

CERN-TH, October 5, 2016

The Chiral Magnetic Effect:

from quark-gluon plasma to Dirac semimetals

D. Kharzeev



Stony Brook University



RIKEN BNL
Research Center



Classical symmetries and Quantum anomalies

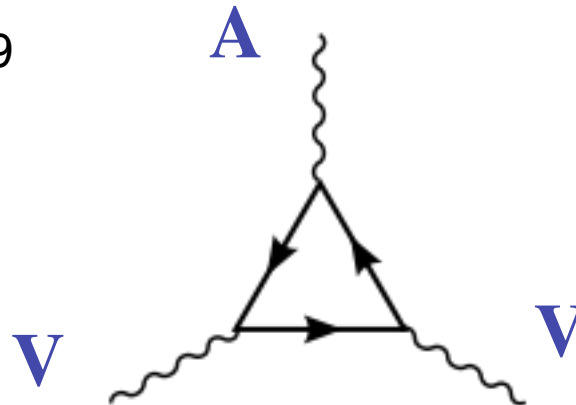
Anomalies: The classical symmetry of the Lagrangian is broken by quantum effects -

examples: chiral symmetry - chiral anomaly $\partial_\mu j_A^\mu = C_A \mathbf{E} \cdot \mathbf{B}$
scale symmetry - scale anomaly

Anomalies imply correlations between currents:

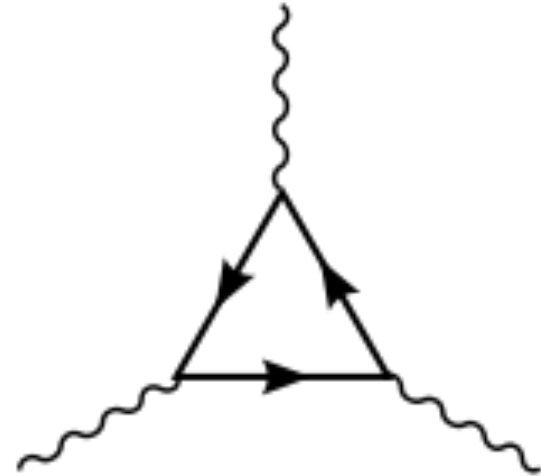
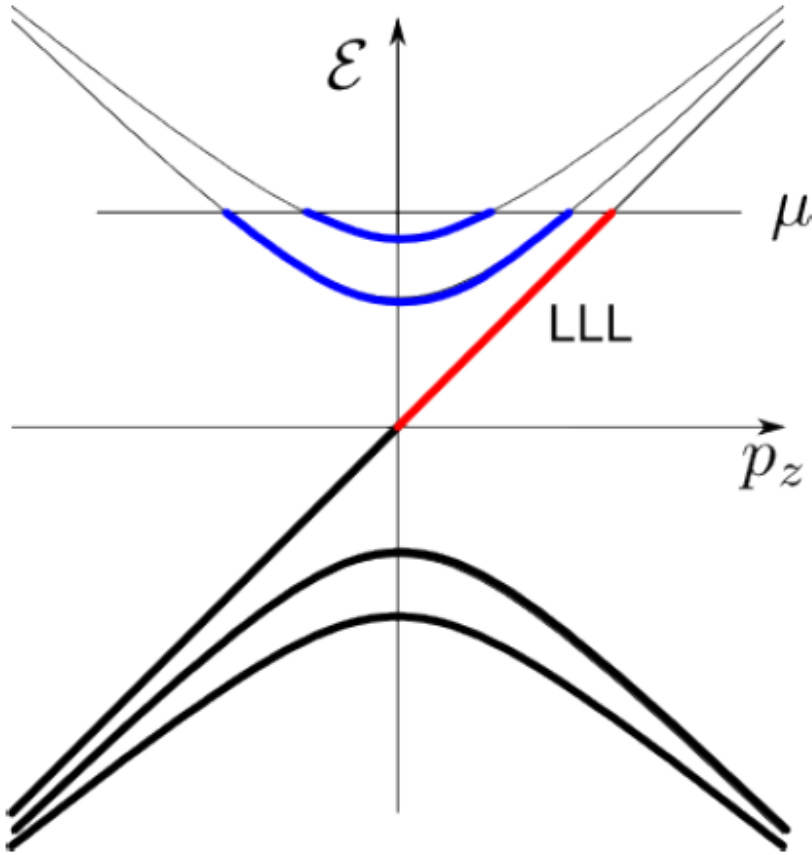
Adler; Bell, Jackiw '69

e.g.
 $\pi^0 \rightarrow \gamma\gamma$
decay



**if A, V are
background fields,
V is generated!**

Chiral anomaly



In classical background fields (E and B), chiral anomaly induces a collective motion in the Dirac sea

Early work on P-odd currents in magnetic field

(see DK, Prog.Part.Nucl.Phys. 75 (2014) 133
for a complete (?) list of references)

A.Vilenkin (1980) “Equilibrium parity-violating current in a magnetic field”;
(1980) “Cancellation of equilibrium parity-violating currents”

H.Nielsen, M.Ninomiya (1983) “ABJ anomaly and Weyl fermions in crystal”

G. Eliashberg (1983) JETP 38, 188

L. Levitov, Yu.Nazarov, G. Eliashberg (1985) JETP 88, 229

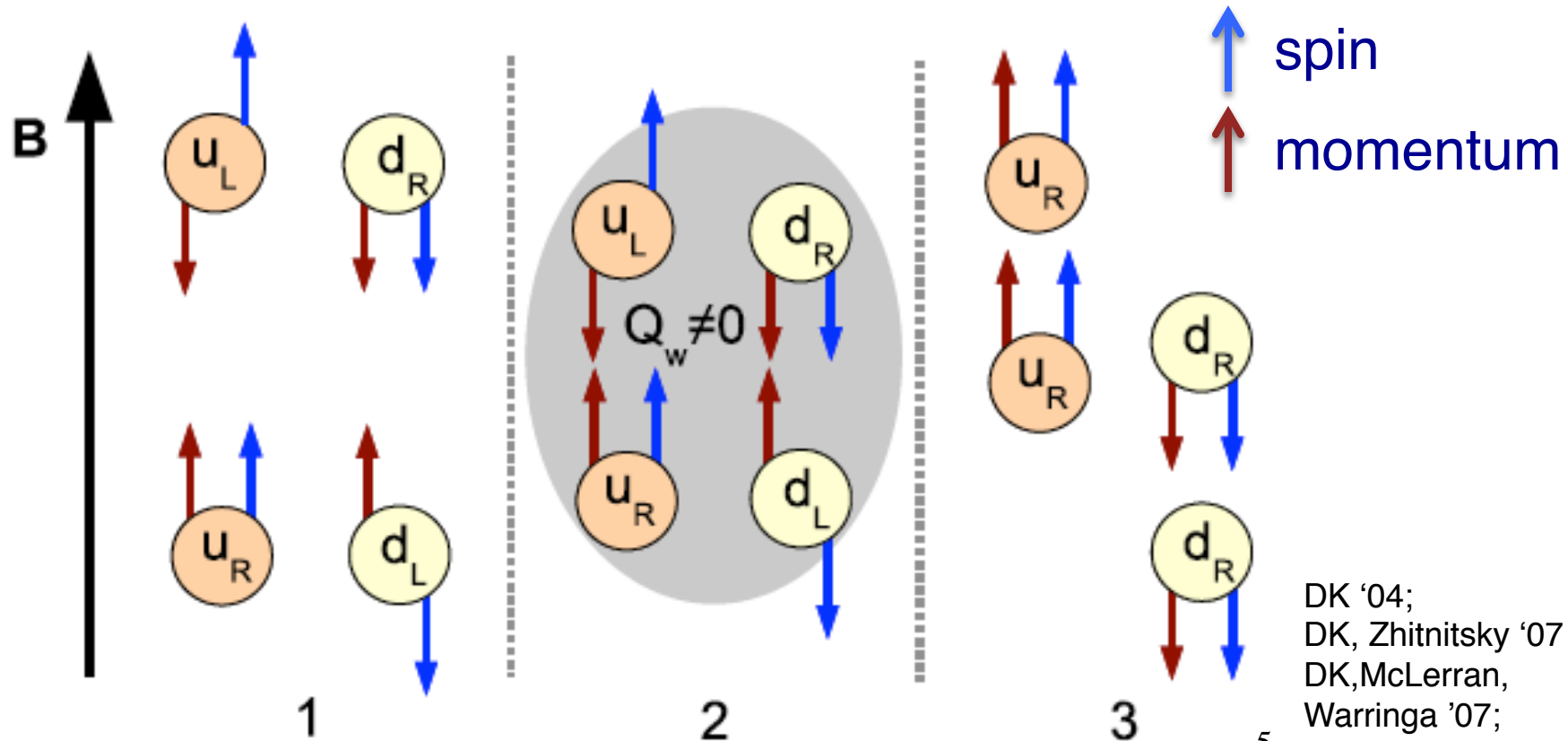
M. Joyce and M. Shaposhnikov (1997) PRL 79, 1193;

M. Giovannini and M. Shaposhnikov (1998) PRL 80, 22

A. Alekseev, V. Cheianov, J. Frohlich (1998) PRL 81, 3503

Chirality in 3D: the Chiral Magnetic Effect

chirality + magnetic field = current



Review: DK, arxiv:1312.3348 (Prog.Part.Nucl.Phys'14)

DK '04;
DK, Zhitnitsky '07
DK, McLerran,
Warringa '07;
Fukushima,
DK, Warringa '08

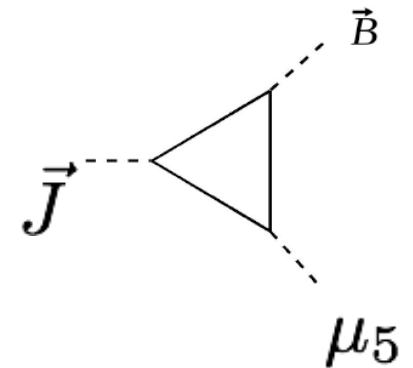
Chiral Magnetic Effect

DK'04; K.Fukushima, DK, H.Warringa, PRD'08;
Review and list of refs: DK, arXiv:1312.3348

Chiral chemical potential is formally equivalent to a background chiral gauge field: $\mu_5 = A_5^0$

In this background, and in the presence of \vec{B} , vector e.m. current is generated:

$$\partial_\mu J^\mu = \frac{e^2}{16\pi^2} \left(F_L^{\mu\nu} \tilde{F}_{L,\mu\nu} - F_R^{\mu\nu} \tilde{F}_{R,\mu\nu} \right)$$



Compute the current through

$$J^\mu = \frac{\partial \log Z[A_\mu, A_\mu^5]}{\partial A_\mu(x)}$$

The result:

$$\vec{J} = \frac{e^2}{2\pi^2} \mu_5 \vec{B}$$

Coefficient is fixed by the axial anomaly, no corrections

Chiral magnetic conductivity: discrete symmetries

P – parity

T – time reversal

$$\vec{J} = \frac{e^2}{2\pi^2} \mu_5 \vec{B}$$

P-odd

T-odd

P-even

T-odd

P-odd

P-odd effect!

T-even

Non-dissipative current!
(topologically protected)

Effect persists in
hydrodynamics!

cf Ohmic
conductivity:

$$\vec{J} = \sigma \vec{E}$$

T-odd,
dissipative

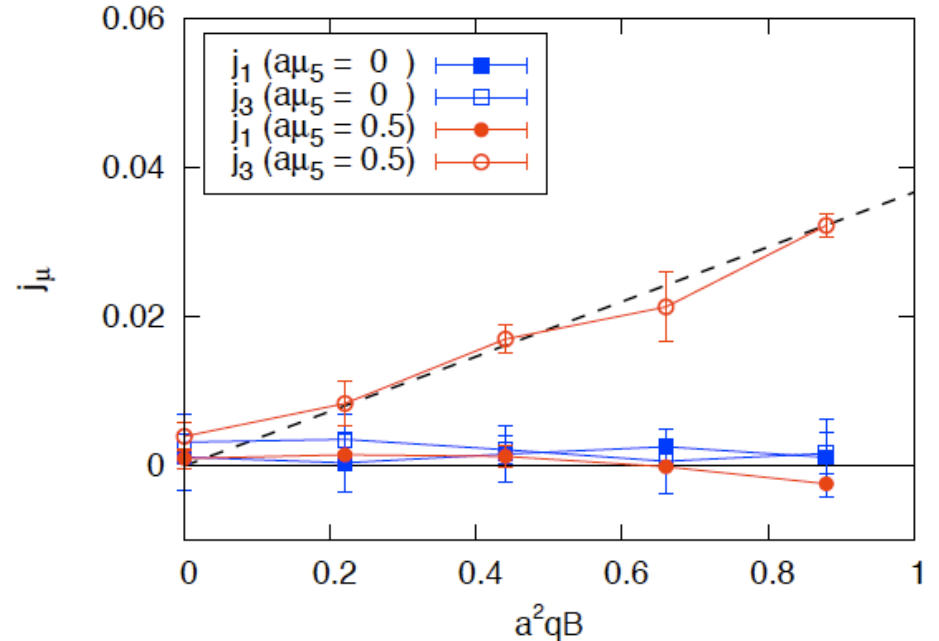
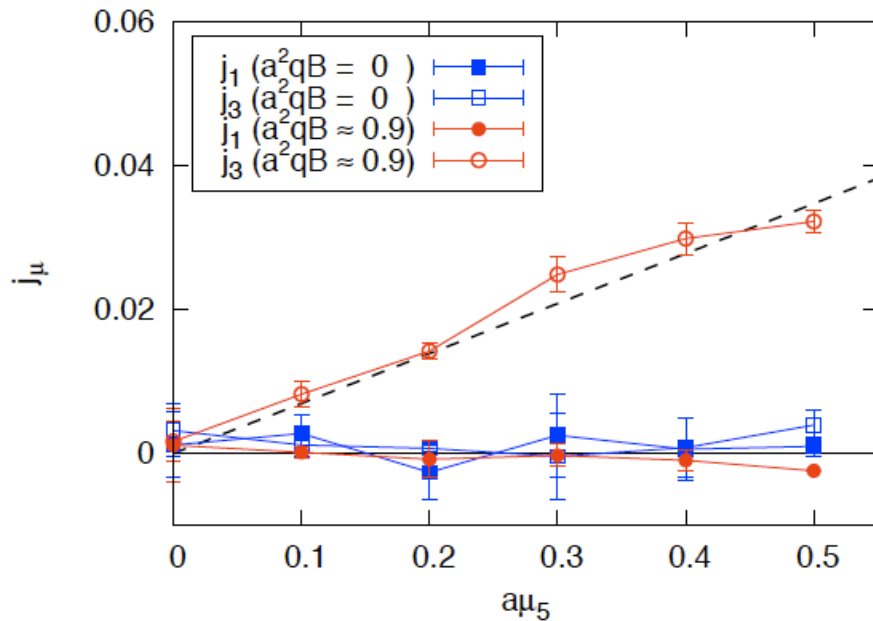
Chiral magnetic effect in lattice QCD with chiral chemical potential

Arata Yamamoto

Department of Physics, The University of Tokyo, Tokyo 113-0033, Japan

(Dated: May 3, 2011)

We perform a first lattice QCD simulation including two-flavor dynamical fermion with chiral chemical potential. Because the chiral chemical potential gives rise to no sign problem, we can exactly analyze a chirally asymmetric QCD matter by the Monte Carlo simulation. By applying an external magnetic field to this system, we obtain a finite induced current along the magnetic field, which corresponds to the chiral magnetic effect. The obtained induced current is proportional to the magnetic field and to the chiral chemical potential, which is consistent with an analytical prediction.



Systematics of anomalous conductivities

Magnetic field

Vorticity

Vector
current

$$\frac{\mu_A}{2\pi^2}$$

$$\frac{\mu\mu_A}{2\pi^2}$$

Axial
current

$$\frac{\mu}{2\pi^2}$$

$$\frac{\mu^2 + \mu_A^2}{4\pi^2} + \frac{T^2}{12}$$

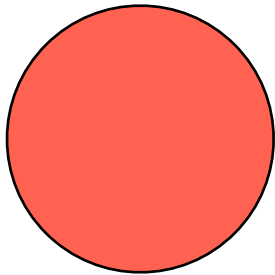
Hydrodynamics and symmetries

- Hydrodynamics: an effective low-energy TOE. States that the response of the fluid to slowly varying perturbations is completely determined by conservation laws (energy, momentum, charge, ...)
- Conservation laws are a consequence of symmetries of the underlying theory
- What happens to hydrodynamics when these symmetries are broken by quantum effects (anomalies of QCD and QED)?

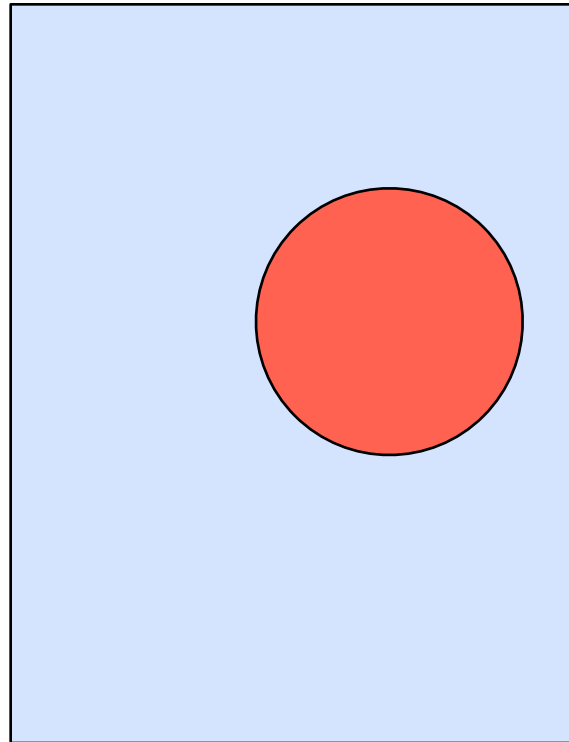
No entropy production from P-odd anomalous terms

DK and H.-U. Yee, 1105.6360; PRD

Entropy grows



$$\partial_{\mu} s^{\mu} \geq 0$$



Mirror reflection:
entropy decreases ?

$$\partial_{\mu} s^{\mu} \leq 0$$

Decrease is ruled
out by 2nd law of
thermodynamics



$$\partial_{\mu} s^{\mu} = 0_{11}$$

Allows to compute analytically 13 out of 18
anomalous transport coefficients in 2nd order
relativistic hydrodynamics

The CME in relativistic hydrodynamics:

The Chiral Magnetic Wave

DK, H.-U. Yee,
arXiv:1012.6026 [hep-th];
PRD

$$\vec{j}_V = \frac{N_c e}{2\pi^2} \mu_A \vec{B}; \quad \vec{j}_A = \frac{N_c e}{2\pi^2} \mu_V \vec{B},$$

CME

Chiral separation

$$\begin{pmatrix} \vec{j}_V \\ \vec{j}_A \end{pmatrix} = \frac{N_c e \vec{B}}{2\pi^2} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} \mu_V \\ \mu_A \end{pmatrix}$$

Propagating chiral wave: (if chiral symmetry
is restored)

$$\left(\partial_0 \mp \frac{N_c e B \alpha}{2\pi^2} \partial_1 - D_L \partial_1^2 \right) j_{L,R}^0 = 0$$

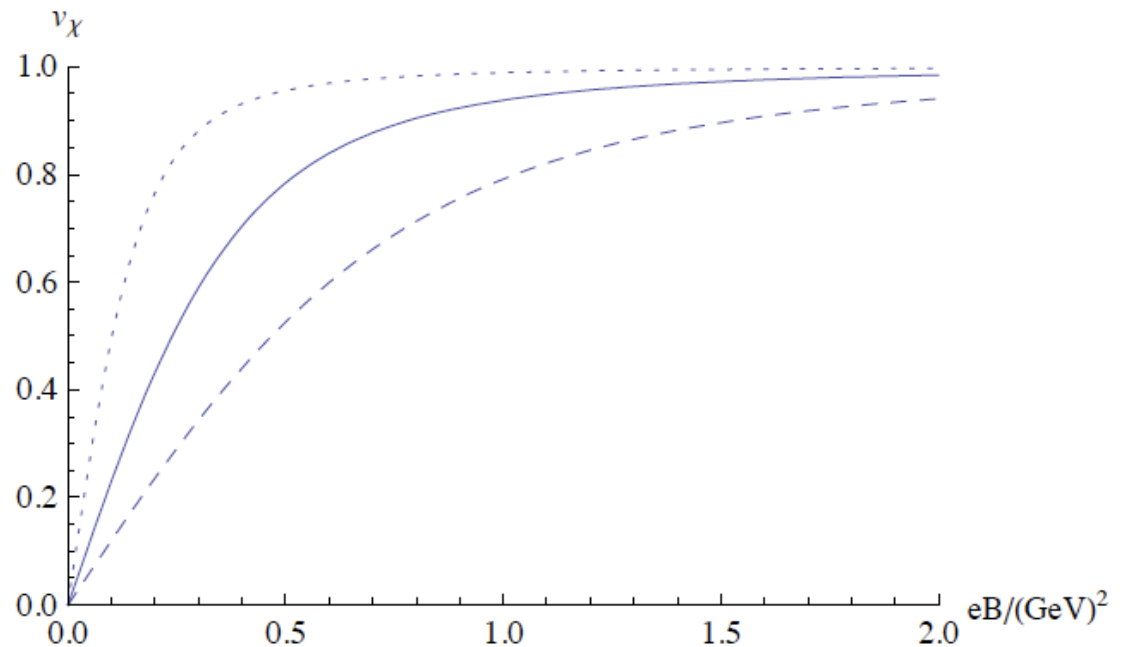
Gapless collective mode is the carrier of CME current in MHD:

$$\omega = \mp v_\chi k - i D_L k^2 + \dots$$



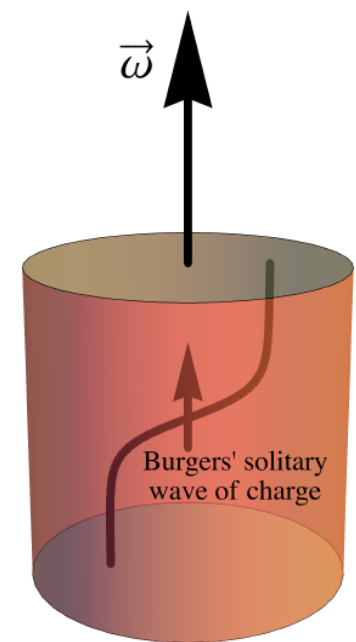
The Chiral Magnetic Wave: oscillations of electric and chiral charges coupled by the chiral anomaly

In strong magnetic field, CMW
propagates with the speed of light!



Anomalous transport and the Burgers' equation

Consider a “hot” system (QGP, DSM) with $\frac{\mu}{T} \ll 1$



The chemical potential is then proportional to charge density:

$$\mu \approx \chi^{-1} \rho + \mathcal{O}(\rho^3)$$

the CME current is

$$J^3 = \frac{ke}{4\pi^2} \left(\chi^{-2} \rho^2 + \frac{\pi^2}{3} T^2 \right) \omega - D \partial_3 \rho + \mathcal{O}(\partial^2, \rho^3)$$

and the charge conservation $\partial_t \rho + \partial_3 J^3 = 0$ leads to

$$\partial_t \rho + C \rho \partial_x \rho - D \partial_x^2 \rho = 0 \quad C = \frac{ke\omega}{2\pi^2 \chi^2} \quad x \equiv x^3$$

The Burgers' equation

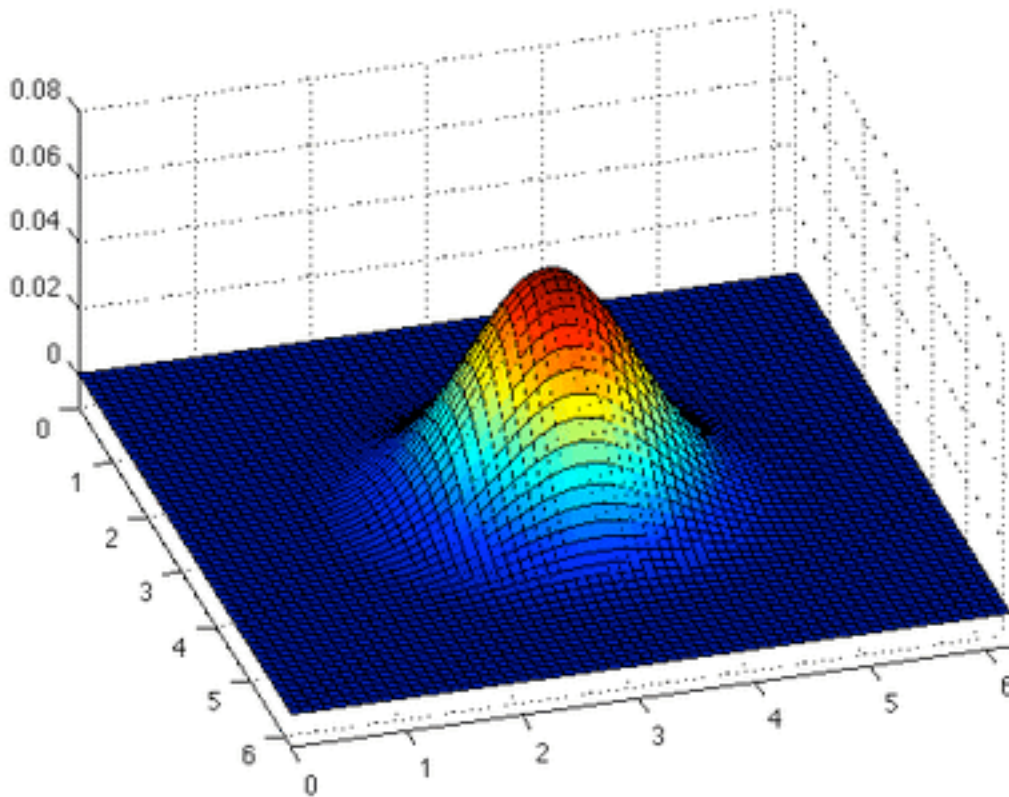
$$\partial_t \rho + C \rho \partial_x \rho - D \partial_x^2 \rho = 0$$



Exactly soluble by
Cole-Hopf
transformation -

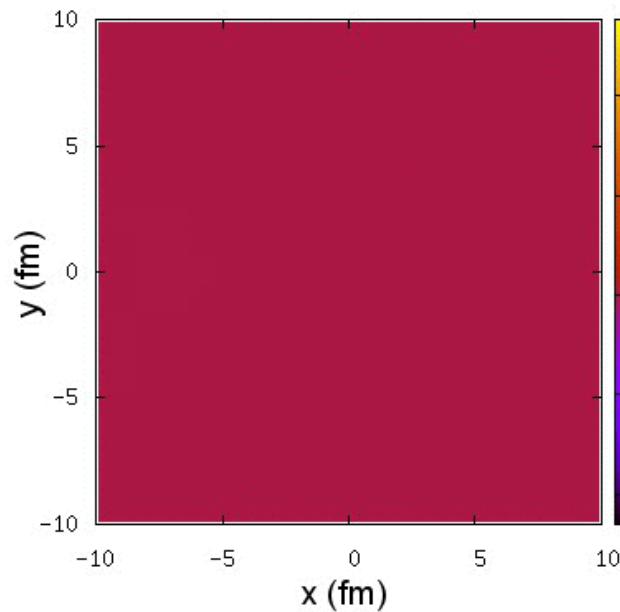
initial value problem,
integrable dynamics

describes shock
waves, **solitons**, ...

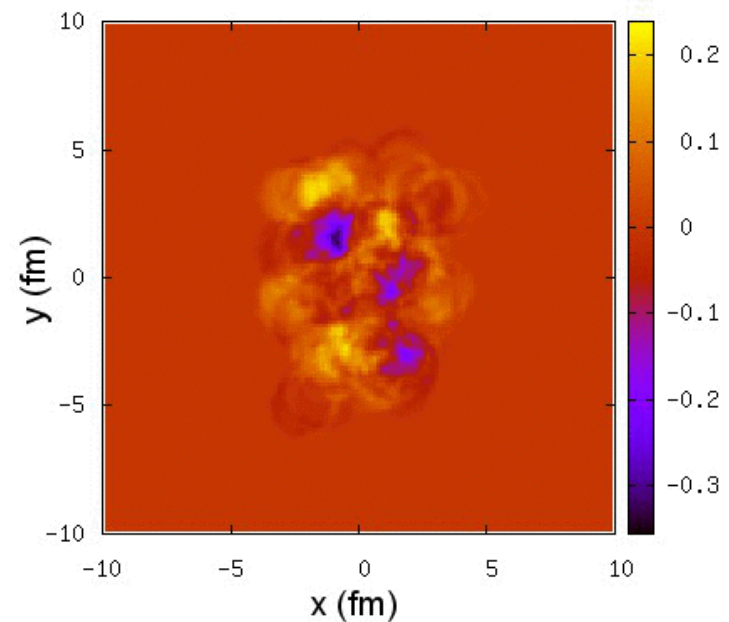


CMHD

Electric charge



Chiral charge

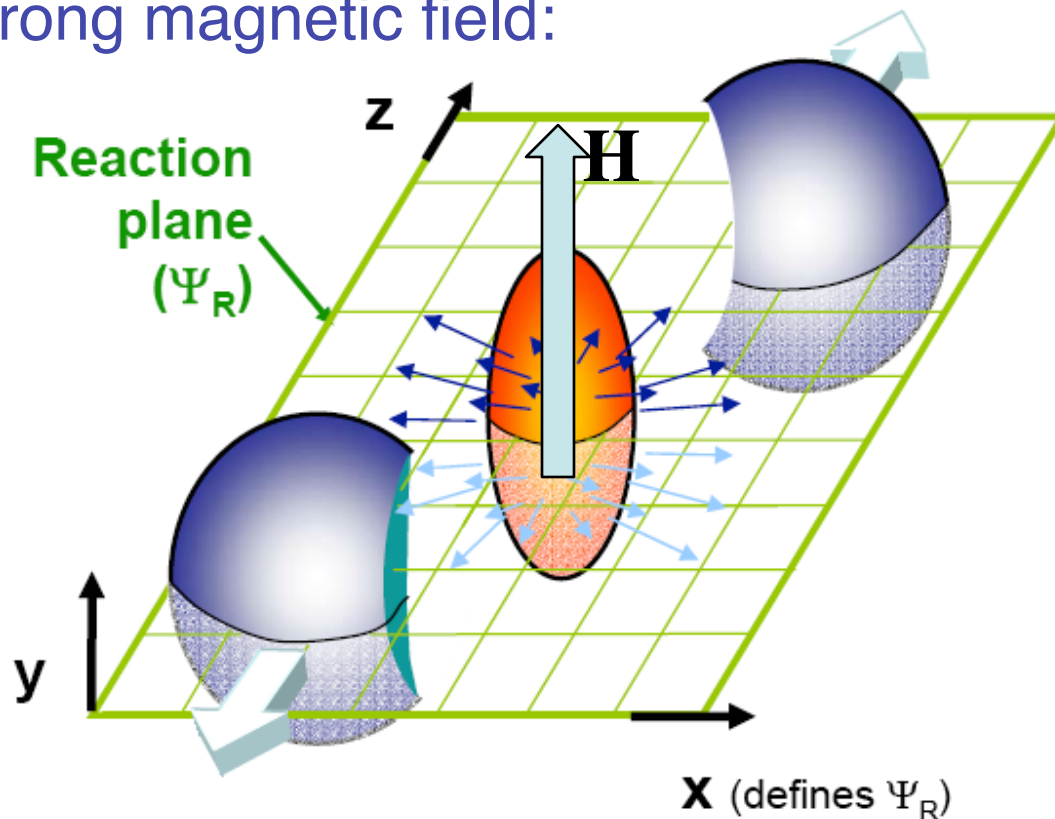


Y.Hirono, T.Hirano, DK, (Stony Brook – Tokyo), arxiv:1412.0311
(3+1) ideal CMHD (Chiral MagnetoHydroDynamics)

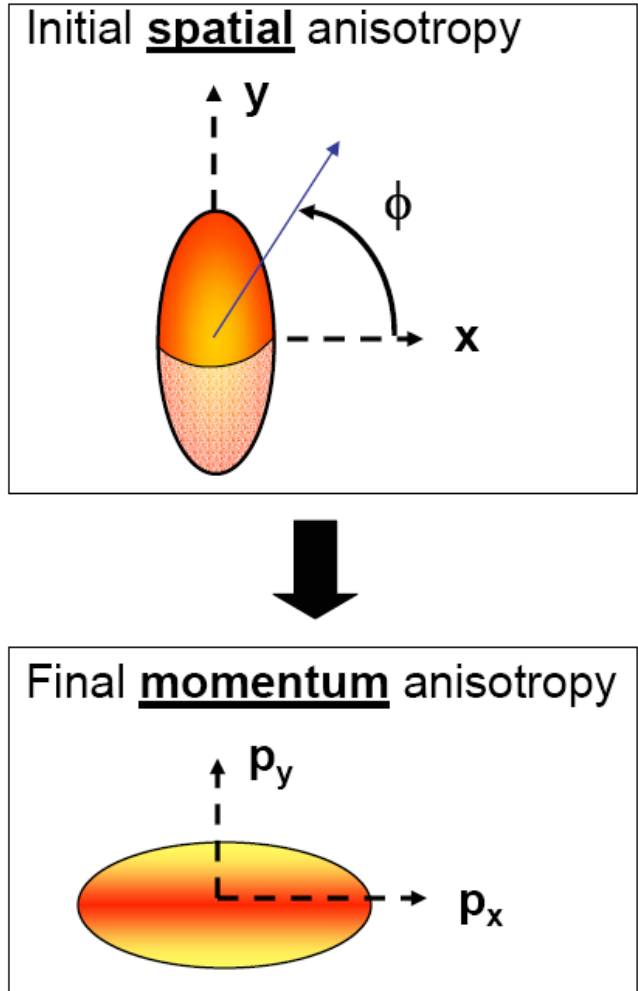
BEST Theory Collaboration (DOE)

Is there a way to observe CME in nuclear collisions at RHIC?

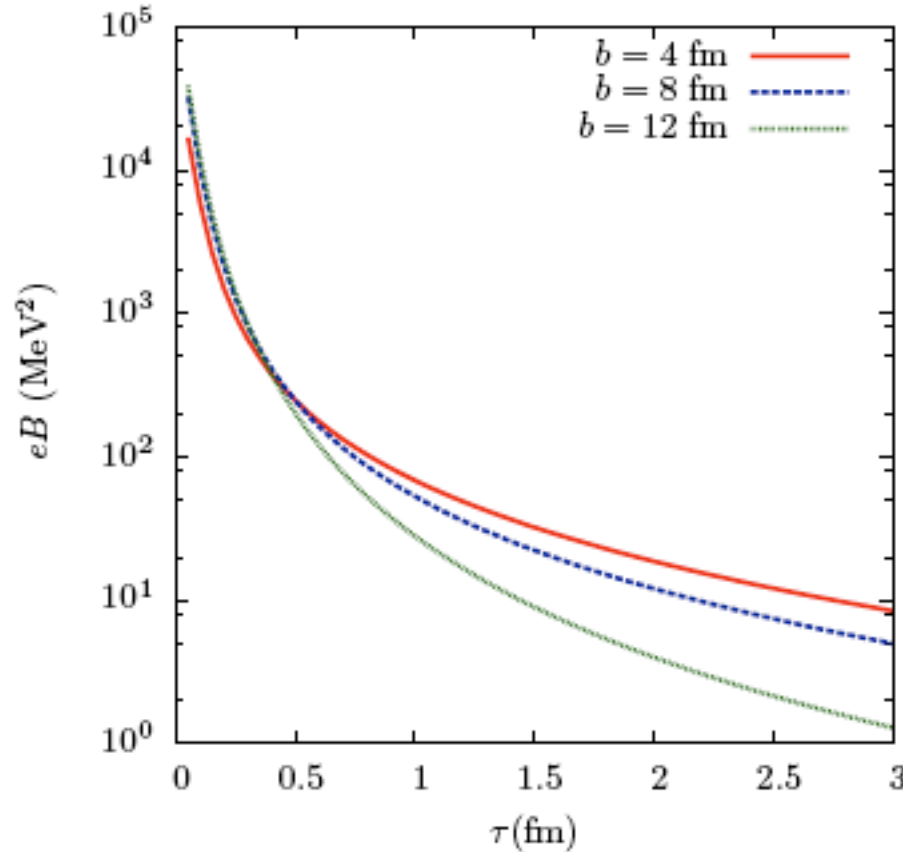
Relativistic ions create
a strong magnetic field:



DK, McLerran, Warringa '07



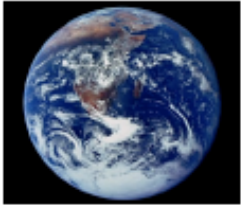
Heavy ion collisions as a source of the strongest magnetic fields available in the Laboratory



DK, McLerran, Warringa,
Nucl Phys A803(2008)227

Fig. A.2. Magnetic field at the center of a gold-gold collision, for different impact parameters. Here the center of mass energy is 200 GeV per nucleon pair ($Y_0 = 5.4$).

Comparison of magnetic fields



The Earth's magnetic field

0.6 Gauss

A common, hand-held magnet

100 Gauss

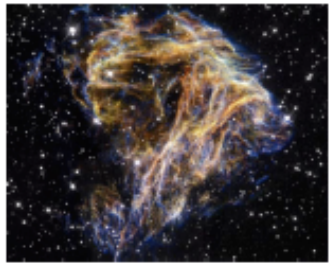


The strongest steady magnetic fields achieved so far in the laboratory

4.5×10^5 Gauss

The strongest man-made fields ever achieved, if only briefly

10^7 Gauss



Typical surface, polar magnetic fields of radio pulsars

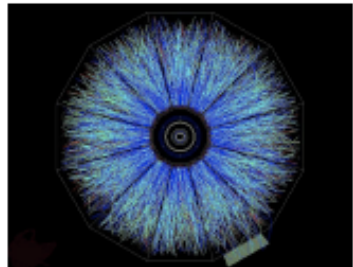
10^{13} Gauss

Surface field of Magnetars

10^{15} Gauss

<http://solomon.as.utexas.edu/~duncan/magnetar.html>

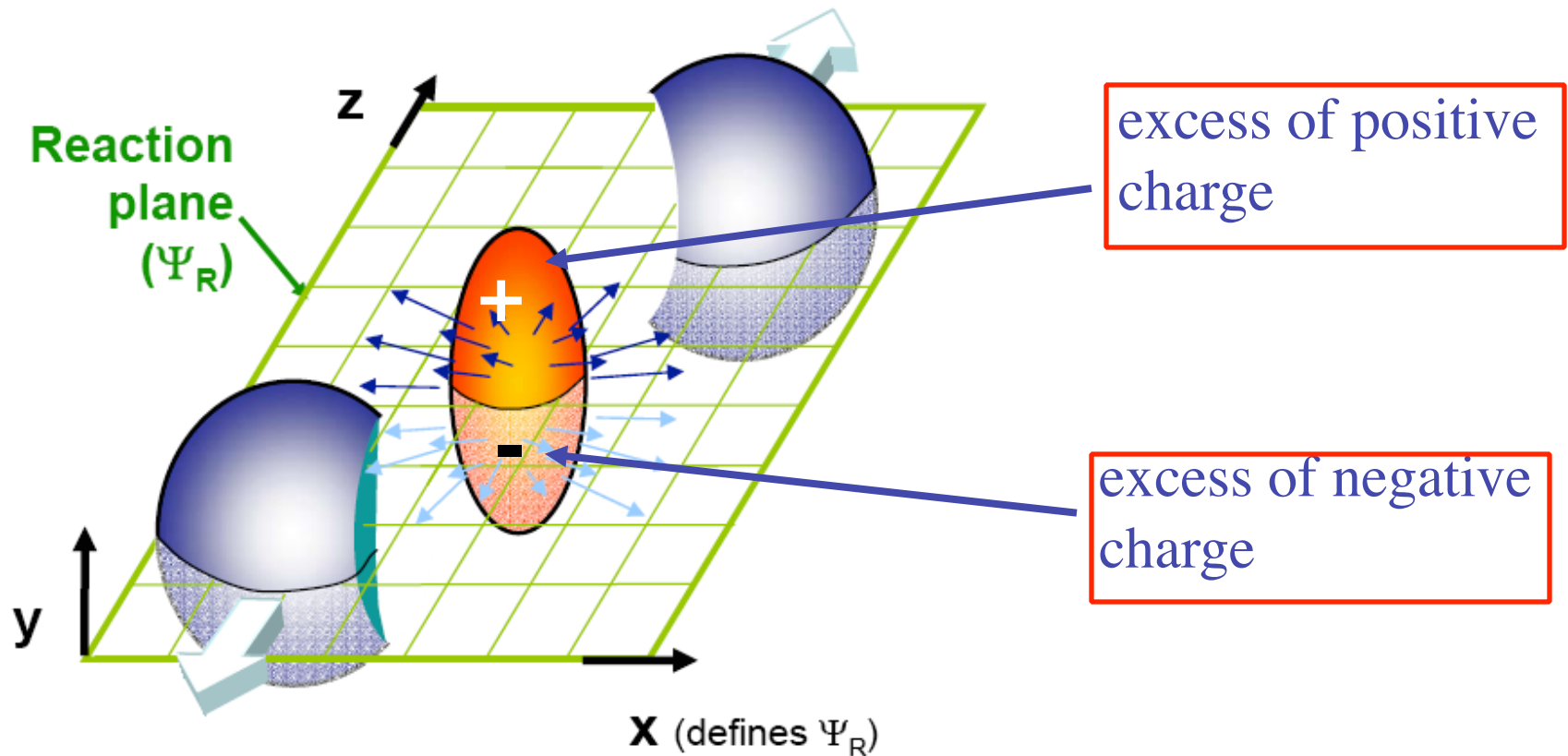
Heavy ion collisions: the strongest magnetic field ever achieved in the laboratory



Off central Gold-Gold Collisions at 100 GeV per nucleon

$$eB(\tau=0.2 \text{ fm}) = 10^3 \sim 10^4 \text{ MeV}^2 \sim 10^{17} \text{ Gauss}$$

Charge asymmetry w.r.t. reaction plane as a signature of chirality imbalance

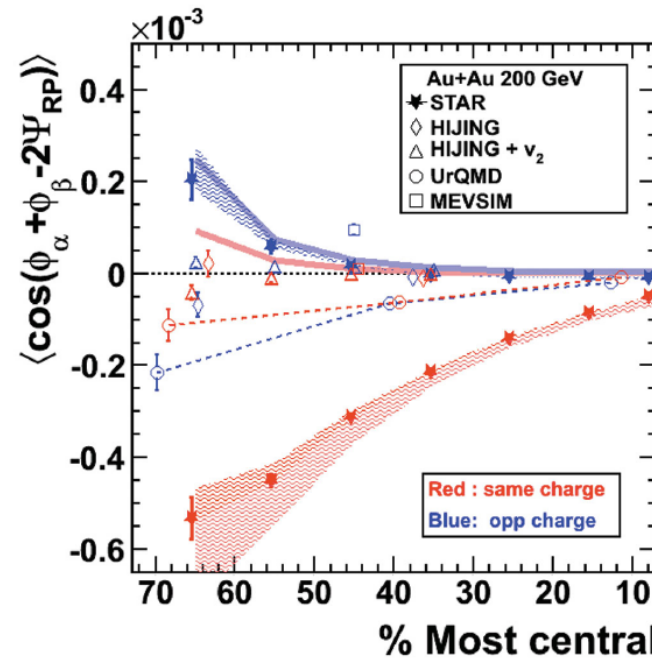
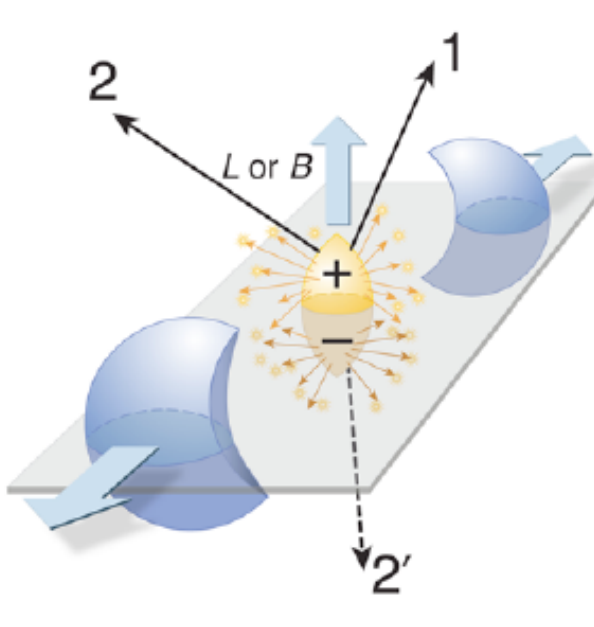


Electric dipole moment due to chiral imbalance



Azimuthal Charged-Particle Correlations and Possible Local Strong Parity Violation

(STAR Collaboration)



S.Voloshin '04

$$\begin{aligned} \gamma &\equiv \langle \cos(\phi_\alpha + \phi_\beta - 2\Psi_{RP}) \rangle = \langle \cos \Delta\phi_\alpha \cos \Delta\phi_\beta \rangle - \langle \sin \Delta\phi_\alpha \sin \Delta\phi_\beta \rangle \\ &= [\langle v_{1,\alpha} v_{1,\beta} \rangle + B_{IN}] - [\langle a_\alpha a_\beta \rangle + B_{OUT}] \approx -\langle a_\alpha a_\beta \rangle + [B_{IN} - B_{OUT}], \end{aligned}$$

NB: P-even quantity (strength of P-odd fluctuations)

– subject to large background contributions

Chiral Magnetic Effect Task Force Report

Vladimir Skokov (co-chair),^{1,*} Paul Sorensen (co-chair),^{2,†} Volker Koch,³

Soeren Schlichting,² Jim Thomas,³ Sergei Voloshin,⁴ Gang Wang,⁵ and Ho-Ung Yee^{6,1}

arxiv:1608.00982

describe the observed signal. Subsequently however, model studies of background effects and discoveries related to the importance of fluctuations in the initial geometry of heavy-ion collisions have shown that much if not all of the original signal reported in 2009 could arise from effects unrelated to the chiral magnetic effect. Further measurements and calculations

See also DK, J. Liao, S. Voloshin, G. Wang,
“Chiral magnetic and vortical effects in high-energy nuclear collisions:
A status report” Prog. Part. Nucl. Phys. 88 (2016) 1

Observation of charge-dependent azimuthal correlations in pPb collisions and its implication for the search for the chiral magnetic effect

The CMS Collaboration*

arxiv:1610.00263

October 2, 2016

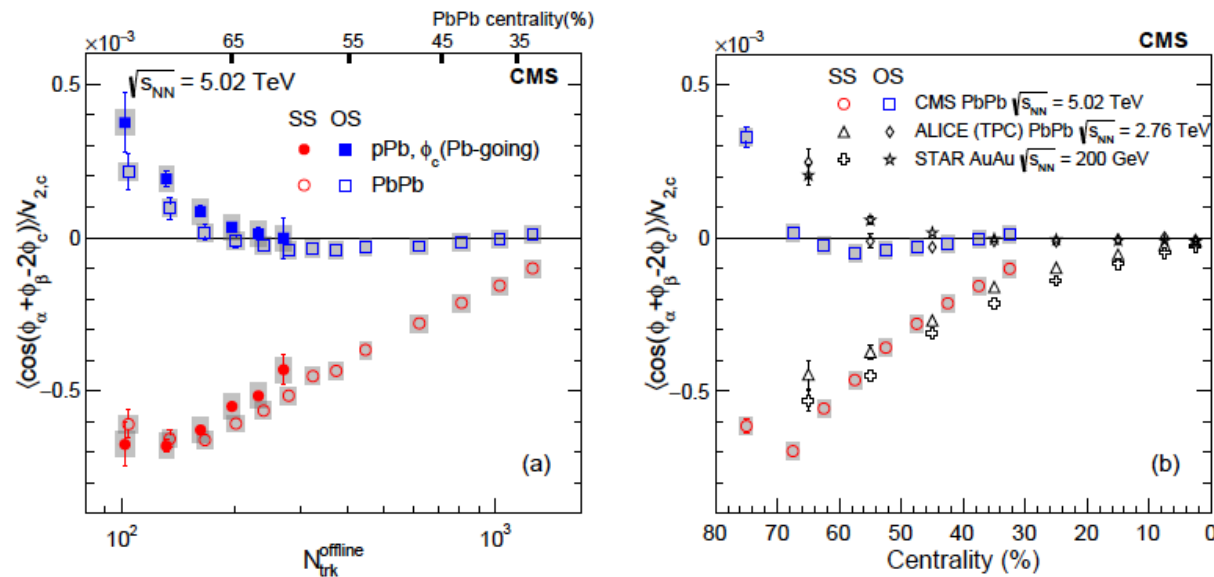


Figure 2: In (a), the same sign (SS) and opposite sign (OS) three-particle correlator averaged over $|\eta_\alpha - \eta_\beta| < 1.6$ as a function of $N_{\text{trk}}^{\text{offline}}$ in pPb and PbPb collisions at $\sqrt{s_{\text{NN}}} = 5.02$ TeV are shown. In (b), the same correlation as a function of centrality is presented in PbPb collisions at $\sqrt{s_{\text{NN}}} = 5.02$ TeV from CMS, at $\sqrt{s_{\text{NN}}} = 2.76$ TeV from ALICE, and in AuAu collisions at $\sqrt{s_{\text{NN}}} = 0.2$ TeV from STAR. Statistical and systematic uncertainties are indicated by the error bars and shaded regions, respectively.

Background everywhere?
(dAu at RHIC!)

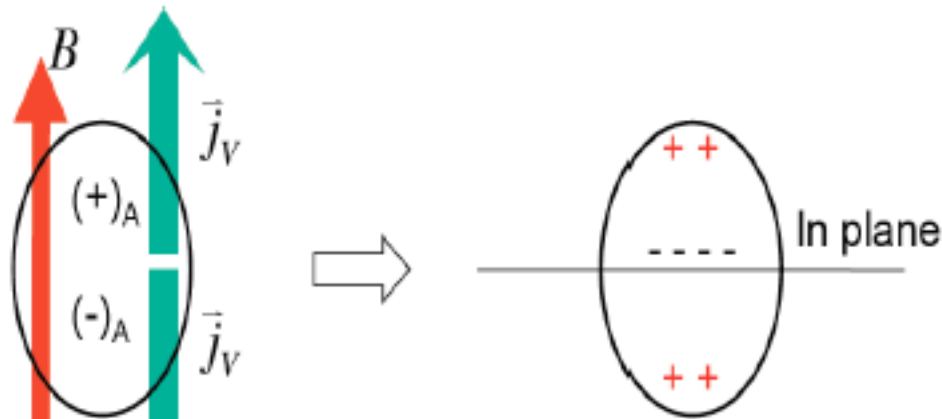
Magnetic field in pA?

Is there a better way to search for CME?

The Chiral Magnetic Wave: controlling the initial state

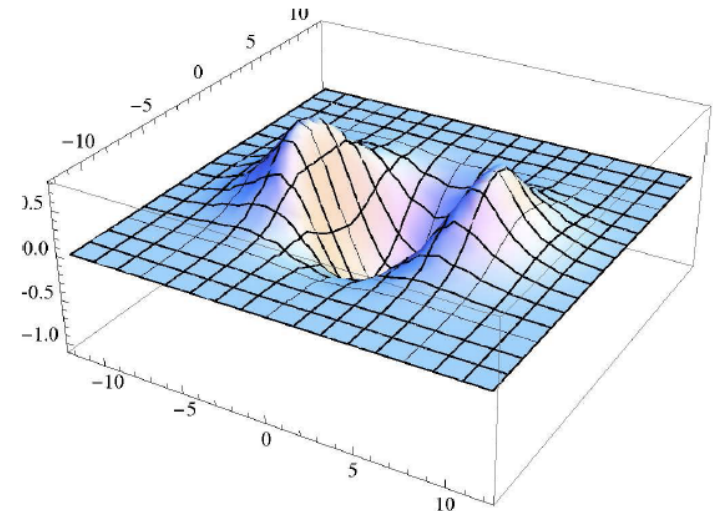
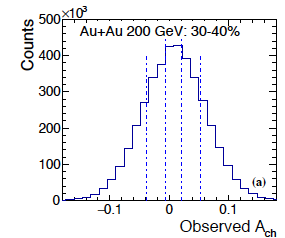
Finite baryon density + CMW = electric quadrupole moment of QGP

Signature - difference of elliptic flows of positive and negative pions determined by total charge asymmetry of the event A :
at $A > 0$, $v_2(-) > v_2(+)$; at $A < 0$, $v_2(+ > v_2(-)$



$$v_2^- - v_2^+ = C + 2\left(\frac{q_e}{\bar{\rho}_e}\right)A_{\pm}$$

$$A_{\pm} = (\bar{N}_+ - \bar{N}_-)/(\bar{N}_+ + \bar{N}_-)$$

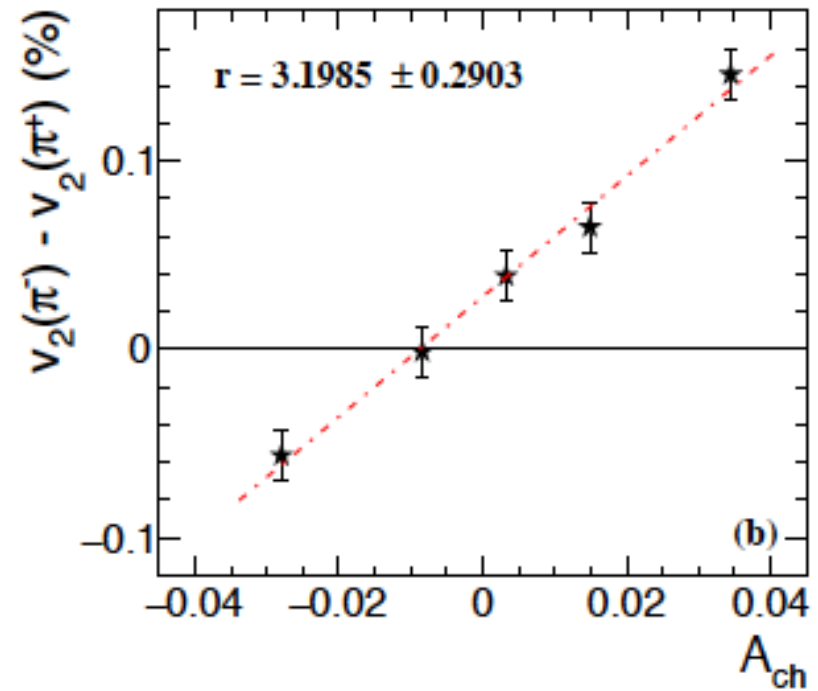
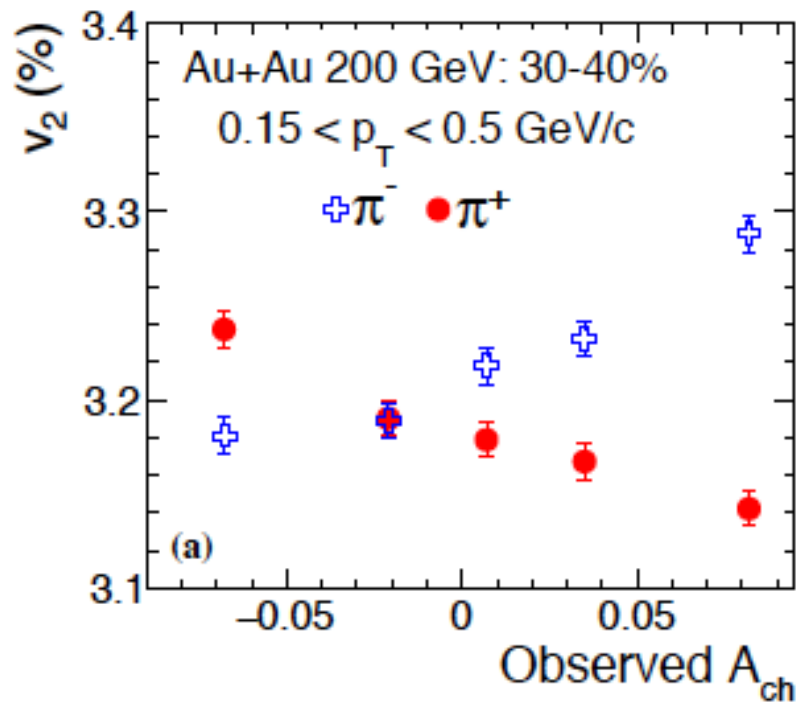


Y.Burnier, DK, J.Liao, H.Yee,
PRL 2011

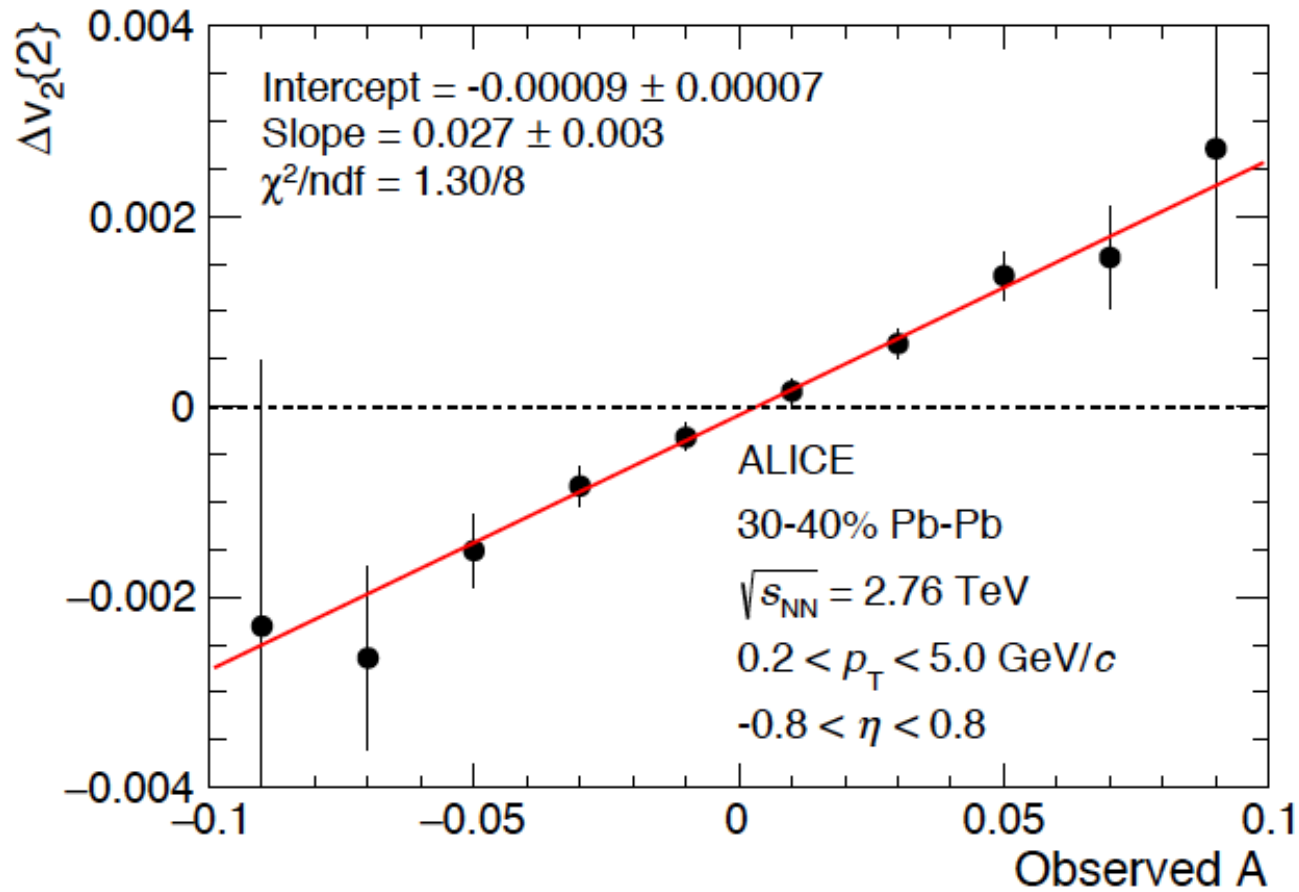
Observation of charge asymmetry dependence of pion elliptic flow and the possible
chiral magnetic wave in heavy-ion collisions

(STAR Collaboration)

arXiv:1504.02175



ALICE Coll. at the LHC



ALICE Coll, Phys. Rev. C93 (2016) 044903

Chiral Magnetic Effect Task Force Report

Vladimir Skokov (co-chair),^{1,*} Paul Sorensen (co-chair),^{2,†} Volker Koch,³
Soeren Schlichting,² Jim Thomas,³ Sergei Voloshin,⁴ Gang Wang,⁵ and Ho-Ung Yee^{6,1}

arxiv:1608.00982

The unique identification of the chiral magnetic effect in heavy-ion collisions would represent one of the highlights of the RHIC physics program and would provide a lasting legacy for the field. The current plan for completing the RHIC mission envisions a second phase of RHIC. We have specifically investigated the case for colliding nuclear isobars (nuclei with the same mass but different charge) and find the case compelling. We recommend that a program of nuclear isobar collisions to isolate the chiral magnetic effect from background sources be placed as a high priority item in the strategy for completing the RHIC mission.

Approved dedicated 2018 CME run at RHIC with
Zr (Z=40), Ru (Z=44) isobars

Broader implications: Dirac semimetals



SOVIET PHYSICS JETP

VOLUME 32, NUMBER 4

APRIL, 1971

POSSIBLE EXISTENCE OF SUBSTANCES INTERMEDIATE BETWEEN METALS AND DIELECTRICS

A. A. ABRIKOSOV and S. D. BENESLAVSKIĬ

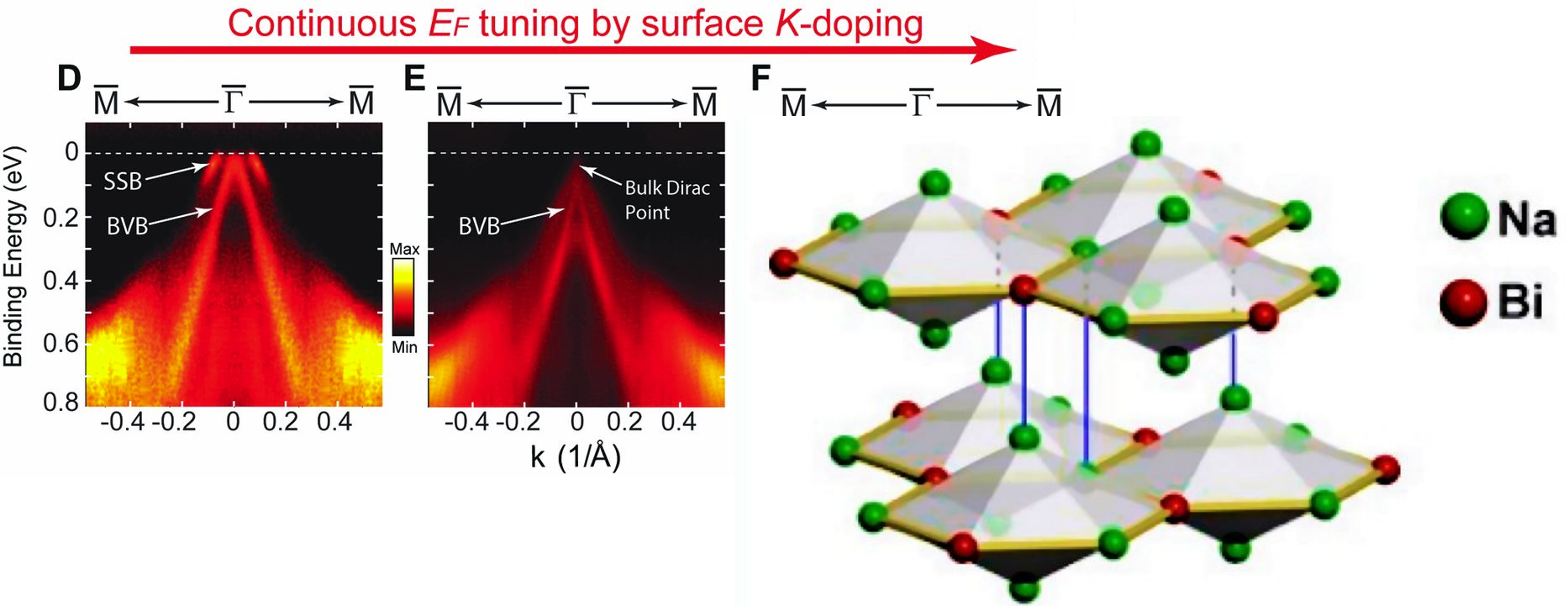
L. D. Landau Institute of Theoretical Physics

Submitted April 13, 1970

Zh. Eksp. Teor. Fiz. 59, 1280—1298 (October, 1970)

The question of the possible existence of substances having an electron spectrum without any energy gap and, at the same time, not possessing a Fermi surface is investigated. First of all the question of the possibility of contact of the conduction band and the valence band at a single point is investigated within the framework of the one-electron problem. It is shown that the symmetry conditions for the crystal admit of such a possibility. A complete investigation is carried out for points in reciprocal lattice space with a little group which is equivalent to a point group, and an example of a more complicated little group is considered. It is shown that in the neighborhood of the point of contact the spectrum may be linear as well as quadratic.

The discovery of Dirac semimetals – 3D chiral materials



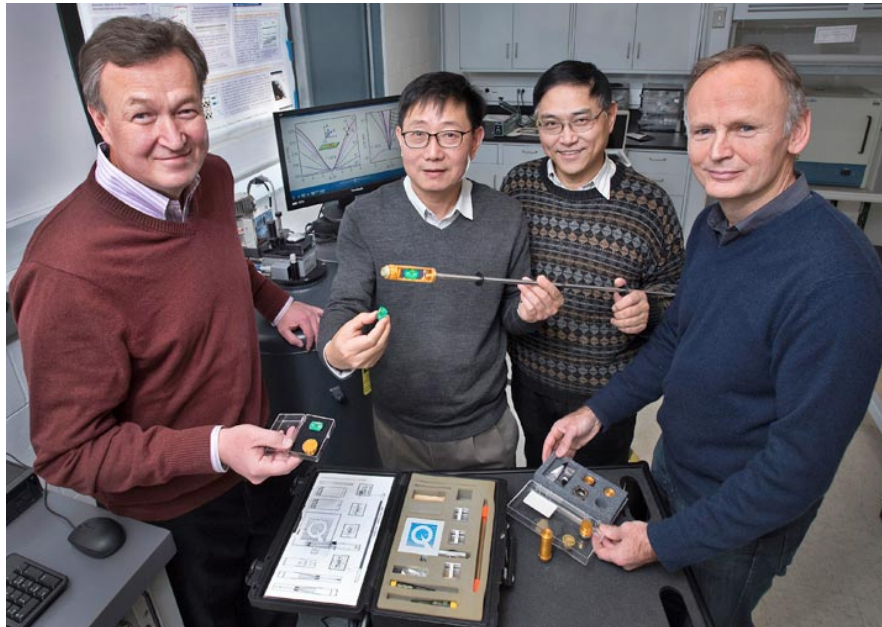
Z.K.Liu et al., Science 343 p.864 (Feb 21, 2014)

CME in condensed matter:

Observation of the chiral magnetic effect in ZrTe_5

Qiang Li,¹ Dmitri E. Kharzeev,^{2,3} Cheng Zhang,¹ Yuan Huang,⁴ I. Pletikosić,^{1,5}
A. V. Fedorov,⁶ R. D. Zhong,¹ J. A. Schneeloch,¹ G. D. Gu,¹ and T. Valla¹

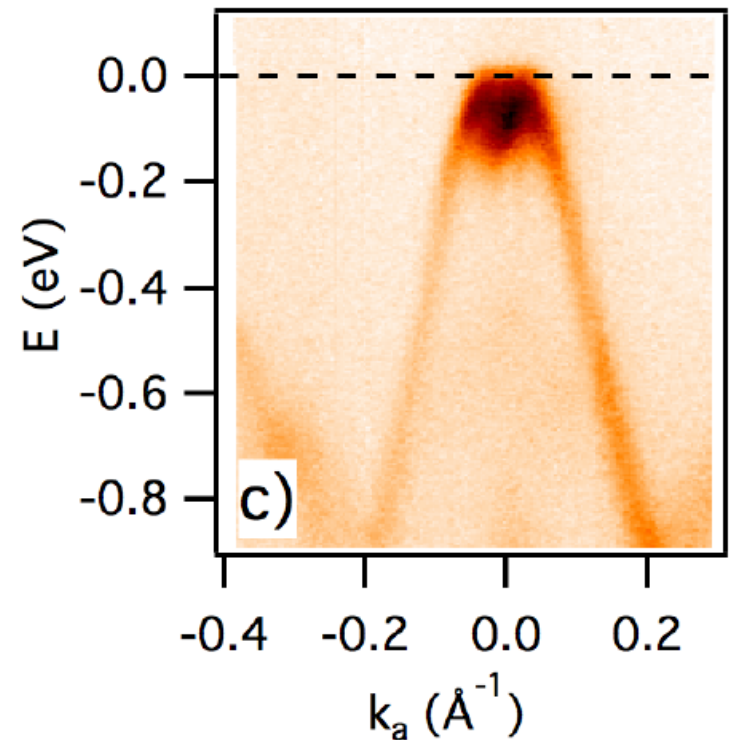
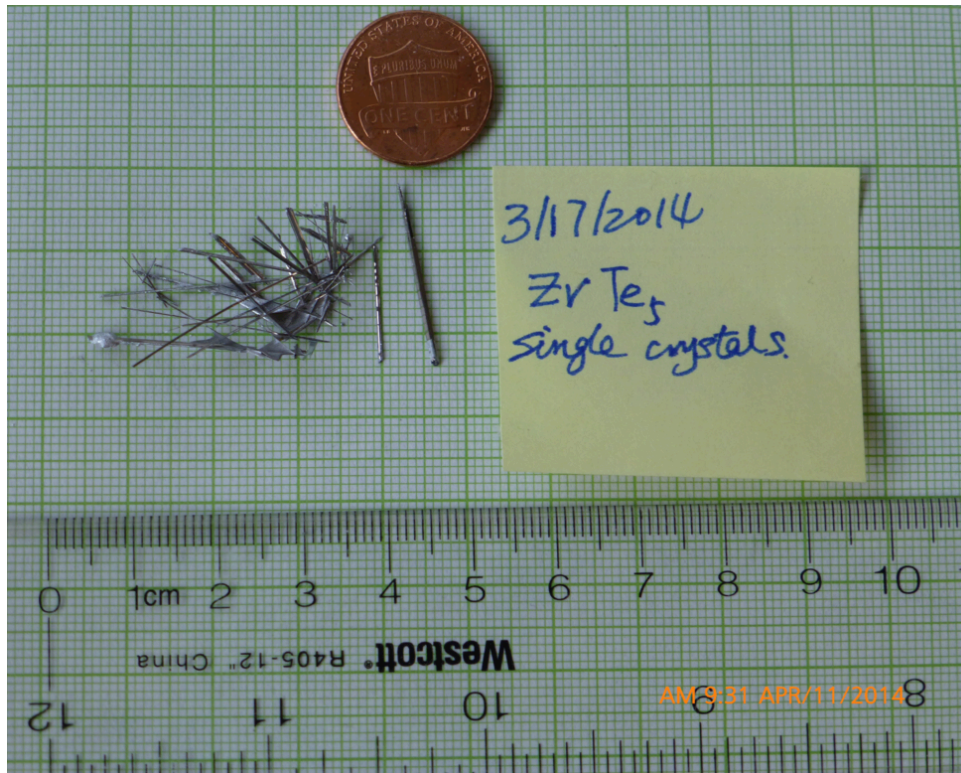
BNL - Stony Brook - Princeton - Berkeley



arXiv:1412.6543 [cond-mat.str-el]³⁰

Observation of the chiral magnetic effect in ZrTe_5

Qiang Li,¹ Dmitri E. Kharzeev,^{2,3} Cheng Zhang,¹ Yuan Huang,⁴ I. Pletikosić,^{1,5}
A. V. Fedorov,⁶ R. D. Zhong,¹ J. A. Schneeloch,¹ G. D. Gu,¹ and T. Valla¹



arXiv:1412.6543 (December 2014); Nature Physics **12**, 550 (2016)

Observation of the chiral magnetic effect in ZrTe_5

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A. V. Fedorov,⁶ R. D. Zhong,¹ J. A. Schneeloch,¹ G. D. Gu,¹ and T. Valla¹

Put the crystal in parallel \mathbf{E} , \mathbf{B} fields – the anomaly
generates chiral charge:

$$\frac{d\rho_5}{dt} = \frac{e^2}{4\pi^2\hbar^2c} \vec{E} \cdot \vec{B} - \frac{\rho_5}{\tau_V}.$$

and thus the chiral chemical potential:

$$\mu_5 = \frac{3}{4} \frac{v^3}{\pi^2} \frac{e^2}{\hbar^2c} \frac{\vec{E} \cdot \vec{B}}{T^2 + \frac{\mu^2}{\pi^2}} \tau_V.$$

Observation of the chiral magnetic effect in ZrTe_5

Qiang Li,¹ Dmitri E. Kharzeev,^{2,3} Cheng Zhang,¹ Yuan Huang,⁴ I. Pletikosić,^{1,5}
A. V. Fedorov,⁶ R. D. Zhong,¹ J. A. Schneeloch,¹ G. D. Gu,¹ and T. Valla¹

so that there is a chiral magnetic current:

$$\vec{J}_{\text{CME}} = \frac{e^2}{2\pi^2} \mu_5 \vec{B}.$$

resulting in the quadratic dependence of CME conductivity on B:

$$J_{\text{CME}}^i = \frac{e^2}{\pi\hbar} \frac{3}{8} \frac{e^2}{\hbar c} \frac{v^3}{\pi^3} \frac{\tau_V}{T^2 + \frac{\mu^2}{\pi^2}} B^i B^k E^k \equiv \sigma_{\text{CME}}^{ik} E^k.$$

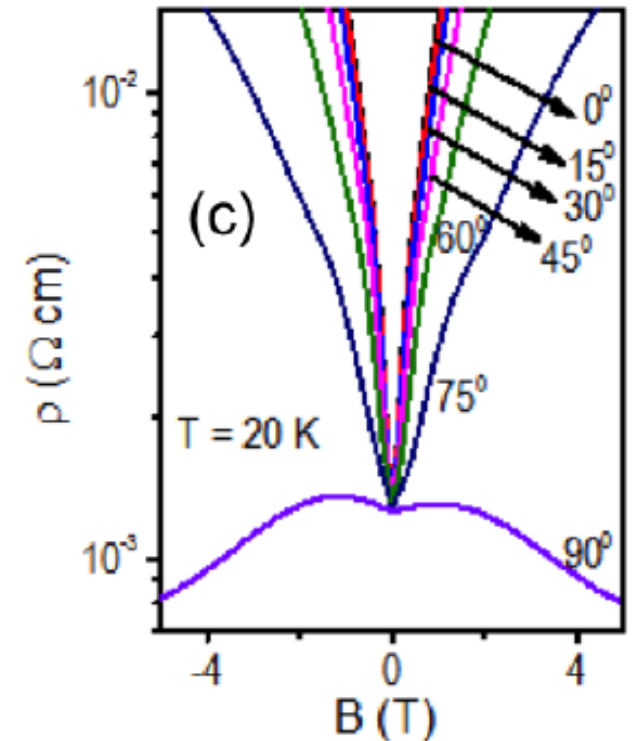
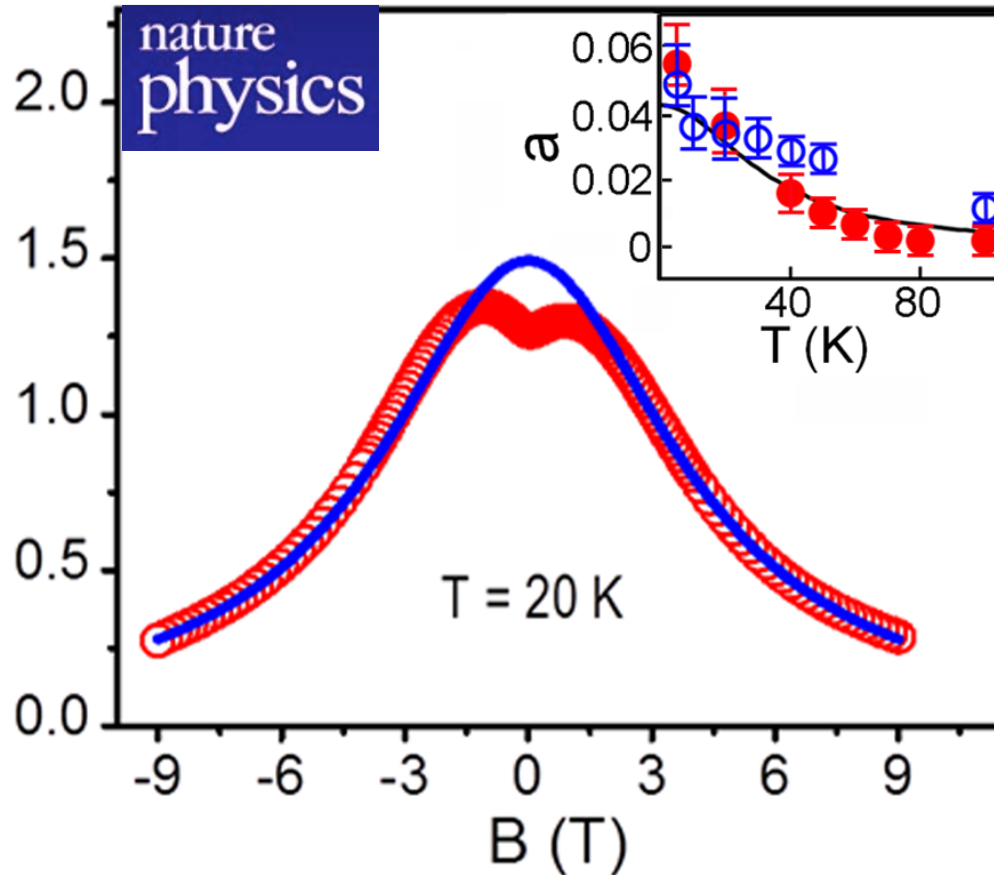
adding the Ohmic one – negative magnetoresistance

Chiral Magnetic Effect Generates Quantum Current

Separating left- and right-handed particles in a semi-metallic material produces anomalously high conductivity

February 8, 2016

Nature Physics **12**, 550 (2016)

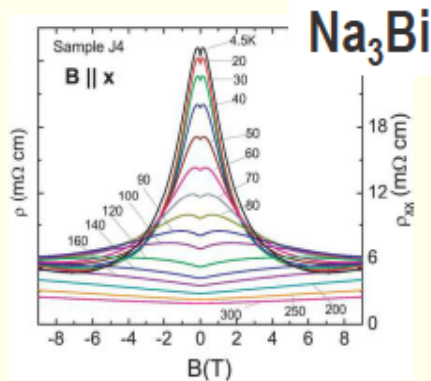


Qiang Li's Distinguished CQM lecture at Simons Center, Feb 19, 2016
on video:

http://scgp.stonybrook.edu/video_portal/video.php?id=2458

Chiral magnetic effect in Dirac/Weyl semimetals

Dirac semimetals:

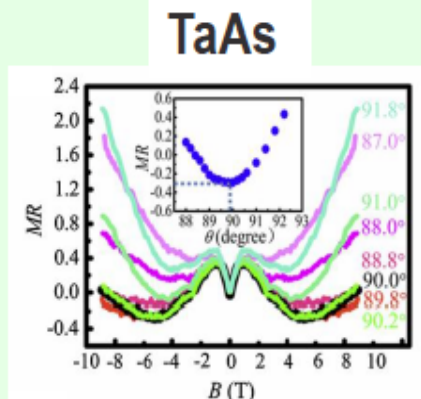


ZrTe₅ - Q. Li, D. Kharzeev, et al (BNL and Stony Brook Univ.)
arXiv:[1412.6543](#); doi:10.1038/NPHYS3648

Na₃Bi - J. Xiong, N. P. Ong et al (Princeton Univ.)
arxiv:[1503.08179](#); Science 350:413,2015

Cd₃As₂ - C. Li et al (Peking Univ. China)
arxiv:[1504.07398](#); Nature Commun. 6, 10137 (2015).

Weyl semimetals



TaAs - X. Huang et al (IOP, China)
arxiv:[1503.01304](#); Phys. Rev. X 5, 031023

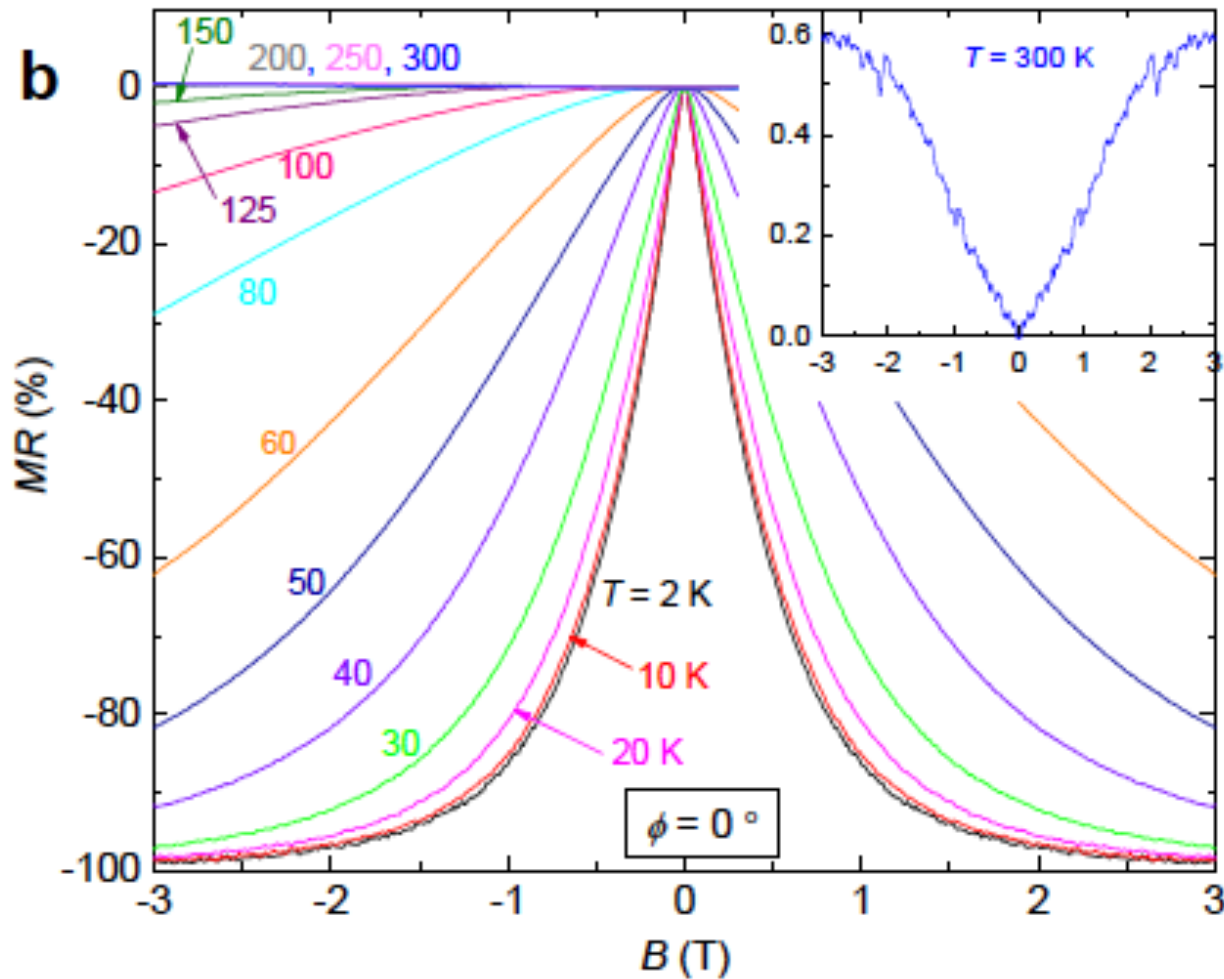
NbAs - X. Yang et al (Zhejiang Univ. China)
arxiv:[1506.02283](#)

NbP - Z. Wang et al (Zhejiang Univ. China)
arxiv:[1504.07398](#)

TaP - Shekhar, C. Felser, B. Yang et al (MPI-Dresden)
arxiv:[1506.06577](#)

Bi_{1-x}Sb_x at x ≈ 0.03 - Kim, et al. "Dirac versus Weyl Fermions in Topological Insulators: Adler-Bell-Jackiw Anomaly in Transport Phenomena. Phys. Rev. Lett., 111, 246603 (2013).

Negative MR in TaAs₂



Y.Luo et al, 1601.05524

Towards the room temperature CME?

CME as a new type of superconductivity

London theory of superconductors, '35:

$$\vec{J} = -\lambda^{-2} \vec{A} \quad \nabla \cdot \vec{A} = 0$$



Fritz and Heinz London

$$\vec{E} = -\dot{\vec{A}}$$

$$\vec{E} = \lambda^2 \dot{\vec{J}}$$

assume that chirality
is conserved:

$$\mu_5 \sim \vec{E} \vec{B} \, t$$

CME:

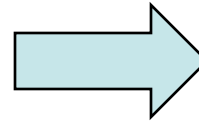
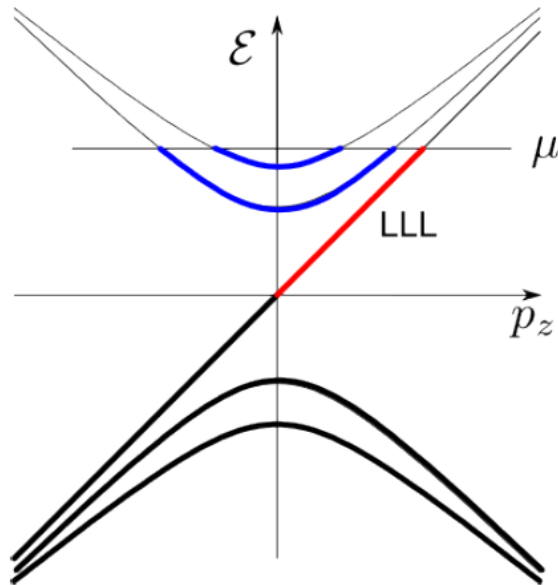
$$\vec{J} \sim \mu_5 \vec{B}$$

for $\vec{E} \parallel \vec{B}$

$$\vec{E} \sim B^{-2} \dot{\vec{J}}$$

superconducting
current, tunable
by magnetic field!

Chirality transfer from fermions to magnetic helicity



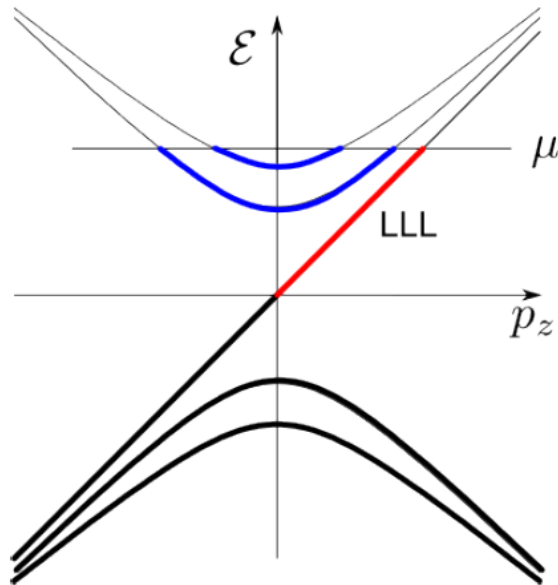
Chandrasekhar-Kendall states
(ApJ, 1957)

$$h_m \equiv \int d^3x \mathbf{A} \cdot \mathbf{B}$$

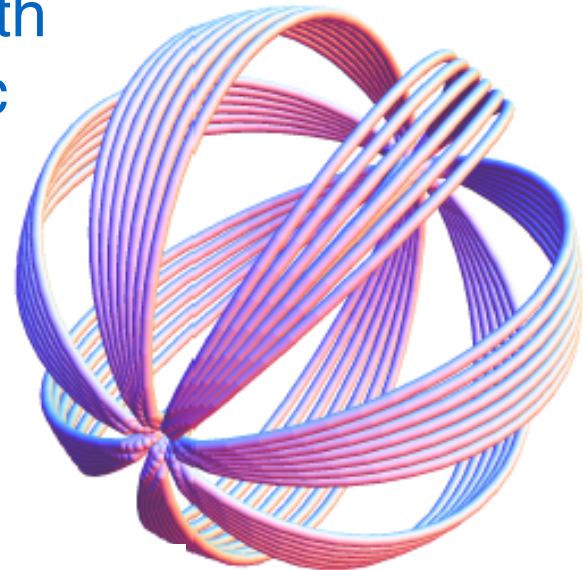
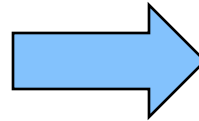
$$\partial_\mu j_A^\mu = C_A \mathbf{E} \cdot \mathbf{B}$$

$$h_0 \equiv h_m + h_F = \text{const} \quad \int d^3x \mathbf{E} \cdot \mathbf{B} = -\frac{1}{2} \frac{\partial h_m}{\partial t}$$

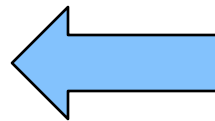
Inverse cascade of magnetic helicity



Instability at $k < C_A \mu_A$ leads to the growth of magnetic helicity



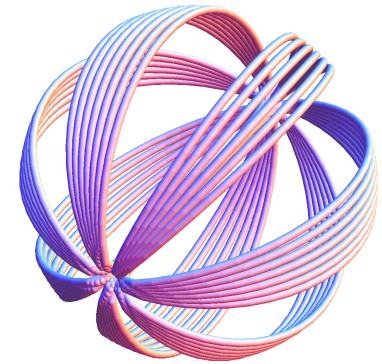
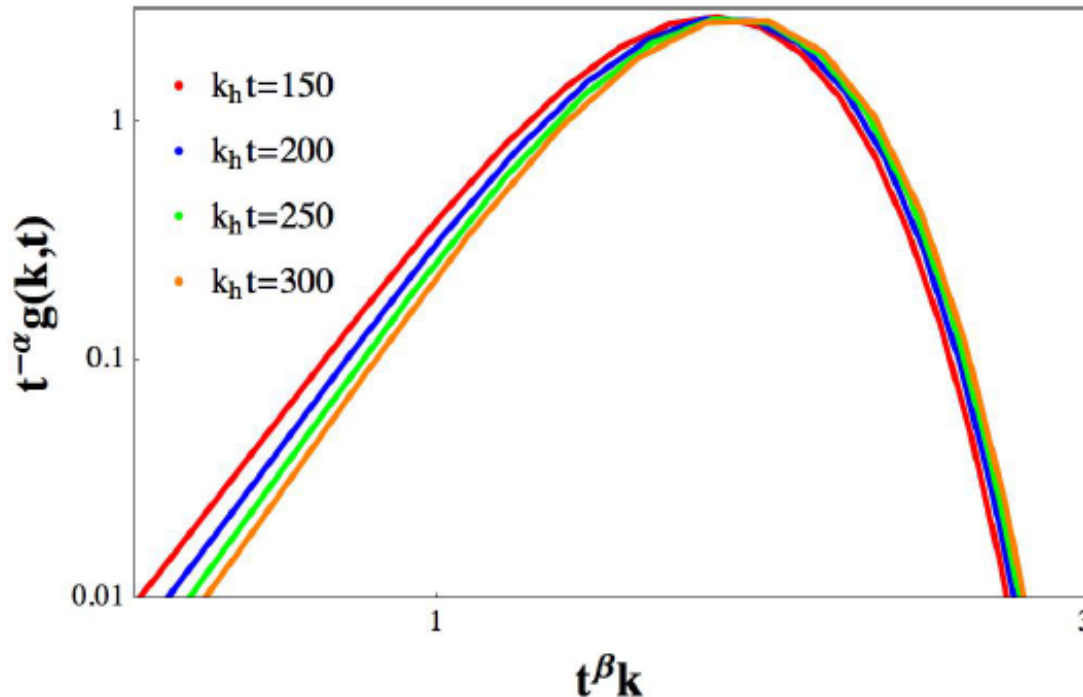
Increase of magnetic helicity reduces μ_A



Inverse cascade:

M.Joyce and M.Shaposhnikov, PRL 79 (1997) 1193;
R.Jackiw and S.Pi, PRD 61 (2000) 105015;
A.Boyarsky, J.Frohlich, O.Ruchayskiy, PRL 108 (2012) 031301;
PRD 92 (2015) 043004;
H.Tashiro, T.Vachaspati, A.Vilenkin, PRD 86 (2012) 105033

Self-similar cascade of magnetic helicity driven by CME

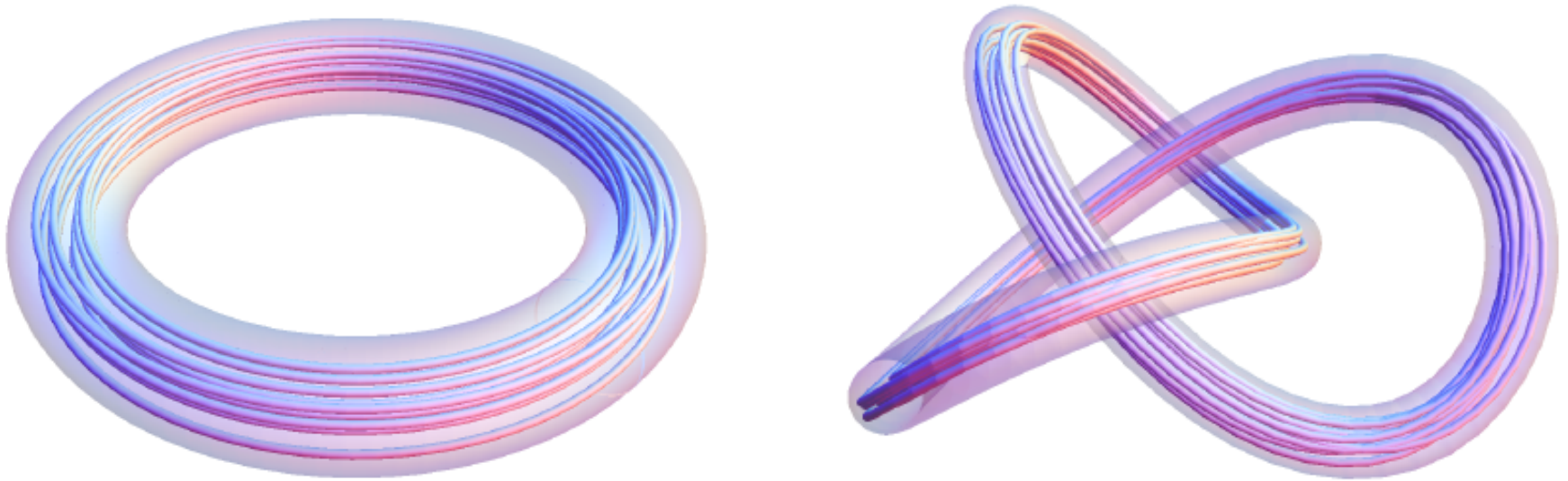


$$g(k, t) \sim t^{\alpha} \tilde{g}(t^{\beta} k) \quad \alpha = 1, \quad \beta = 1/2$$

Y. Hirono, DK, Y. Yin, Phys.Rev.D92 (2015) 125031;
N. Yamamoto, Phys.Rev.D93 (2016) 125016

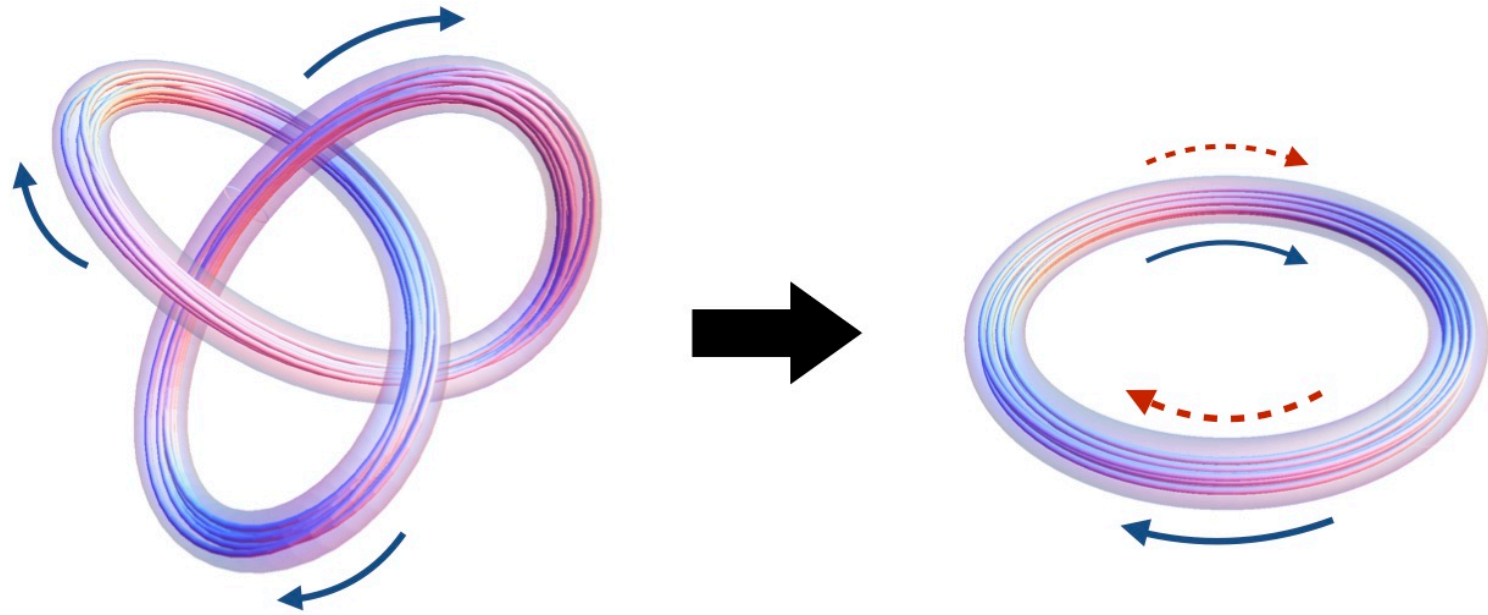
Quantized CME from knot reconnections

Y. Hirono, DK, Y. Yin, arXiv:1606.09611; PRL, in press



Consider a tube (unknot) of magnetic flux, with chiral fermions localized on it.

To turn it into a (chiral) knot, we need a magnetic reconnection.
What happens to the fermions during the reconnection?



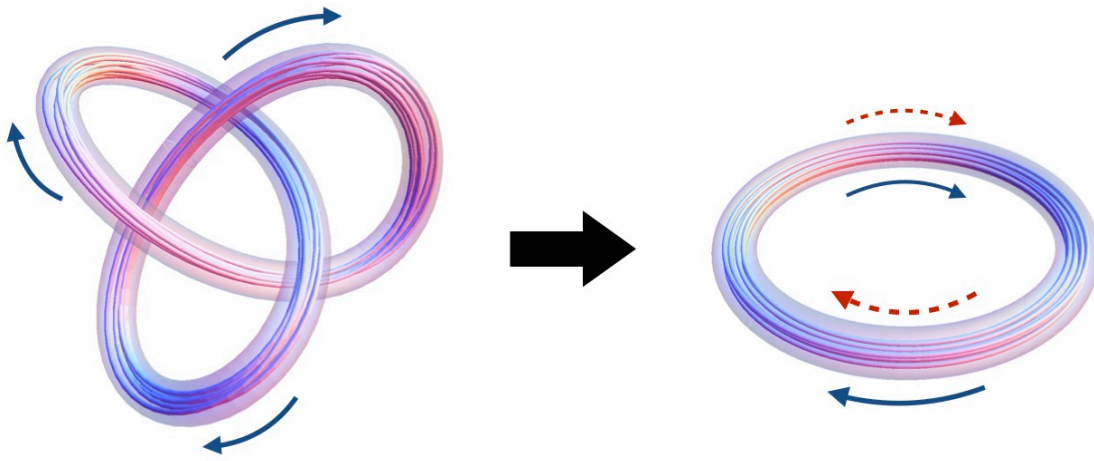
Changing magnetic flux through the area spanned by the tube will generate the electric field (Faraday's induction):

$$\frac{d}{dt}\Phi_B = - \oint_C \mathbf{E} \cdot d\mathbf{x}$$

The electric field will generate electric current of fermions (chiral anomaly in 1+1 D):

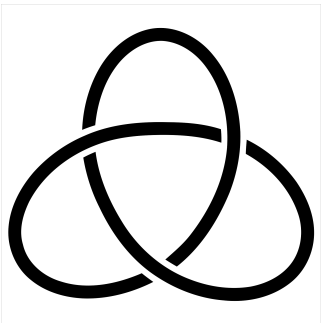
$$\Delta J = \Delta J_R + \Delta J_L = \frac{q^3 \Phi^2}{2\pi^2 L}$$

Y. Hirono, DK, Y. Yin,
arXiv:1606.09611;
PRL, in press



Helicity change per magnetic reconnection is $\Delta\mathcal{H} = 2\Phi^2$.

Multiple magnetic reconnections leading to non-chiral knots do not induce net current (need to break left-right symmetry).



For N_+ positive and N_- negative crossings on a planar knot diagram, the total magnetic helicity is:

$$\mathcal{H} = 2(N_+ - N_-)\Phi^2$$

The total current induced by reconnections to a chiral knot:

$$J = \frac{q^3 \mathcal{H}}{4\pi^2 L}$$

Summary

