FALKO DULAT

HIGH PRECISION DETERMINATION OF THE GLUON FUSION CROSS SECTION AT THE LHC

THE N3LOTEAM

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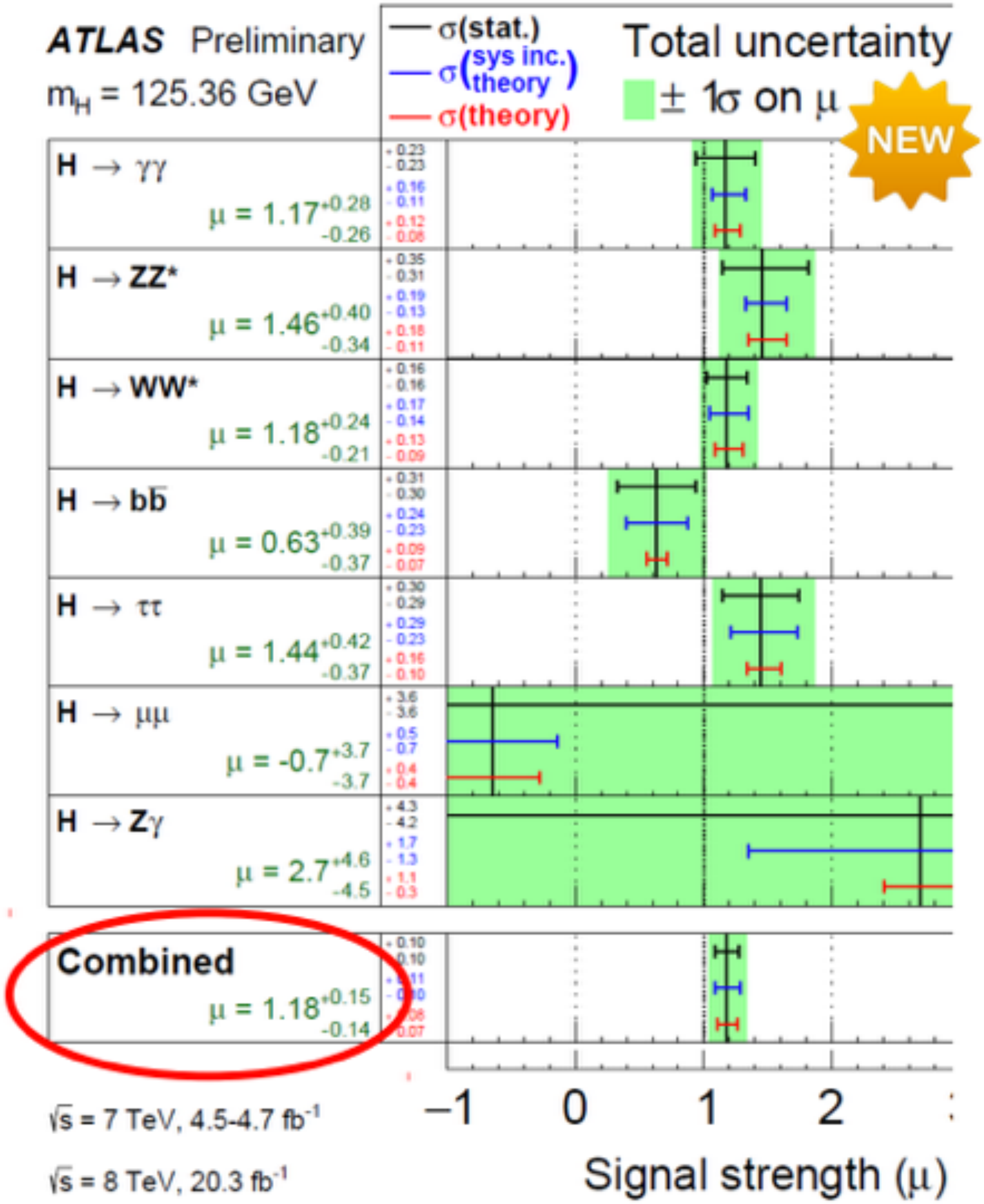
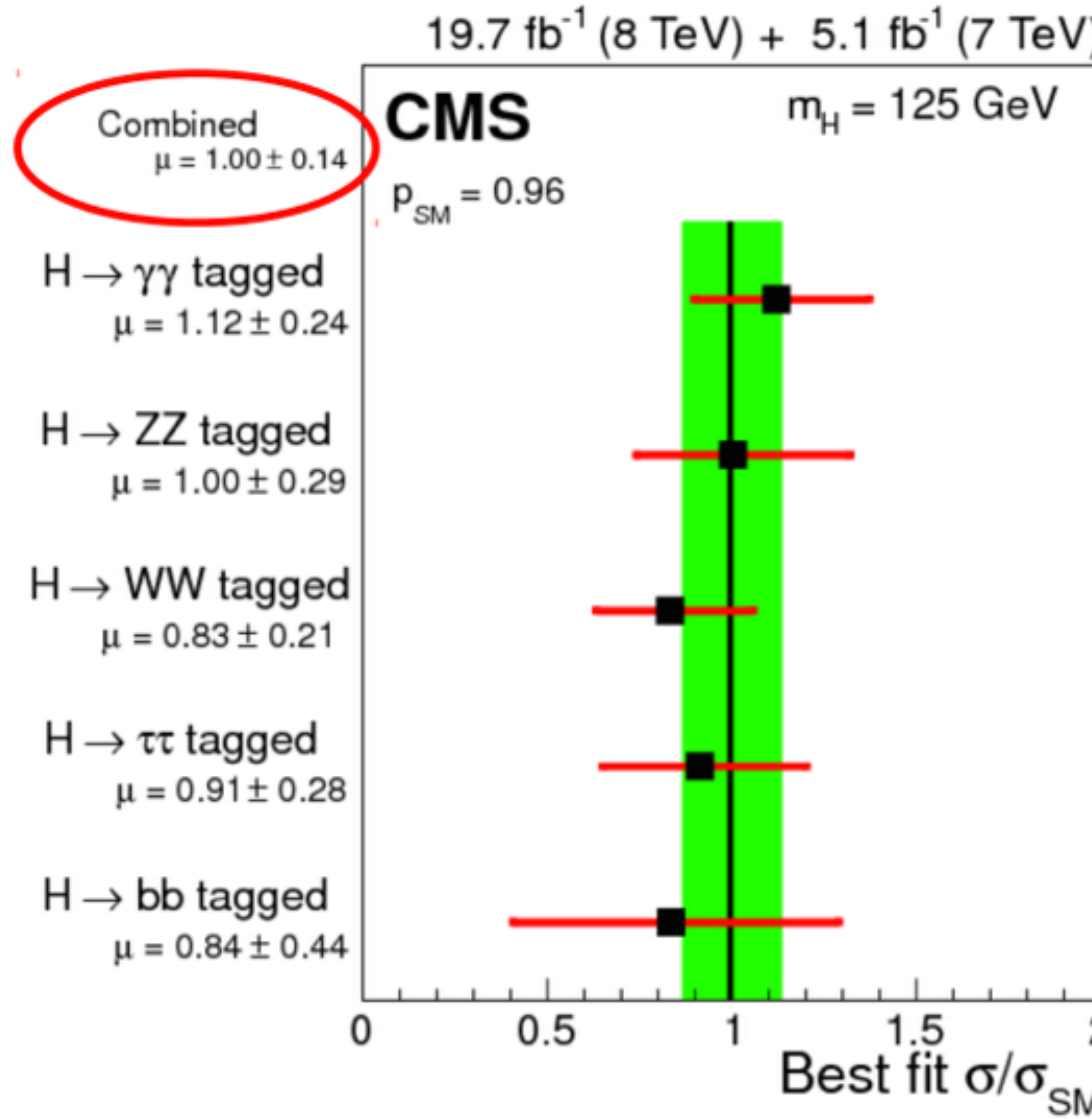
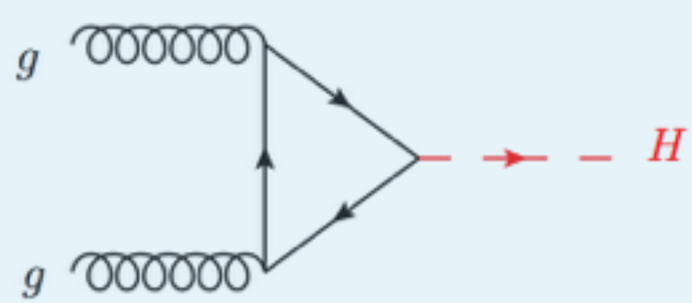
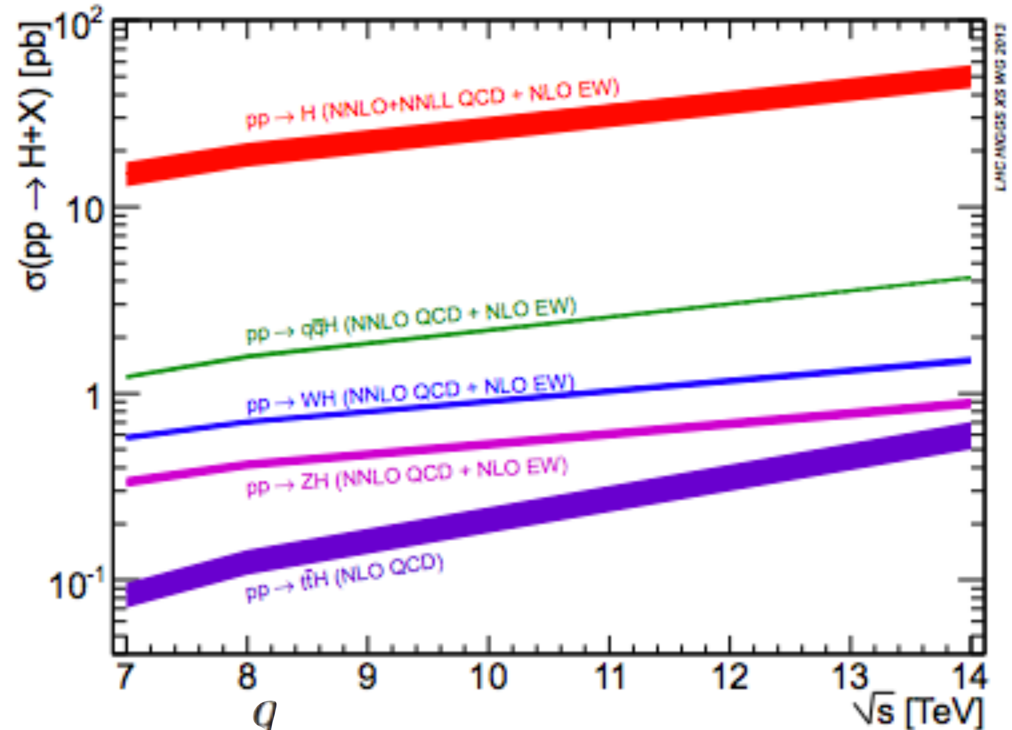

ATLAS Preliminary
m_H = 125.36 GeV
σ(stat.)
σ(sys inc. theory)
σ(theory)
Total uncertainty
± 1σ on μ
NEW
H → γγ
μ = 1.17 +0.28 -0.26
H → ZZ*
μ = 1.46 +0.40 -0.34
H → WW*
μ = 1.18 +0.24 -0.21
H → bb̄
μ = 0.63 +0.39 -0.37
H → ττ
μ = 1.44 +0.42 -0.37
H → μμ
μ = -0.7 +3.7 -3.7
H → Zγ
μ = 2.7 +4.6 -4.5
Combined
μ = 1.18 +0.15 -0.14
√s = 7 TeV, 4.5-4.7 fb⁻¹
√s = 8 TeV, 20.3 fb⁻¹
−1
0
1
2
Signal strength (μ)

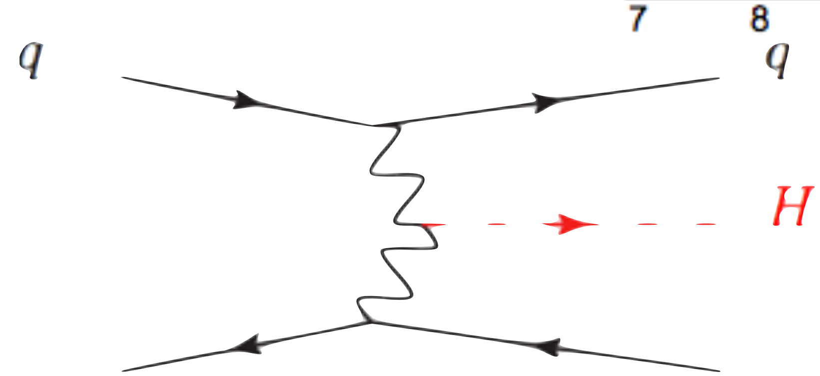

19.7 fb⁻¹ (8 TeV) + 5.1 fb⁻¹ (7 TeV)
CMS
m_H = 125 GeV
p_{SM} = 0.96
Combined
μ = 1.00 ± 0.14
H → γγ tagged
μ = 1.12 ± 0.24
H → ZZ tagged
μ = 1.00 ± 0.29
H → WW tagged
μ = 0.83 ± 0.21
H → ττ tagged
μ = 0.91 ± 0.28
H → bb tagged
μ = 0.84 ± 0.44
0
0.5
1
1.5
2
Best fit σ/σ_SM



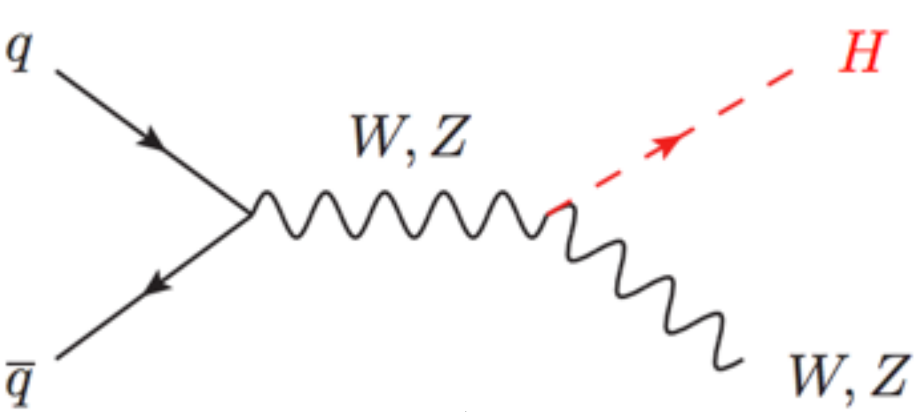
87%



LHC@14TEV



7%



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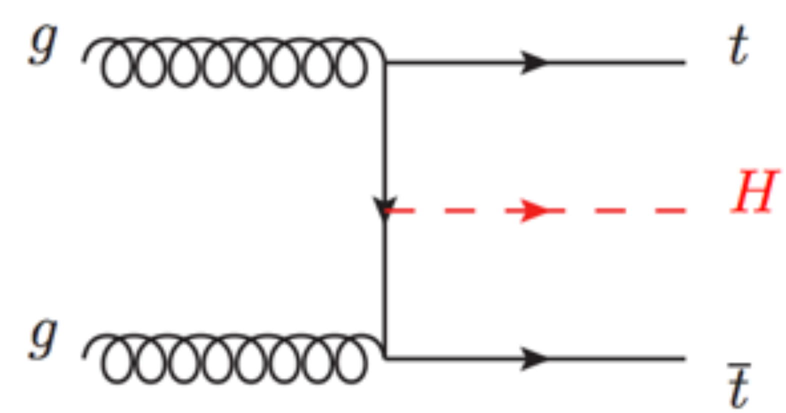

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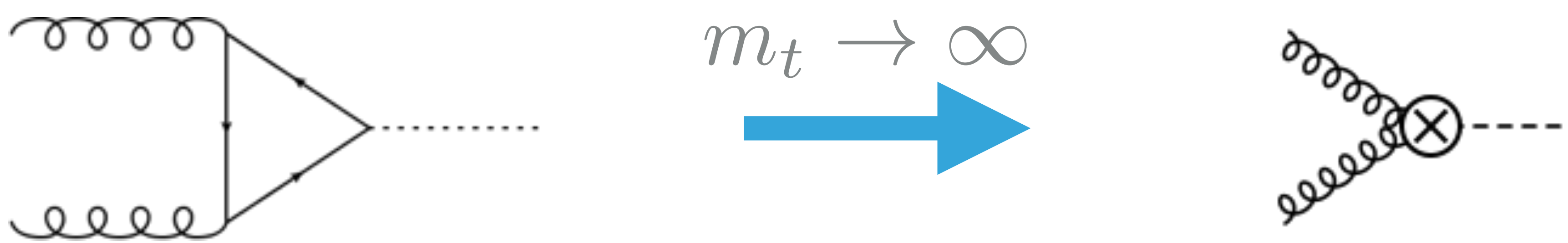
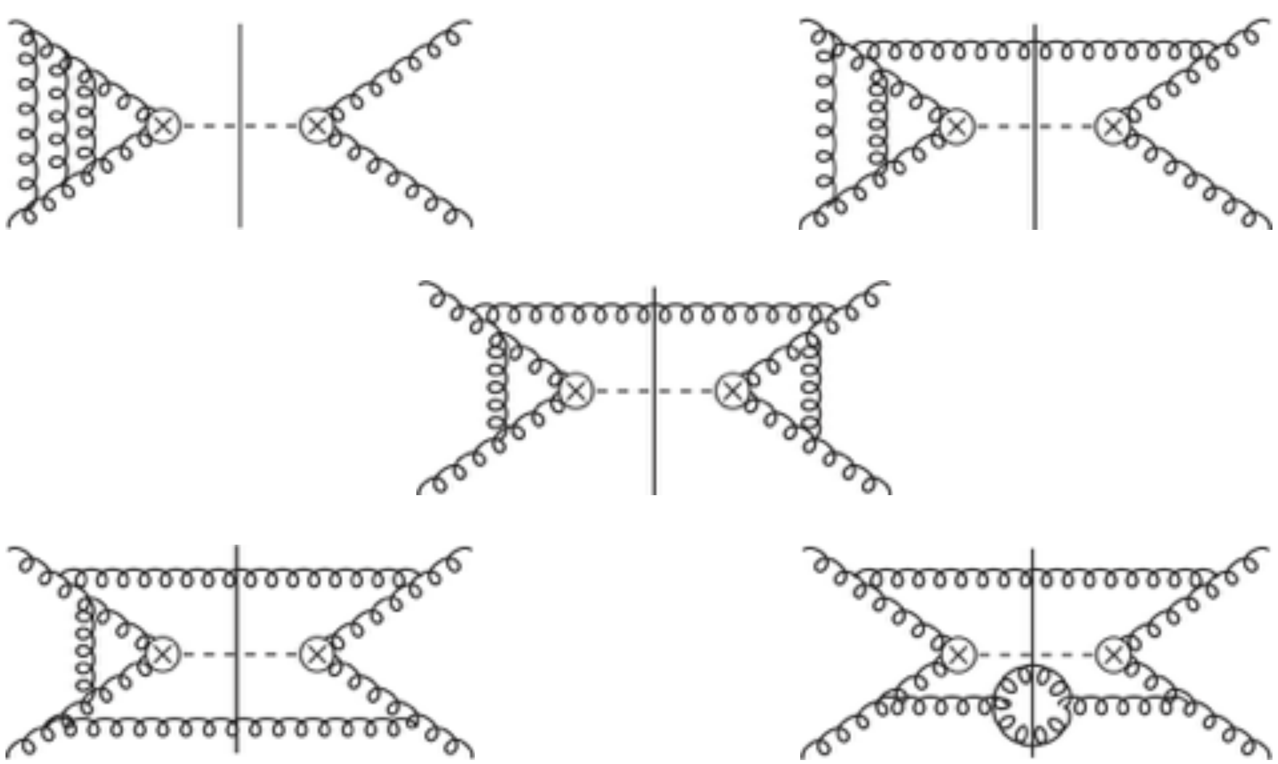


$$\hat{\sigma} = \alpha_s^2 \sigma^{\mathrm{LO}} + \alpha_s^3 \sigma^{\mathrm{NLO}} + \alpha_s^4 \sigma^{\mathrm{NNLO}} + \alpha_s^5 \sigma^{\mathrm{N3LO}}$$

✓ ✓ ✓ ✓

Partonic cross section is a function of the threshold variable $z = \frac{m_h^2}{s}$

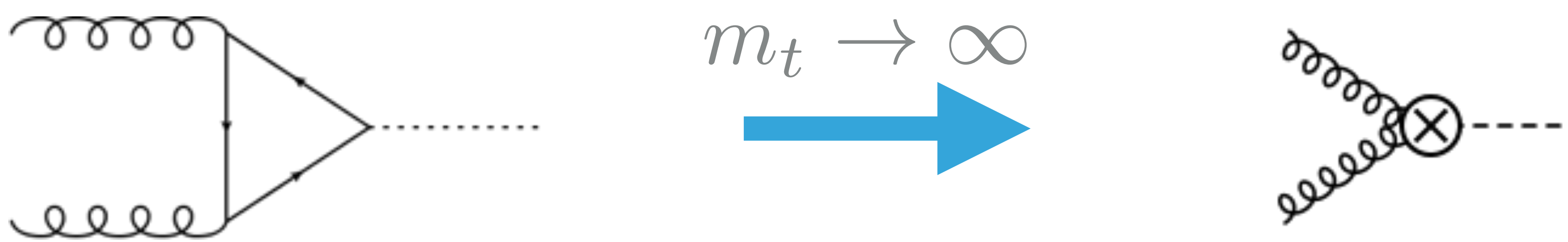


$$\hat{\sigma} = \hat{\sigma}(z)$$

Measure of energy available for radiation

$$\sigma = \mathcal{L}(z) \otimes \hat{\sigma}(z)$$

FIRST N3LO CALCULATION FOR HADRON COLLIDERS



The heavy top effective theory receives corrections due to the finite top mass already at LO

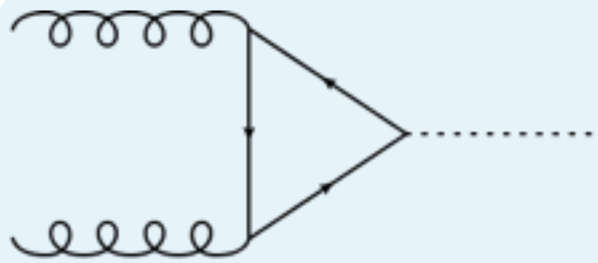
Good phenomenological approximation to rescale the effective theory cross sections with LO top dependence

$$K_{\mathrm{LO}} = \frac{\sigma^{\mathrm{LO}}_{\mathrm{exact}}}{\sigma^{\mathrm{LO}}_{\mathrm{EFT}}}$$

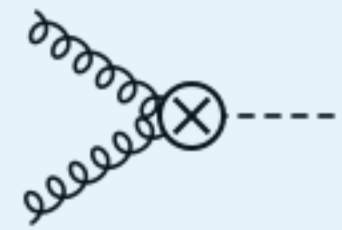
$\sqrt{S}$	13TeV
m_h	125GeV
PDF	PDF4LHC15_nnlo_100
$a_s(m_Z)$	0.118
$m_t(m_t)$	162.7 ($\overline{MS}$)
$m_b(4.18GeV)$	4.18 ($\overline{MS}$)
$m_c(3GeV)$	0.986 ($\overline{MS}$)
$\mu = \mu_R = \mu_F$	62.5 ($= m_h/2$)

$$\sigma^{\mathrm{NLO}}_{\mathrm{rEFT}} = K_{\mathrm{LO}} \times \sigma^{\mathrm{NLO}}_{\mathrm{EFT}} = 37.00\mathrm{pb} \longleftrightarrow \sigma^{\mathrm{NLO}}_{\mathrm{exact}} = 37.76\mathrm{pb}$$

BETTER THAN 0.6%



$m_t \rightarrow \infty$

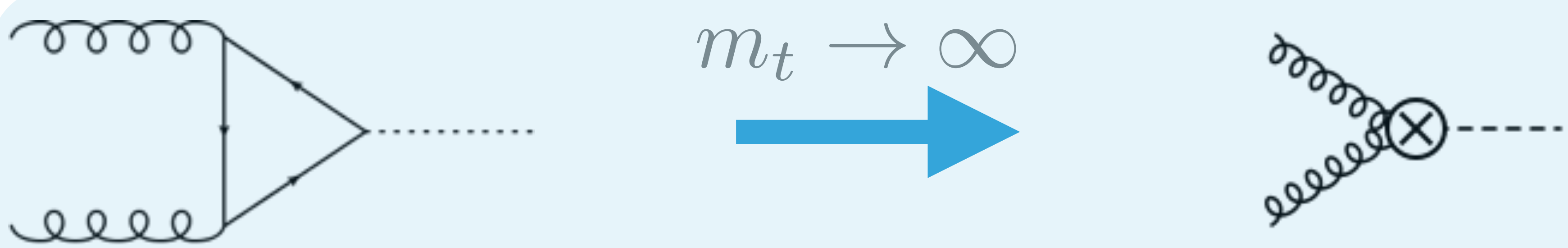


$$\hat{\sigma} = \alpha_s^2 \sigma^{\mathrm{LO}} + \alpha_s^3 \sigma^{\mathrm{NLO}} + \alpha_s^4 \sigma^{\mathrm{NNLO}} + \alpha_s^5 \sigma^{\mathrm{N3LO}}$$

✓ ✓ ✓ ✓

?

Experiments



$$\hat{\sigma} = \alpha_s^2 \sigma^{\mathrm{LO}} + \alpha_s^3 \sigma^{\mathrm{NLO}} + \alpha_s^4 \sigma^{\mathrm{NNLO}} + \alpha_s^5 \sigma^{\mathrm{N3LO}}$$

✓ ✓ ✓ ✓

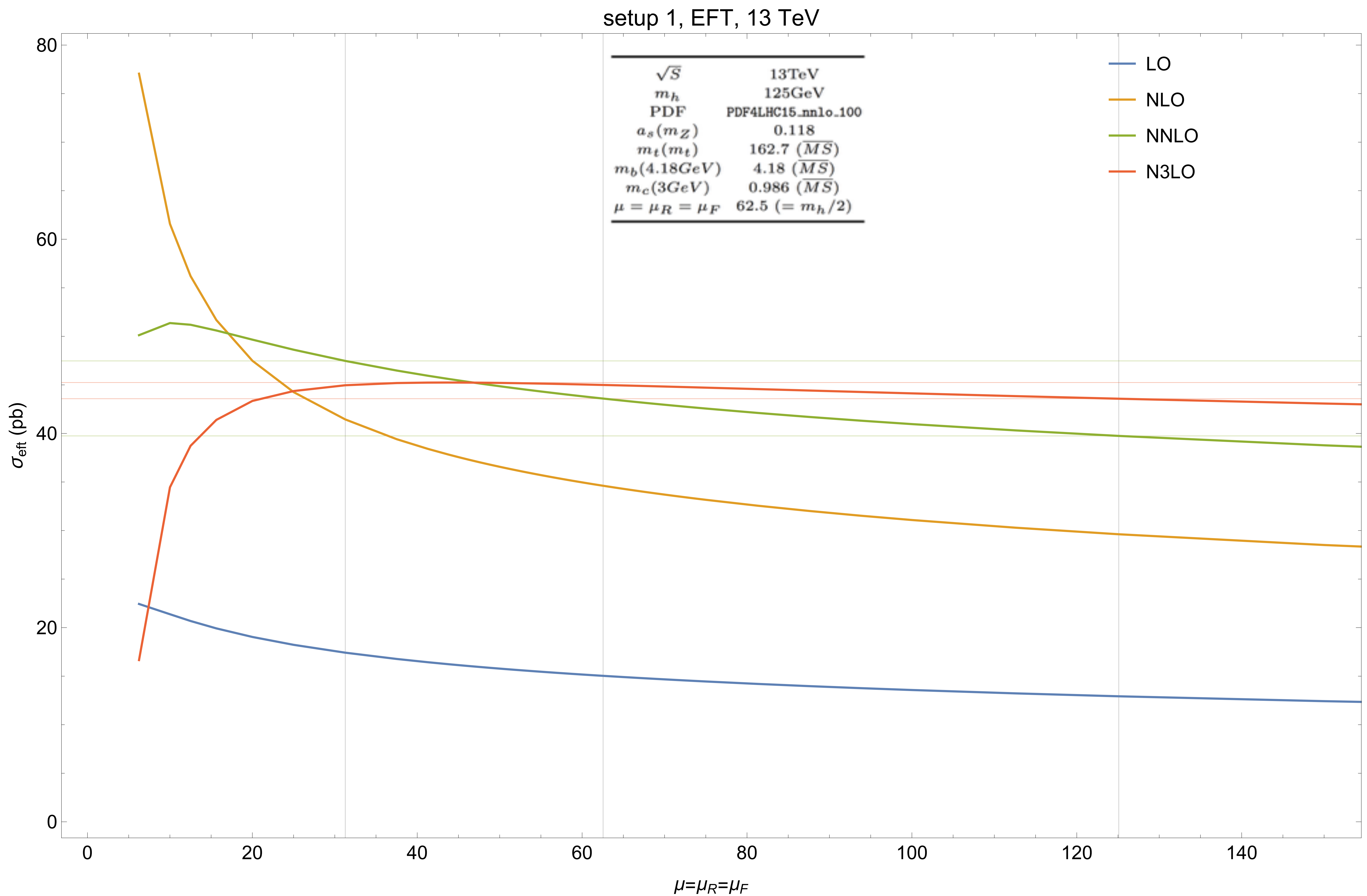
UNCERTAINTY OF EFT

FINITE QUARK MASSES

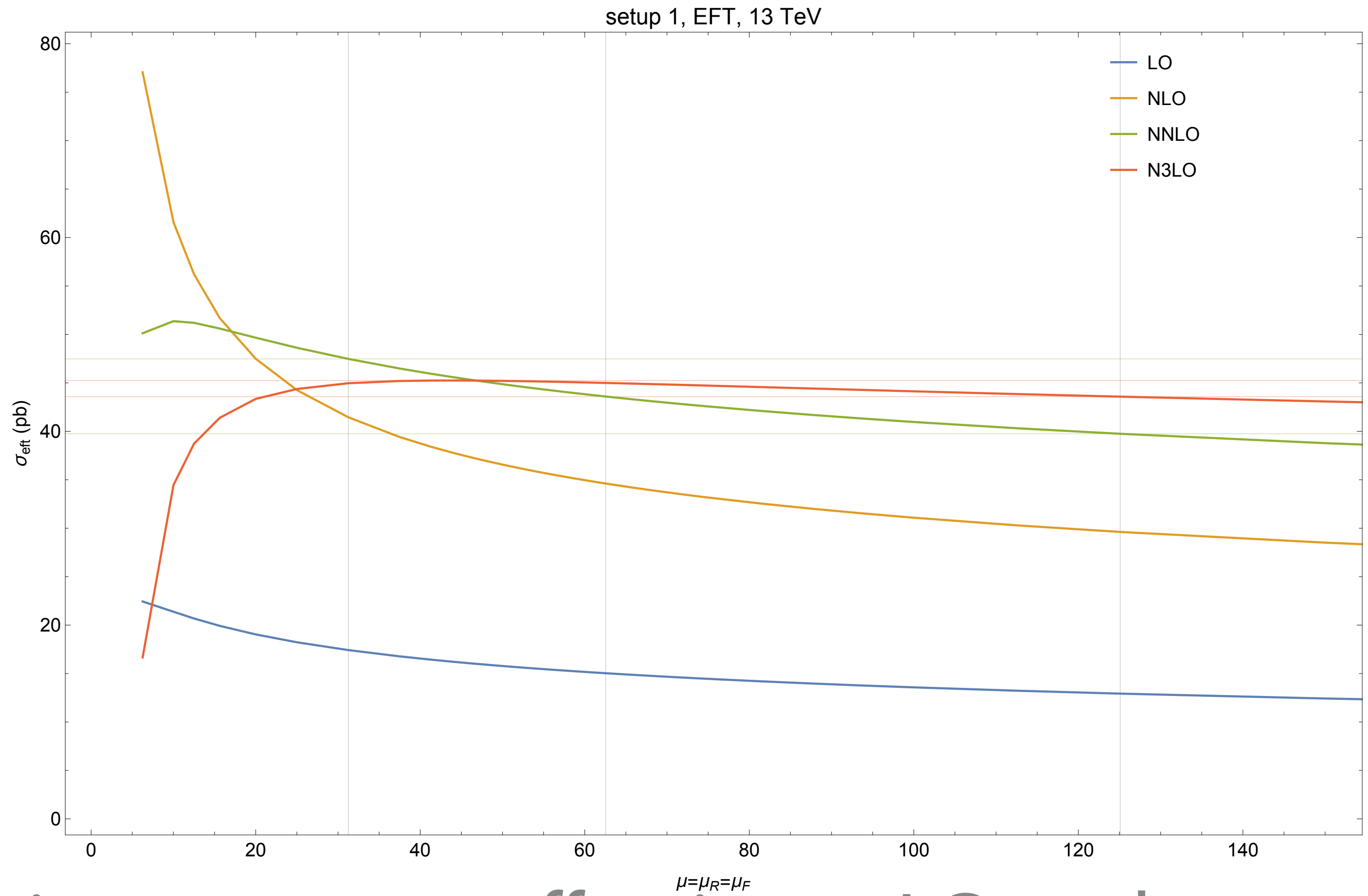
CONVOLUTION WITH PDFS

Experiments

SCALE VARIATION UNCERTAINTY


setup 1, EFT, 13 TeV
$\sqrt{S}$ 13TeV
m_h 125GeV
PDF PDF4LHC15_nnlo_100
$a_s(m_Z)$ 0.118
$m_t(m_t)$ 162.7 ($\overline{MS}$)
$m_b(4.18GeV)$ 4.18 ($\overline{MS}$)
$m_c(3GeV)$ 0.986 ($\overline{MS}$)
$\mu = \mu_R = \mu_F$ 62.5 ($= m_h/2$)
LO
NLO
NNLO
N3LO
80
60
40
20
0
σ_{eft} (pb)
0
20
40
60
80
100
120
140
$\mu=\mu_R=\mu_F$

Estimate missing higher orders (MHO) from scale variation



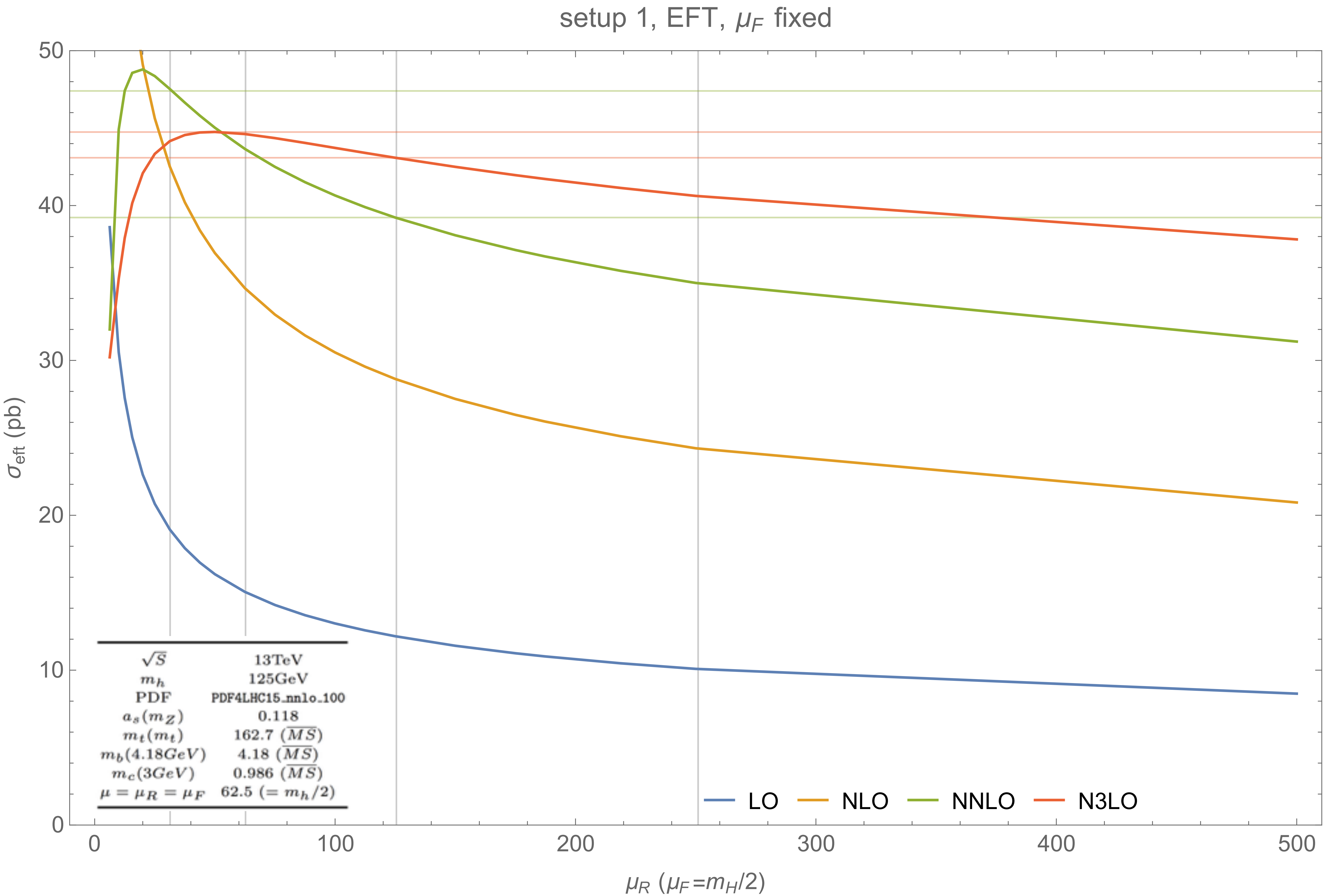
Vary scale in interval $\left[\frac{m_H}{4}, m_H\right]$

Estimating MHO from scale variations not very effective at LO and NLO because of larger corrections

Perturbative series seems to stabilize from NNLO on

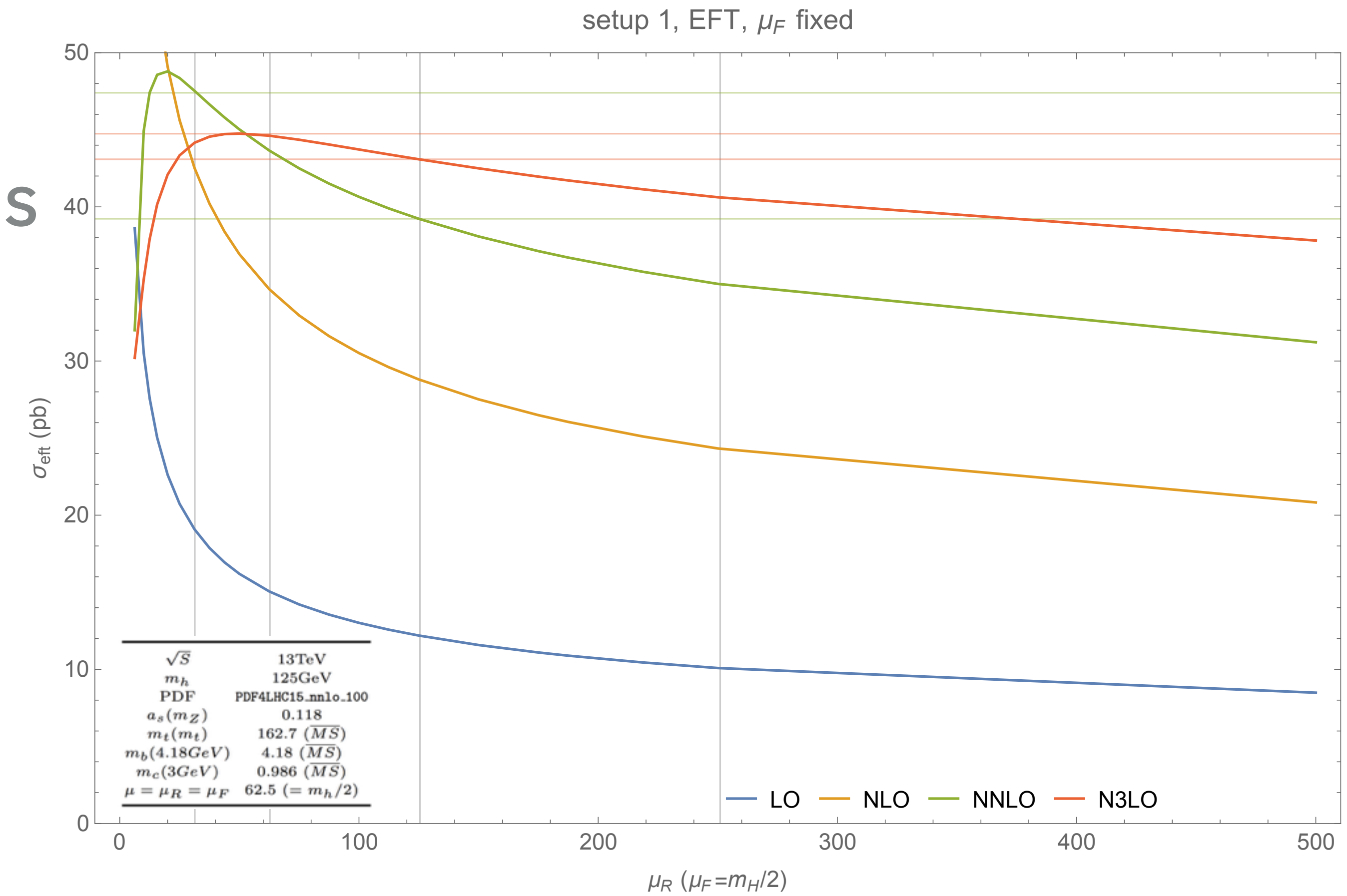
$$\pm\frac{\sigma_{\max}-\sigma_{\min}}{\sigma_{\max}+\sigma_{\min}}$$

LO	$\pm 14.8\%$
NLO	$\pm 16.6\%$
NNLO	$\pm 8.8\%$
$\mathrm{N^3LO}$	$\pm 1.6\%$


setup 1, EFT, μ_F fixed
$\sigma_{\rm eft}$ (pb)
μ_R (μ_F=m_H/2)
0
10
20
30
40
50
0
100
200
300
400
500
LO
NLO
NNLO
N3LO
$\sqrt{S}$ 13TeV
m_h 125GeV
PDF PDF4LHC15_nnlo_100
$a_s(m_Z)$ 0.118
$m_t(m_t)$ 162.7 ($\overline{MS}$)
$m_b(4.18GeV)$ 4.18 ($\overline{MS}$)
$m_c(3GeV)$ 0.986 ($\overline{MS}$)
$\mu = \mu_R = \mu_F$ 62.5 (= $m_h/2$)

Faster convergence for low scales

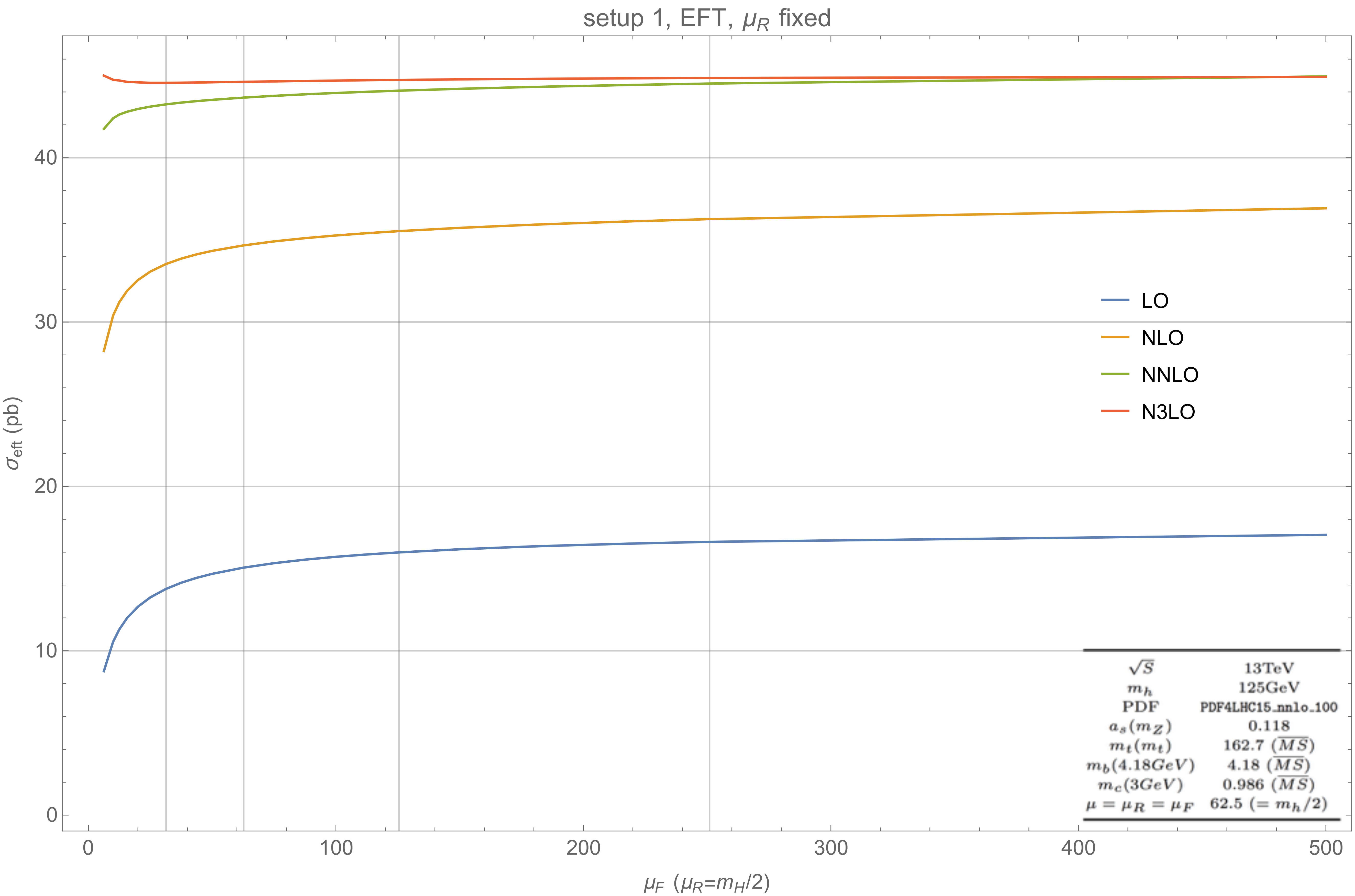
Vary scale in interval $\left[\frac{m_H}{4}, m_H\right]$



Scale variation relative to average cross section

$$\pm\frac{\sigma_{\max} - \sigma_{\min}}{\sigma_{\max} + \sigma_{\min}}$$

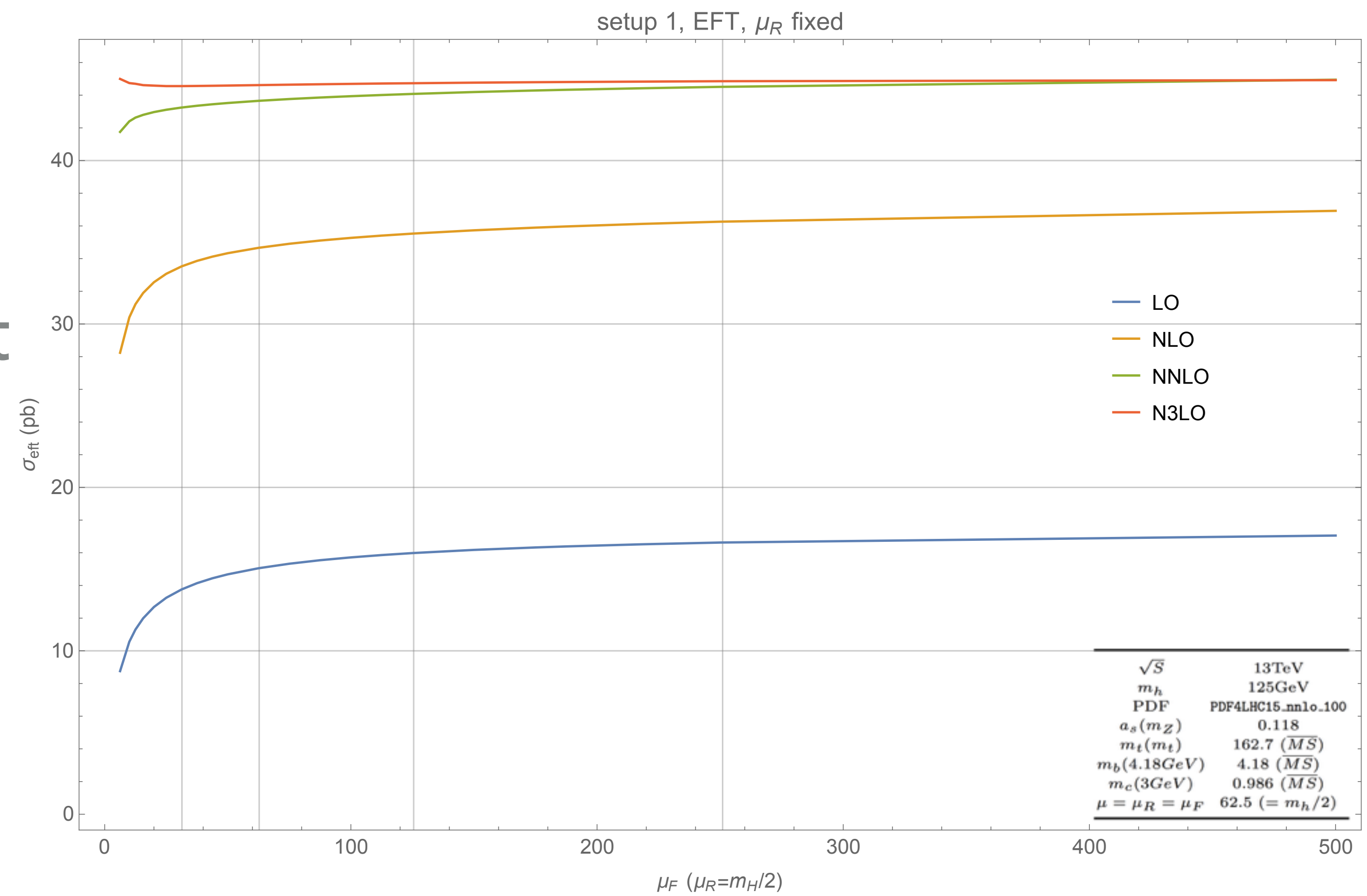
LO	$\pm 22.0\%$
NLO	$\pm 19.2\%$
NNLO	$\pm 9.5\%$
$\mathrm{N^3LO}$	$\pm 1.9\%$


setup 1, EFT, μ_R fixed
σ_{eft} (pb)
μ_F (μ_R=m_H/2)
0
100
200
300
400
500
10
20
30
40
LO
NLO
NNLO
N3LO
$\sqrt{S}$ 13TeV
m_h 125GeV
PDF PDF4LHC15_nnlo_100
$a_s(m_Z)$ 0.118
$m_t(m_t)$ 162.7 ($\overline{MS}$)
$m_b(4.18GeV)$ 4.18 ($\overline{MS}$)
$m_c(3GeV)$ 0.986 ($\overline{MS}$)
$\mu = \mu_R = \mu_F$ 62.5 ($= m_h/2$)

Traditionally:

PDFs at LO are not constant but evolved with one-loop splitting functions

Improved LO agreement with data



This approach would require the unknown four loop splitting functions at N3LO

CONSISTENT FIXED ORDER QCD

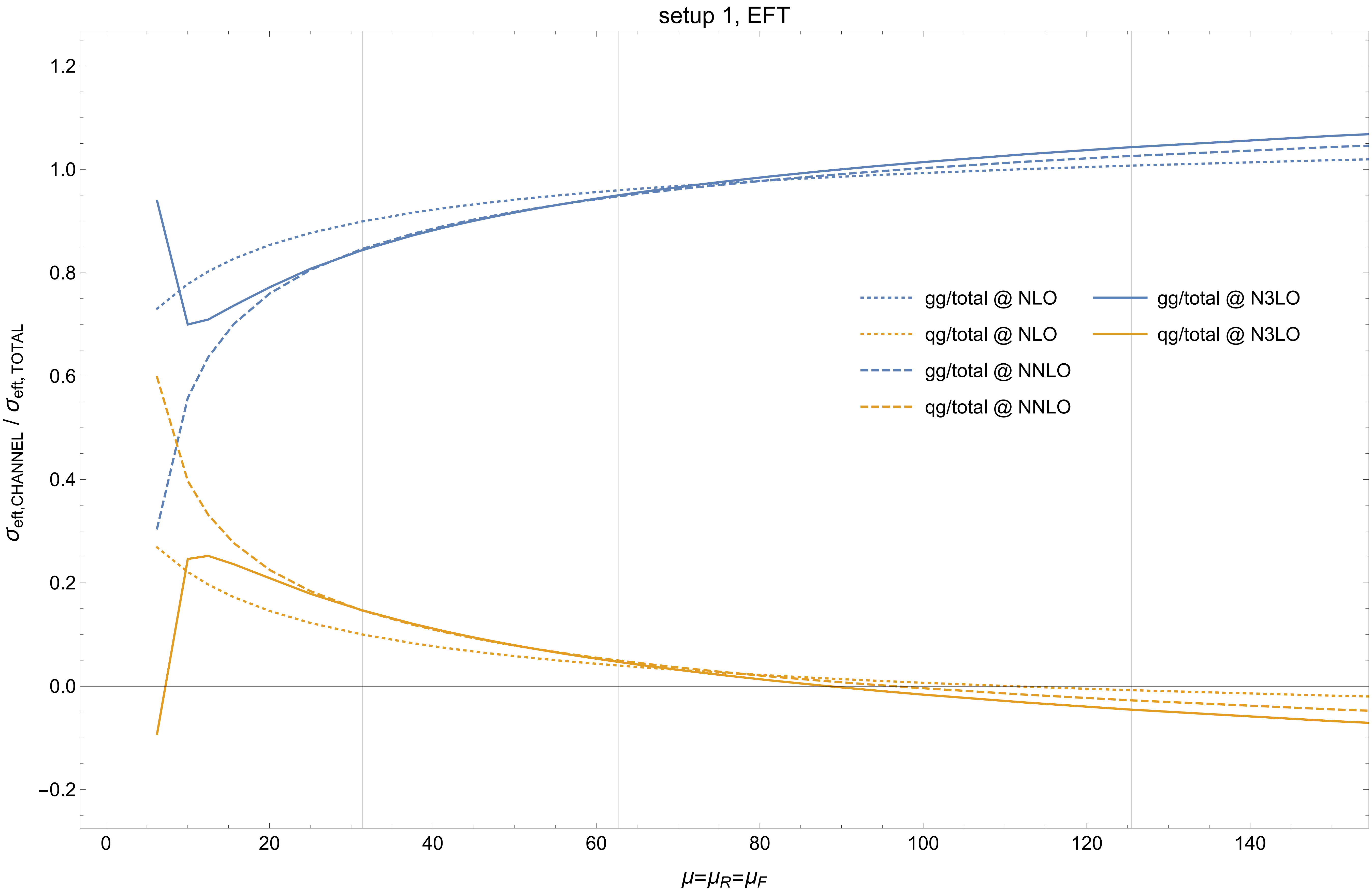
Improved Phenomenology

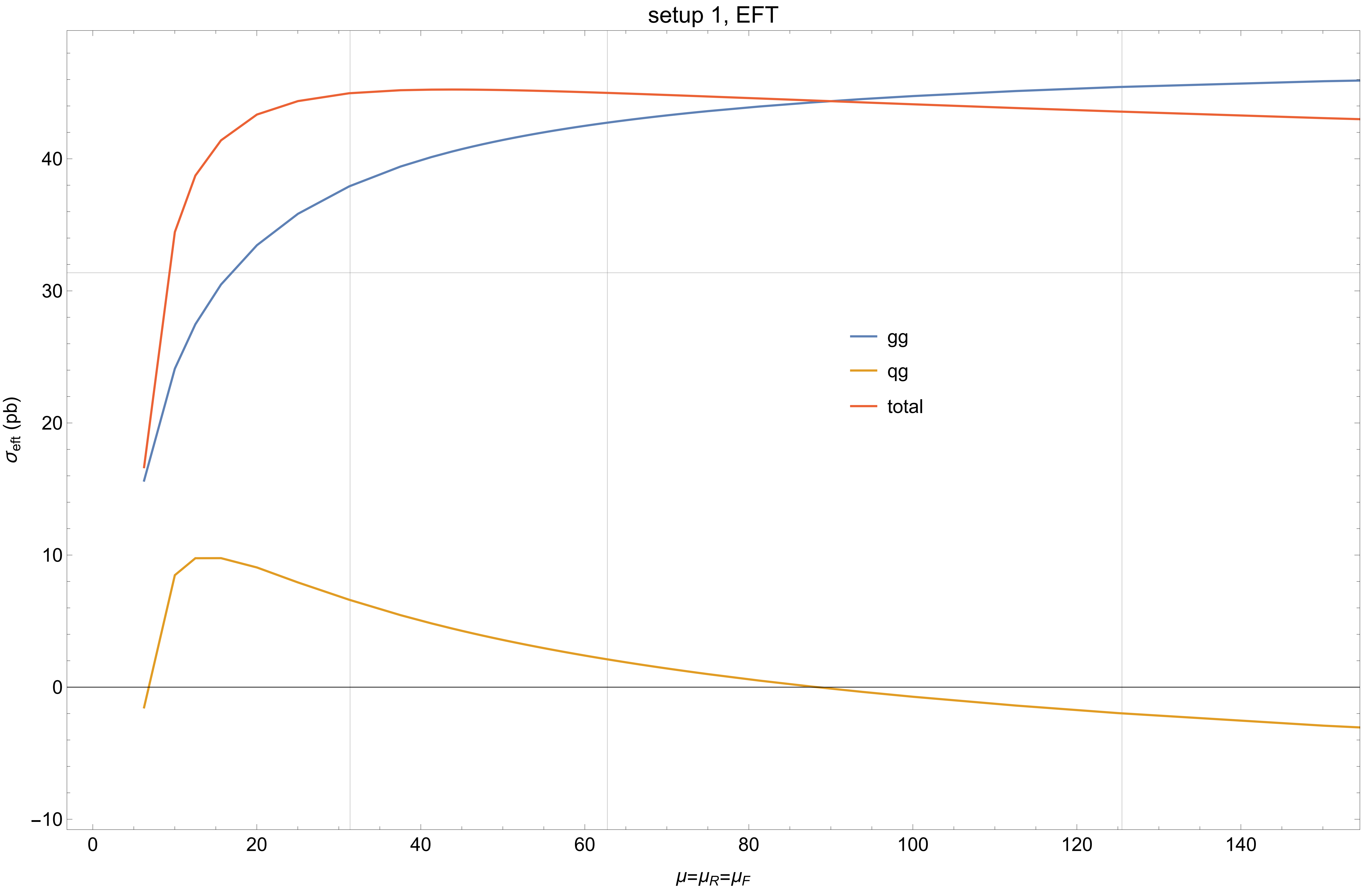
Order	Traditional	Minimal
LO	$P^{(0)}$	const.
NLO	$P^{(1)}$	$P^{(0)}$
NNLO	$P^{(2)}$	$P^{(1)}$
N^3LO	$P^{(3)}$	$P^{(2)}$

Unreliable

Reliable
Not yet possible

Reliable

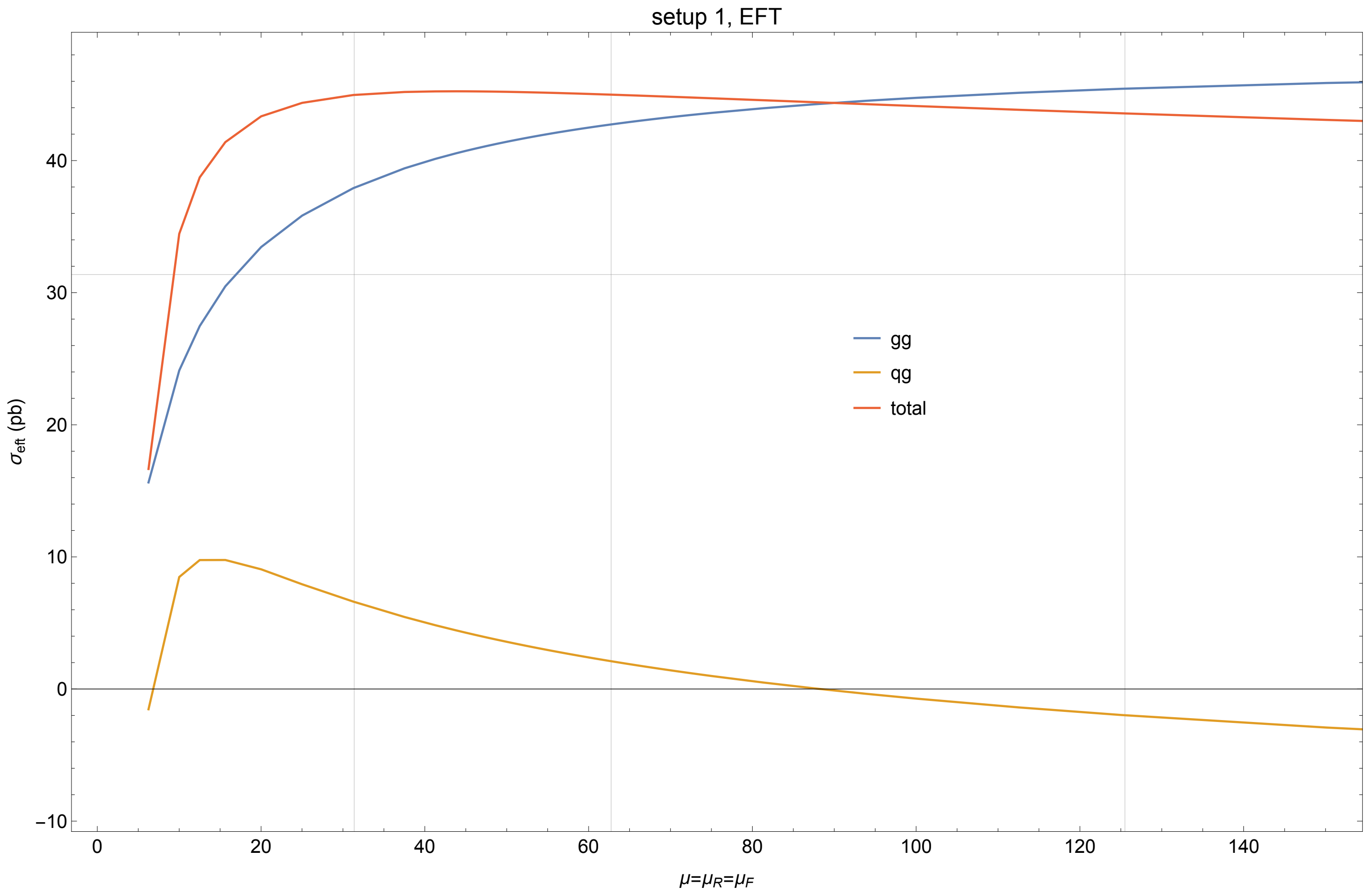

setup 1, EFT
gg/total @ NLO
qg/total @ NLO
gg/total @ NNLO
qg/total @ NNLO
gg/total @ N3LO
qg/total @ N3LO
$\sigma_{\text{eft,CHANNEL}} / \sigma_{\text{eft, TOTAL}}$
$\mu=\mu_R=\mu_F$
1.2
1.0
0.8
0.6
0.4
0.2
0.0
−0.2
0
20
40
60
80
100
120
140


setup 1, EFT
gg
qg
total
σ_{eft} (pb)
$\mu=\mu_R=\mu_F$
-10
0
10
20
30
40
0
20
40
60
80
100
120
140

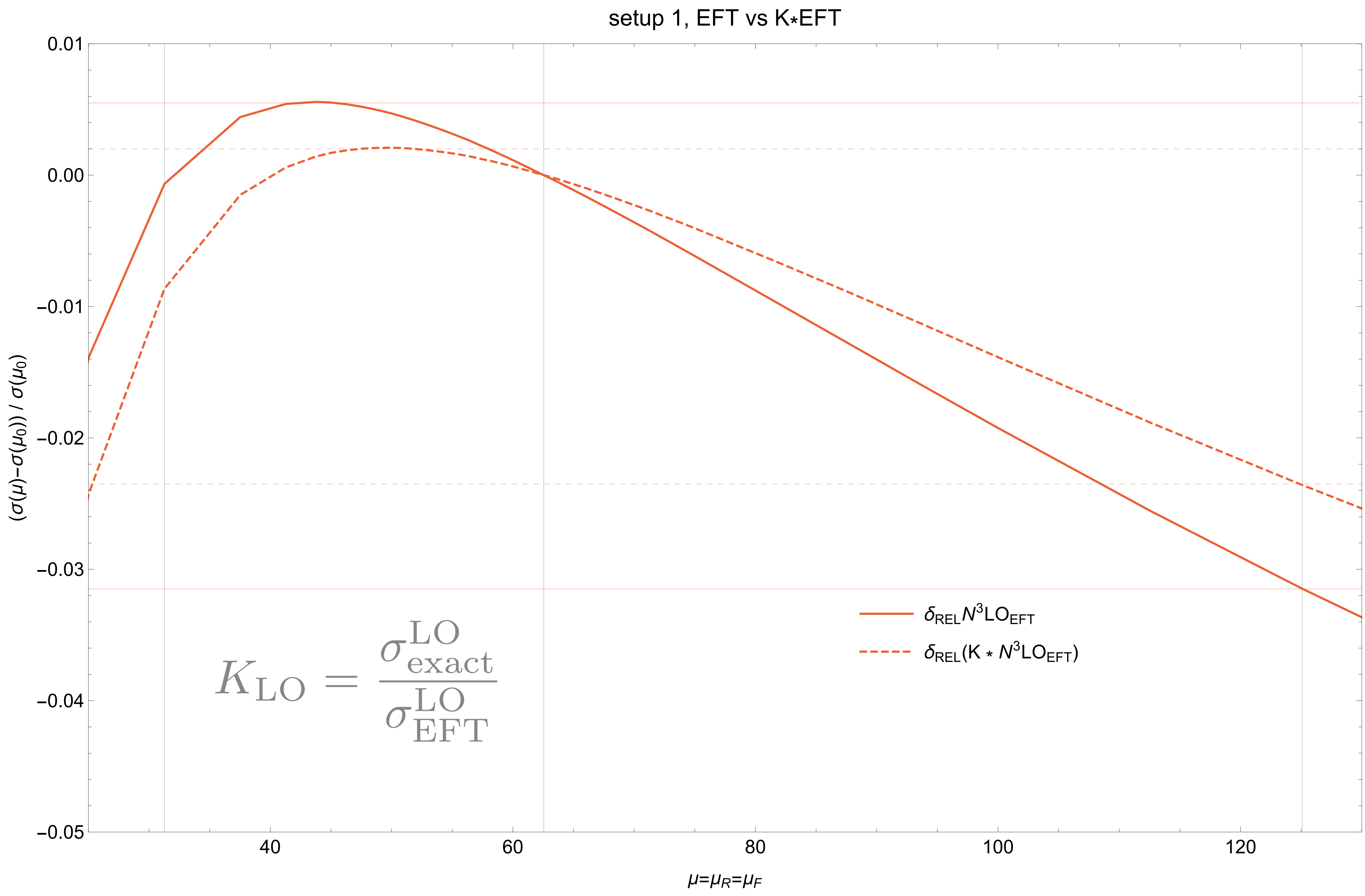
qg channel is important for scale variation

Cross section is stabilized by the qg channel at lower scales

Approximations based on the soft term miss this effect



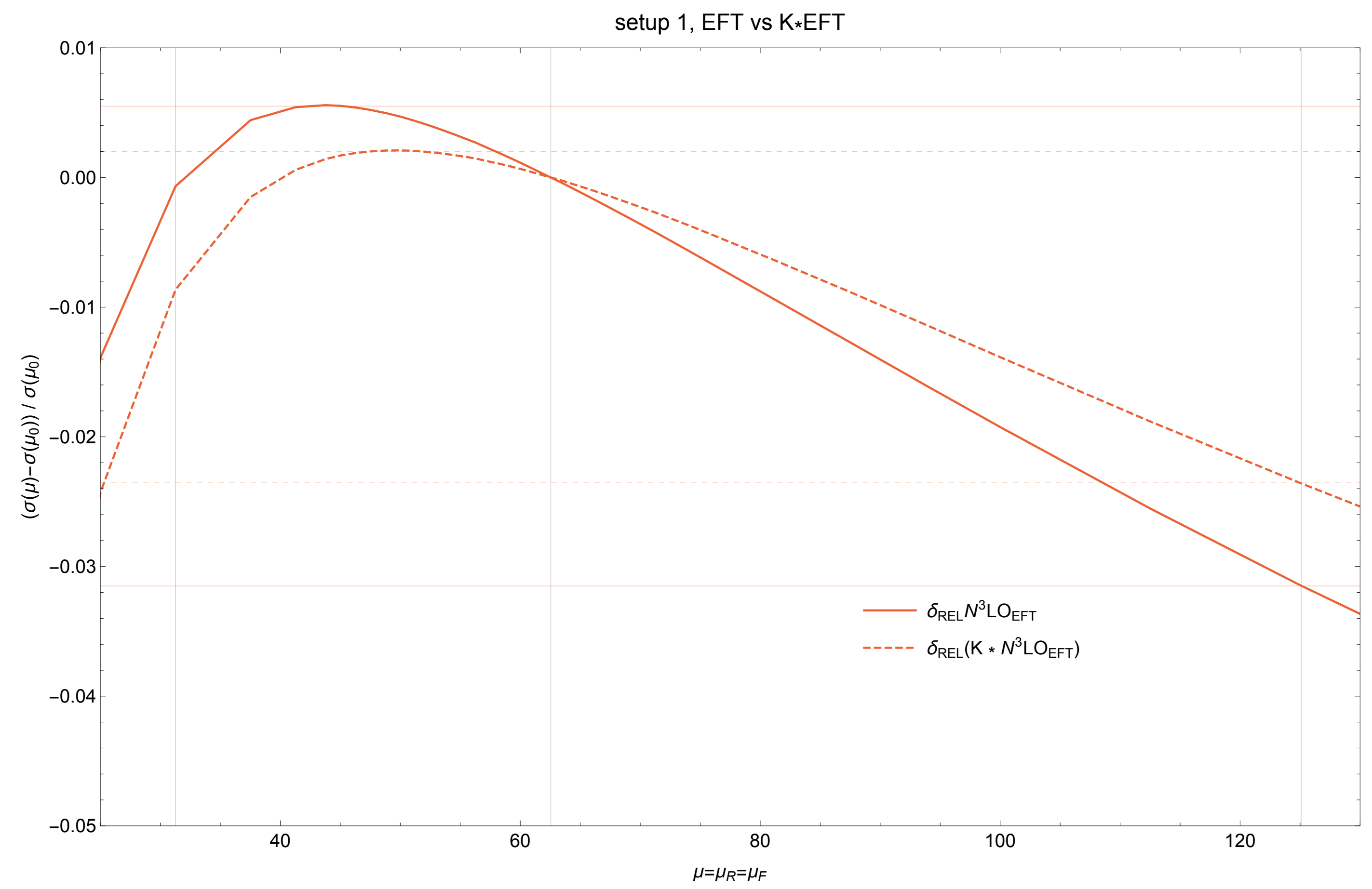
Scale variation receives correction from rescaling of the EFT



$$K_{\mathrm{LO}} = \frac{\sigma^{\mathrm{LO}}_{\mathrm{exact}}}{\sigma^{\mathrm{LO}}_{\mathrm{EFT}}}$$

Scale dependence is improved by inclusion of the rescaled LO

Running of the top mass in MSbar compensates partially the running of the cross section



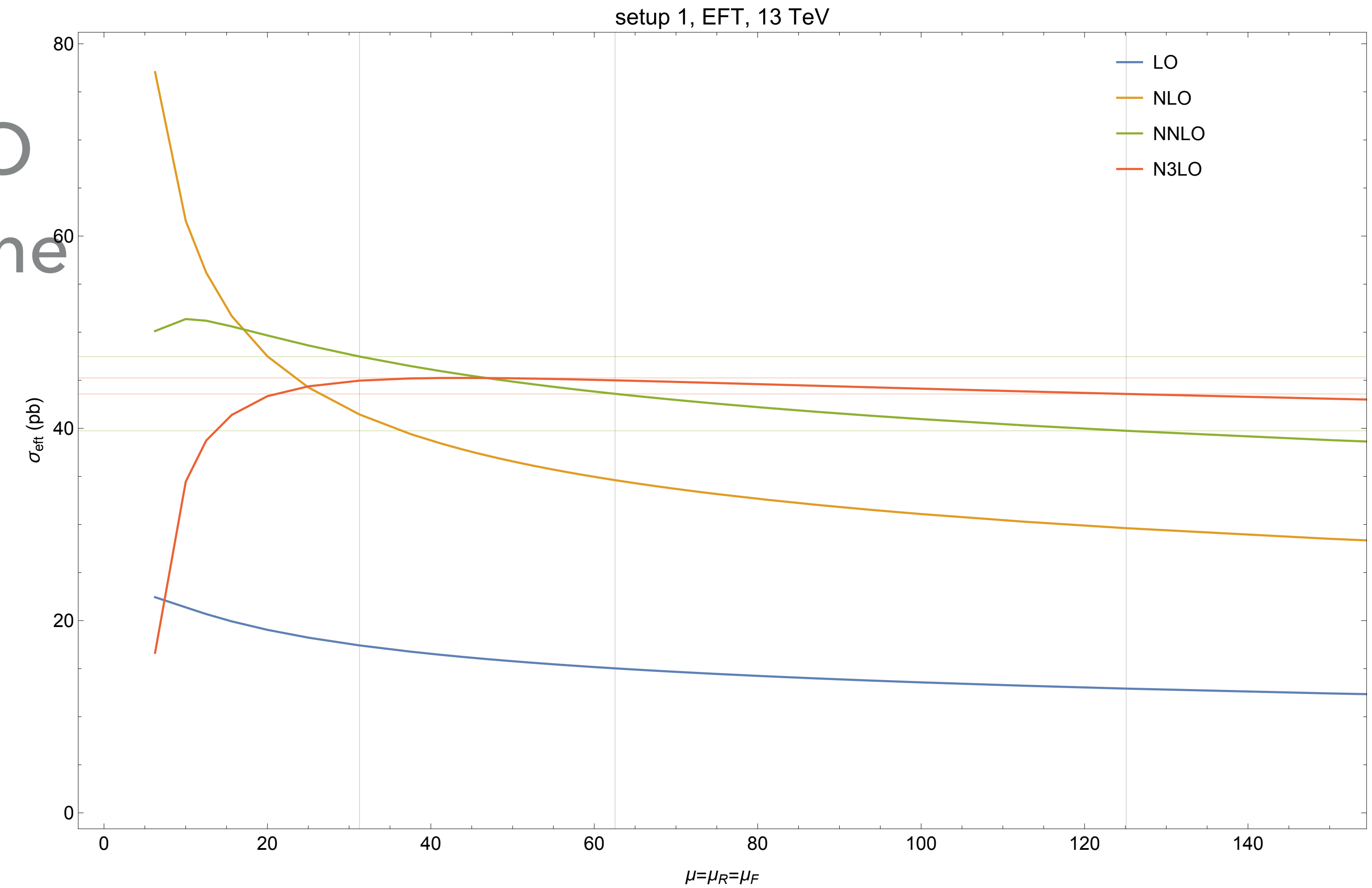
Scale uncertainty from variation with all channels in $\left[\frac{m_H}{4}, m_H\right]$

$$\delta_{\text{scale}} = {}^{+0.13}_{-1.24}\,\text{pb} = {}^{+0.30}_{-2.79}\,\%$$

IS SCALE VARIATION A GOOD ESTIMATOR

Scale variation at LO and NLO notoriously underestimates the corrections from MHO

Is scale variation at N3LO a good estimator?



We need to understand the convergence of the perturbative series

Reshuffle orders in the perturbative expansions

USE FACTORIZATION TO RESHUFFLE PERTURBATIVE ORDERS

Two types of factorization in inclusive Higgs cross section

$m_{\text{top}} \to \infty$	EFT	FACTORIZE WILSON COEFFICIENT
$z \to 1$	soft limit	RESUMMATION

Wilson coefficient receives QCD corrections

$$\mathcal{O}\left(\alpha_s^n\right)$$

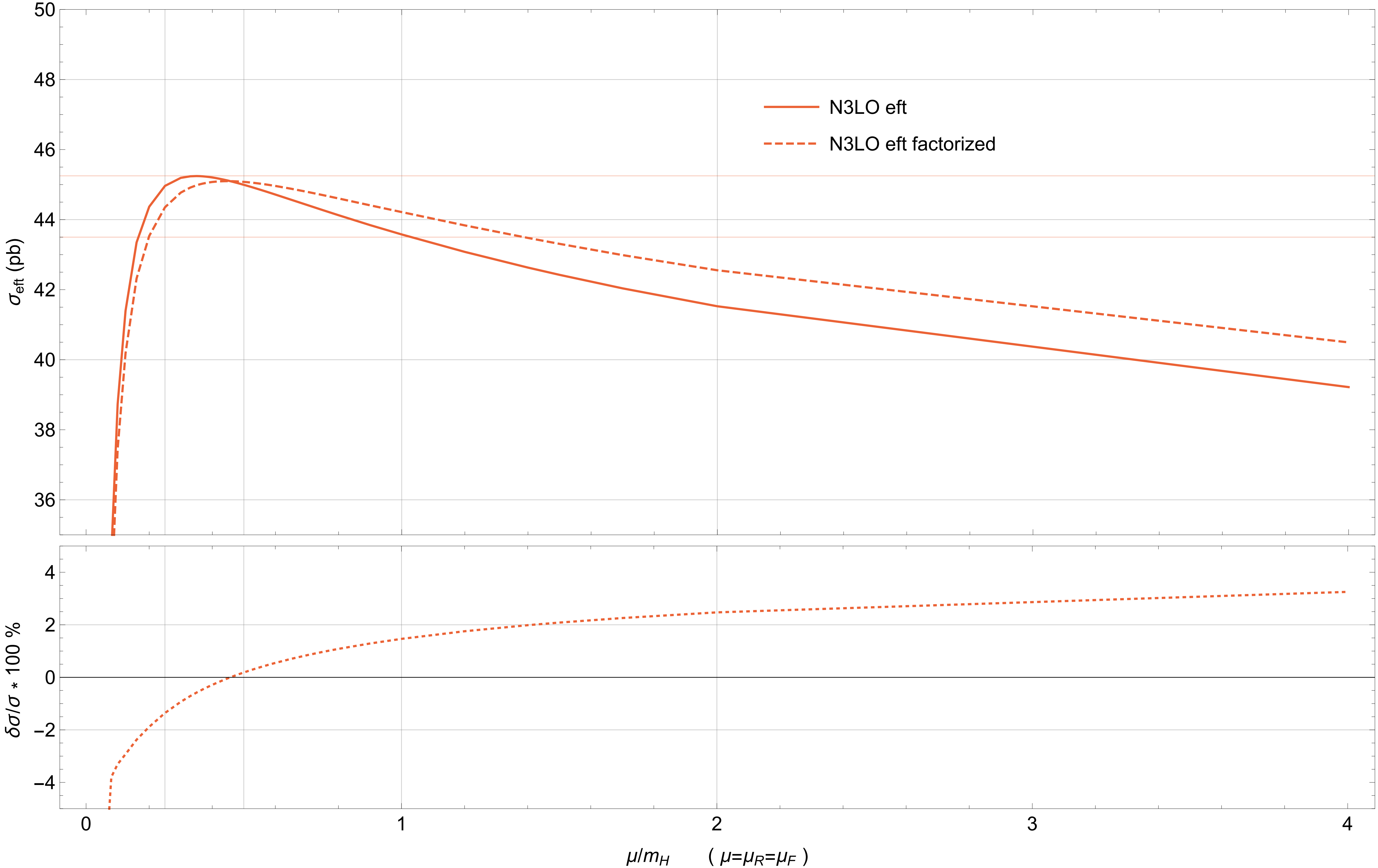
$$\hat{\sigma} = \left|C(\alpha_s)\right|^2 \times \eta(\alpha_s)$$

PERTURBATIVE SERIES

Conventional approach: Expand the product to $\mathcal{O}\left(\alpha_s^n\right)$

Alternative approach: Keep terms of up to $\mathcal{O}\left(\alpha_s^{2n}\right)$ in the product

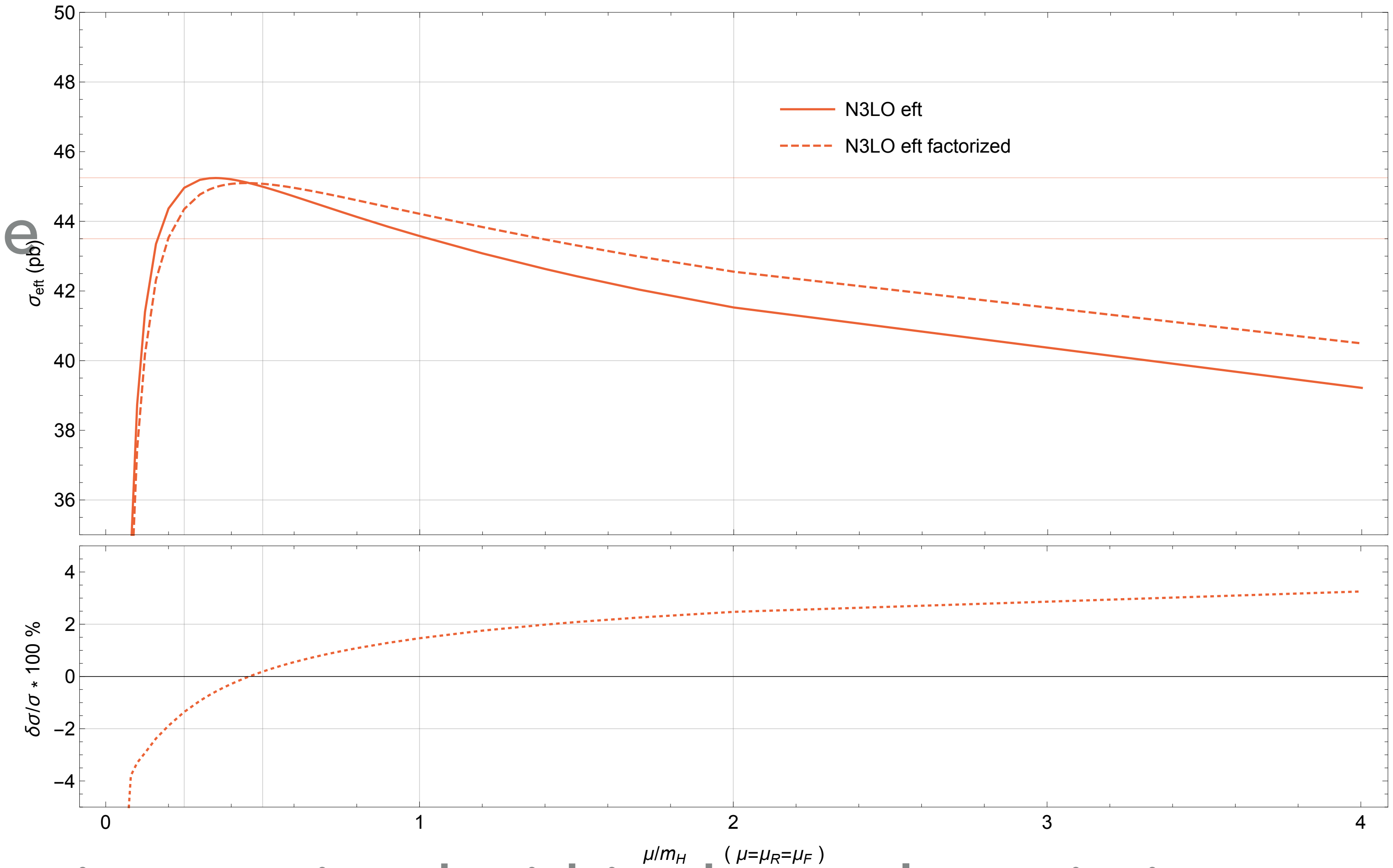
Captures some pieces of higher order cross sections



N3LO eft
N3LO eft factorized
σ_{eft} (pb)
50
48
46
44
42
40
38
36
4
2
0
−2
−4
$\delta\sigma/\sigma * 100$ %
0
1
2
3
4
μ/m_H (μ=μ_R=μ_F)

Factorizing the Wilson coefficient reduces the scale dependence

Both approaches exactly equivalent at the preferred scale mh/2



Change of the scale variation is contained within the scale variation in the conventional approach

Scale variation in the conventional approach is a more conservative estimator

Cross section factorizes in the soft limit in Mellin space $z \to 1$

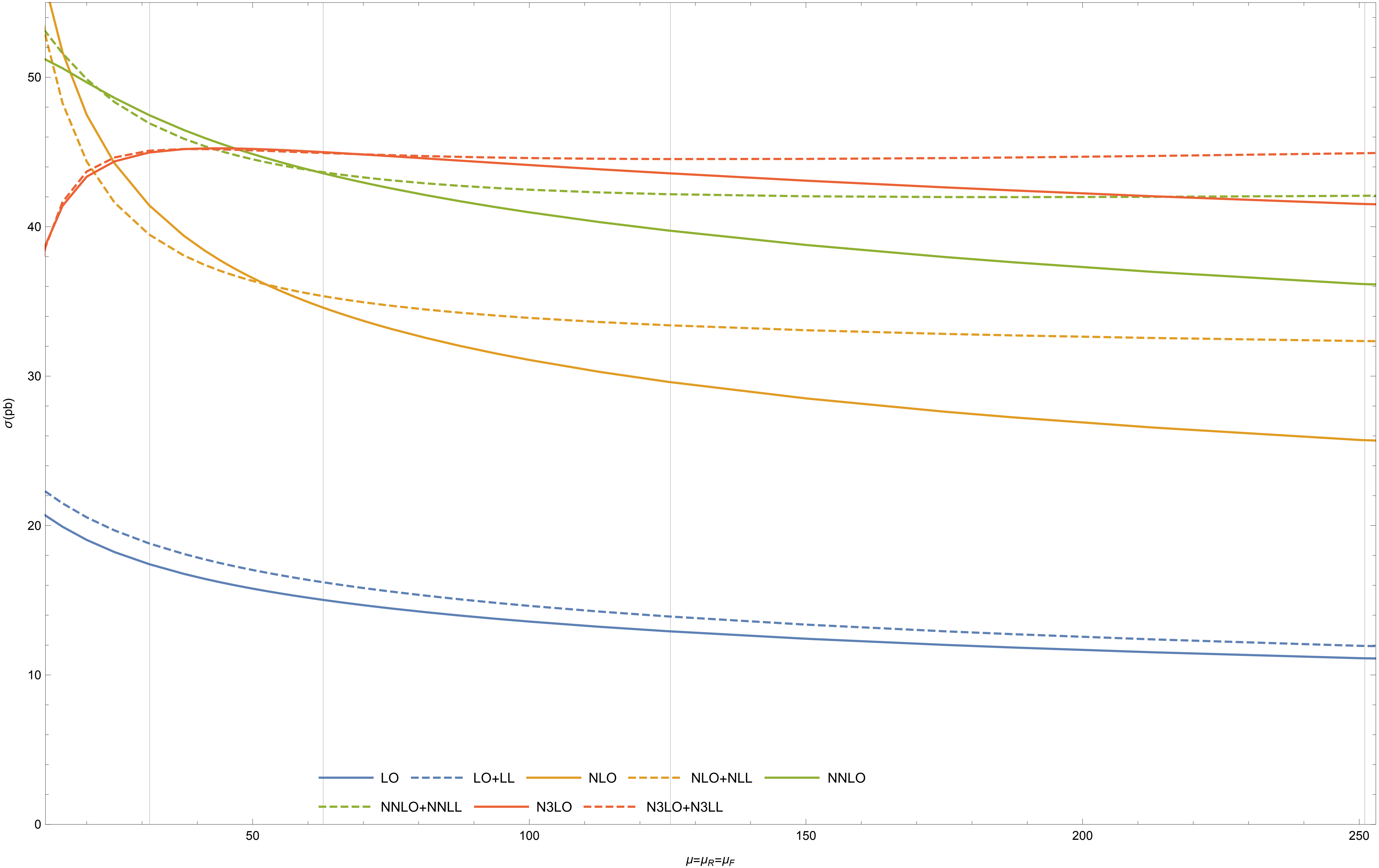
$$\sigma_{res} = \alpha_s^2(\mu_r)\underbrace{e^{\mathcal{G}(\alpha_s,\mu_r,\mu_f)}}_{\text{EXPONENTIATE DIVERGENT TERMS}}\underbrace{C_{gg}(\alpha_s(\mu_r),\mu_r,\mu_s)}_{\text{FINITE PIECES}}$$

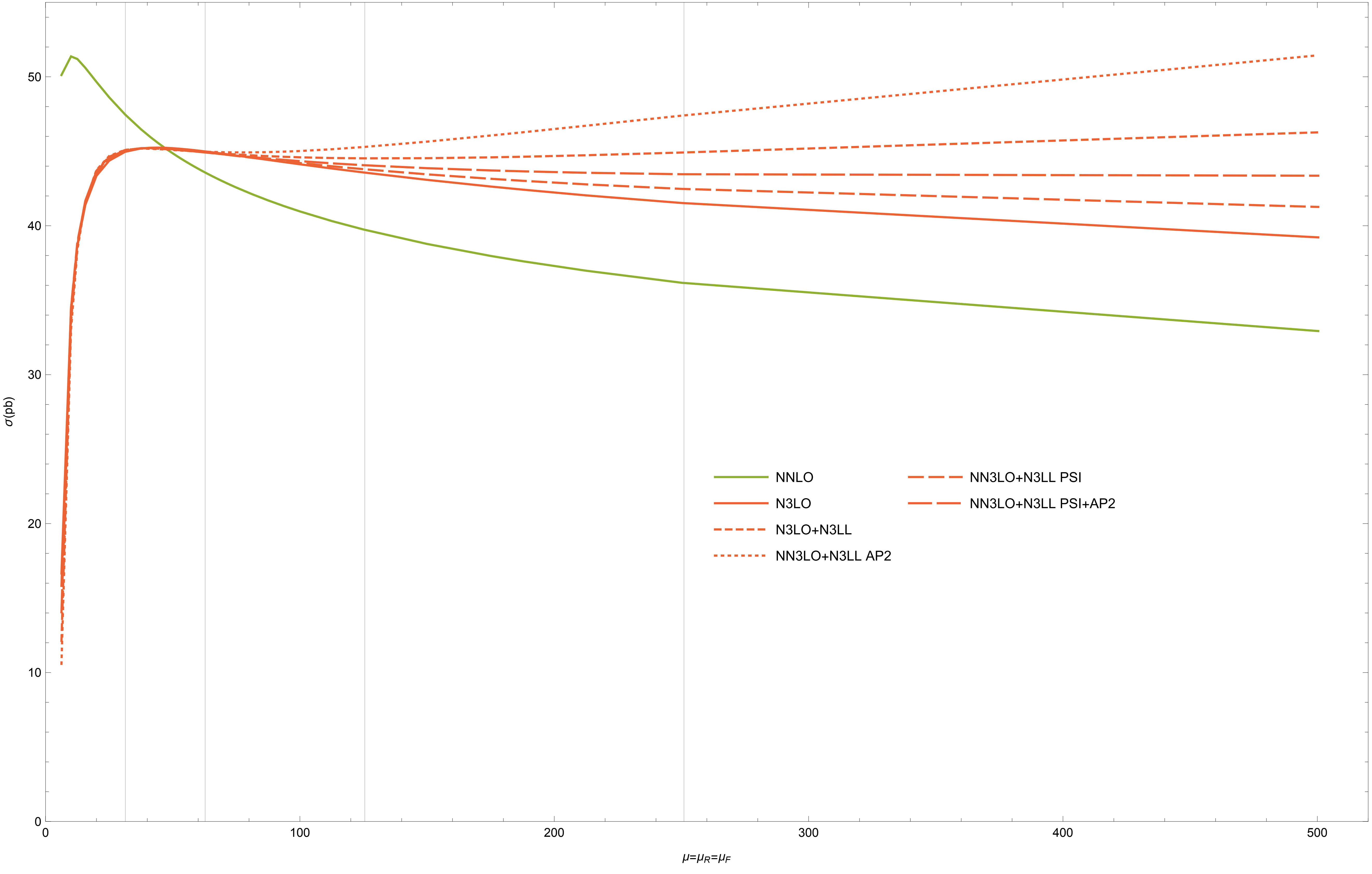
Exponentiate universal emission of soft gluons

MELLIN SPACE

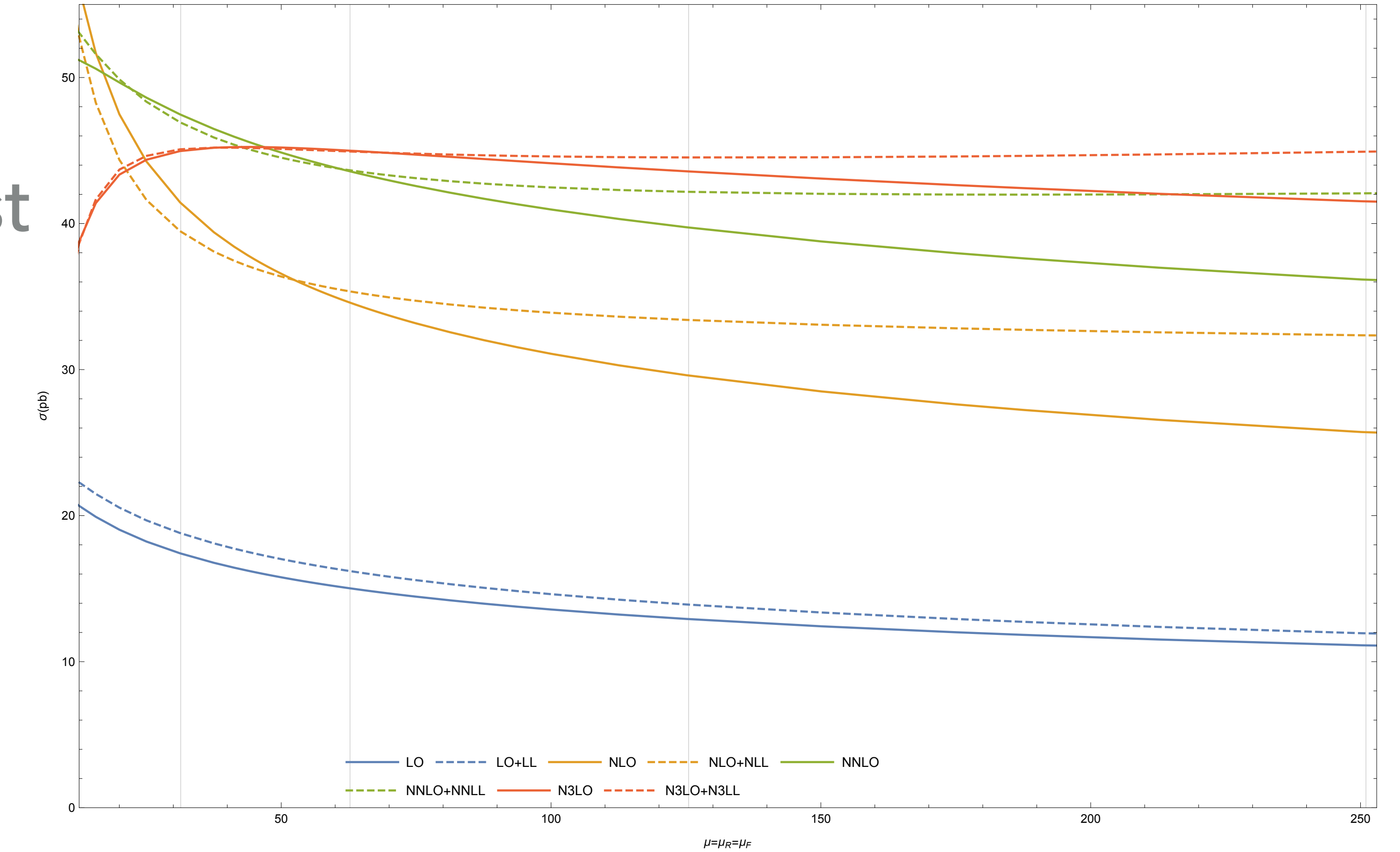
Captures the n most leading threshold logarithms

Different resummation prescriptions differ by subleading terms


σ(pb)
μ=μ_R=μ_F
LO
LO+LL
NLO
NLO+NLL
NNLO
NNLO+NNLL
N3LO
N3LO+N3LL
0
10
20
30
40
50
50
100
150
200
250


σ(pb)
μ=μR=μF
NNLO
N3LO
N3LO+N3LL
NN3LO+N3LL AP2
NN3LO+N3LL PSI
NN3LO+N3LL PSI+AP2

Leading logarithms not the most important piece numerically

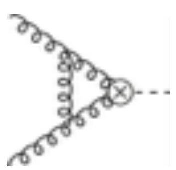


Formally equivalent prescriptions differ strongly at high scales

No correction from resummation at the preferred scale mh/2

Resummation correction within the preferred interval contained inside scale variation

Cross section factorizes in the soft limit in Laplace space $z \to 1$



$$\sigma_{SCET} = H \times S$$

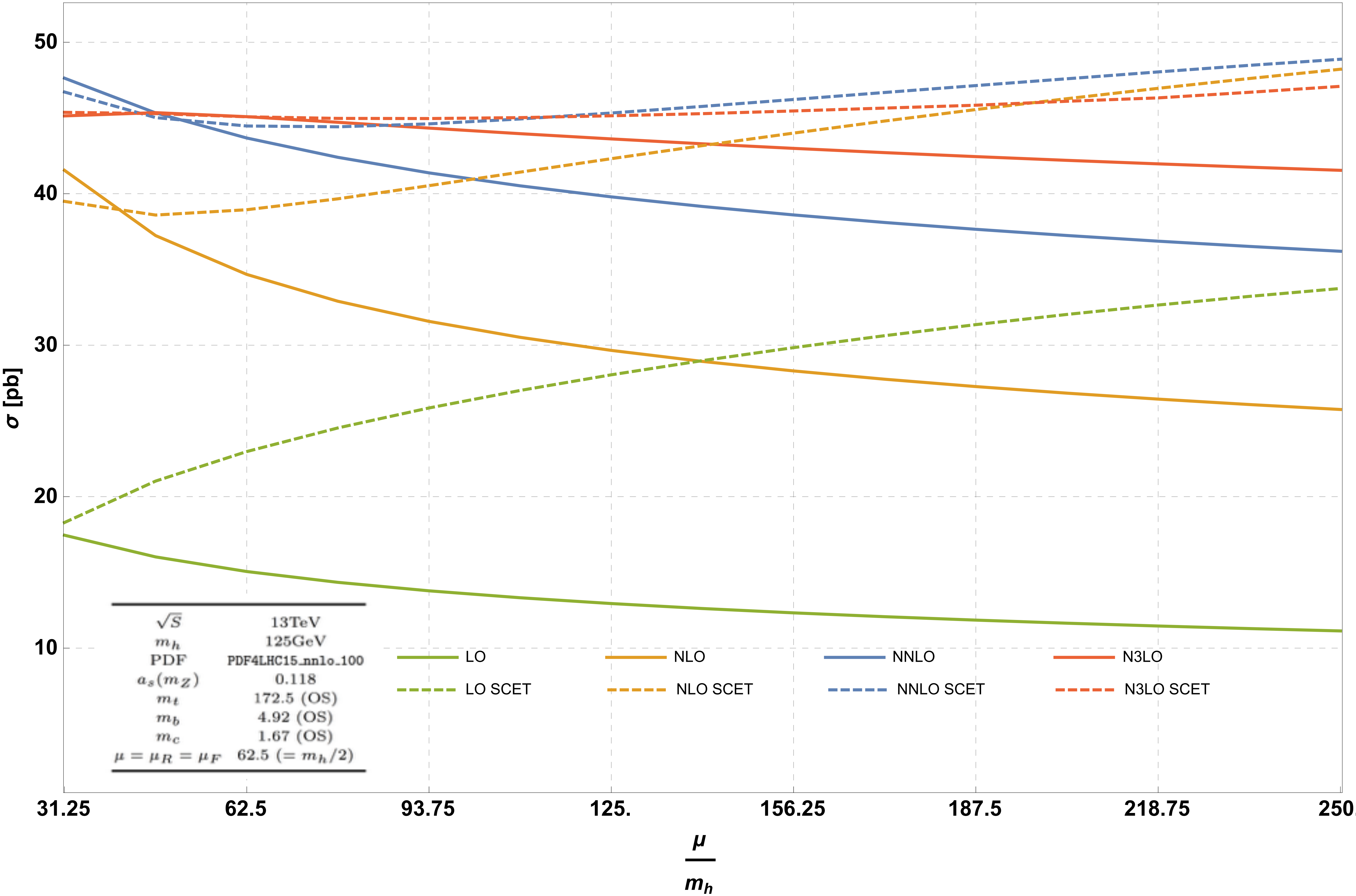
HARD FUNCTION

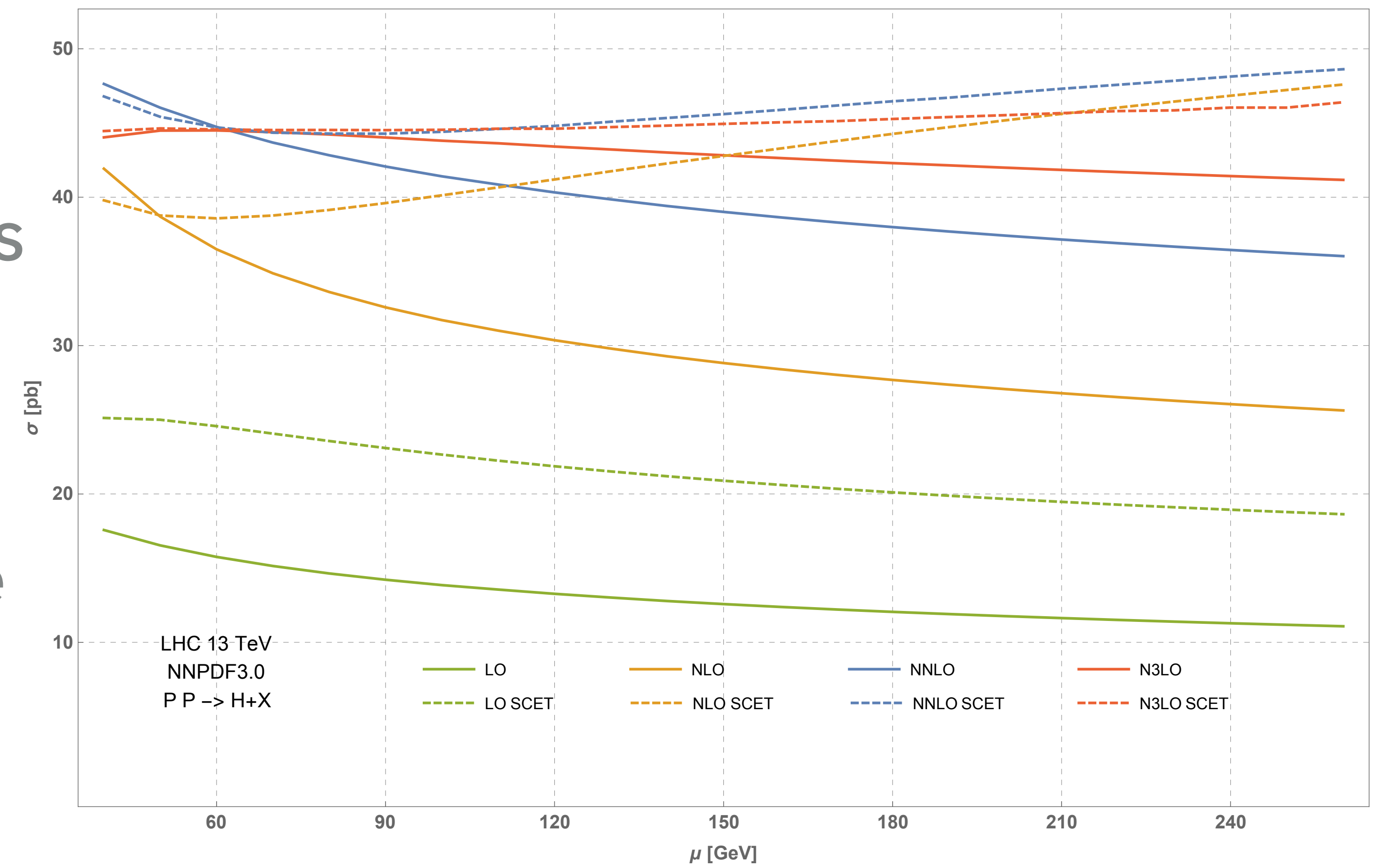
WILSON COEFFICIENTS OF SCET

SOFT FUNCTION

Solve RGE for the wilson coefficients

Reduces the scale dependence of the cross section


50
40
30
20
10
σ [pb]
31.25
62.5
93.75
125.
156.25
187.5
218.75
250.
μ / m_h
√S 13TeV
m_h 125GeV
PDF PDF4LHC15_nnlo_100
α_s(m_Z) 0.118
m_t 172.5 (OS)
m_b 4.92 (OS)
m_c 1.67 (OS)
μ = μ_R = μ_F 62.5 (= m_h/2)
LO
NLO
NNLO
N3LO
LO SCET
NLO SCET
NNLO SCET
N3LO SCET



Large corrections for high scales

No correction at preferred scale mh/2

Scale variation captures the correction from SCET in the preferred interval

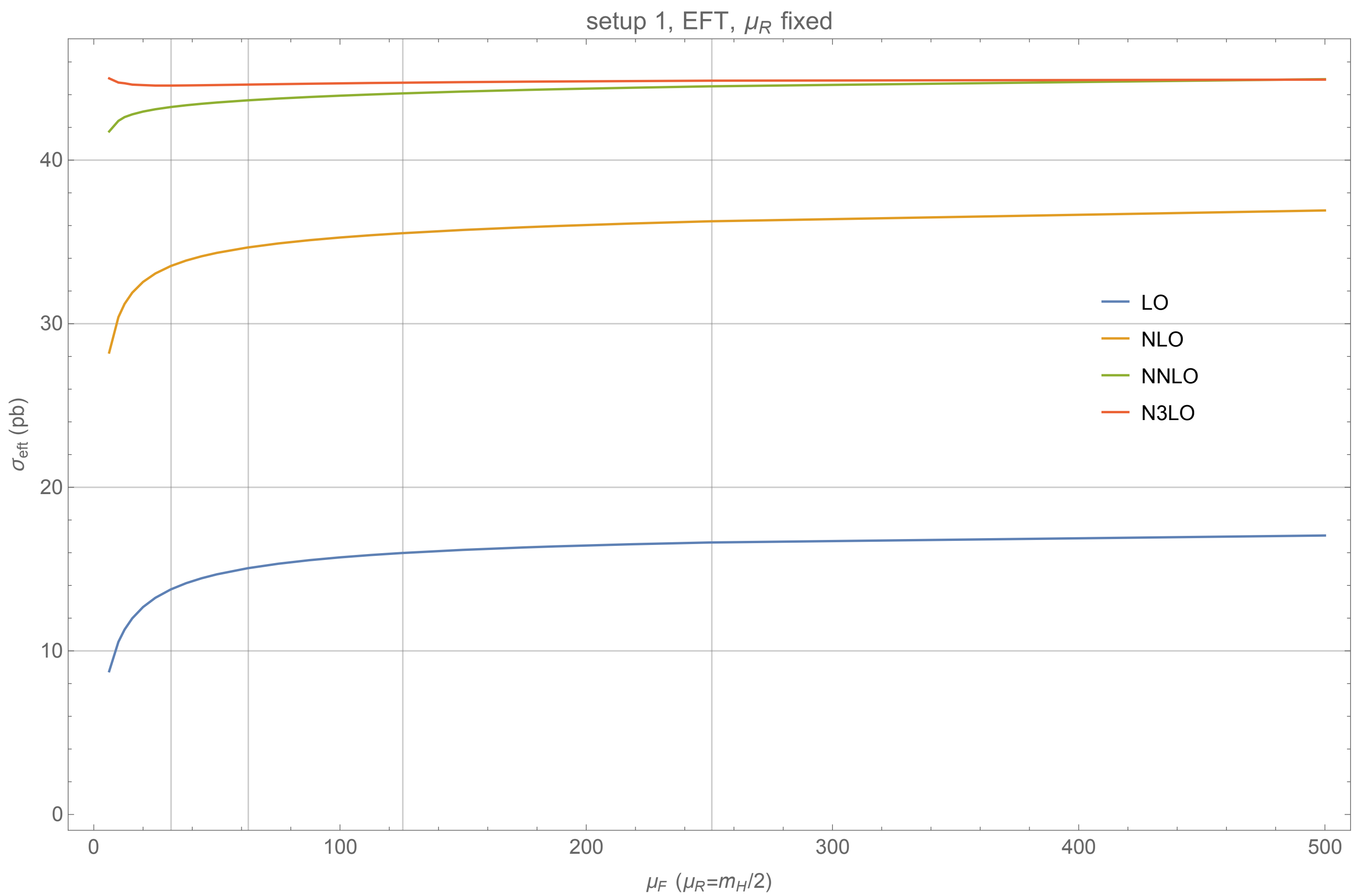
Scale variation is a good conservative estimator of missing higher order effects

MISSING N3LO PDFS

We use NNLO PDFs

Contain data extracted using (almost) NNLO calculations

We should be using be using N3LO PDFs

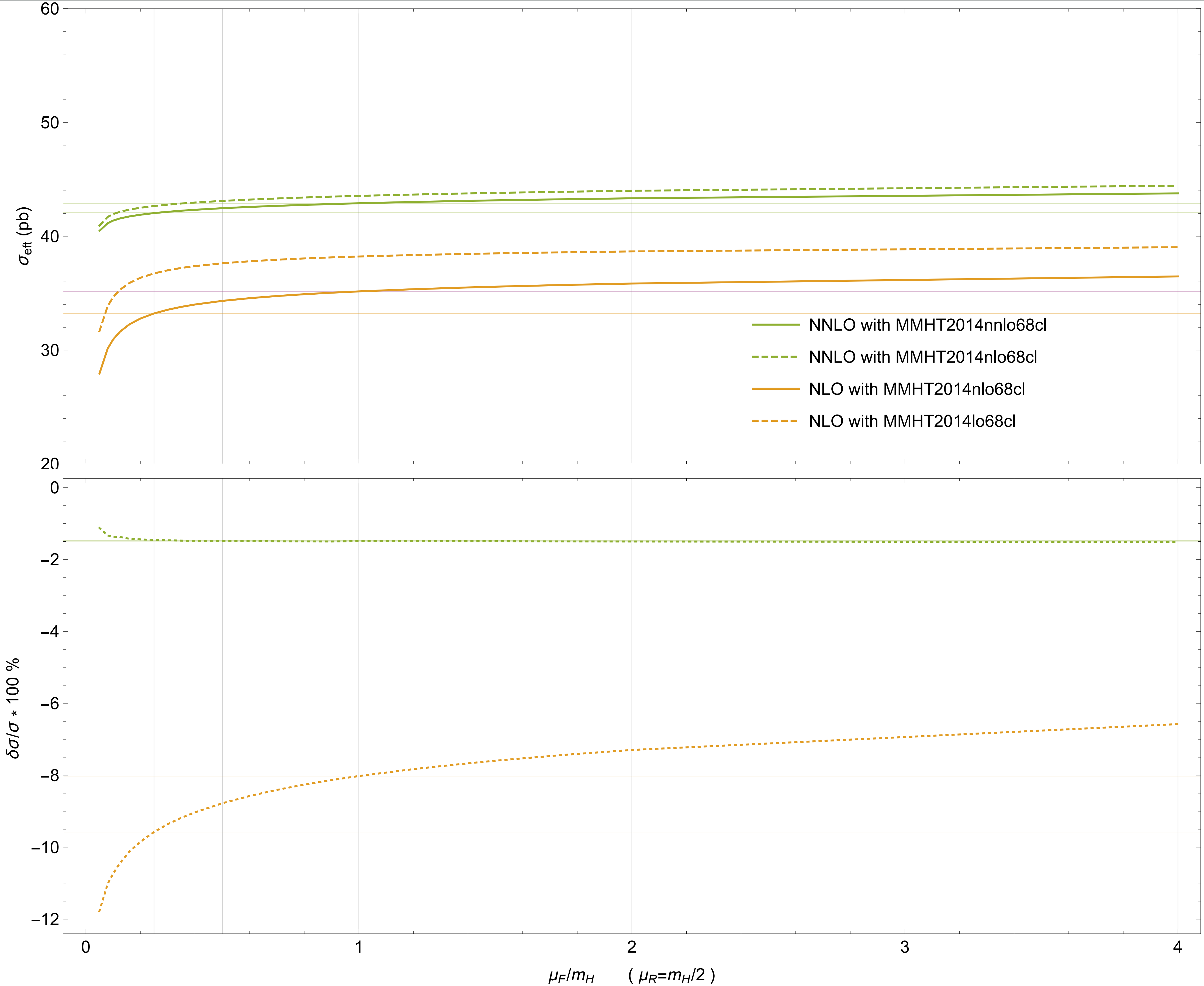


Missing N3LO corrections in the extraction processes

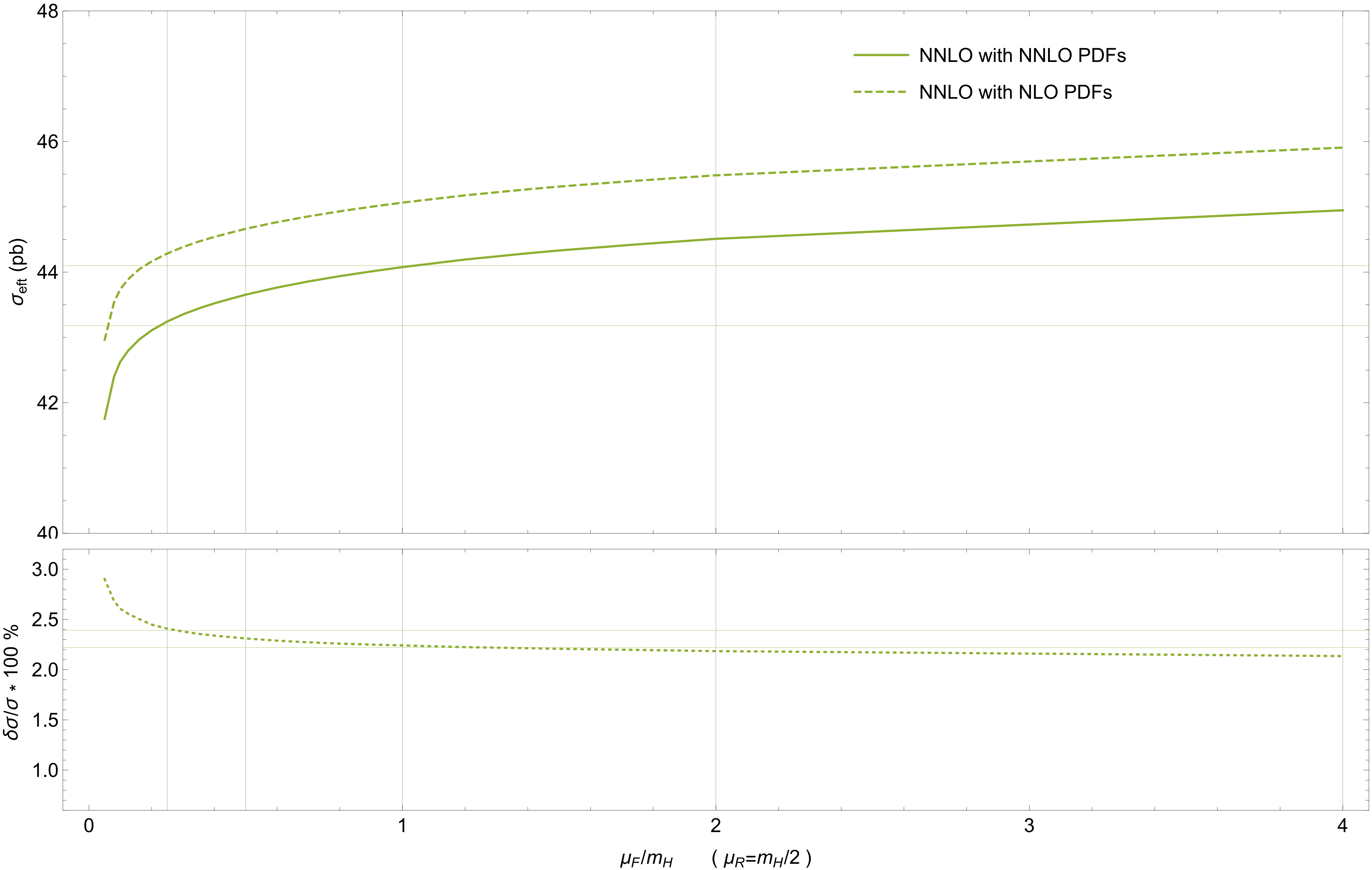
This uncertainty is not accounted for by the PDF uncertainties

Estimate the effect of higher orders in the extraction processes

MISSING N3LO PDFS



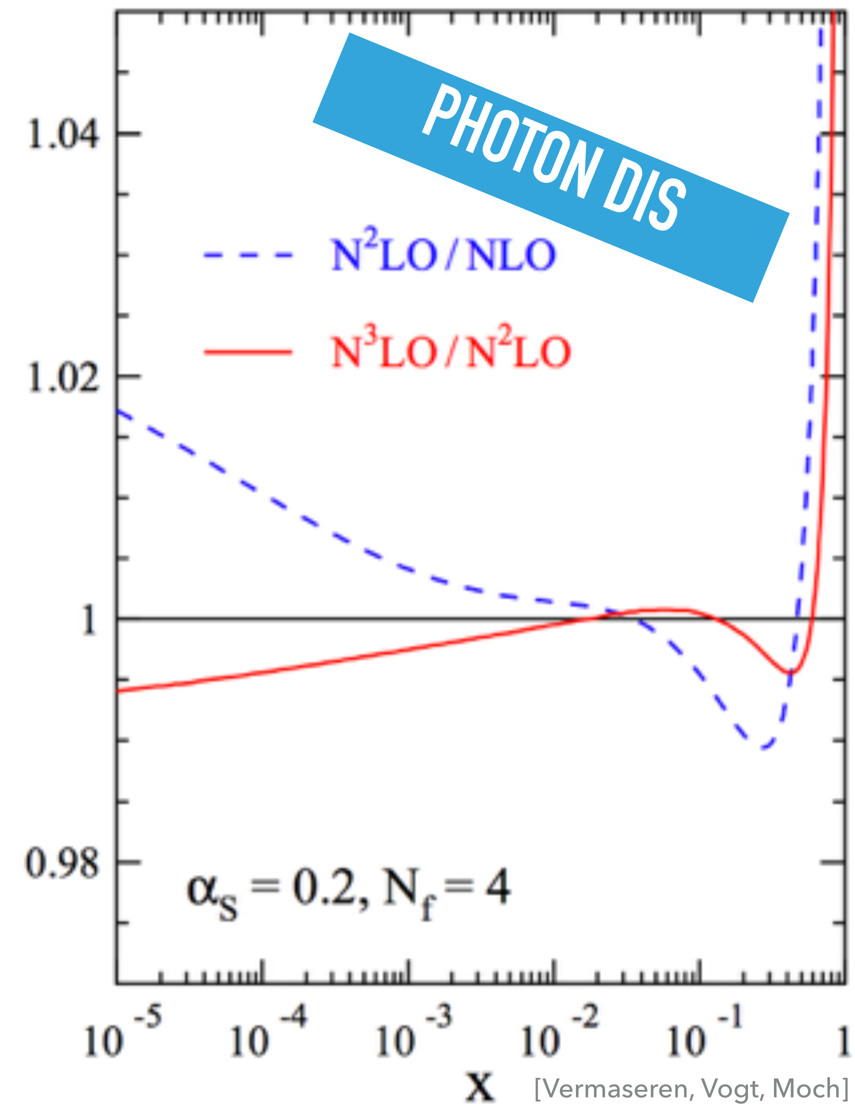
MISSING N3LO PDFS



Estimation based on the change from NLO PDFs to NNLO PDFs at NNLO

Conservative estimator, N3LO corrections likely smaller

DIS coefficients are smaller at N3LO



[Vermaseren, Vogt, Moch]

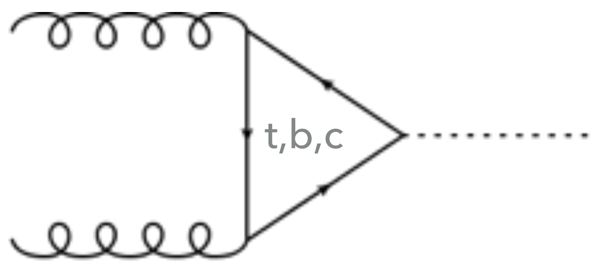
$$\delta_{\rm pdfTh} = \pm\frac{1}{2} \times \frac{\sigma^{\rm NNLO}_{\rm NNLO\ PDF} - \sigma^{\rm NNLO}_{\rm NLO\ PDF}}{\sigma^{\rm NNLO}_{\rm NLO\ PDF}} \sigma^{\rm N^3LO}_{\rm NNLO\ PDF}$$

CONSISTENT WITH PDF FROM HIGGS @ N3LO ESTIMATE

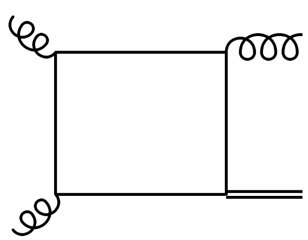
$$\delta_{\rm pdfTh} = \pm 0.55{\rm pb} = \pm 1.15\%$$

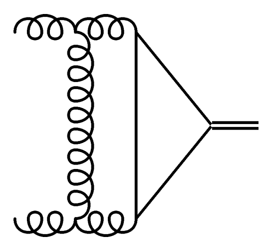
FINITE QUARK MASS EFFECTS

$$m_t \neq \infty \qquad\qquad m_b \neq 0$$



We know LO fully, including interferences

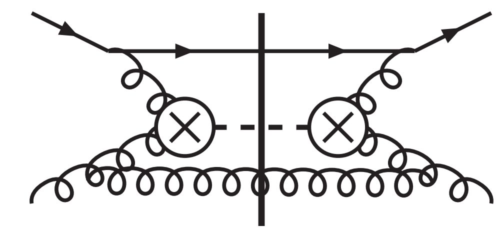




NLO also known fully, including interferences [Djouadi, Graudenz, Spira, Zerwas; Aglietti, Bonciani, Degrassi, Vicini; ...]

σ^{LO}_{eft}	15.13	σ^{NLO}_{eft}	34.81
$\sigma^{LO}_{eft;R}$	16.08	$\sigma^{NLO}_{eft;R}$	37.00
$\sigma^{LO}_{ex.;t}$	16.08	$\sigma^{NLO}_{ex;t}$	36.76
$\sigma^{LO}_{ex.;t+b}$	15.02	$\sigma^{NLO}_{ex;t+b}$	35.09
$\sigma^{LO}_{ex.;t+b+c}$	14.90	$\sigma^{NLO}_{ex;t+b+c}$	34.91

$\sqrt{S}$	13TeV
m_h	125GeV
PDF	PDF4LHC15_nnlo_100
$a_s(m_Z)$	0.118
$m_t(m_t)$	162.7 ($\overline{MS}$)
$m_b(4.18GeV)$	4.18 ($\overline{MS}$)
$m_c(3GeV)$	0.986 ($\overline{MS}$)
$\mu = \mu_R = \mu_F$	62.5 ($= m_h/2$)



No exact mass effects starting from NNLO

We rescale the effective theory with the exact LO k-factor at NNLO and N3LO

$$K_{\mathrm{LO}} = \frac{\sigma^{\mathrm{LO}}_{\mathrm{exact}}}{\sigma^{\mathrm{LO}}_{\mathrm{EFT}}} \approx 1.062$$

$$\sigma^{\mathrm{N^3LO}}_{\mathrm{rEFT}} = K_{\mathrm{LO}} \times \sigma^{\mathrm{N^3LO}}_{\mathrm{EFT}}$$

At NNLO corrections beyond rescaled EFT as 1/mt expansion

[Harlander, Mantler, Marzani, Ozeren]

We add these corrections to the rescaled gg and qg channels

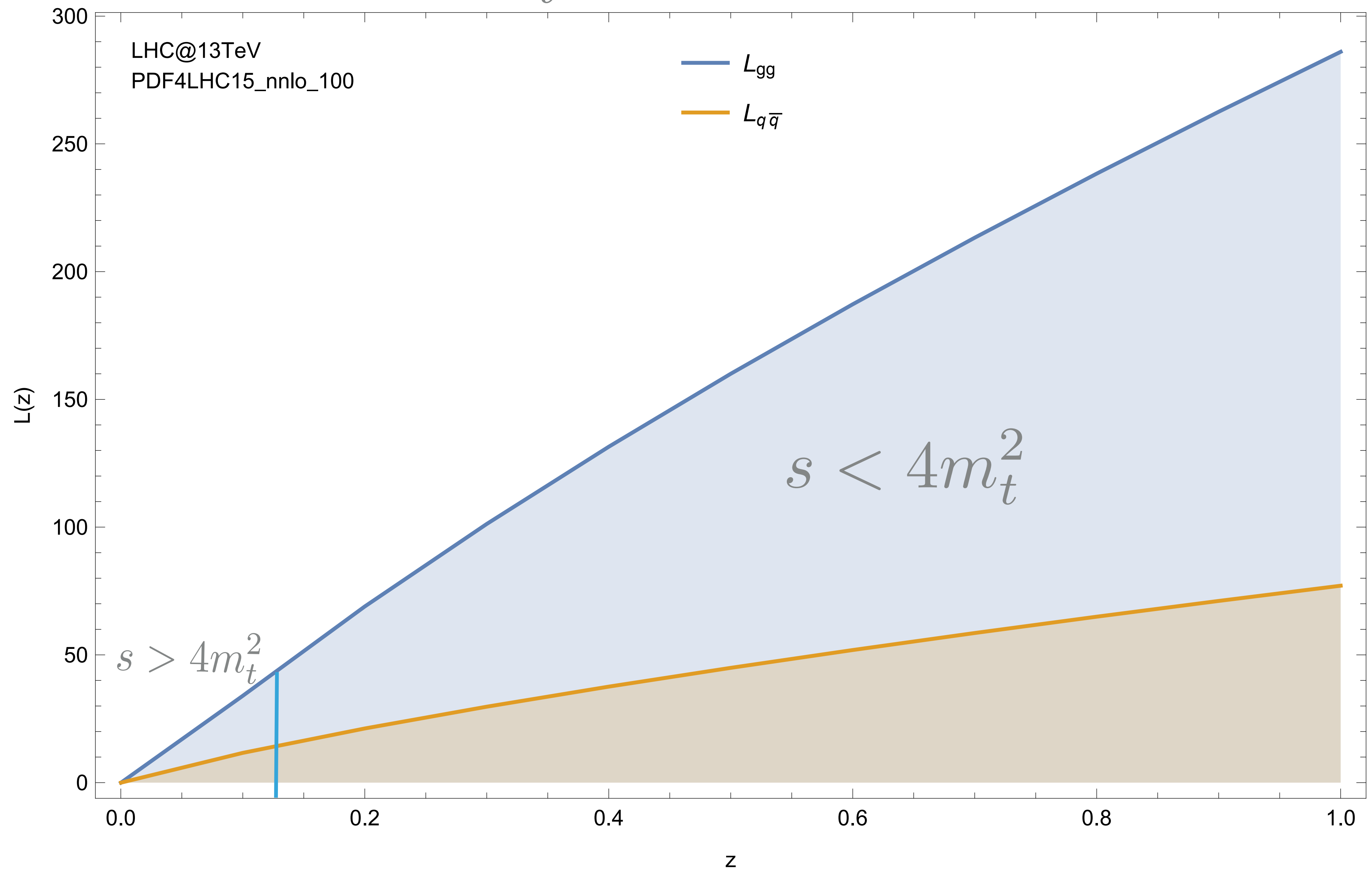
$$gg \ \sim 1.2\%$$
$$qg \ \sim -0.5\%$$

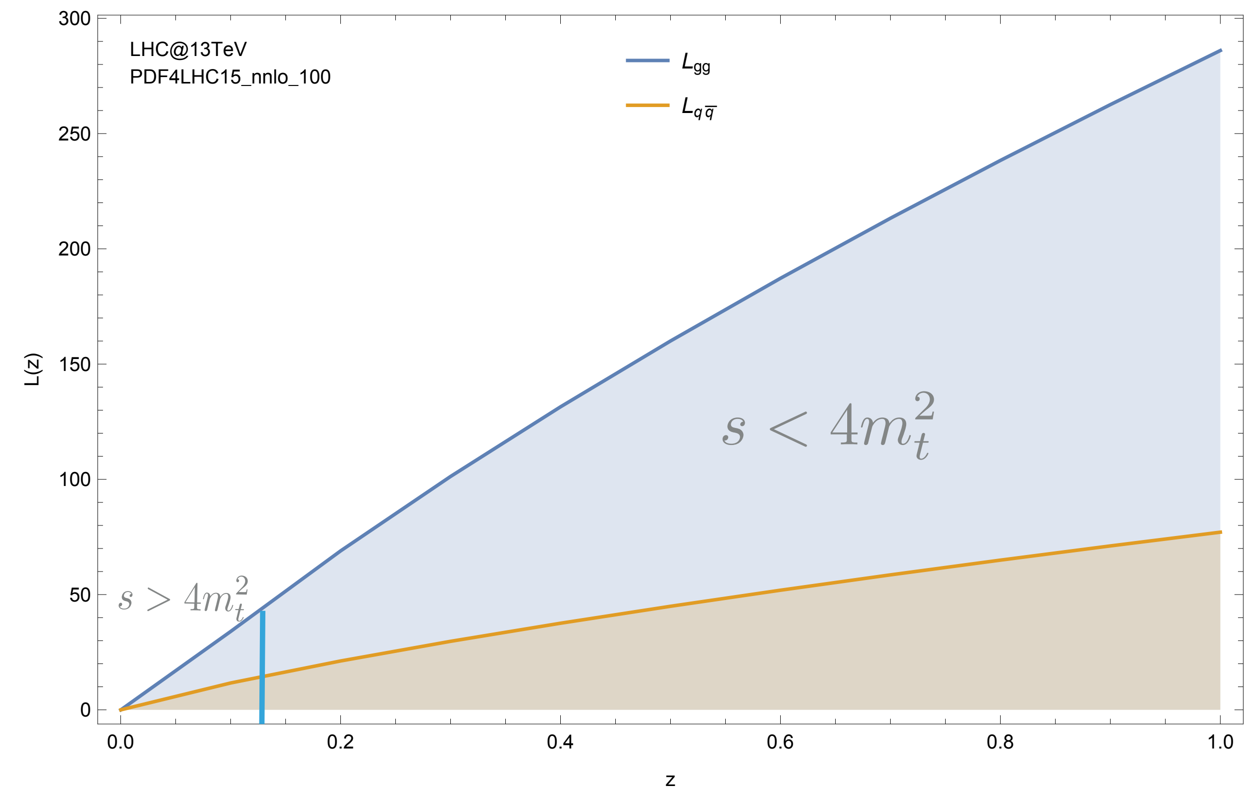
Expansion at NNLO is an expansion in $$\frac{\hat{s}}{4m_t^2} = \frac{m_h^2}{4m_t^2}\frac{1}{z}$$

Potentially problematic because z is integrated over

Expansion of order 1 for $$z < \frac{m_h^2}{4m_t^2} \approx 0.13$$

Only luminosity suppressed





Expansion can be matched to the BFKL limit $z \to 0$

BFKL limit at NNLO is only known to leading log accuracy

BFKL is missing the constant piece of uncontrolled size

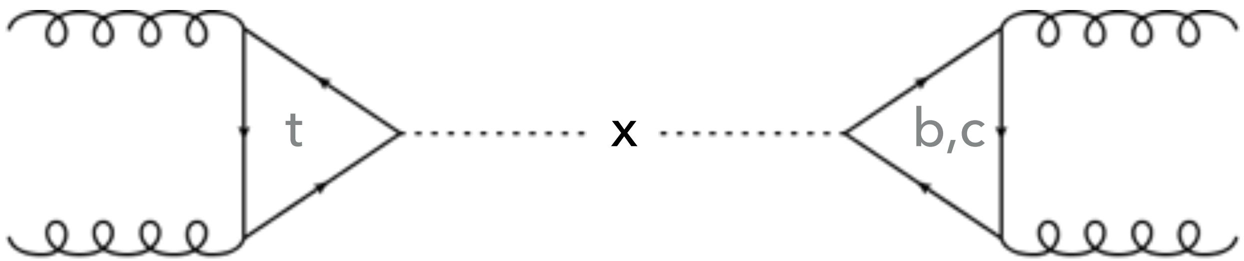
A proper inverse expansion would be useful

The corrections come with a matching uncertainty

$$\delta_{1/mt} = \pm 0.54\text{pb} = \pm 1\%$$

[Harlander, Mantler, Marzani, Ozeren]

Contributions from light quarks at LO and NLO



$$\mathrm{LO}:\ -7.3\%\ \ \mathrm{NLO}:\ -5.6\%$$

t-b interference not known at NNLO

We estimate the uncertainty as

$$\delta_{tb} = \Delta^{\mathrm{NNLO}}_{\mathrm{rEFT}} \frac{\Delta^{\mathrm{NLO}}_{t+b} - \Delta^{\mathrm{NLO}}_{t}}{\Delta^{\mathrm{NLO}}_{t}}$$

$$\delta_{\mathrm{tb}} = \pm 0.38\mathrm{pb} = \pm 0.7\%$$

Quark masses are renormalization scheme dependent

EFT wilson coefficient also depends on the scheme at NNLO

For the top these effects cancel

The rescaling coefficient is scheme dependent

$$K_{\mathrm{LO}} = 1.062 \;\; \mathrm{vs} \;\; K_{\mathrm{LO};OS} = 1.066$$

For the top these effects cancel

Scheme change for the top changes the cross section by 0.1%

2.1% with bottom and charm

OS scheme not recommended for bottom and charm

	$\overline{MS}$		OS
$\sigma^{LO}_{ex;t+b+c}$	14.90[1]	$\sigma^{LO}_{ex;t+b+c}$	16.12[1]
$\sigma^{NLO}_{ex;t}$	36.76[1]	$\sigma^{NLO}_{ex;t}$	36.80[1]
$\sigma^{NLO}_{ex;t+b}$	35.09[1]	$\sigma^{NLO}_{ex;t+b}$	34.63[1]
$\sigma^{NLO}_{ex;t+b+c}$	34.91[1]	$\sigma^{NLO}_{ex;t+b+c}$	34.15 [1]

We follow the HXSWG recommendation for the quark mass parametric uncertainties

Quark mass uncertainties are clearly negligible < 0.17% at NLO

$\delta m_t = 1 GeV$	$\sigma^{NLO}_{ex;t+b+c}$	34.91[1]
$m_t + \delta m_t$	$\sigma^{NLO}_{ex;t+b+c}$	34.85[1]
$m_t - \delta m_t$	$\sigma^{NLO}_{ex;t+b+c}$	34.93[1]

If we triple the b uncertainty the effect is still below 0.35% at NLO

$\delta m_b = 0.03 GeV$	$\sigma^{NLO}_{ex;t+b+c}$	34.91[1]
$m_b + \delta m_b$	$\sigma^{NLO}_{ex;t+b+c}$	34.89[1]
$m_b - \delta m_b$	$\sigma^{NLO}_{ex;t+b+c}$	34.92[1]

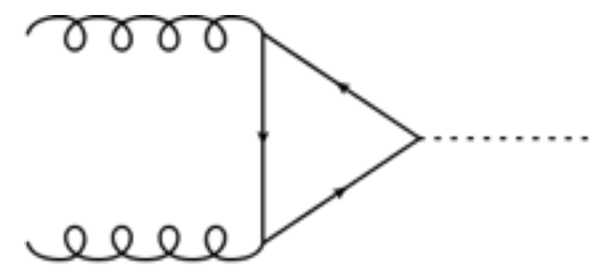
The effect of the t uncertainty on the rescaling coefficient is below 0.1%

$\delta m_c = 0.026$	$\sigma^{NLO}_{ex;t+b+c}$	34.91[1]
$m_c + \delta m_c$	$\sigma^{NLO}_{ex;t+b+c}$	34.90[1]
$m_c - \delta m_c$	$\sigma^{NLO}_{ex;t+b+c}$	34.91[1]

ELECTROWEAK CORRECTIONS

Electroweak corrections to LO process are known

$\mathcal{O}(\alpha\alpha_s^2)$



5.2% corrections to the LO cross section

[Actis, Passarino, Sturm, Uccirati]

Exact EW corrections to the NLO QCD correction are unknown

Mixed corrections due to light quarks are computed in an EFT

$\mathcal{O}(\alpha\alpha_s^3)$

Light quarks account for 80% of the LO EW correction

Leads to 5.1% correction at NLO and 5% correction at NNLO

[Anastasiou, Boughezal, Petriello]

$$C_{\rm QCD} \to C_{\rm QCD} + \lambda_{\rm EW}\left(1 + C_{1w}\alpha_s + C_{2w}\alpha_s + \ldots\right)$$

Almost complete factorisation

EXACT

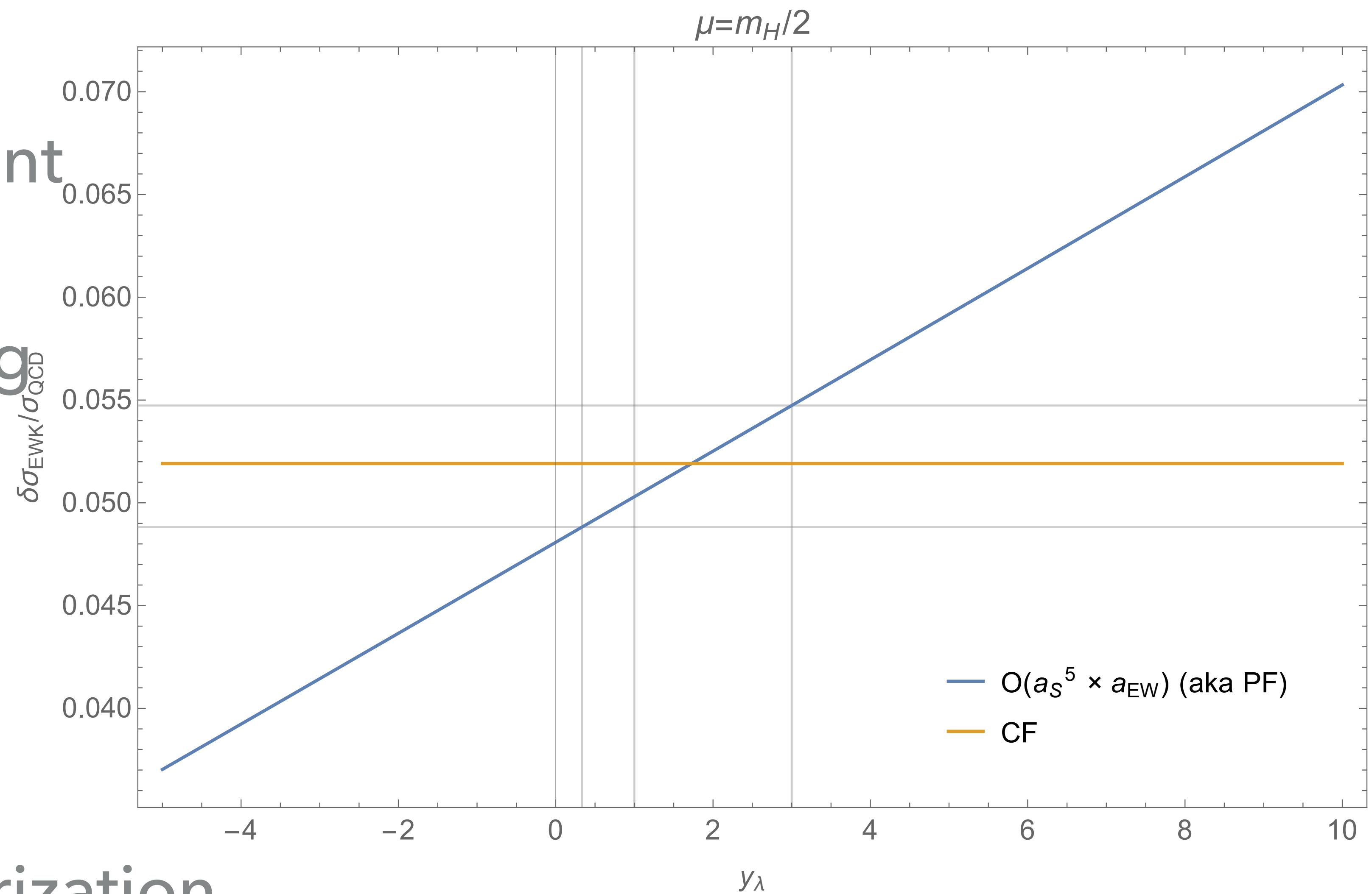
LIGHT QUARKS

UNKNOWN

Estimate uncertainty by varying the wilson coefficient

1% uncertainty from varying by a factor in [-3,+6]

Alternative argumentation



Calculation based on factorization

Hard part of the NLO QCD cross section is ~40%

Calculation misses the hard part of the corrections

$$\delta_{\mathrm{EW}} = \pm 0.48\mathrm{pb} = \pm 1$$

PDF+ALPHA_S UNCERTAINTY

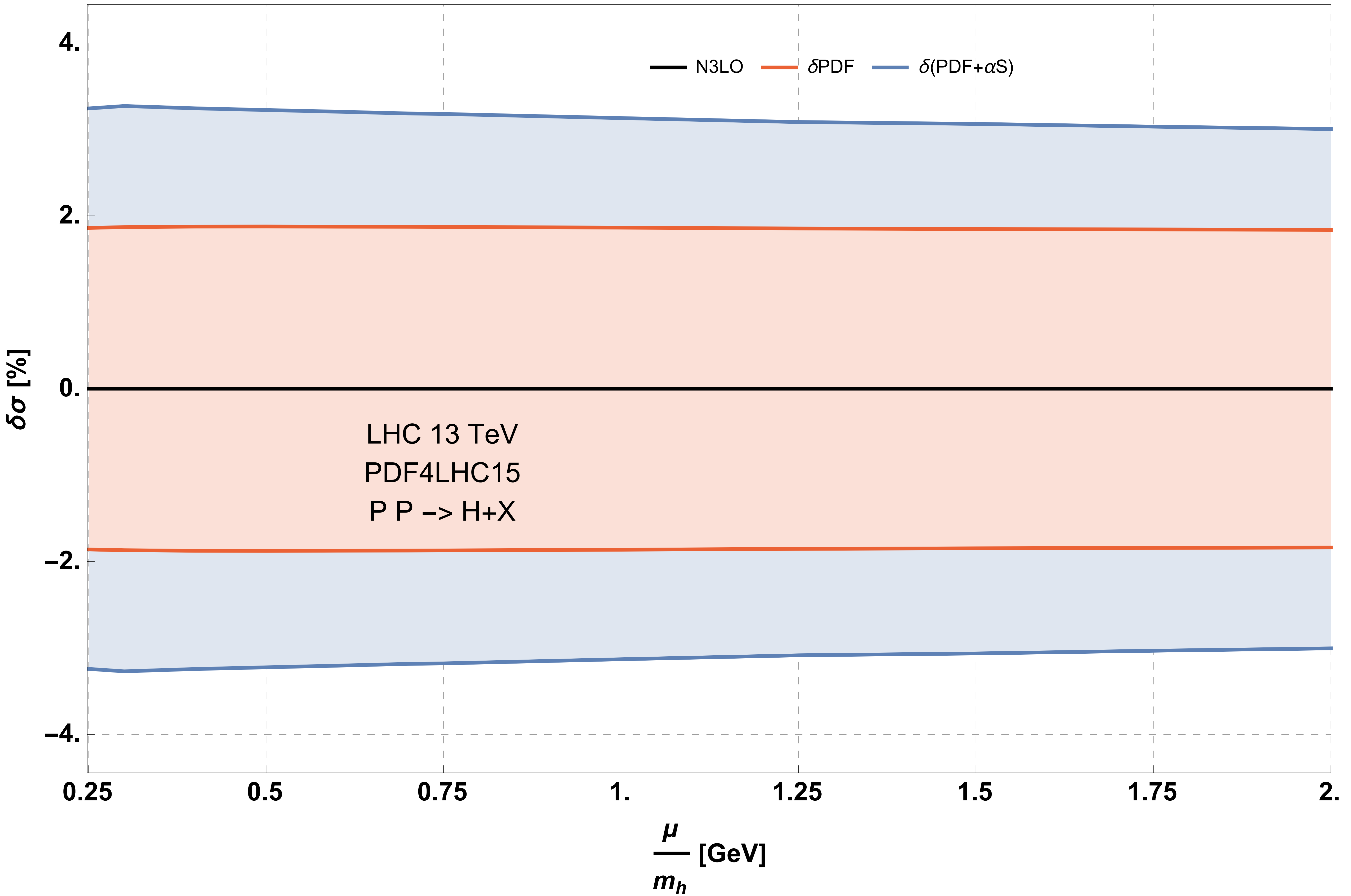
We follow the PDF4LHC recommendation for the PDF and alpha_s treatment

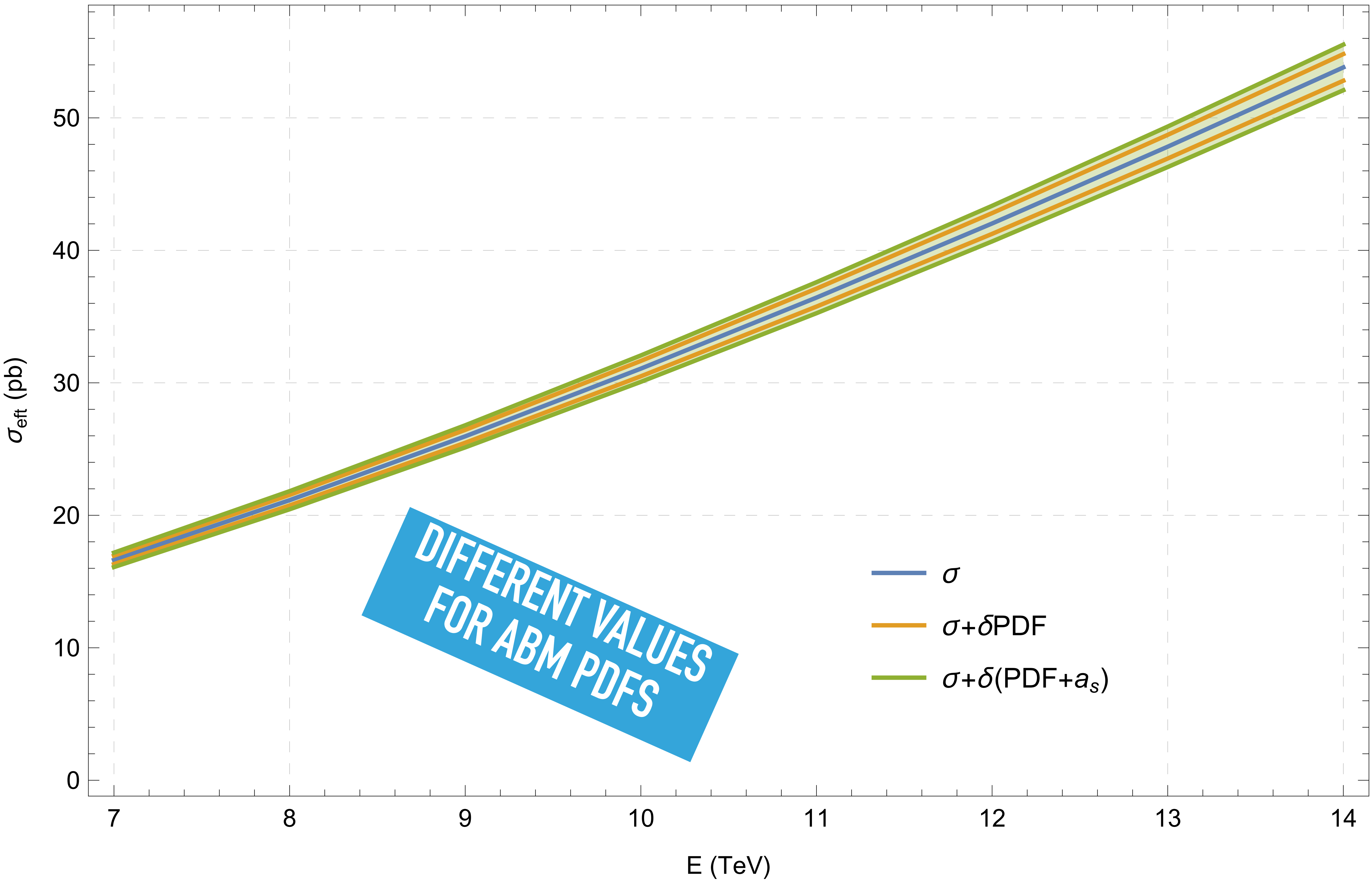
We use the Hessian PDF sets for the determination of the PDF uncertainty

$$\delta_{\mathrm{PDF}} = \sqrt{\sum_{k=1}^{N} (\sigma^{(k)} - \sigma^{(0)})^2}$$

$$\delta_{\alpha_s} = \frac{1}{2}\left(\sigma(\alpha_s = \alpha_s^0 + \Delta\alpha_s) - \sigma(\alpha_s = \alpha_s^0 - \Delta\alpha_s)\right)$$

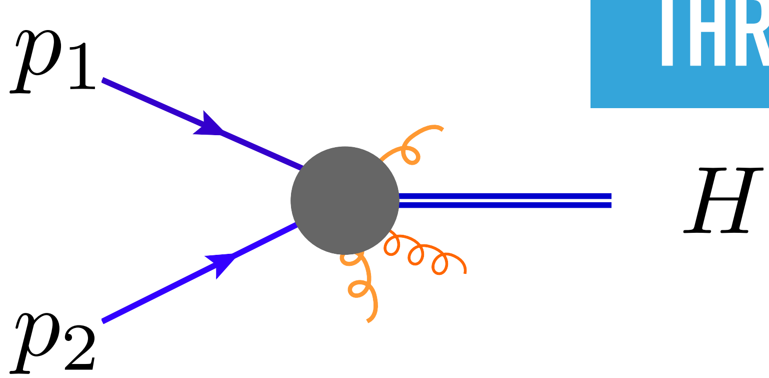
$$\alpha_s^0(m_Z) = 0.1180 \qquad \Delta\alpha_s = 0.0015$$


N3LO
δPDF
δ(PDF+αS)
LHC 13 TeV
PDF4LHC15
P P –> H+X
δσ [%]
4.
2.
0.
−2.
−4.
0.25
0.5
0.75
1.
1.25
1.5
1.75
2.
μ / m_h [GeV]



$\sigma_{\rm eft}$ (pb)
0
10
20
30
40
50
7
8
9
10
11
12
13
14
E (TeV)
σ
$\sigma+\delta$PDF
$\sigma+\delta$(PDF+a_s)
DIFFERENT VALUES FOR ABM PDFS

TRUNCATION UNCERTAINTY



THRESHOLD LIMIT

$$\hat{s} \sim m_h \Leftrightarrow z \sim 1$$

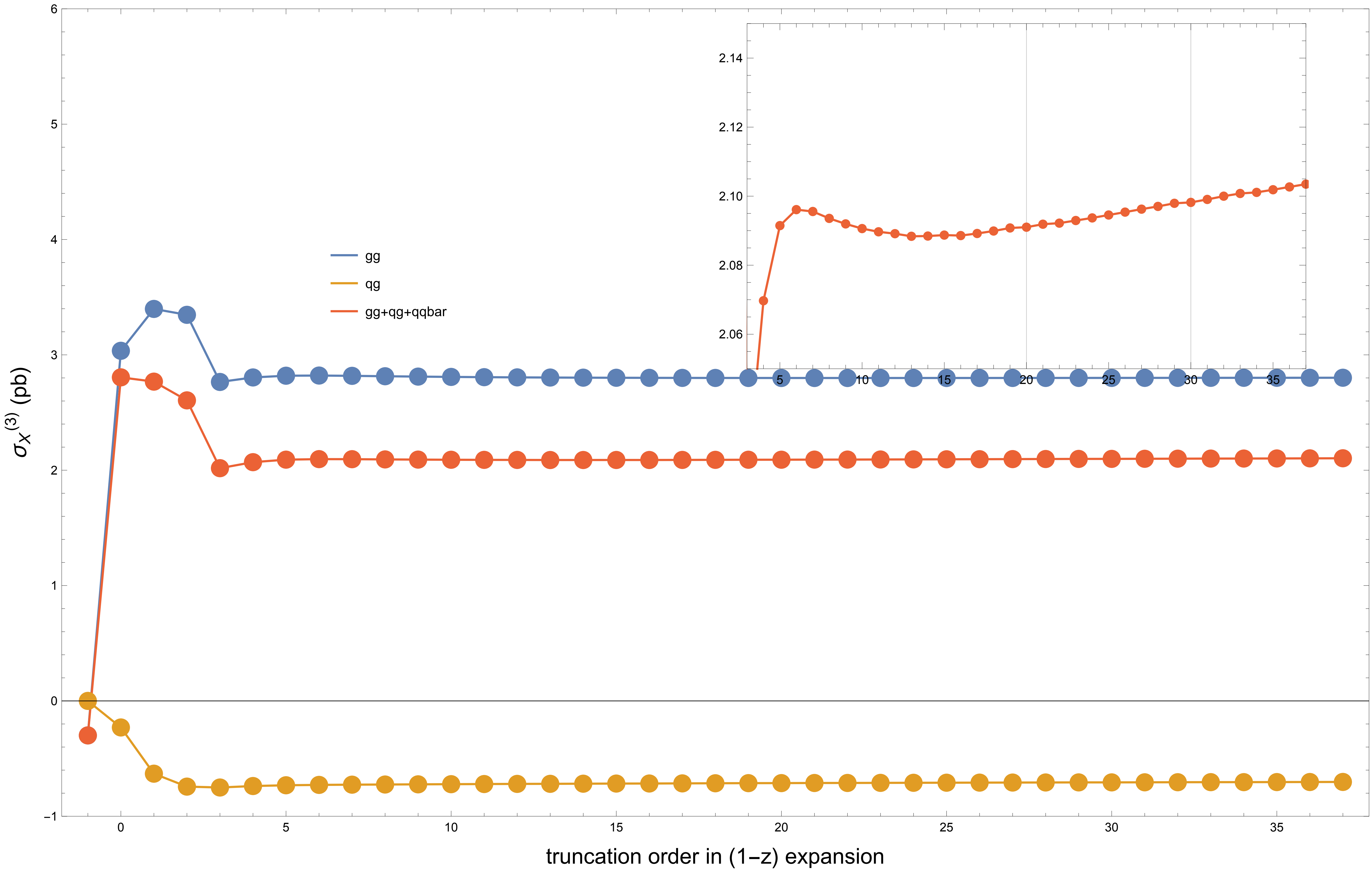
All available threshold energy is used to produce the Higgs

Any radiation has to be soft → soft limit of the cross section

Possible to systematically expand around the limit

$$\hat{\sigma}(z) = \sigma^{\mathrm{SV}} + \sigma^{0} + (1-z)\sigma^{1} + \ldots$$

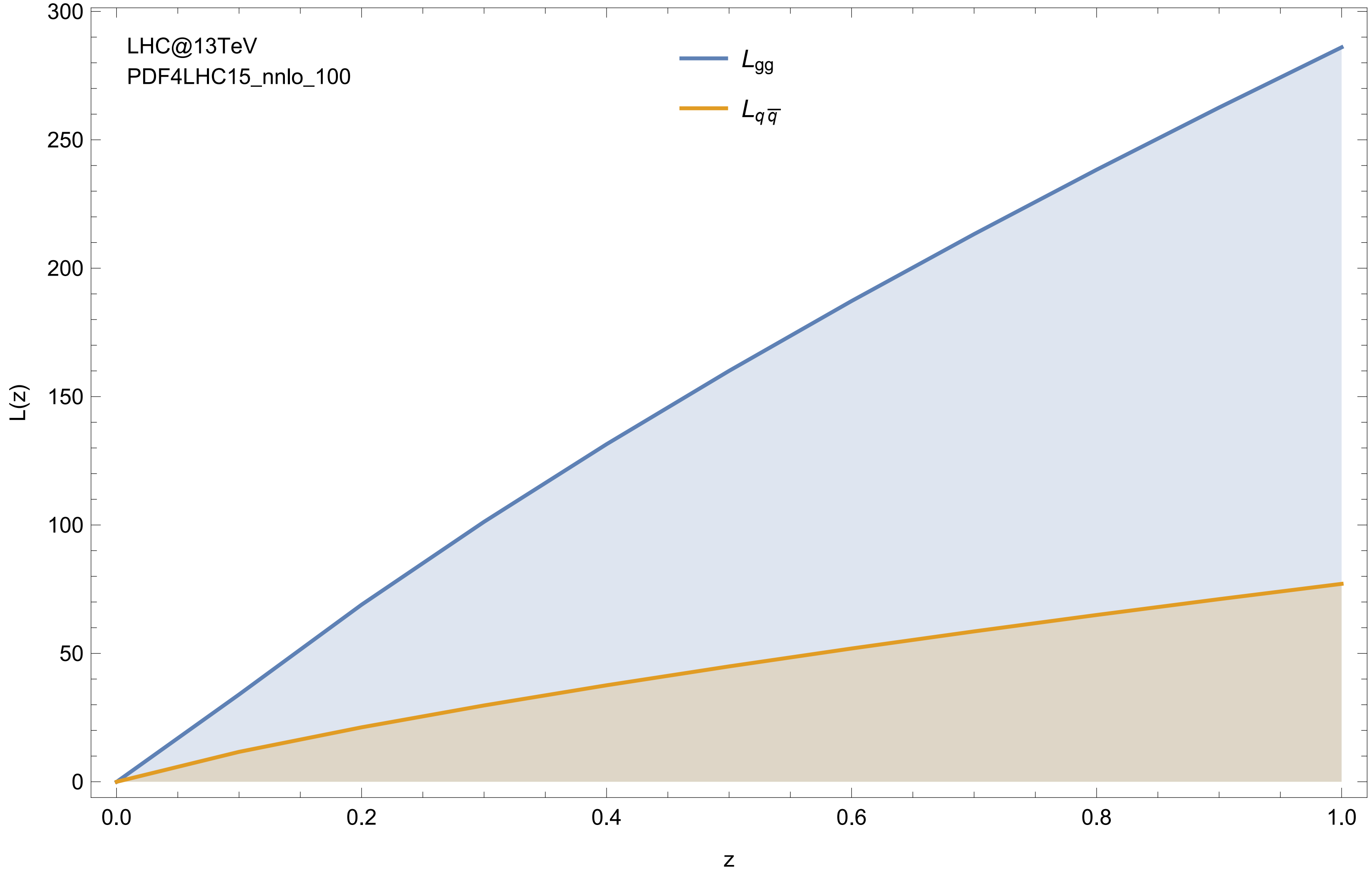
z is not fixed, is this a good expansion? $\sigma = \mathcal{L}(z) \otimes \hat{\sigma}(z)$

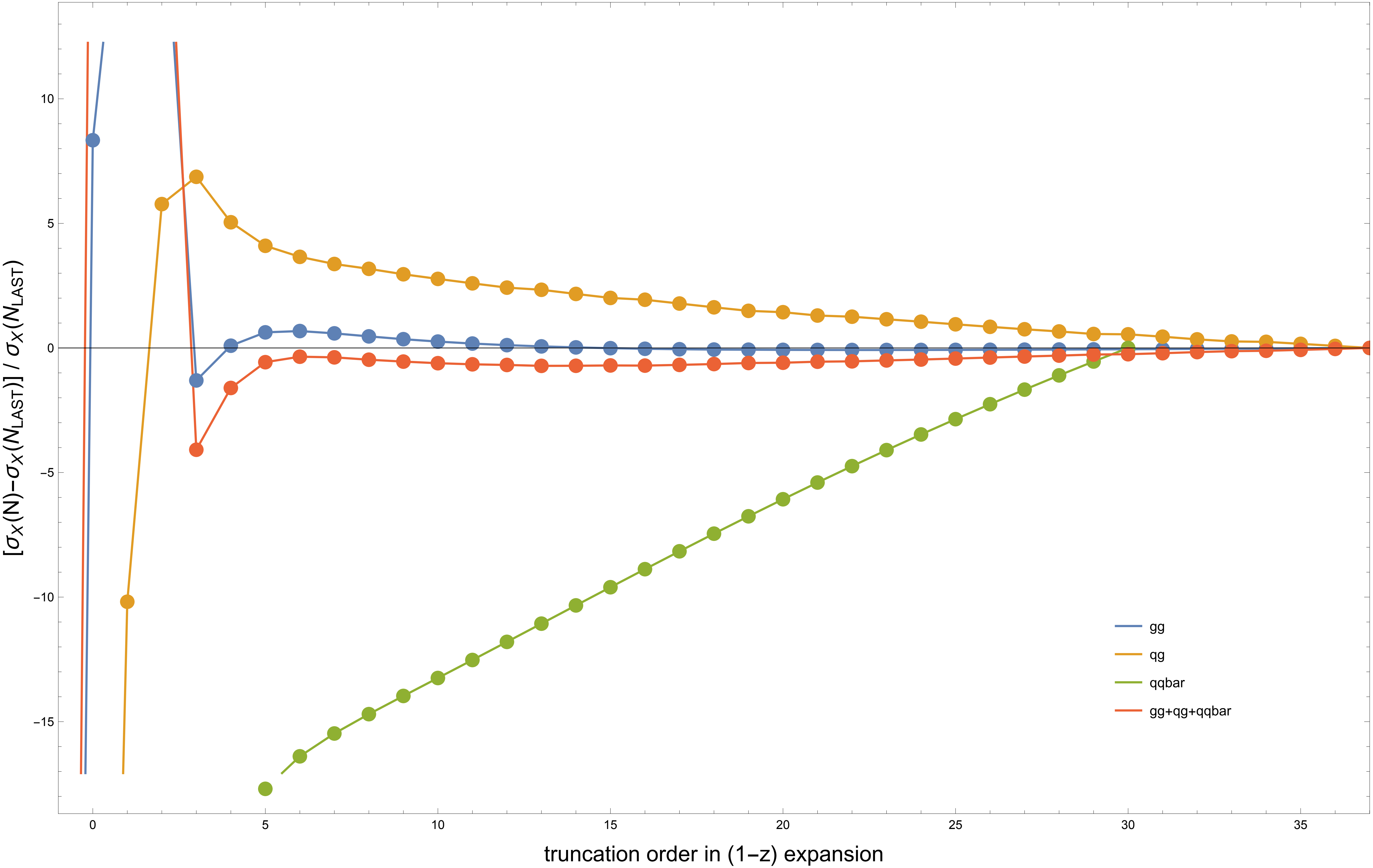

gg
qg
gg+qg+qqbar
$\sigma_X^{(3)}$ (pb)
truncation order in (1−z) expansion

37+ TERMS

EXACT

$$\hat{\sigma}(z) = f_0(z) + \log(1-z)f_1(z) + \log^2(1-z)f_2(z) + \log^3(1-z)f_3(z) + \log^4(1-z)f_4(z) + \log^5(1-z)f_5(z)$$




$[\sigma_X(N)-\sigma_X(N_{LAST})] / \sigma_X(N_{LAST})$
10
5
0
−5
−10
−15
0
5
10
15
20
25
30
35
truncation order in (1−z) expansion
gg
qg
qqbar
gg+qg+qqbar

$$\sigma = \mathcal{L}(z) \otimes \boxed{\hat{\sigma}(z)}$$

EXPANSION

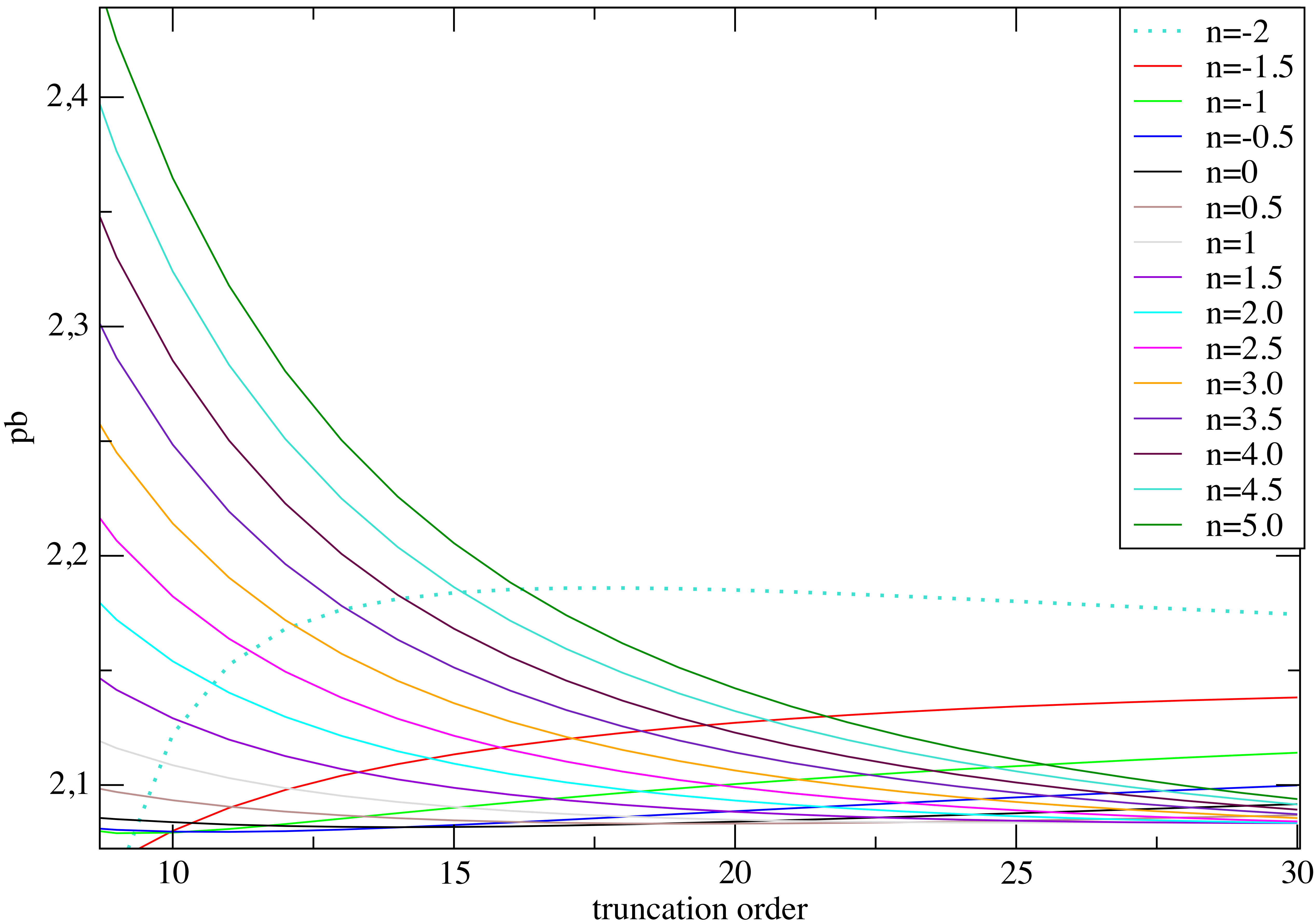
Redefine expansion slightly

$$\sigma = (\mathcal{L}(z) z^n) \otimes \boxed{\left(\frac{\hat{\sigma}(z)}{z^n}\right)}$$

Formally equivalent

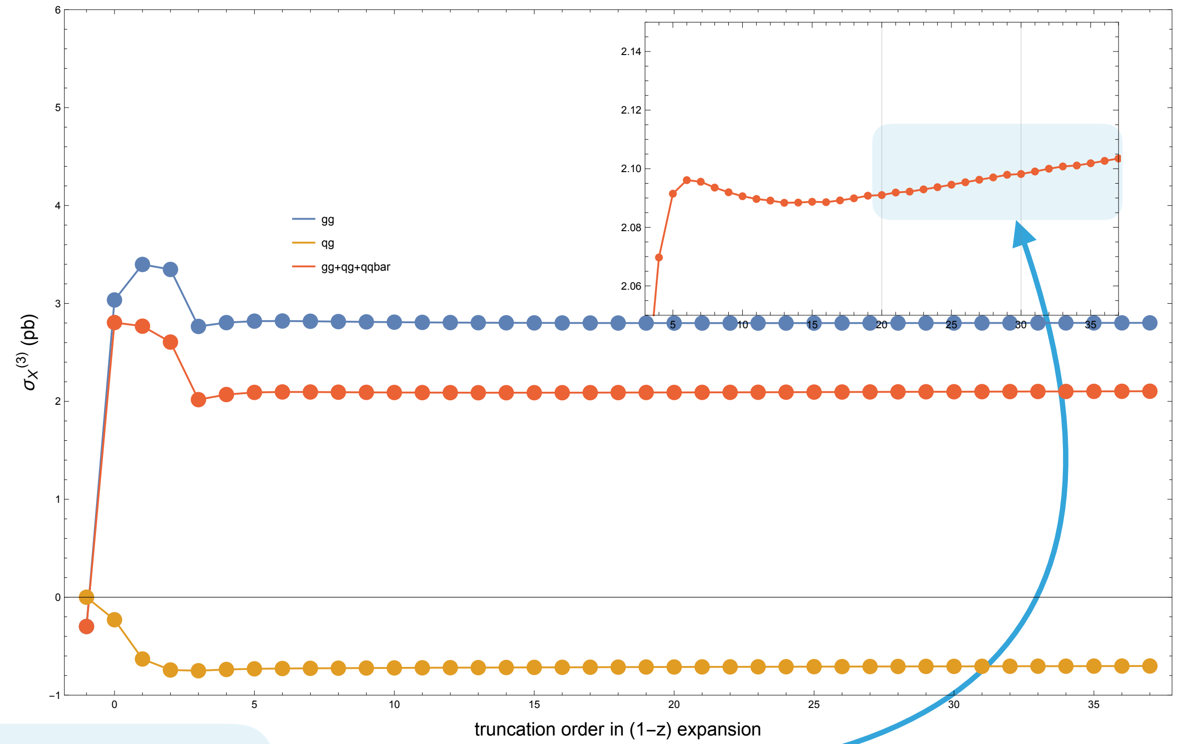
Reshuffles some orders

Exactly equivalent for infinitely many terms in the expansion


n=-2
n=-1.5
n=-1
n=-0.5
n=0
n=0.5
n=1
n=1.5
n=2.0
n=2.5
n=3.0
n=3.5
n=4.0
n=4.5
n=5.0
2,4
2,3
2,2
2,1
pb
10
15
20
25
30
truncation order

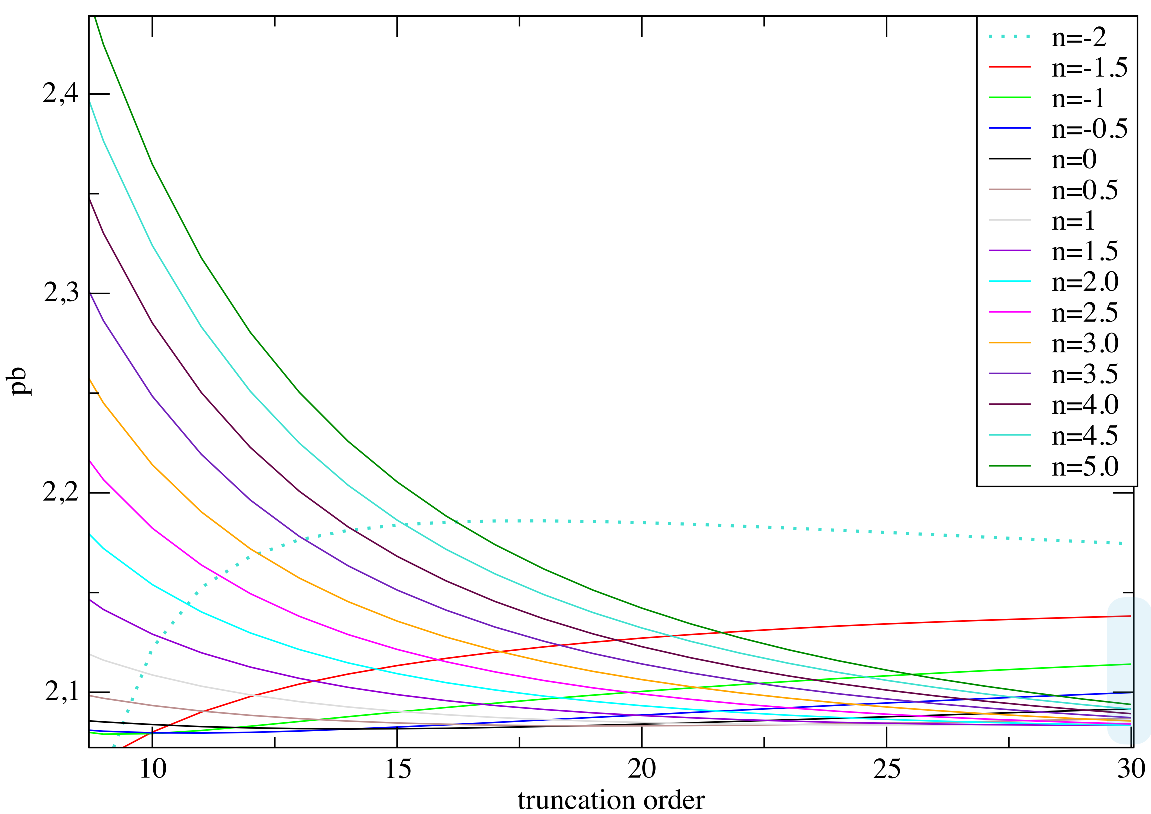
13 TEV

Estimate truncation uncertainty from progression of the series



CONSERVATIVE FACTOR

$$\delta_{\text{trunk}} = 10 \times \left(\sigma^{(3)}_{\text{total}}(30) - \sigma^{(3)}_{\text{total}}(20)\right) = 0.29\text{pb} = 0.6\%$$



Consistent with spread from different expansions

Consistent with analysis in Mellin space

CONCLUSION

13 TEV

σ/pb	δ_{PDF}/pb	δ_{α_s}/pb	δ_{scale}/pb	δ_{trunc}/pb	δ_{pdfTH}/pb	δ_{EW}/pb	δ_{tb}/pb	δ_{1/m_t}/pb
48.48	± 0.90	± 1.26	$^{+0.09}_{-1.11}$	± 0.12	± 0.56	± 0.48	± 0.34	± 0.48
48.48	$\pm 1.86\%$	$\pm 2.60\%$	$^{+0.2}_{-2.3}\%$	$\pm 0.25\%$	$\pm 1.15\%$	$\pm 1.00\%$	$\pm 0.70\%$	$\pm 1.00\%$

ADD IN QUADRATURE

ADD LINEARLY

$$\sigma = \left(48.48 \pm 1.55\ ^{+2.08}_{-3.10}\right)\ \mathrm{pb}$$

$$\sigma = 48.48\mathrm{pb} \pm 3.19\ ^{+4.29\%}_{-6.40\%}$$

$$\sigma = 48.48\ ^{+2.60}_{-3.47}\mathrm{pb} = 48.48\mathrm{pb}\ ^{+5.36\%}_{-7.15\%}$$

σ/pb	δ_{PDF}/pb	δ_{α_s}/pb	δ_{scale}/pb	δ_{trunc}/pb	δ_{pdfTH}/pb	δ_{EW}/pb	δ_{tb}/pb	δ_{1/m_t}/pb
48.48	± 0.90	± 1.26	$^{+0.09}_{-1.11}$	± 0.12	± 0.56	± 0.48	± 0.34	± 0.48
48.48	$\pm 1.86\%$	$\pm 2.60\%$	$^{+0.2}_{-2.3}\%$	$\pm 0.25\%$	$\pm 1.15\%$	$\pm 1.00\%$	$\pm 0.70\%$	$\pm 1.00\%$

Great effort to reduce the scale uncertainty

Now it is time to work on other sources of uncertainty

Full massive calculation at NNLO will drastically reduce the uncertainty

PDFs at N3LO will also reduce the uncertainty considerably