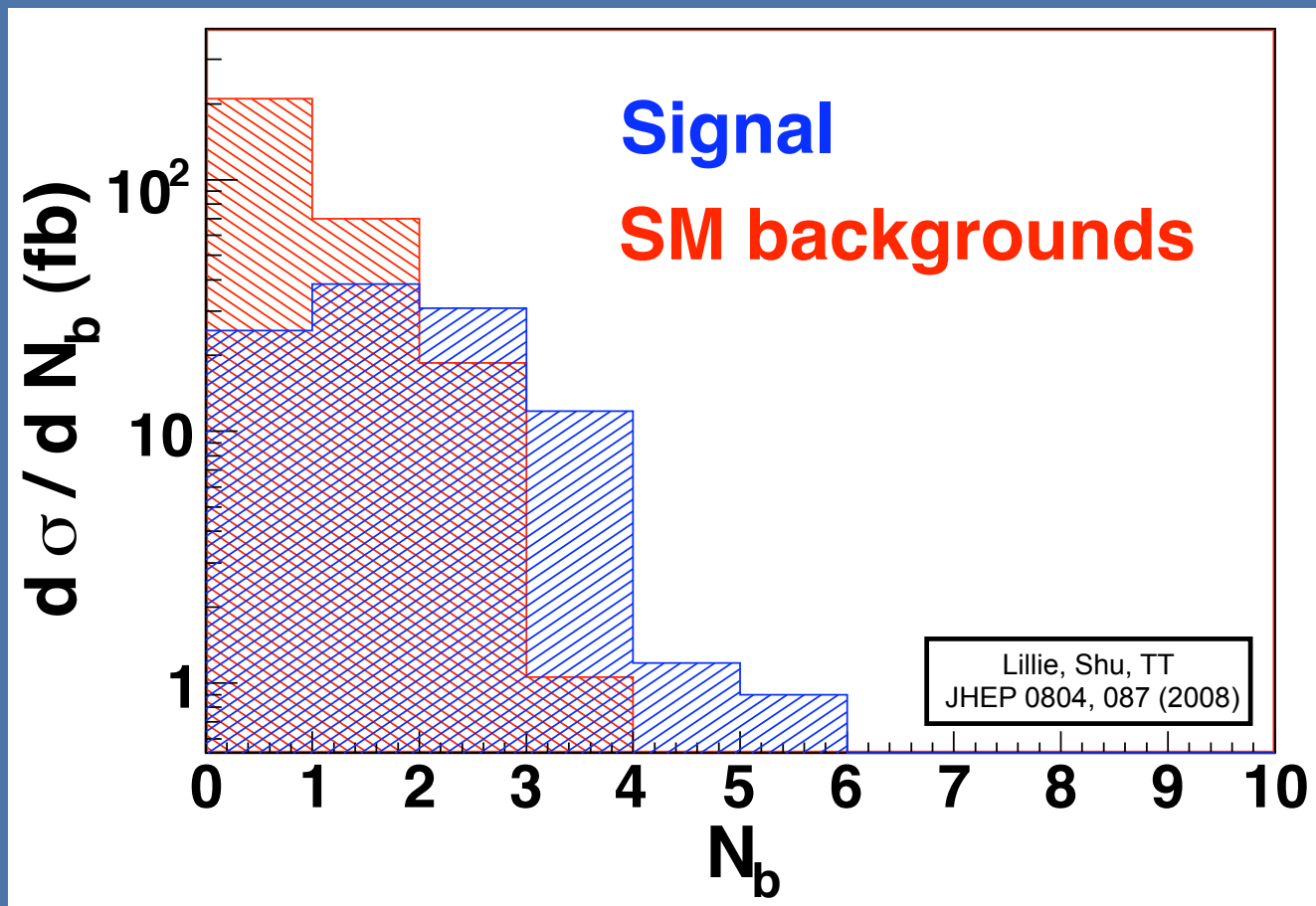
# Number of Jets



Our tops aren't tremendously boosted.

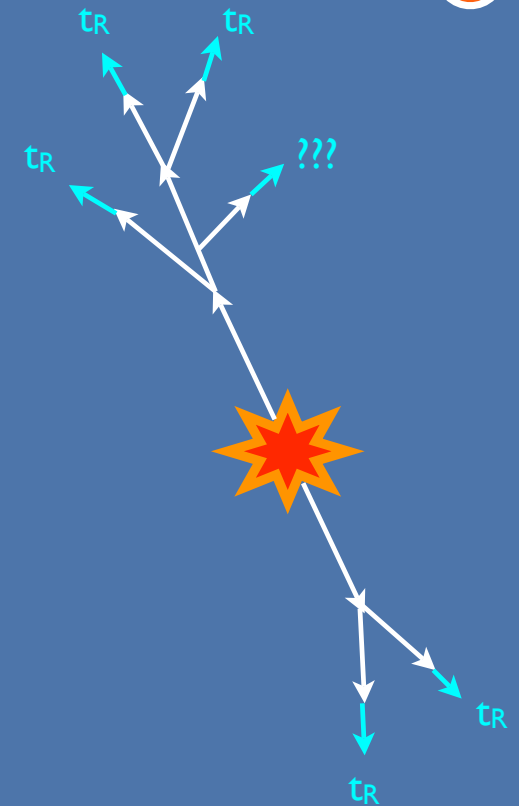


# Number of b-tags



# Future Directions?

- With a low compositeness scale, we might even be able to see the constituents directly.
- If we imagine the highest energies the LHC can probe (over the course of its life-time), even more exotic phenomena can emerge.
- For example, if we produce constituents in a regime where they are energetic and weakly coupled, maybe we can see them “hadronize” (into top quarks) or even “shower”. The result could be jets of high momentum top quarks.
- Could the LHC even reconstruct such an event? I have no idea, but it would be a lot of fun to try!





# Conclusions



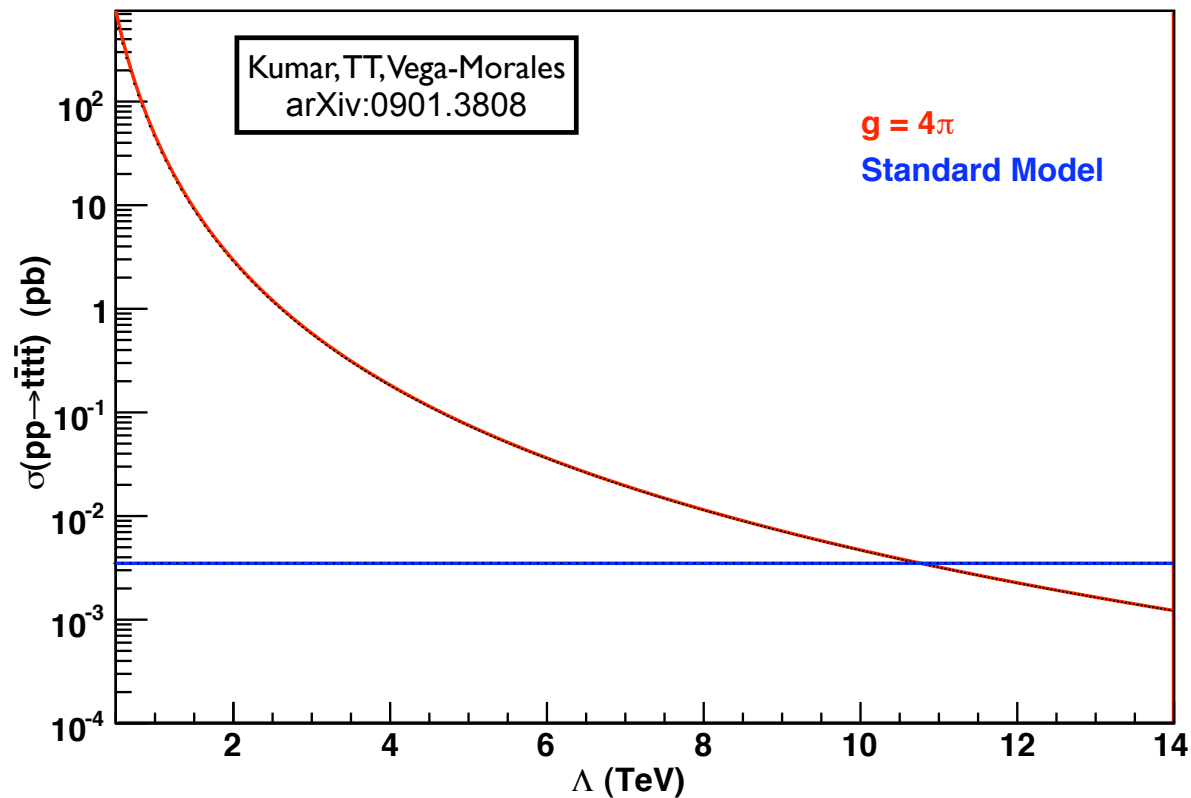
- The top quark is the newest component of the Standard Model. It is important to understand it as well as possible, and our hazy current understanding could lead to surprises!
- Top observables have become routine at the Tevatron but can be challenging at the LHC. There's a lot of room to improve our techniques to detect it in unusual or difficult circumstances.
- Composite models are hard to quantify, but easily lead to new signatures! It's fun to explore them!



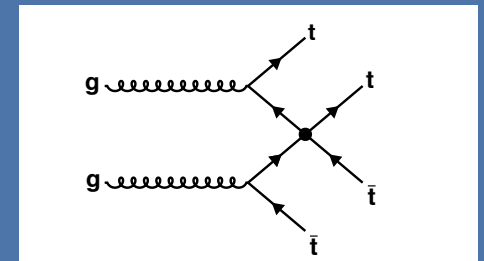
A decorative graphic on a blue background. It features a central white rounded rectangle containing the text "Bonus Material". Surrounding this rectangle are several circles: a large orange circle on the left, a white circle above it, a green circle below it, a green circle on the right, and a large blue circle at the bottom right. All circles have a white outline.

# Bonus Material

# Four tops: Operator Language



We can also describe four top production in an operator language at the LHC.



Similar results in : Pomarol, Serra PRD78, 074026 (2008)





**Can we see  
Compositeness at the  
LHC?**



# Compositeness?



- A better first question would be to explain what I mean when I say “compositeness”.
    - The physical picture I have in mind is that some or all of the fields of the Standard Model might be revealed to have internal structure.
    - A good picture is the proton: from far away, it looks point-like, but up close it is made out of quarks.
    - If the SM fields were weakly bound states, we would notice, because it would be relatively easy to rip them apart.
    - So I will focus on the case when some of the SM fields are strongly bound states, arising from some new confined force.
- 
- 

# The quick answer is...

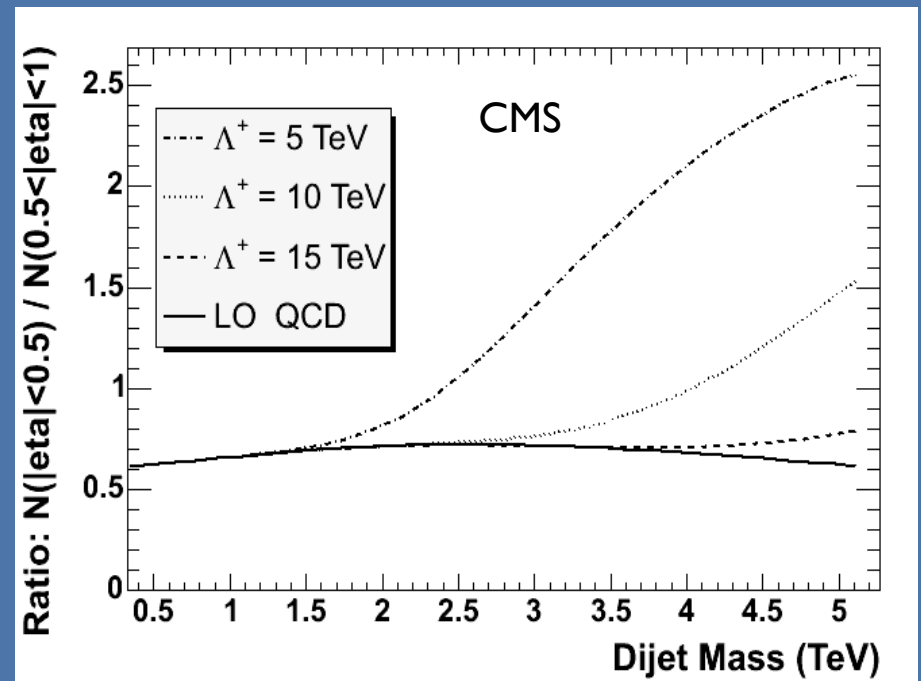


Yes.

Using the Eichten-Lane-Peskin parameterization in terms of higher dimensional operators, the LHC will probe (some) operators up to scales of order 10's of TeV.

$$\frac{g^2}{\Lambda^2} [\bar{q}\gamma^\mu q] [\bar{q}\gamma_\mu q]$$

Eichten, Lane, Peskin PRL50, 811 (1983)



Wow!

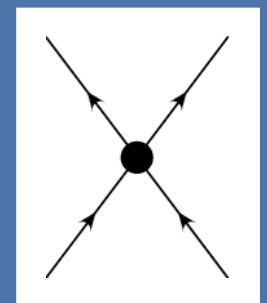
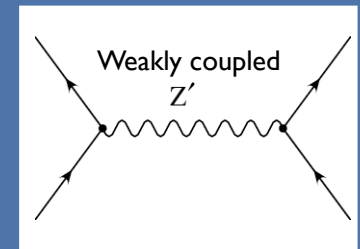


# That's great, but...



Higher dimensional operators are as much a sign of compositeness as they are of any kind of high scale new physics. We tend to refer to them as coming from compositeness mostly because we have no idea what else to do with compositeness.

It would be even better to see some phenomena which we could associate with compositeness and not other types of new physics.





# Constituents



- If the SM is partially or completely composite, we should identify the known particles with the lightest of the composites - the “pions”.
- Beyond contact interactions, we could look for:
  - Higher resonances - the “rhos”, “nucleons”, etc...
  - Constituents - the “quarks”!
- The question is : “Does the Standard Model work so well that we can already guess that there is no hope to see the constituents (“preons”) at the LHC?”





# Constraints



- Using the existing constraints on contact interactions we can (at least roughly) answer the question.

for example: 
$$\frac{g^2}{\Lambda^2} [\bar{q}\gamma^\mu q] [\bar{q}\gamma_\mu q]$$

- Any sector for which  $\Lambda \gg E_{\text{LHC}}$  will be very difficult for the LHC to resolve at the level of constituents.
- A sector for which  $\Lambda \sim E_{\text{LHC}}$  will potentially be visible (at least we can hope for a few resonances).

What I'll do now is run through different sectors of SM and assess the compositeness bounds on each one.



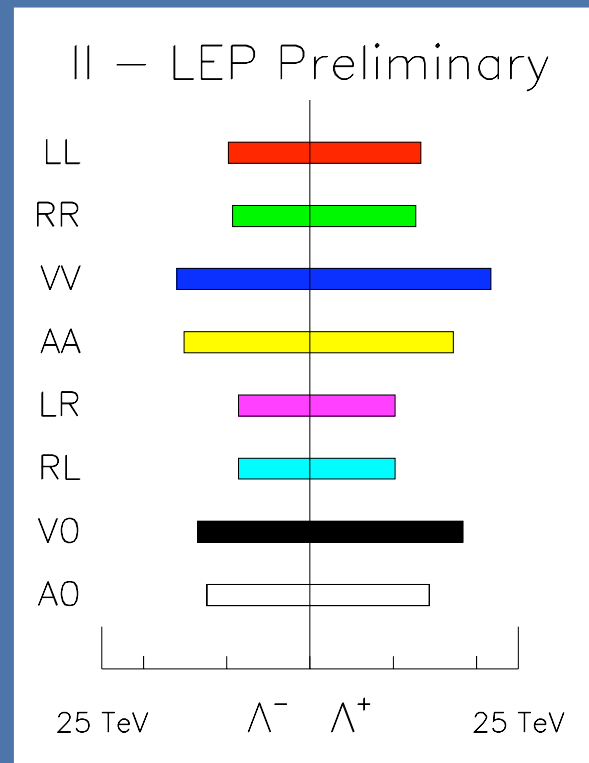
# Leptons at LEP-II

The LEP EWWG uses LEP-II data to put strong bounds on operators involving leptons.

$$\frac{4\pi}{(1+\delta)\Lambda^2} \sum_{i,j=R,L} \bar{e}_i \gamma_\mu e_i \bar{f}_j \gamma^\mu f_j$$

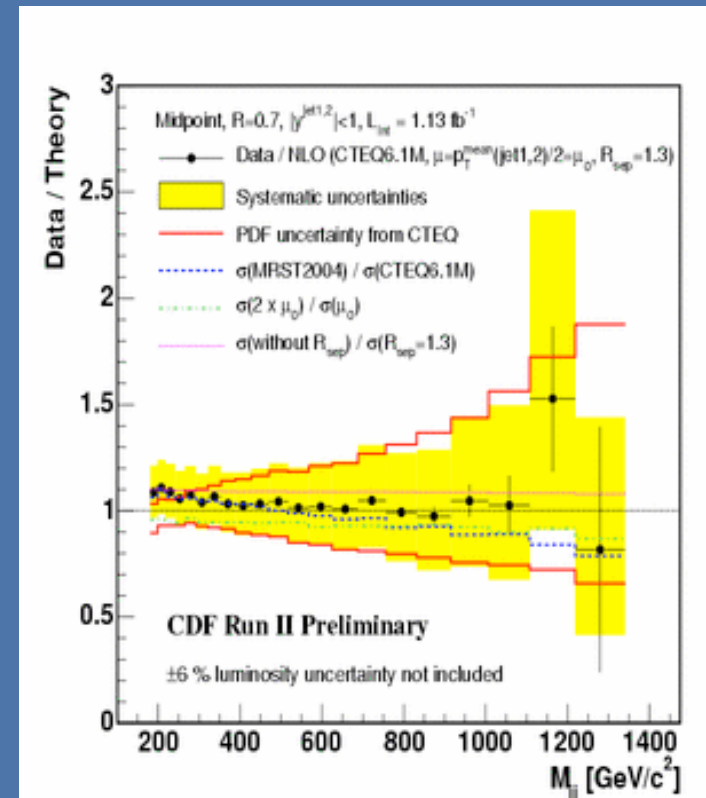
Their analysis derives a limit of about  $\Lambda \gtrsim 10$  TeV.

$$\delta = \begin{cases} 1 & f = e \\ 0 & f \neq e \end{cases}$$



# Light Quarks at Tevatron

- Operators involving four light quarks can contribute to dijet production.
- Neither CDF nor D0 have run II published limits on contact interactions, though one can guess their size from the data.



$$\sigma \simeq \sigma_{SM} \left( 1 + (4\pi)^2 \frac{E^2}{\Lambda^2} \right)$$

$$\Lambda \gtrsim 5 \text{ TeV}$$

# Higgs at LEP/SLD

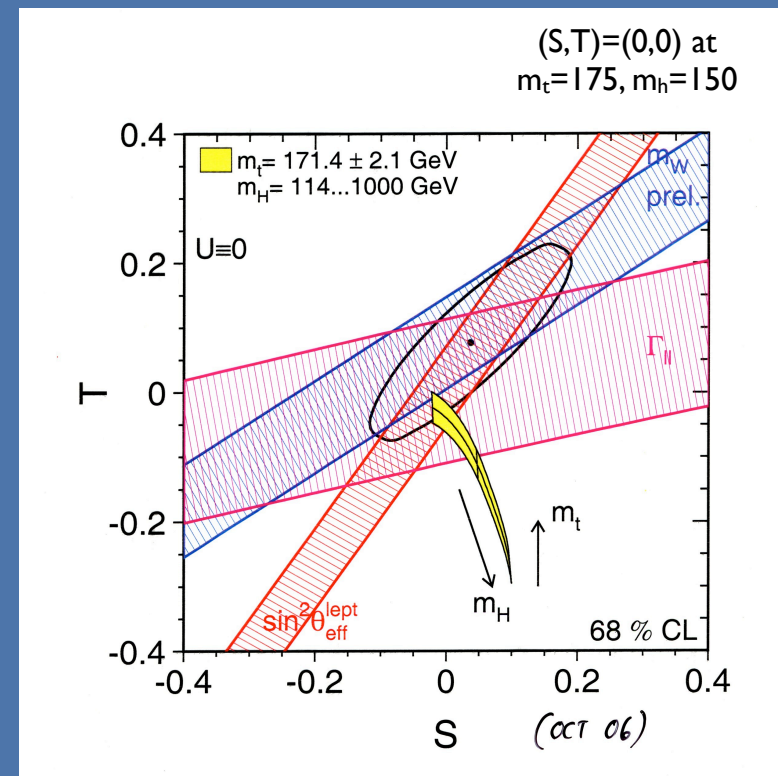
● Precision EW measurements limit Higgs operators.

○ Custodial isospin violating (T-parameter)

$$\Lambda \gtrsim 30 \text{ TeV}$$

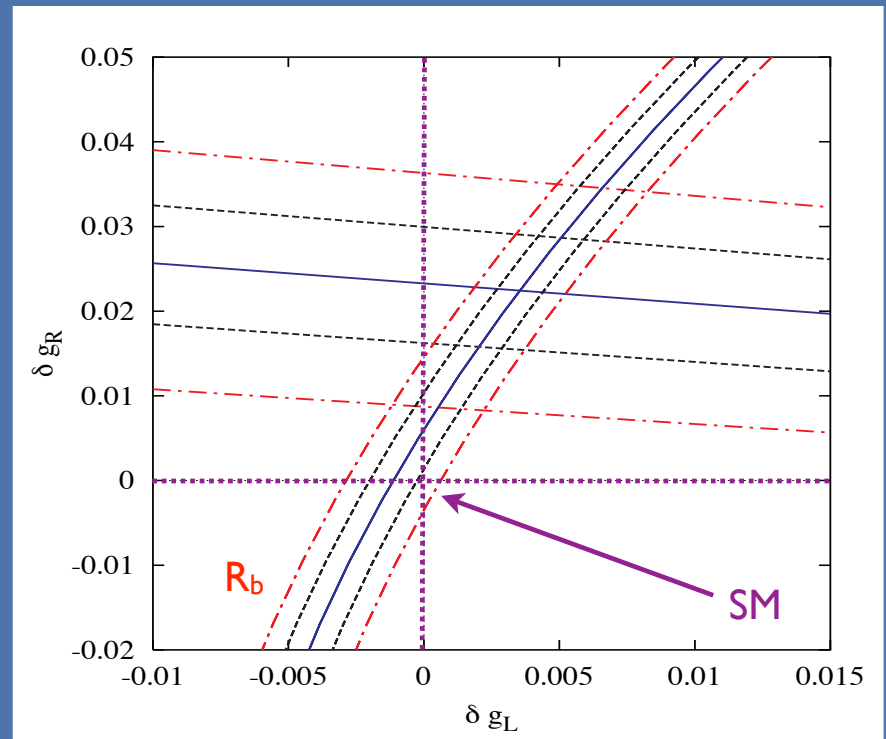
○ Custodial isospin preserving (S-parameter)

$$\Lambda \gtrsim 3 \text{ TeV}$$



# Heavy Quarks

- Precision Electroweak measurements also limit the deviations allowed in the bottom sector.
- Which also limits the scale of compositeness possible for the left-handed top.
- $b_R$  is more subtle, because of the  $A_b^{\text{FB}}$  puzzle.



Choudhury, Wagner, Tait PR65, 053002 (2002)

$$\Lambda \gtrsim 5 \text{ TeV}$$

# A Composite Top IS a good idea with a Composite Higgs!

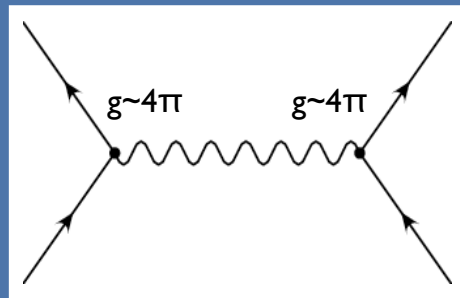


- If both top (left- and right-) and Higgs are composite, we can explain the large top Yukawa as a residual of the strong dynamics. Top-color is an example of a model which works this way, and helps shore up the difficulty technicolor models have with the large top mass.
- RS shows how this can work in the extra-dimensional dual picture by using wave function overlaps.
- A variant of the Fat Higgs includes top in the strong dynamics to explain the large top mass and solve the SUSY little hierarchy problem through the same mechanism.

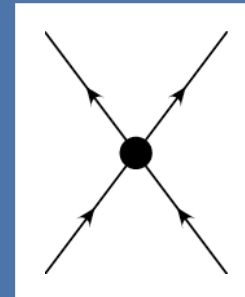


# Naive...

- To simplify the discussion, let's consider only a single sector of the SM to be composite at a time.



$$\sim \frac{g^2}{M_V^2}$$



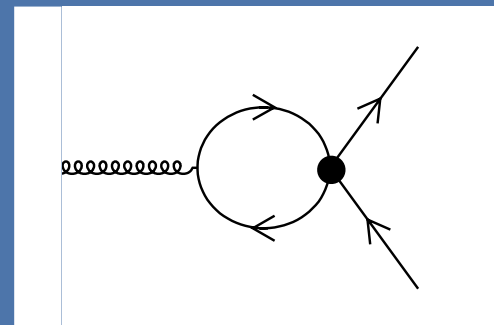
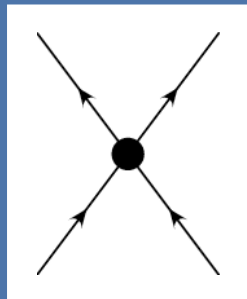
- If the coupling were literally  $4\pi$ , clearly perturbation theory would be in trouble, but this should work to estimate the limit on the scale of the strong physics.
- I'll ignore flavor and CP violating operators that are tightly constrained by low energy measurements.

# Mixed Operators

- I deal with operators mixing composite and fundamental fields by assuming they are induced by loops involving only the composite fields.
- (For  $g \sim 4\pi$  this will reproduce NDA estimates).

- For example:

$$\sim \frac{g^2}{\Lambda^2}$$



$$\sim \frac{g^2}{16\pi^2 \Lambda^2}$$

- The analogy with the pions is:
 
$$\frac{\Lambda}{g} \leftrightarrow f_\pi$$

$$\Lambda \leftrightarrow M_\rho$$



# Composite $t_R$



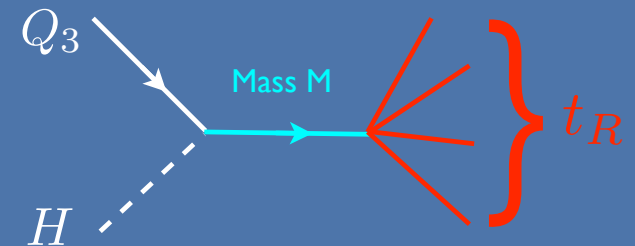
- A composite massless fermion isn't familiar from QCD. We can't engineer it to be a Goldstone boson in analogy with the pions.
- 't Hooft's anomaly matching argument suggests that if we arranged for  $t_R$  (and only  $t_R$ ) to be needed to maintain anomalies, it should appear as a light state.
- We could certainly build supersymmetric theories where we have enough control over the low energy dynamics to result in a light super-multiplet. The SUSY isn't buying you much beyond control over the low energy effective theory (and maybe a solution to the hierarchy problem).
- I have toy models, but none of them are compelling enough for me to present them as "the" theory.



# Composite $t_R$

- A composite right-handed top quark is not really useful in terms of understanding any deep question.
- From the point of view of the top mass, having the the top composite and not the Higgs argues for a suppression of the top mass.

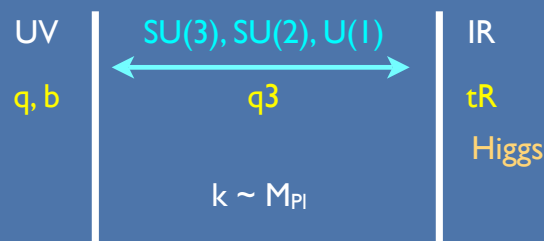
$$y_t \sim \left( \frac{\Lambda}{M} \right)^n$$



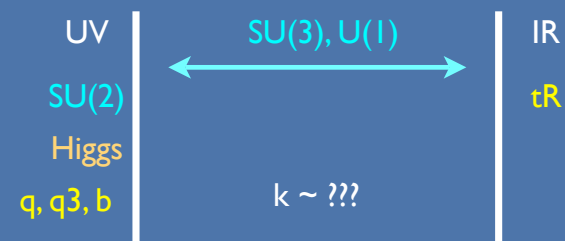
- This isn't a real problem (it just argues for more new physics close to the compositeness scale), but it is part of the over-all context.

# Resonances

- To go past the operator description and think about resonances, we need to make some assumptions about the underlying theory. So things become necessarily more model-dependent.
- The Randall-Sundrum models with the SM in the bulk are a good place to start.
- RS models are highly constrained by precision EW observables, but the structure that is constrained is mostly related to the RS solution to the hierarchy problem. We can imagine constructions like:



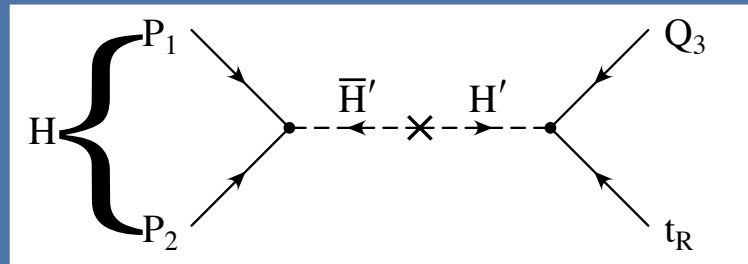
“Standard RS”



“Composite top RS”

# The SUSY Fat Top

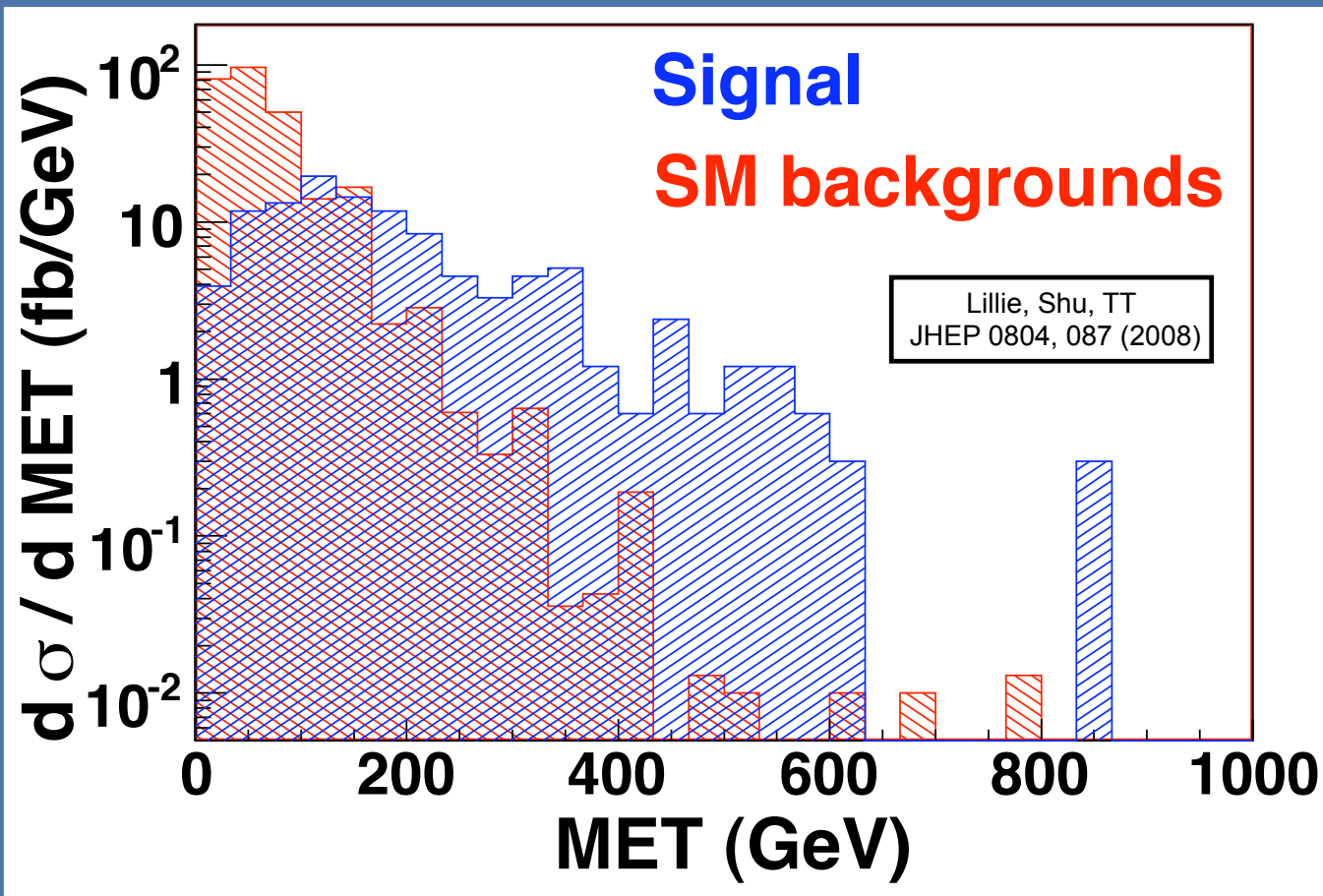
- A composite top can be helpful to explain the large top mass.
- One concrete example arises in some extensions of supersymmetric “Fat Higgs” models. These models have a composite Higgs in order to raise the prediction for the Higgs mass above the MSSM expectation.
- It’s a neat idea, but it makes it difficult to realize a large top mass, because now the Yukawa interactions arise from higher dimensional operators.



Delgado, TT,  
JHEP 0507:023,2005

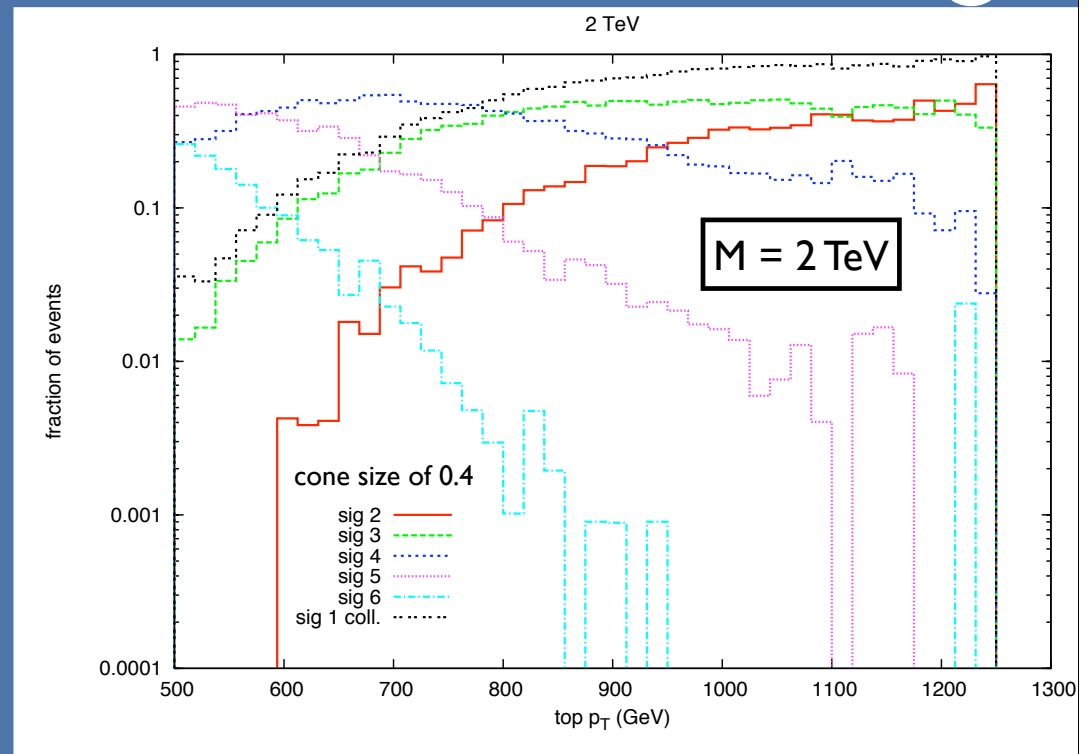
- The Fat Top variant addresses this problem by making the top composite as well, with the top Yukawa arising as a residual of the strong confining dynamics which produced the top and Higgs.

# Missing ET



# High Energy Tops

- To detect these resonances, we need to be able to reconstruct highly boosted top quarks.
- At high  $p_T$ , tops decay into more collimated jets of particles. It can be challenging to identify them as tops.
- Existing studies rely on the “rare” events with enough well separated top decays, taking a hit in efficiency.



Lillie, Randall, Wang JHEP 0709:074,2007

See also: Agashe, Belyaev, Krupovnickas, Perez, Virzi, hep-ph/0612015  
Baur, Orr, arXiv:0707.2066 [hep-ph]



# Worse at Larger Masses

