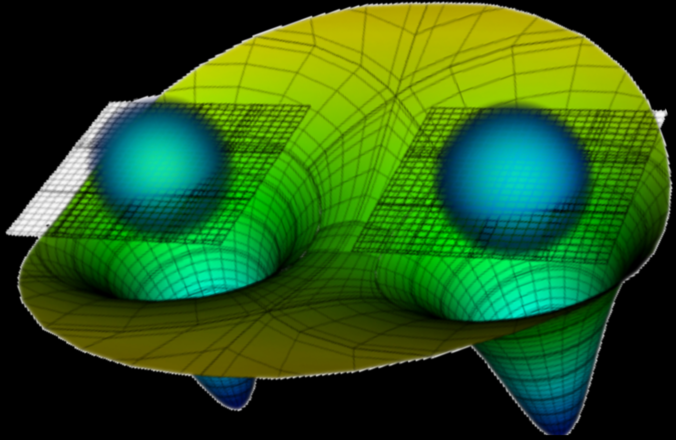
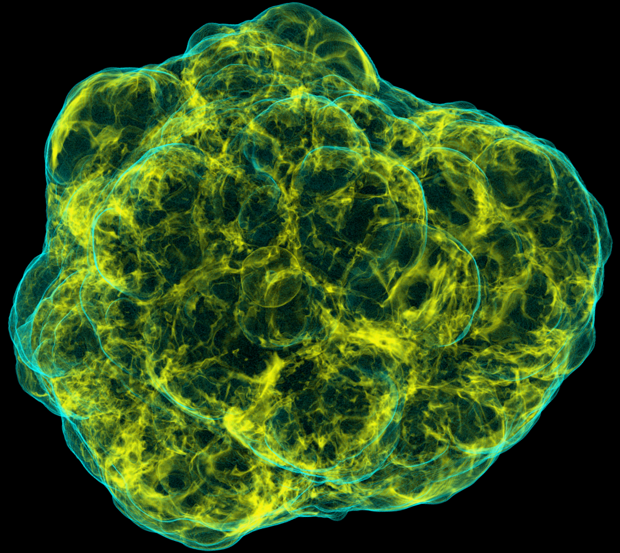
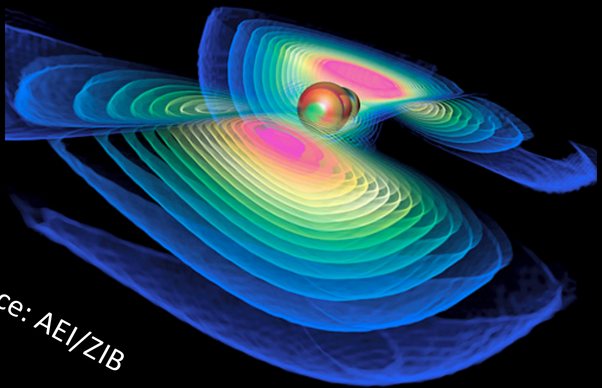
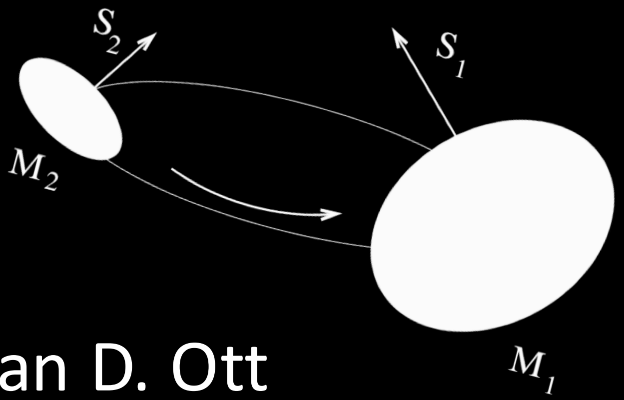
Simulation and Modeling of Gravitational Wave Sources







Source: AEI/ZIB



Christian D. Ott
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Simulating eXtreme Spacetimes (SXS) Collaboration

Caltech
NSF
Sherman Fairchild Foundation
BURKE INSTITUTE

Gravitational Wave Emission

- GWs (in GR!) are to lowest-order **quadrupole waves**.
- Emitted by accelerated aspherical bulk mass-energy motions.
- "Slow-motion" "weak-field" quadrupole approximation:

$$h_{jk}^{TT}(t,\vec{x}) = \left[\frac{2}{c^4}\frac{G}{|\vec{x}|}\ddot{I}_{jk}\left(t-\frac{|\vec{x}|}{c}\right)\right]^{TT}$$

dimensionless GW "strain" (displacement)

mass quadrupole moment

$$\frac{G}{c^4} \approx 10^{-49}\,\mathrm{s}^2\,\mathrm{g}^{-1}\,\mathrm{cm}^{-1}$$

First Numerical Estimate: $M \equiv \text{"aspherical mass"}$

$$I_{jk} = \int \rho x_j x_k d^3x \qquad \frac{d^2}{dt^2} I \sim \mathcal{O}(Mv^2) \qquad h \sim \frac{2G}{c^4 D} M v^2$$

$$M = 1 M_\odot \quad v = 0.1c$$
$$D = 10\,\mathrm{kpc}$$

$$\longrightarrow \quad h \sim 10^{-19}$$

(adv. LIGO:
~4 x 10^{-24} @ 200 Hz)

GW Emission

- **GWs** are very weak and interact weakly with matter.
  - No human-made sources (of detectable GWs):

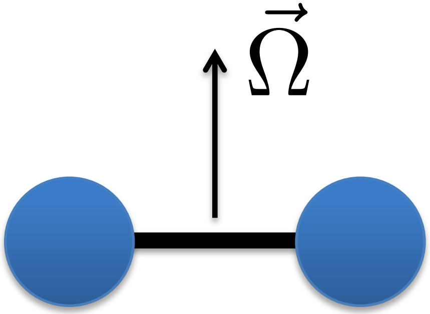
Example: $M = 1000\,\mathrm{kg}$

$$R = 10\,\mathrm{m}$$

$$\Omega = 100\,\mathrm{Hz}$$

$$D = 100\,\mathrm{m}$$

(detector distance)



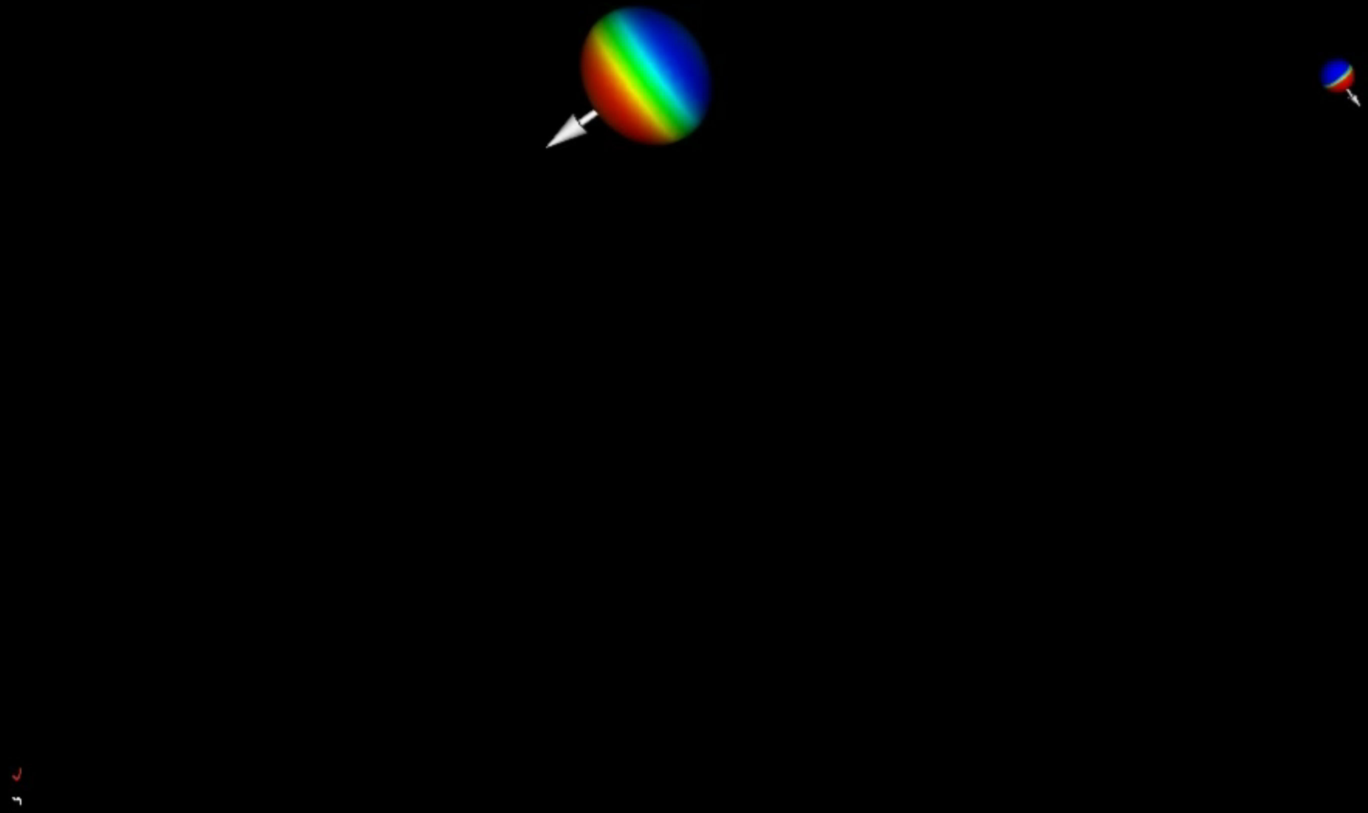
-> GW strain amplitude: $h \sim 10^{-37}$

(adv. LIGO: ~4 x 10^{-24} @ 200 Hz)

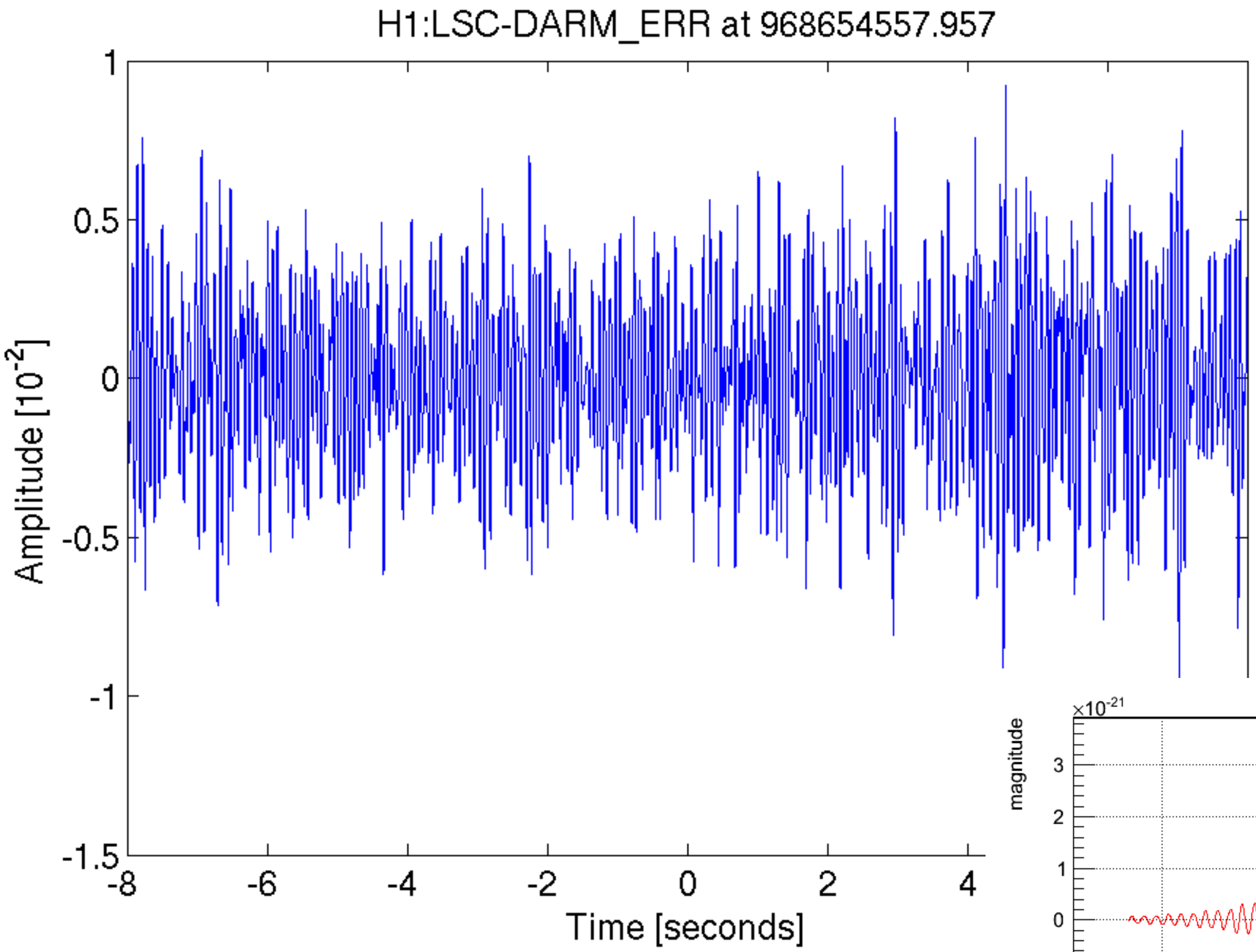
GW Emission

- **GWs** are very weak and interact weakly with matter.
  - No human-made sources.

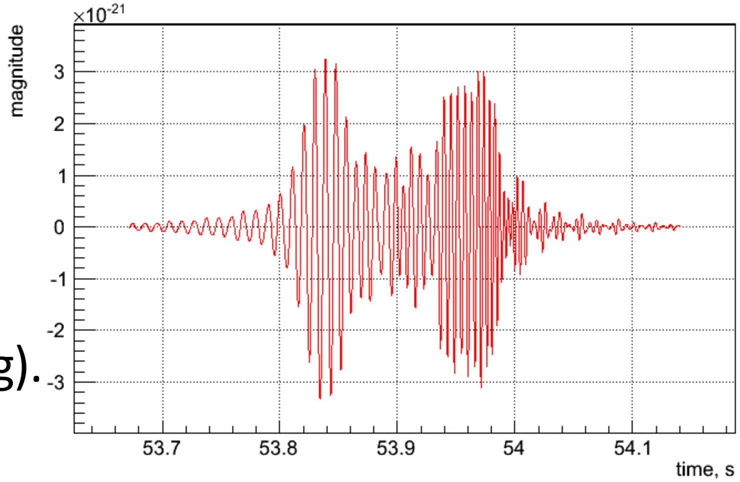
GW generator,
TAPIR group,
Caltech



Why Simulation and Modeling?



GW data stream
+ mock signal at
SNR 10



mock GW signal

Need signal predictions for:

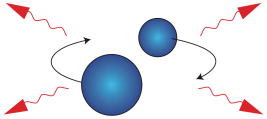
-> Detection of weak signals (matched filtering).

-> Estimation of source parameters & physics.

-> Tests of General Relativity.

Simulation vs. Modeling of GWs

(both are needed)



Simulation

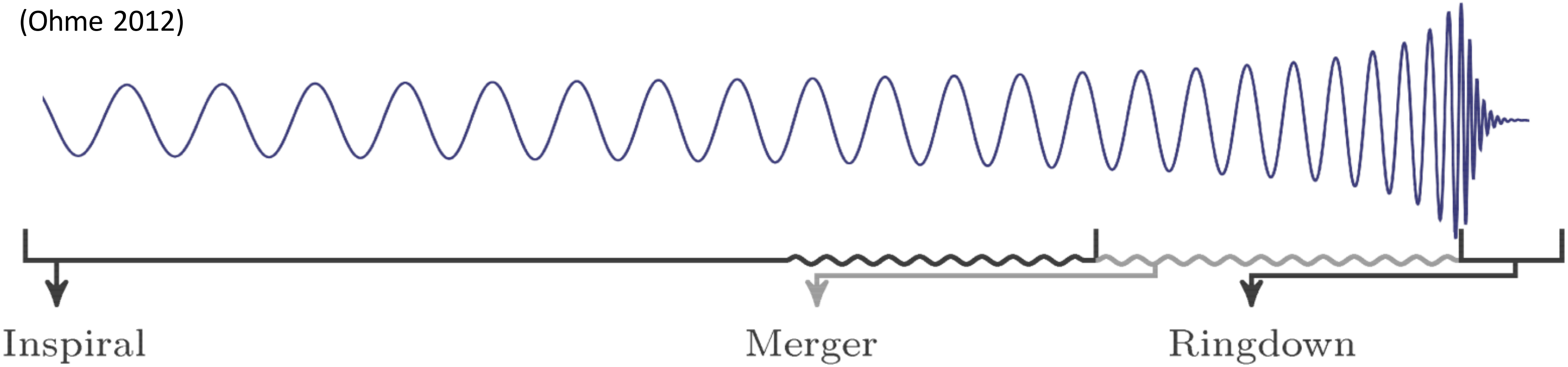
- From first principles.
- Is self-consistent and depends on few free parameters.
- Makes as few approximations as possible.
- Typically involves PDEs.
- Extremely computationally expensive. Sometimes prohibitively expensive.
- Yields reliable predictions (modulo systematics).

Modeling

- From phenomenological, approx. / perturbative model.
- Depends on many free parameters.
- Often tuned / calibrated based on simulations.
- Typically involves ODEs.
- Computationally inexpensive.
- Yields predictions whose reliability must be tested with simulations.

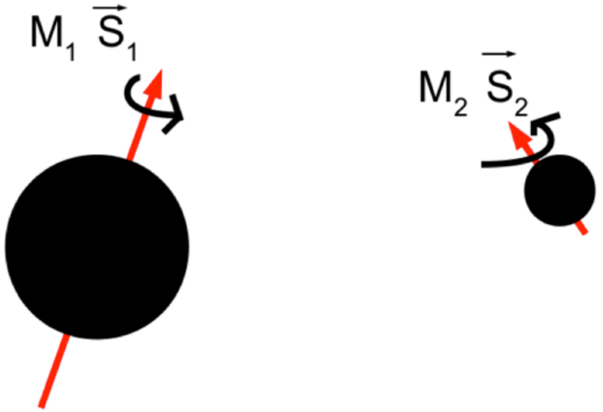
GW Signal Types, Simulation & Modeling

Coalescence Signals (Compact Binary Coalescence [CBC])



(Ohme 2012)

- (Relatively) simple signal morphology.
- Can be well modeled / simulated; ideal for matched filtering.
- BH+BH (BBH), NS+NS, NS+BH.



(J. Blackman)

GW Signal Types, Simulation & Modeling

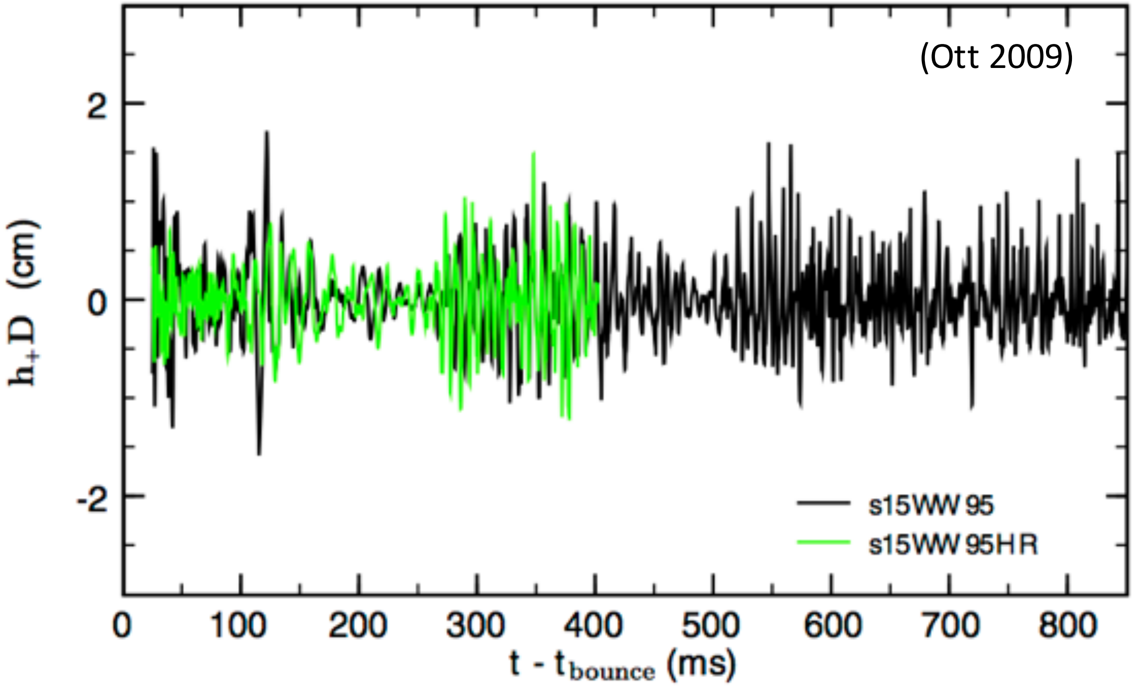
Coalescence Signals (Compact Binary Coalescence [CBC])

Bursts

Examples:

Core-collapse supernovae

Postmerger NS+NS



- Complex signal morphology.
- Hard or impossible to model, difficult to simulate.
- Chaotic signal components (e.g., due to turbulence).
- Matched filtering generally not applicable.

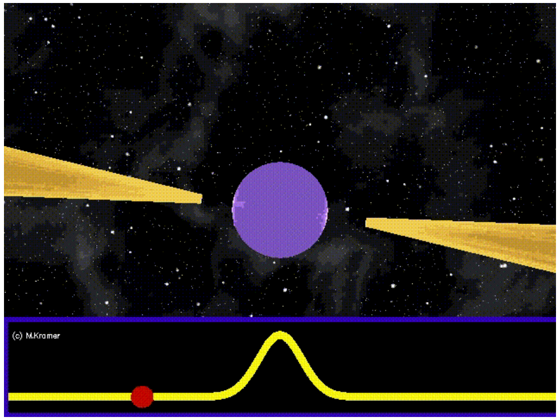
GW Signal Types, Simulation & Modeling

Coalescence Signals (Compact Binary Coalescence [CBC])

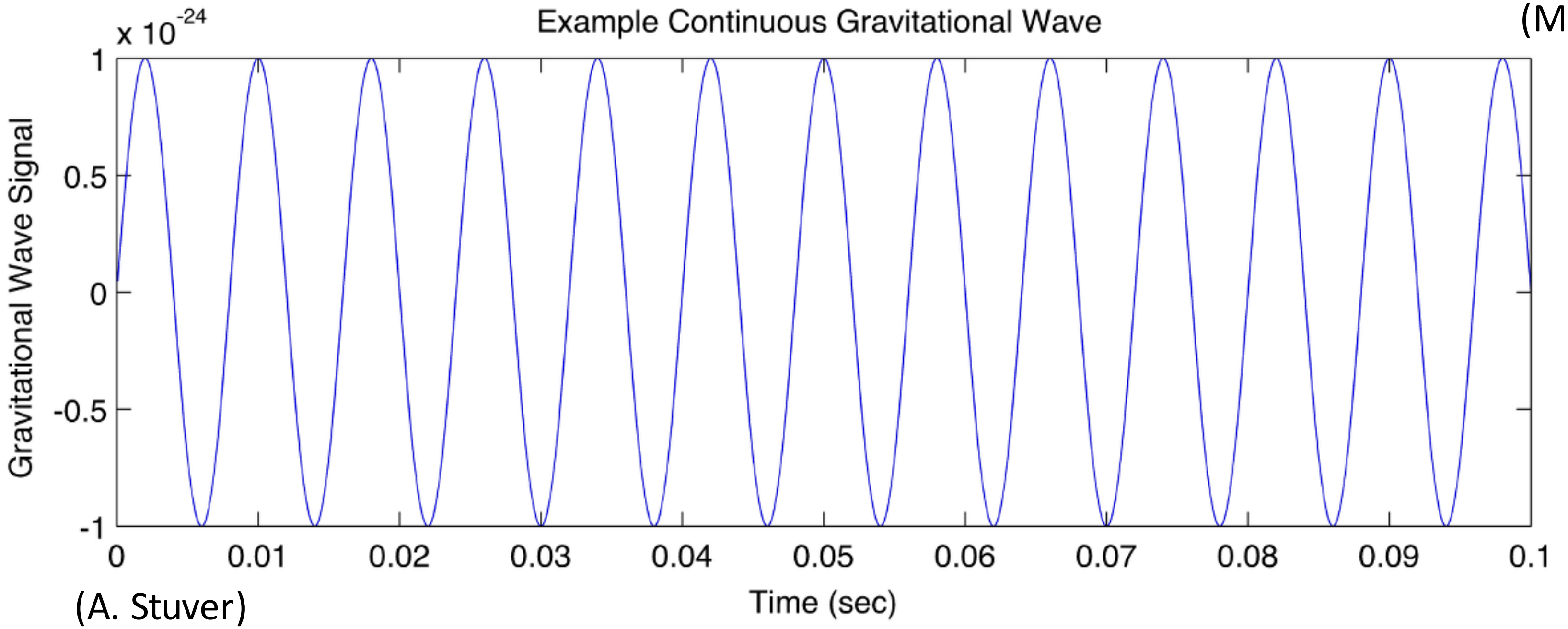
Bursts

Continuous Waves

- Well modeled, highly periodic signals due to small deformations of spinning NSs.



(M. Kramer)



(A. Stuver)

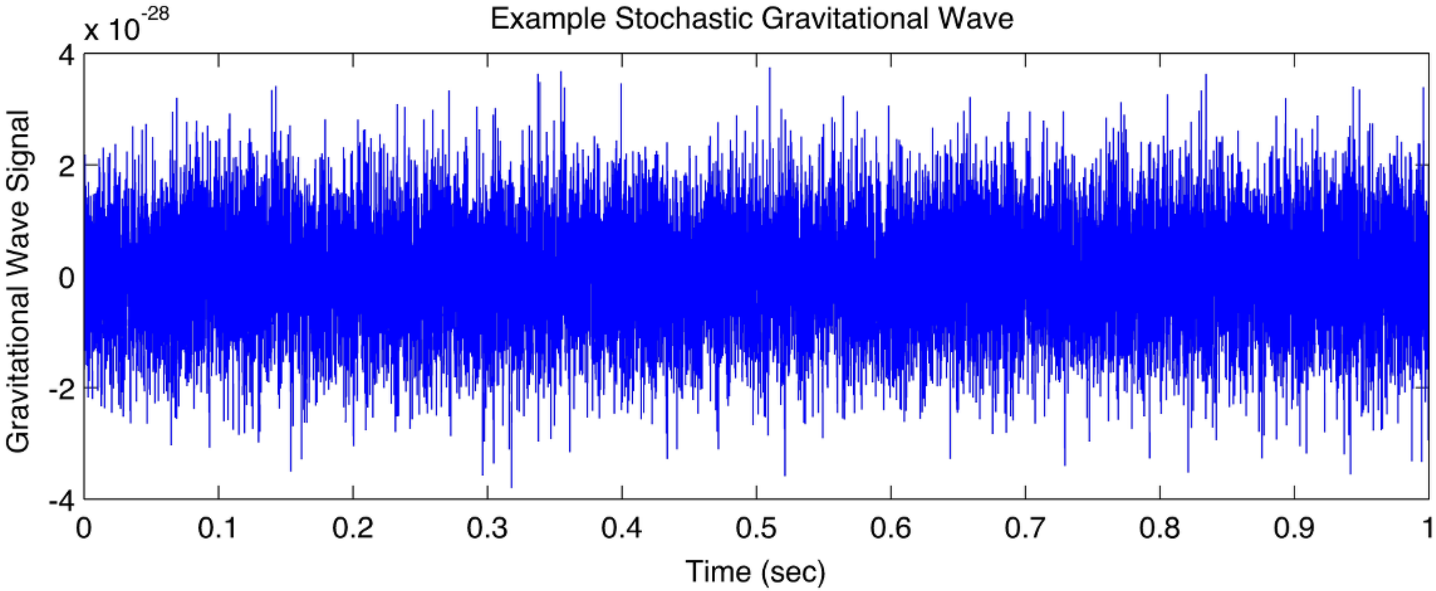
GW Signal Types, Simulation & Modeling

Coalescence Signals (Compact Binary Coalescence [CBC])

Bursts Continuous Waves

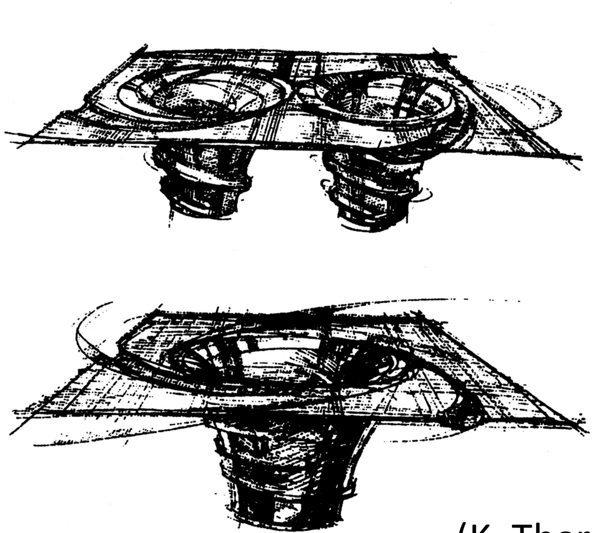
Stochastic Backgrounds

- Cosmological: Big Bang, inflation
- Astrophysical: superposition of cosmol. population of CBC/burst events.
- Stochastic – no detailed h(t) prediction possible.

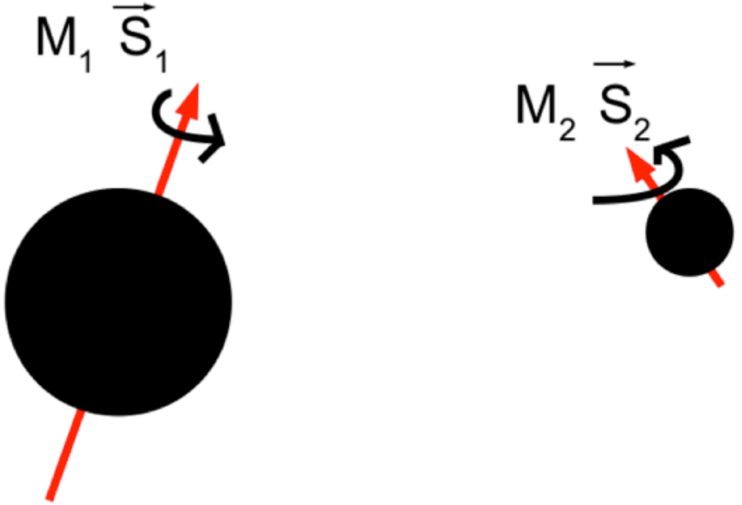


Example 1: Coalescing BH+BH Pairs

Pure gravity! $G^{\mu\nu} = 0$



(K. Thorne)

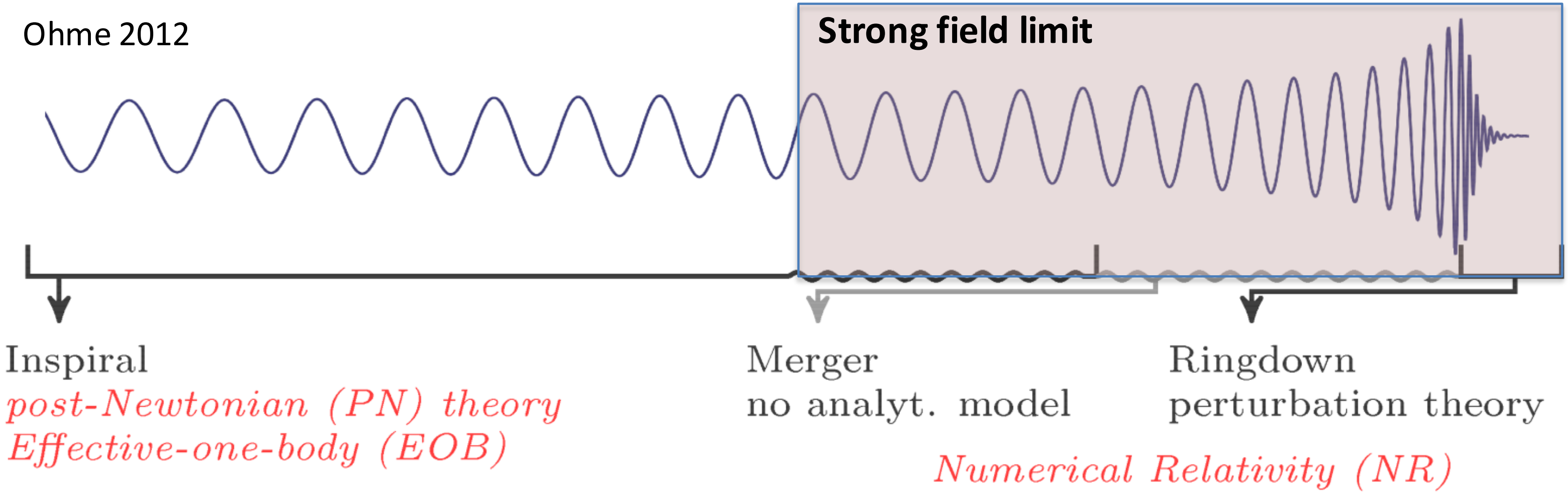


(J. Blackman)

Parameter space

- Black hole masses m_1, m_2
- Spin vectors $\vec{\chi_1}$ and $\vec{\chi_2}$, $\|\vec{\chi_i}\| = \|\vec{S_i}\|/m_i^2 < 1$
- Total mass $M = m_1 + m_2$ can be scaled out, leaving 7 parameters
- A moderately dense covering of the parameter space would require $\sim 10^7$ waveforms! <- this is why modeling is needed!

Binary Black Hole Coalescence



Ohme 2012

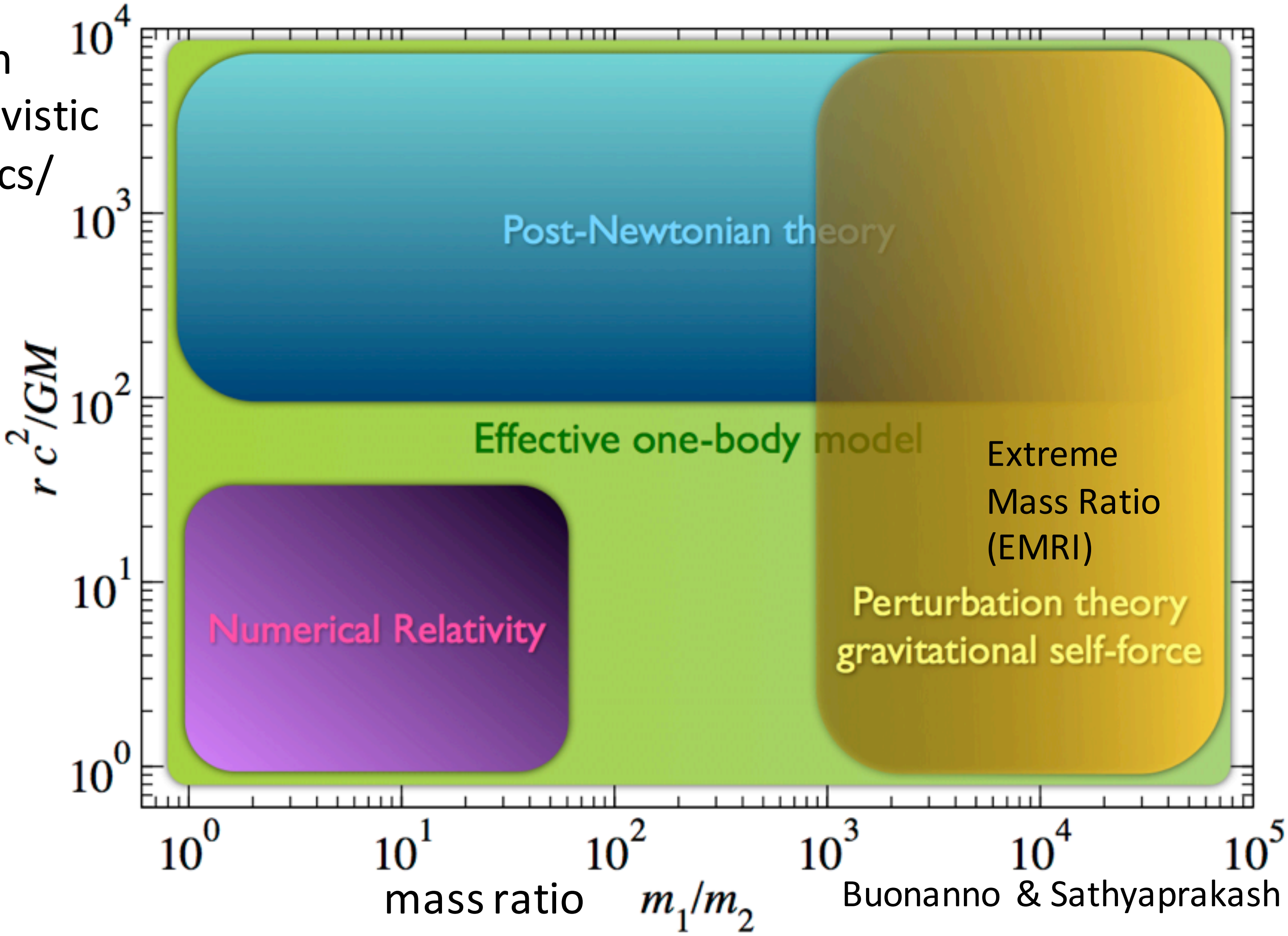
- Simulation: Numerical Relativity – direct integration of field eqns.
- Modeling:
  - Post-Newtonian (PN) approximants (expansion in v/c). Only inspiral. Fails in strong-field regime.
  - Effective-one-body (EOB) and “Phenom”-type PN models: Fits of PN inspiral, merger, ringdown. Calibrated on NR simulations.
  - NR “surrogate models” via reduced-order modeling.

Binary Black Hole Coalescence

Validity of methods

strength
of relativistic
dynamics/
gravity.

$$\frac{c^2}{v^2} \sim$$

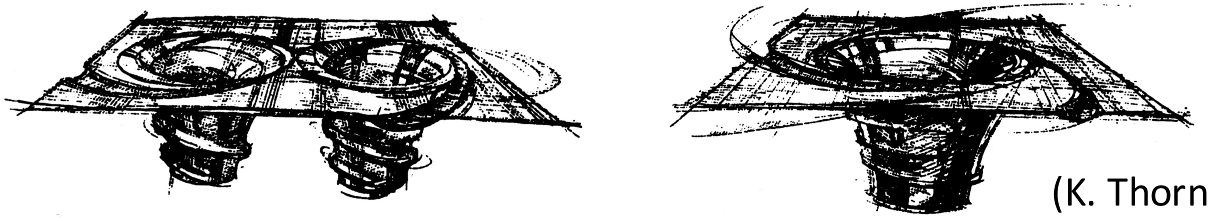


Extreme
Mass Ratio
(EMRI)

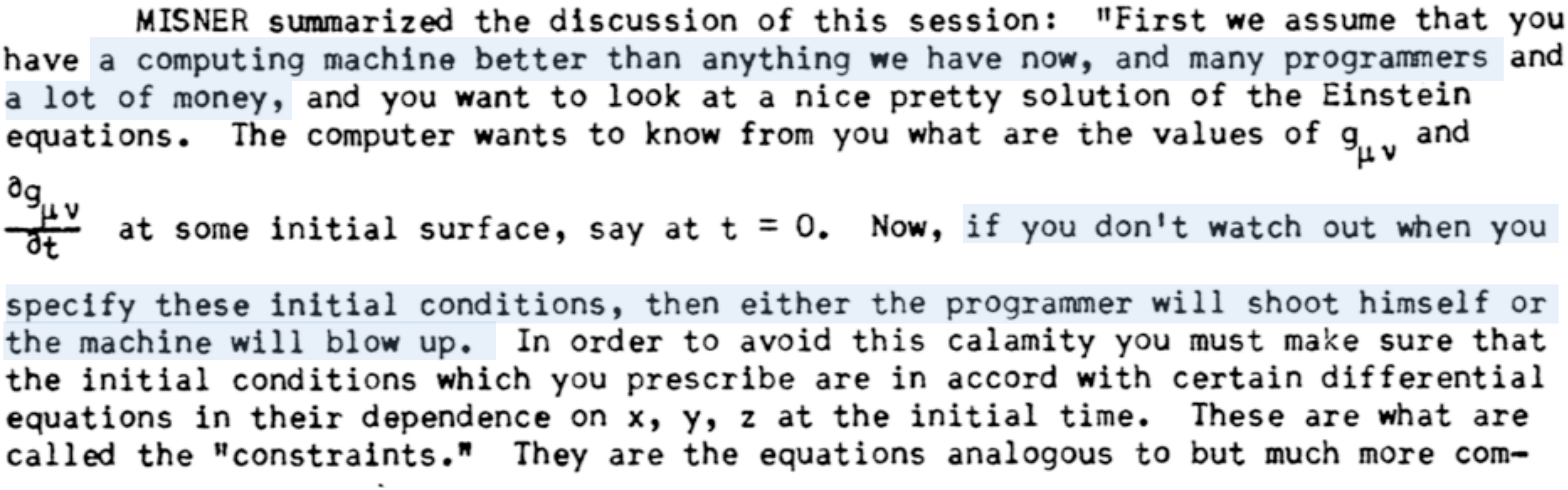
mass ratio

Buonanno & Sathyaprakash 14

Numerical Relativity Simulations



(K. Thorne)

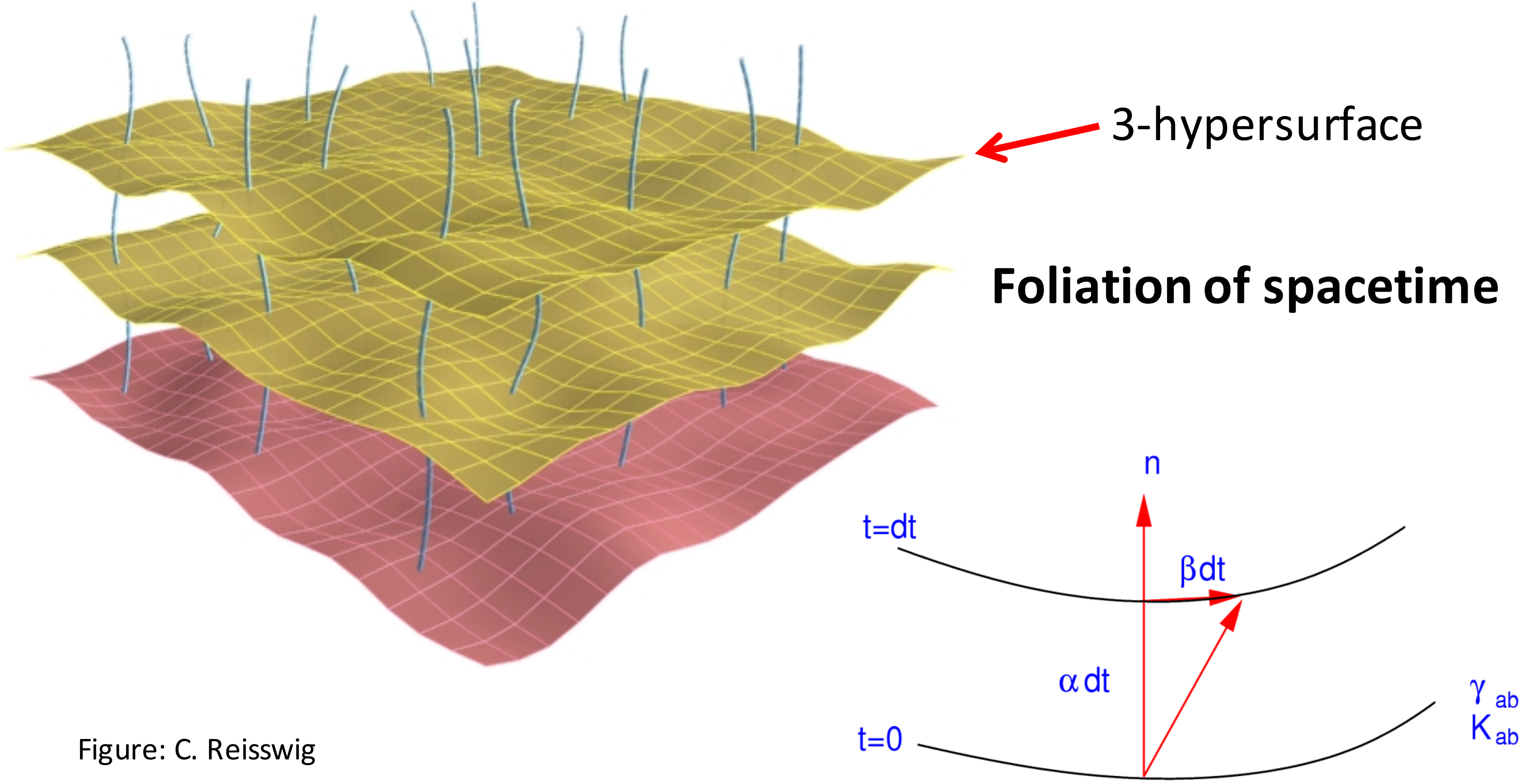


MISNER summarized the discussion of this session: "First we assume that you have a computing machine better than anything we have now, and many programmers and a lot of money, and you want to look at a nice pretty solution of the Einstein equations. The computer wants to know from you what are the values of $g_{\mu\nu}$ and $\frac{\partial g_{\mu\nu}}{\partial t}$ at some initial surface, say at t = 0. Now, if you don't watch out when you specify these initial conditions, then either the programmer will shoot himself or the machine will blow up. In order to avoid this calamity you must make sure that the initial conditions which you prescribe are in accord with certain differential equations in their dependence on x, y, z at the initial time. These are what are called the "constraints." They are the equations analogous to but much more com-

Proceedings of the GR1 Conference on the role of gravitation in physics
University of North Carolina, Chapel Hill [January 18-23, 1957] (via P. Laguna & D. Shoemaker)

-> It took until 2005 (Pretorius, Campanelli+, Baker+) to simulate first BBH merger!

Numerical Relativity



Foliation of spacetime

Figure: C. Reisswig

$$G^{\mu\nu} = \frac{8\pi G}{c^4} T^{\mu\nu}$$

- 12 first-order hyperbolic *evolution* equations.
- 4 elliptic *constraint* equations
- 4 coordinate gauge degrees of freedom: α, β^i.

Numerical Relativity

$$G^{\mu\nu} = \frac{8\pi G}{c^4} T^{\mu\nu}$$

Key issues

- Initial conditions must satisfy Einstein equations.
- No unique way to formulate evolution equations.
- Gauge freedom – how choose gauge conditions?
- Need combination of evolution equations + gauges that yield to numerically stable simulations.

BSSN Formulation Nakamura+87, Shibata & Nakamura 95, Baumgarte & Shapiro 99

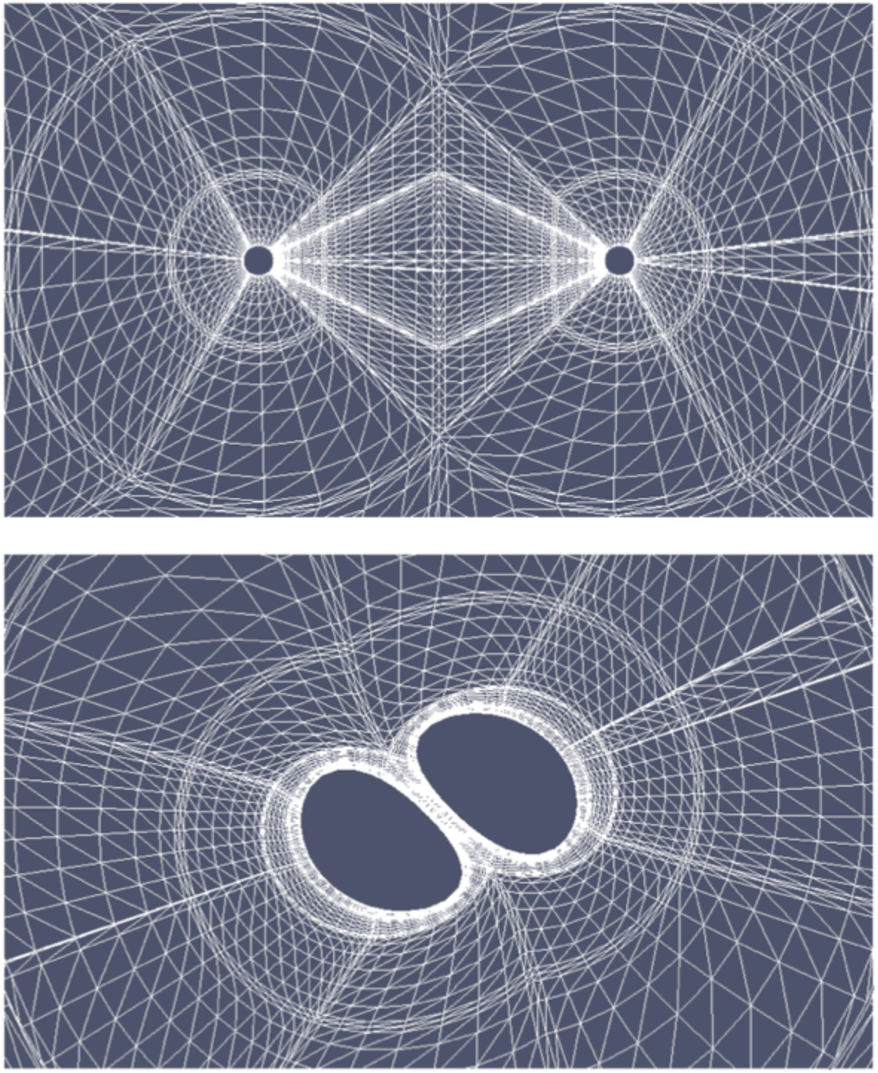
- Conformal-traceless reformulation of Arnowitt-Deser-Misner 59, York 79.
- Additional evolution equations, conditionally **strongly hyperbolic**.
- Sensitive to gauge choice; good gauges known.
- Most widely used evolution system today.

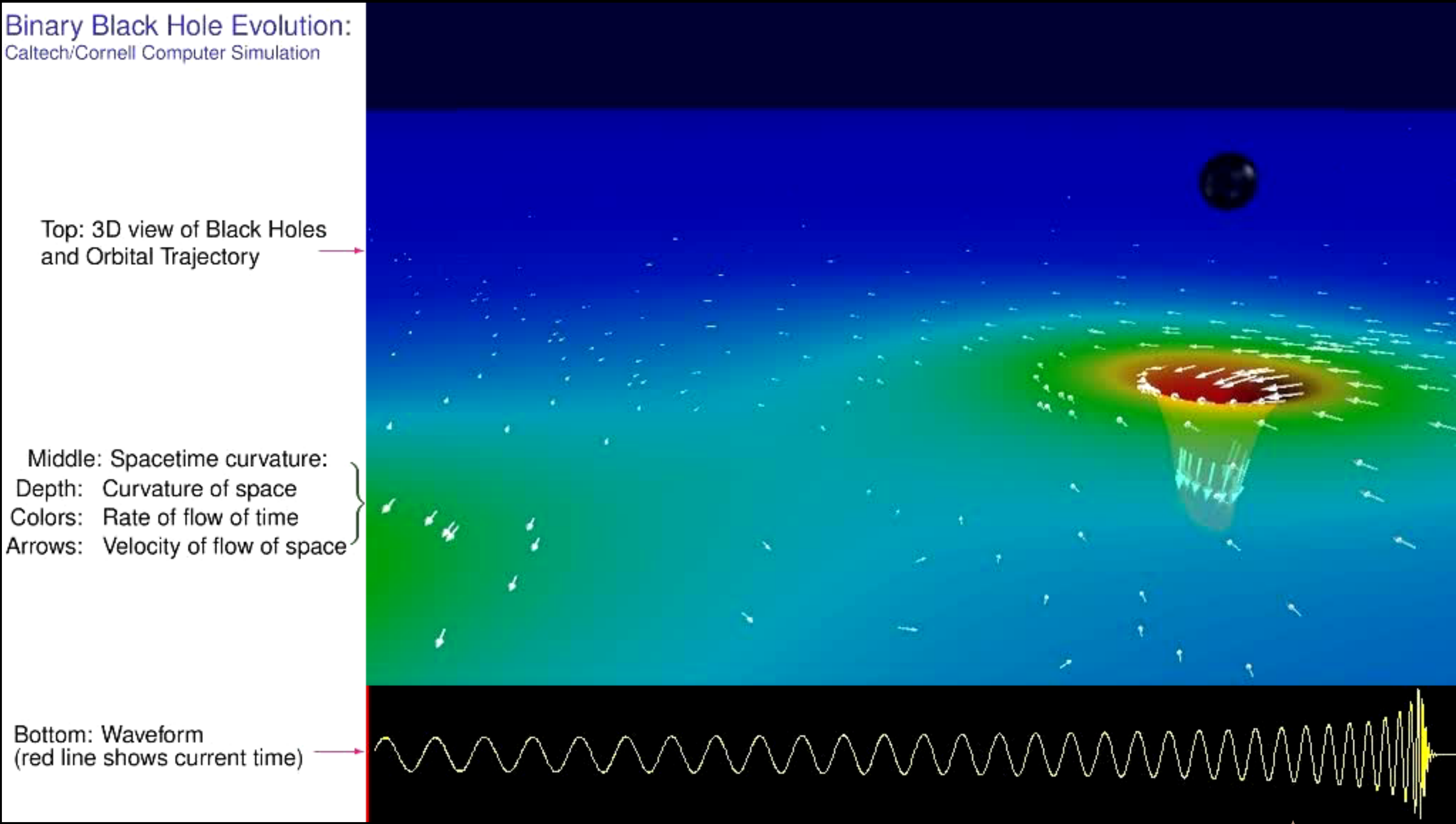
Generalized Harmonic Formulation Friedrich 85, Pretorius 05, Lindblom+ 06

- Choice of coordinates so that evolution equations wave-equation like. **Symmetric hyperbolic**.
- Sensitive to gauge choices, horizon boundary conditions.
- Used primarily by Caltech/Cornell SXS code **SpEC**.

Example Computational Approach: SpEC

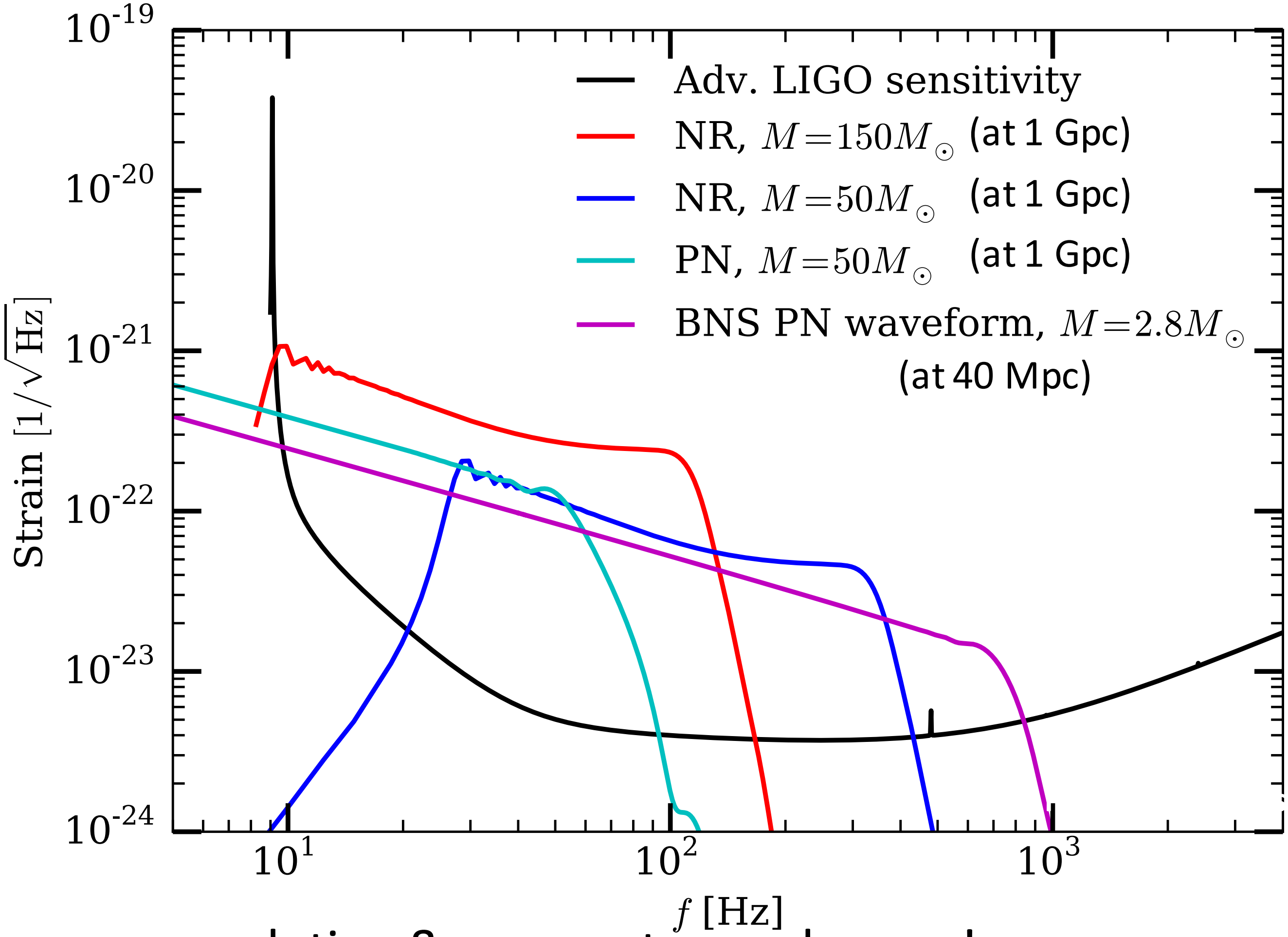
- **Sp**ectral **E**instein **C**ode: **SpEC**
  Caltech-Cornell-CITA-Fullerton
  Simulating eXtreme Spacetimes
  Collaboration (SXS)
- Generalized harmonic formulation.
- Explicit multi-domain, multi-frame pseudo-spectral methods. C++.
- Severely scaling limited > 48 cores.
  1 simulation with 40 orbits:
  3-6 months on 48 cores.
- Proprietary (closed source).
  More info on
  http://www.black-holes.org





Pfeiffer/Scheel

BBH & Advanced LIGO/Virgo



- BBH Source population & parameters unknown!
- Present EOB/Phenom models calibrated for moderate (mostly aligned) spins, mass ratios m1/m2 ~ 1 – 1:10.
- NR simulations needed for rest of parameter space (high spin, precession).

Complete Waveforms: Problems

- 7D parameter space – at least 10^7 simulations needed.
- Many cycles in sensitivity band: $N \sim \frac{4}{2\pi} \times 10^4 \left(\frac{M}{M_\odot}\right)^{-5/3}$
  O(100) for 5+5 $M_\odot$

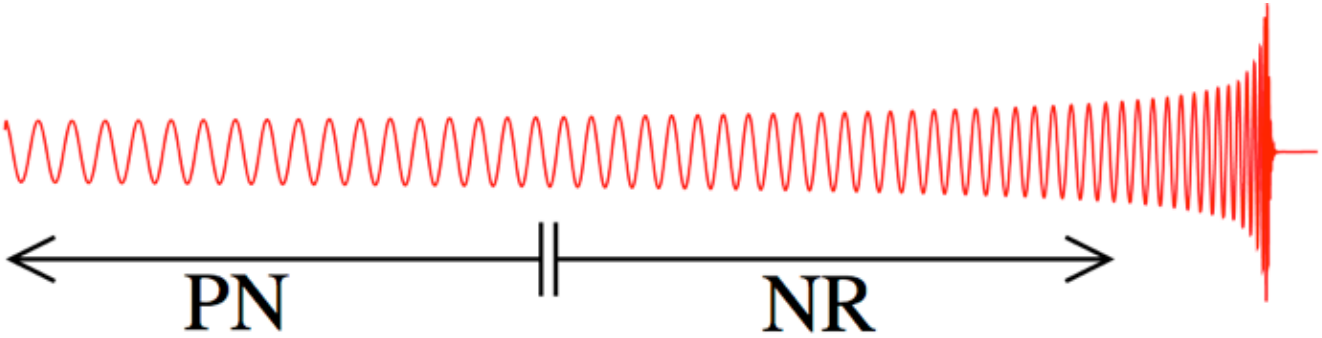
  -> Impossible with numerical relativity simulations!

Complete Waveforms: Solutions

- Many cycles in sensitivity band:
  (~130 for 5+5 $M_\odot$)

$$N \sim \frac{4}{2\pi} \times 10^4 \left(\frac{M}{M_\odot}\right)^{-5/3}$$

Solution: “**Hybridization**”



Further problem:
# of required NR cycles unknown; dependent on system parameters.

Complete Waveforms: Solutions

- 7D parameter space – at least 10^7 simulations needed.

Solution: “**Surrogate Model**” **via Reduced-Order Modeling**

Basic Idea:

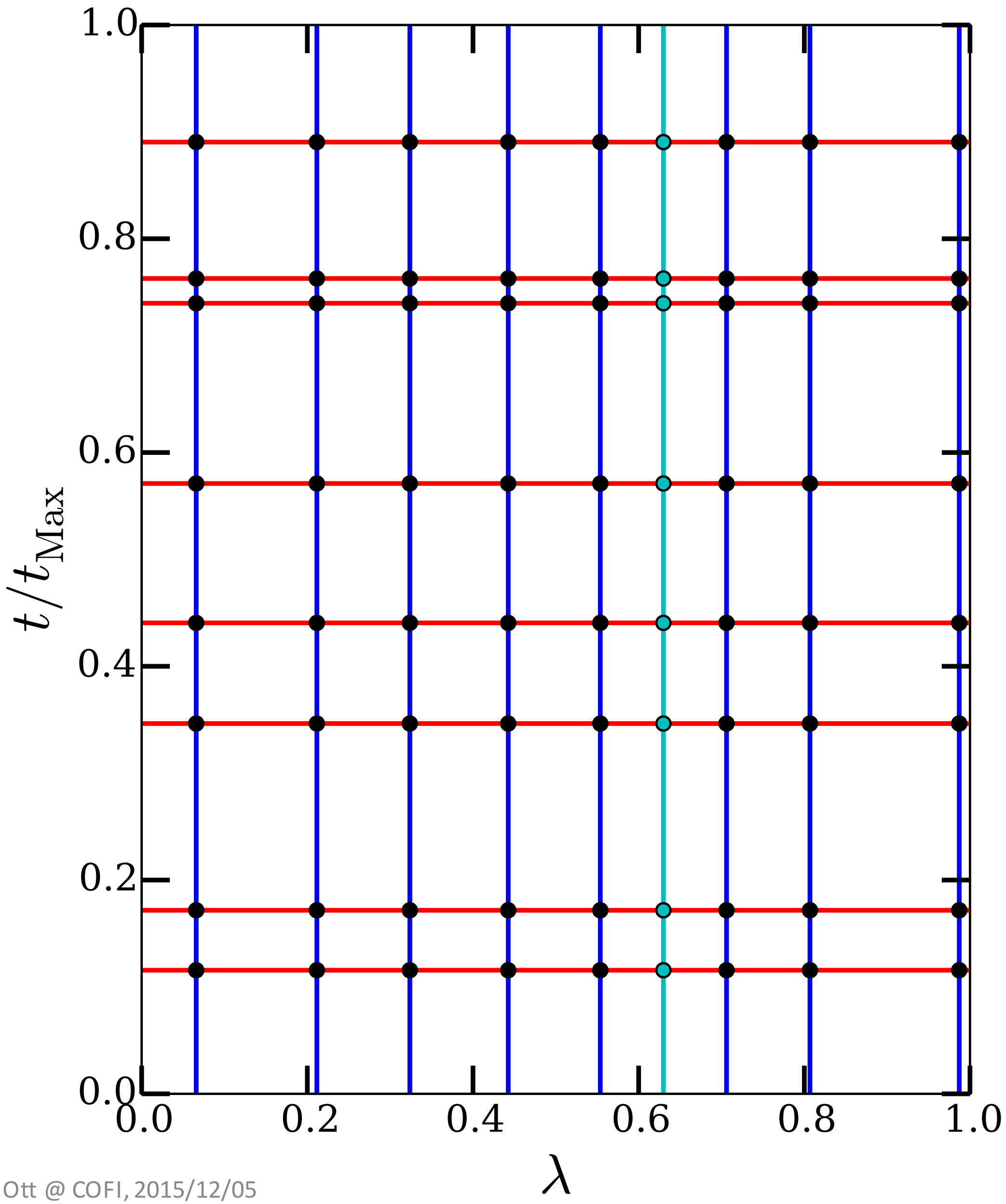
(1) Intelligently & sparsely sample parameter space with O(1,000) numerical relativity simulations.
(2) Interpolate between waveforms to obtain waveform for any set of BBH parameters.

Goal: Build model that is as good as NR and
can be a substitute for NR simulations (surrogate).

Numerical Relativity Surrogate Models

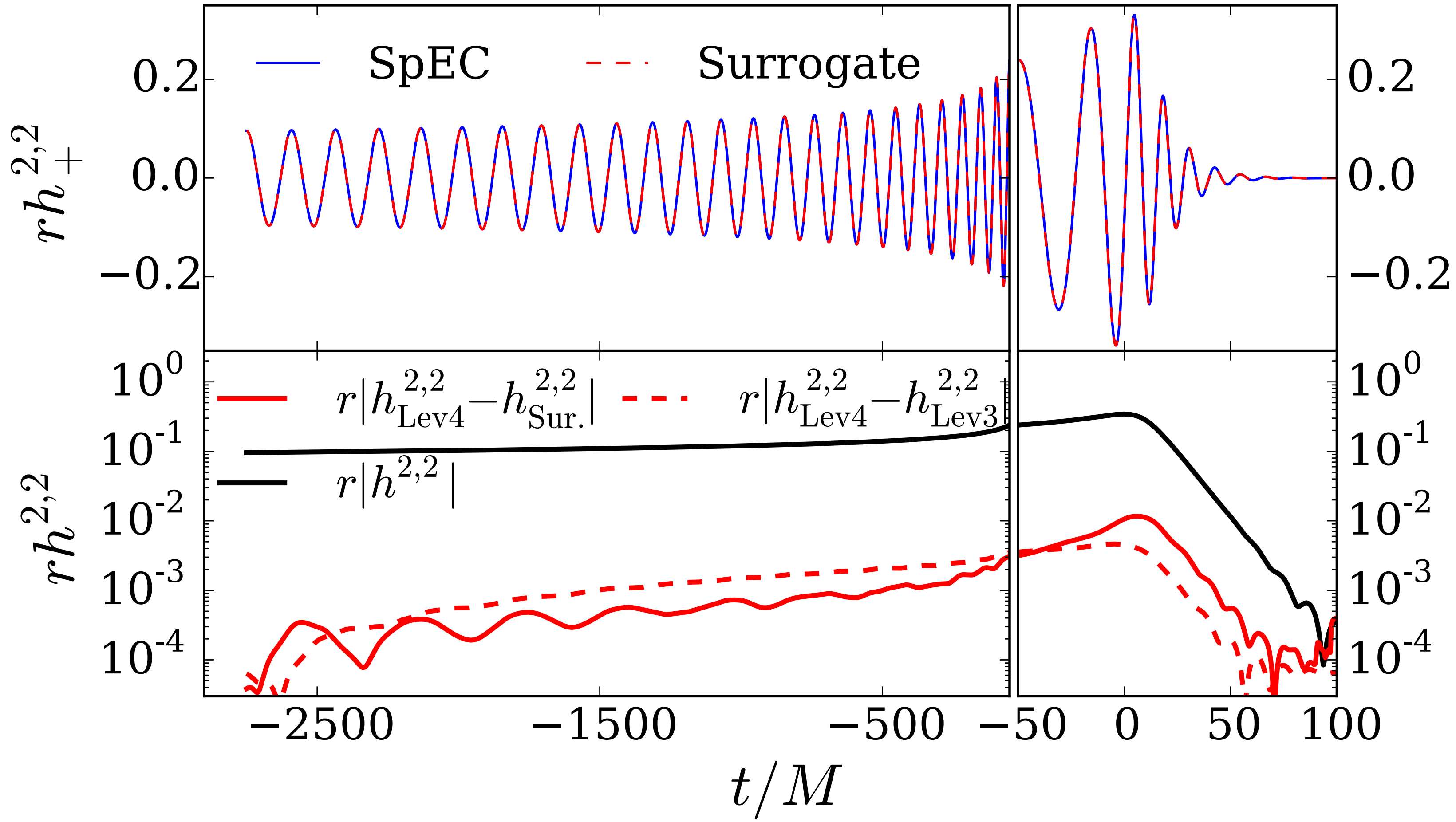
(by Jonathan Blackman)

- Have *N* reduced basis waveforms (blue lines)
- Fit data at *N* empirical time nodes (red lines) using known data (black dots)
- Evaluate fits at arbitrary parameter(s) λ (cyan dots)
- Use empirical interpolant to uniquely determine new data (cyan line)



Numerical Relativity Surrogate Models

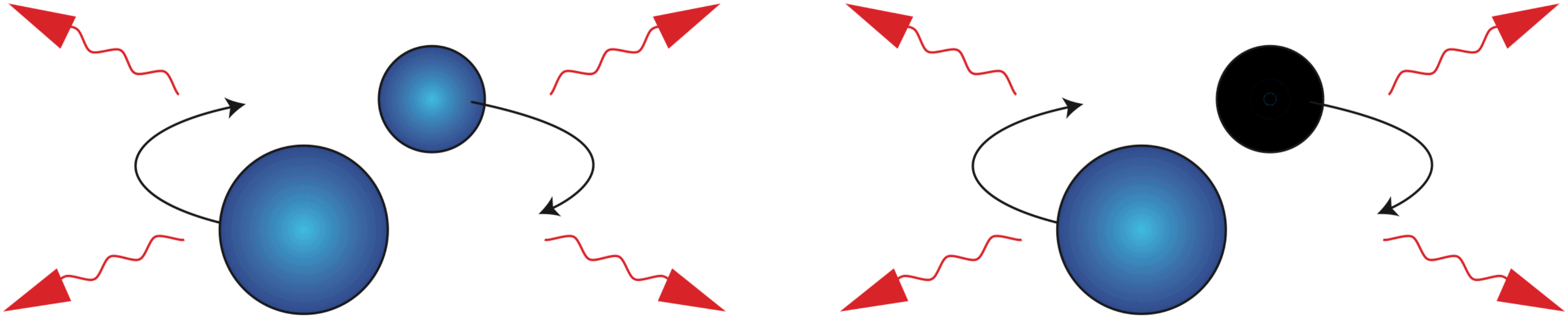
(by Jonathan Blackman, Blackman+15)



1D surrogate model (mass ratio).

Work on multi-D surrogate models in progress.

Example 2: NSNS and BHNS Mergers



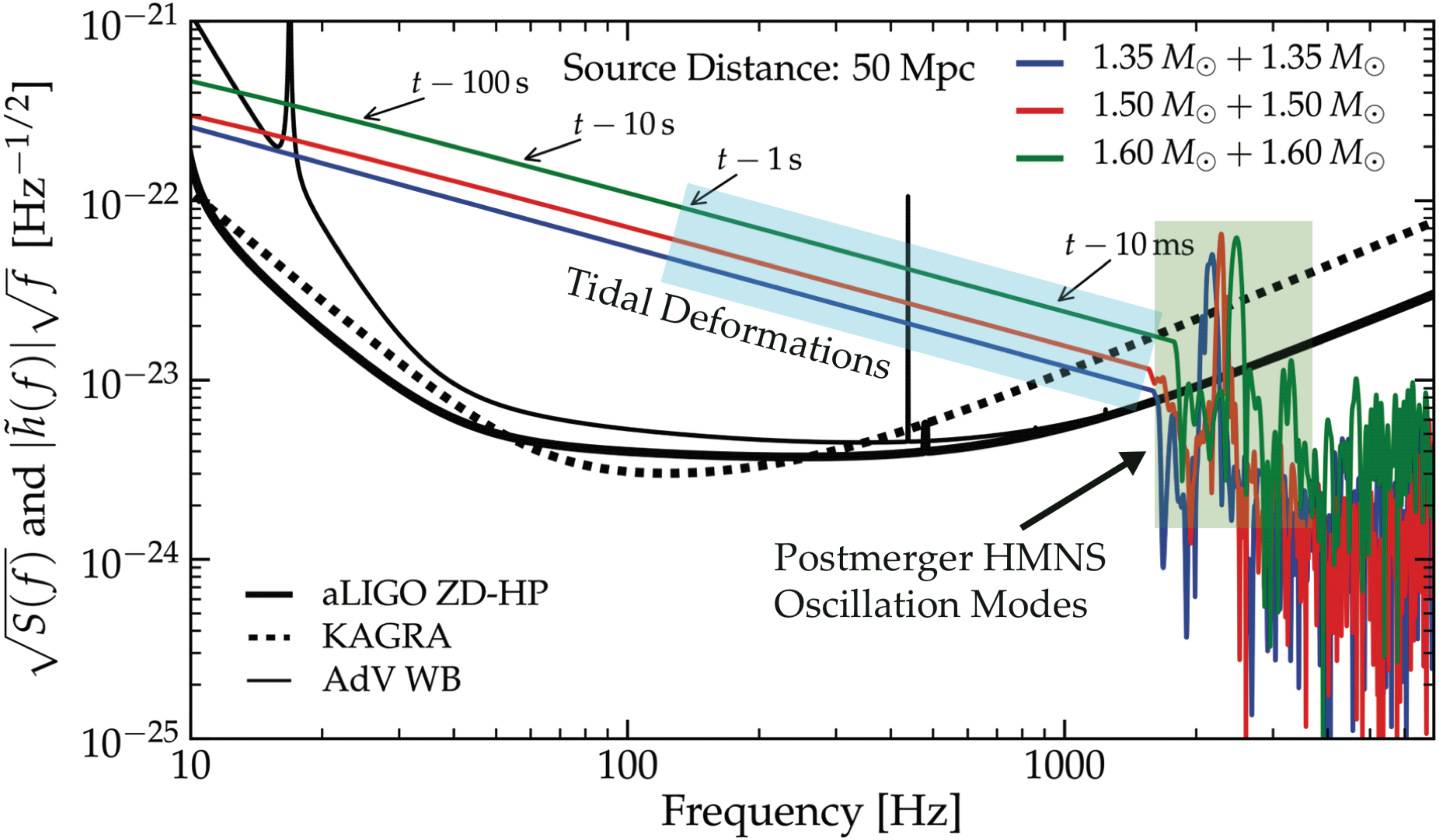
credit: D. Tsang

$M_1 \sim M_2 \sim 1.4\ M_{Sun}$
-> galactic NSNS binaries!

$M_{BH} \sim$ 7-10 x M_{NS} (Belczynski+'10)
(but no BHNS systems known)

- Harder: must simulate also matter (and magnetic fields)
  -> (magneto)-hydrodynamics, neutrinos, nuclear EOS.
- But: lower mass
  -> PN approx. valid for much/most(NSNS) of inspiral.

NSNS in the Advanced Detector Band

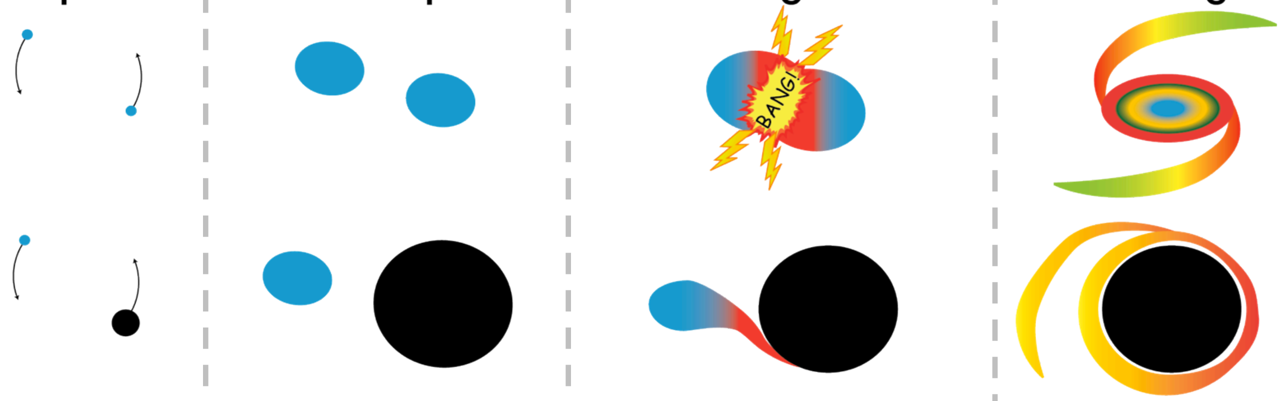


- Potential to constrain nuclear equation of state.

Multi-Physics, Multi-Messenger Astrophysics

Nuclear Equation of State (EOS)   **Crust Physics & Superfluidity (SF)**

Neutrinos/Neutrino Interactions   **Nuclear Reactions & Opacities**

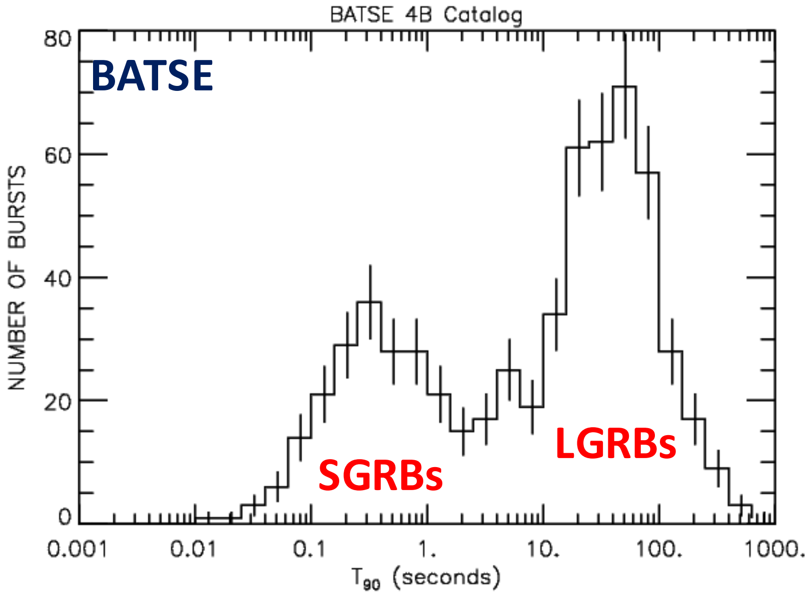


Inspiral	Late Inspiral	Merger	Postmerger		Late Time (t >~1s)
Inspiral	Late Inspiral	Merger/Disruption	Postmerger	GRB?	EM afterglow/ counterpart
	Crust/SF				
	EOS	hot EOS	hot EOS	hot EOS	hot EOS
		Neutrinos	Neutrinos	Neutrinos	Neutrinos
			Nuclear	Nuclear	Nuclear

Gamma-Ray Bursts

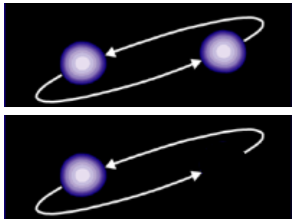
[Reviews: e.g. Woosley & Bloom ‘06, Piran ‘05, Meszaros ’05]

- Two general groups of GRBs:
  Long and Short
- Favored model:
  Beamed Ultrarelativistic outflow
  emitting γ-rays.

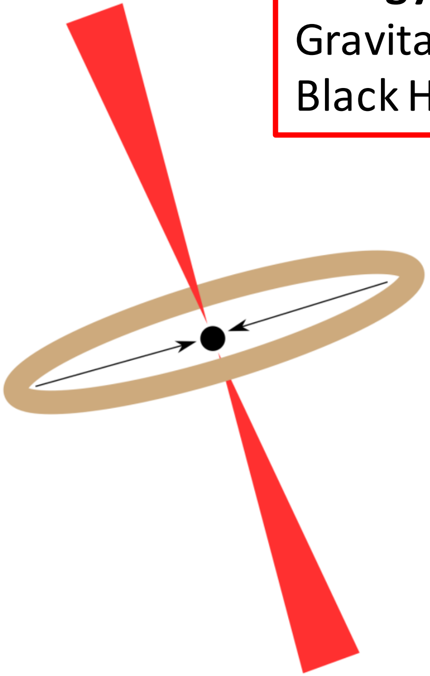


Simplistic Engine Picture:

NS-NS / NS-BH merger



Massive H/He-poor Star



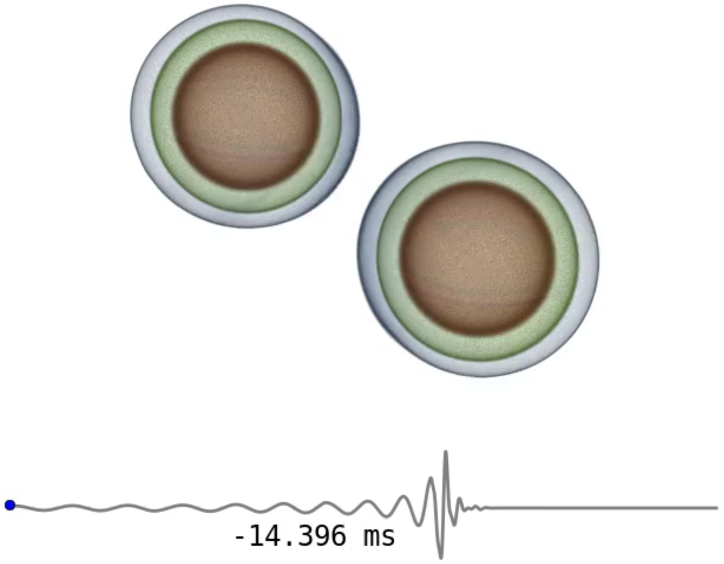
Energy sources:
Gravitational energy (accretion)
Black Hole/NS spin energy.

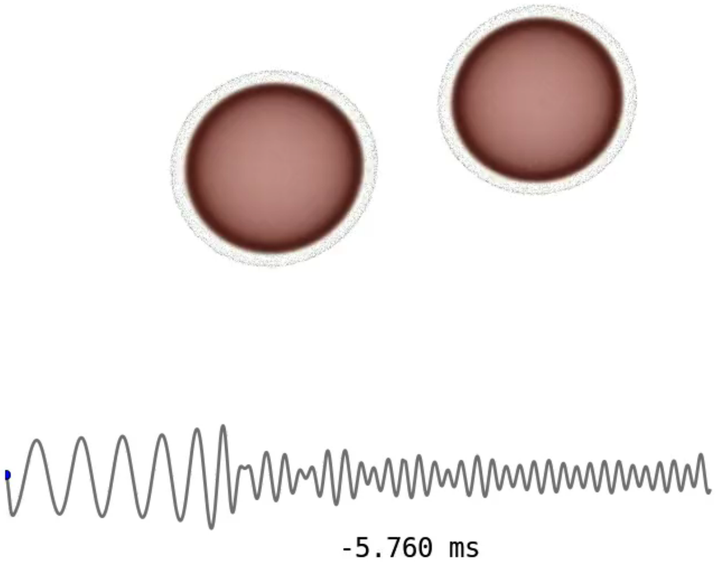
Disk Mass:
~0.1 M_{Sun} → **SGRB**

Disk Mass:
~1 M_{Sun} → **LGRB**

Mediating Processes:
Neutrino Pair Annihilation
Magnetohydrodynamics

NSNS Simulations: Outcomes

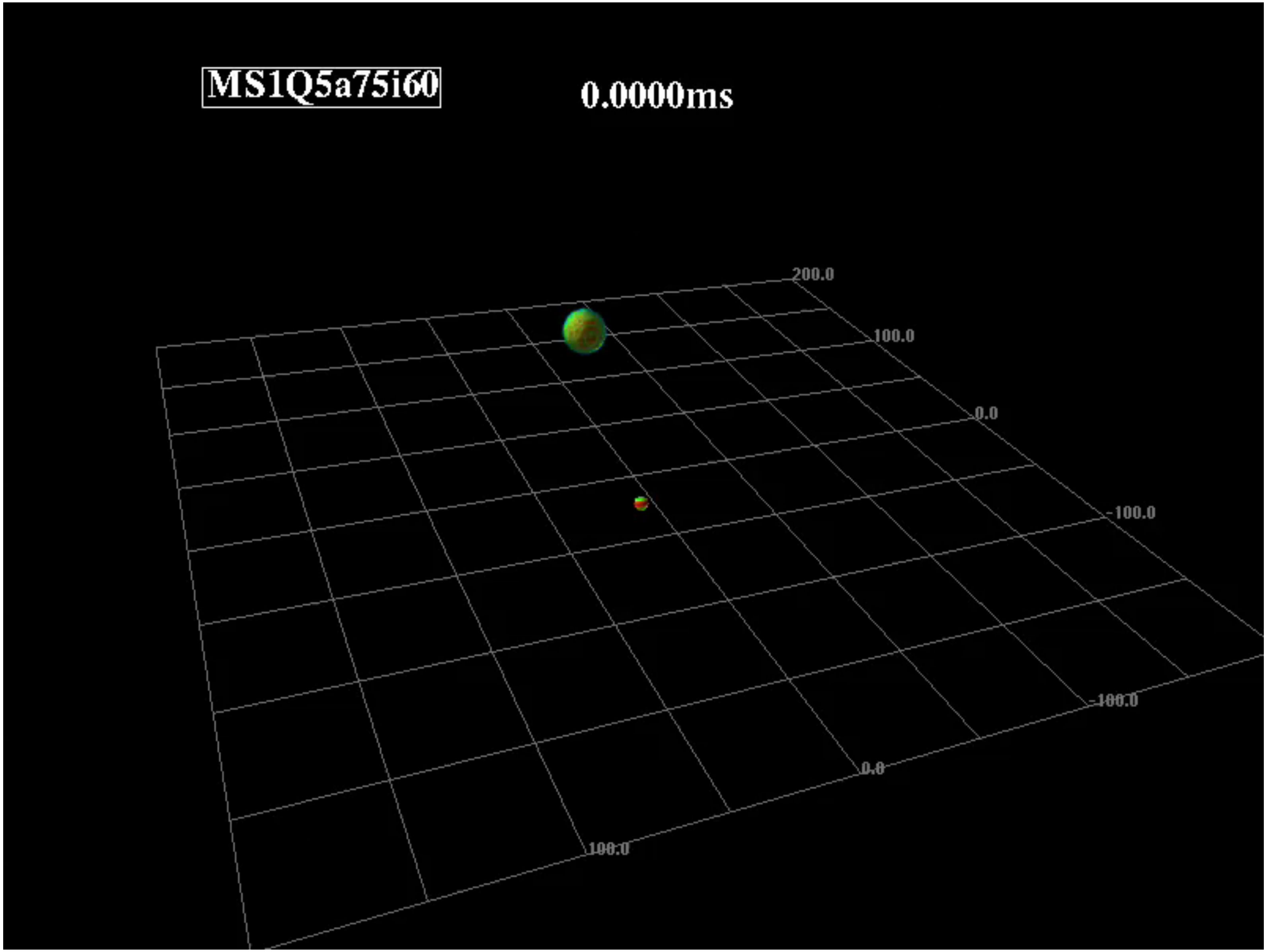




Sensitivity to system mass, mass ratio, and nuclear EOS.

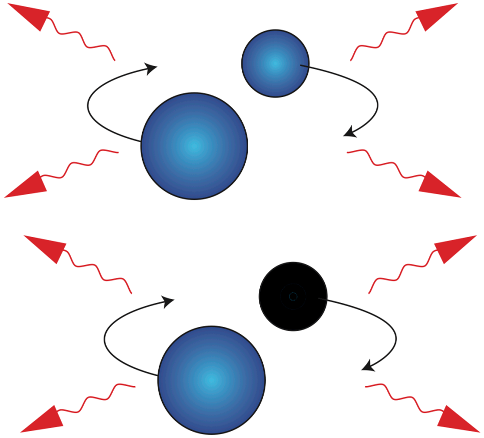
BHNS Merger Scenario

Kyohei Kawaguchi



NSNS/NSBH Modeling and Simulation

credit: D. Tsang



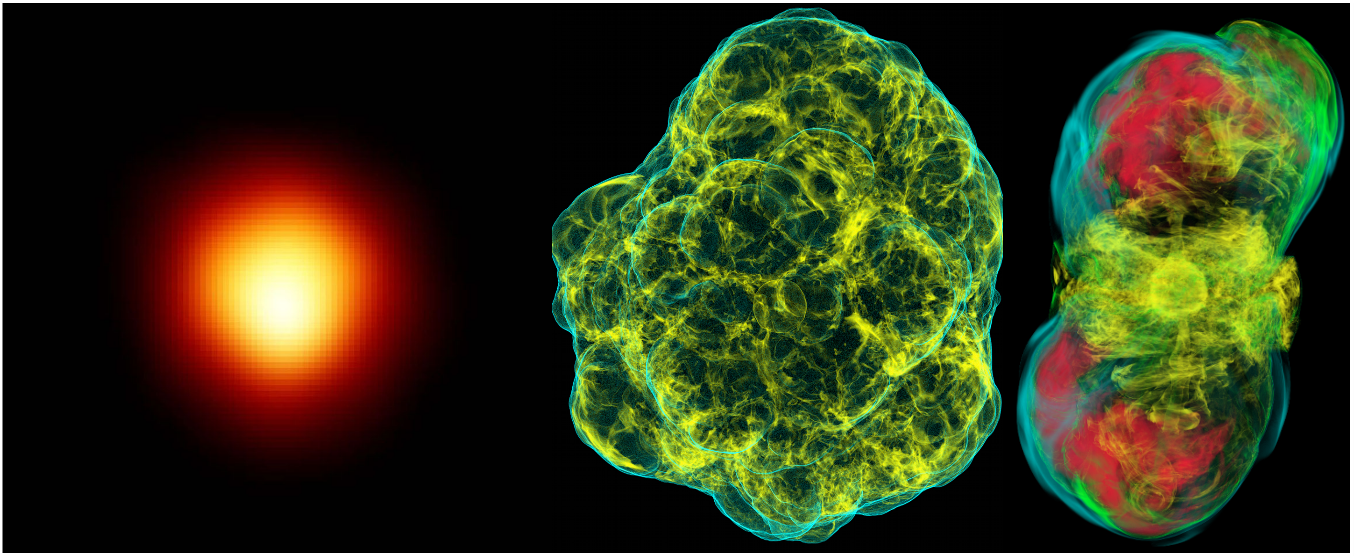
- **NSNS**:
  PN approximation valid through inspiral, multi-physics NR+GR(M)HD simulation for merger/postmerger evolution.

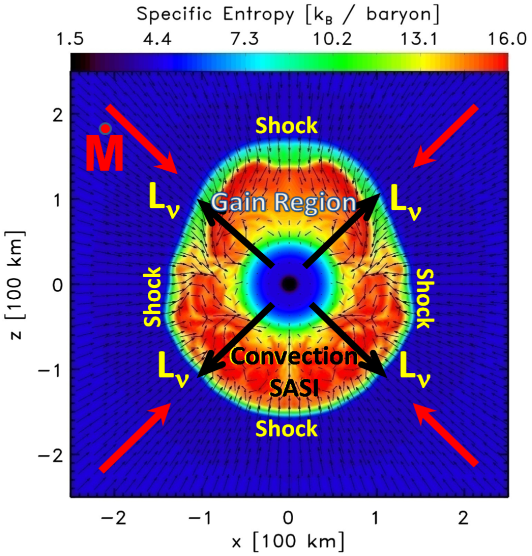
- **BHNS**:
  PN approximation valid in inspiral if mass ratio M_{BH}/M_{NS} small and BH spin small.

  But: most likely BH spin large, M_{BH}/M_{NS} > ~7:1.
  -> need long NR+GR(M)HD BHNS inspiral simulations.

Example 3: Core-Collapse Supernovae

- Explosions of massive stars: **Gravity bombs**.





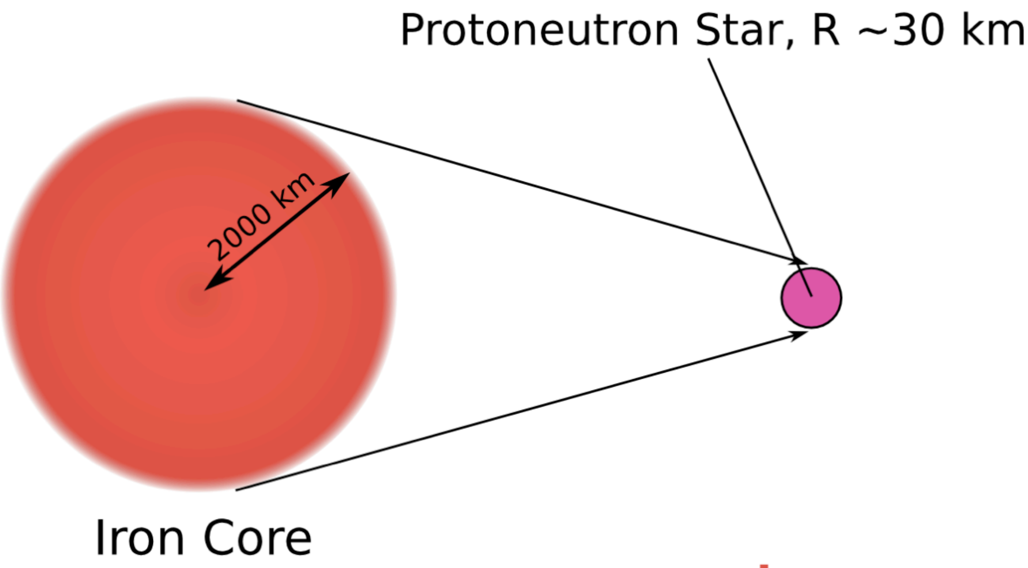
Core-Collapse Supernovae:

Explosions of Massive Stars $8M_\odot \lesssim M \lesssim 130M_\odot$

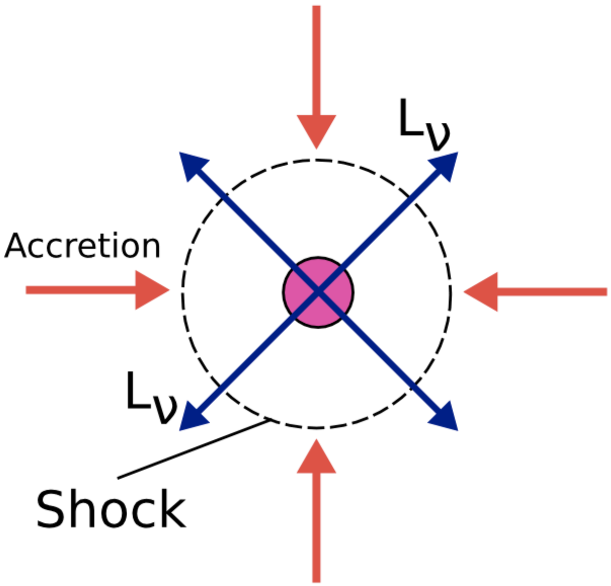
© Anglo-Australian Observatory

Supernova 1987A
Large Magellanic Cloud
Progenitor:
BSG Sanduleak -69° 220a, ≈18 M_{SUN}

Reminder: Core Collapse Basics



Reviews:
Bethe’90
Janka+’12



Nuclear equation of state (EOS) stiffens at nuclear density.

Inner core (~0.5 M_{Sun})
-> protoneutron star core.
Shock wave formed.

Outer core accretes onto shock & protoneutron star with O(1) $M_{\odot}$/s.

-> **Shock stalls at ~100 km, must be “revived” to drive explosion.**

Core-Collapse Supernova Energetics

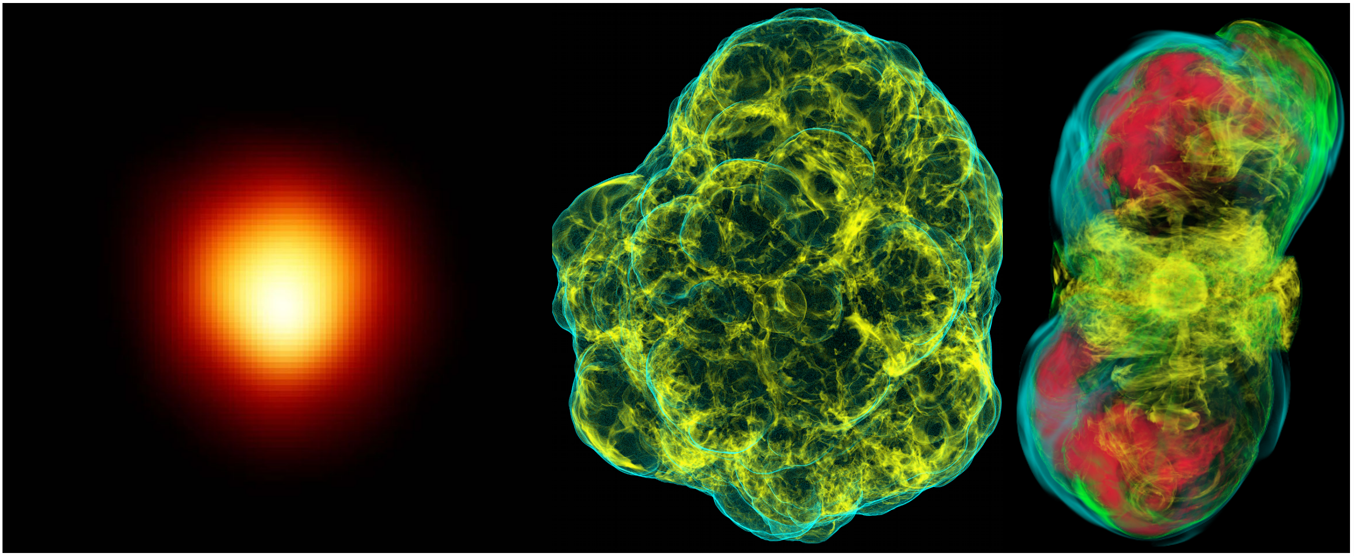
- Collapse to a neutron star: ~3 x 10^{53} erg = 300 [B]ethe **gravitational energy** (≈0.15 $M_{Sun}c^2$).
  -> Any explosion mechanism must tap this reservoir.

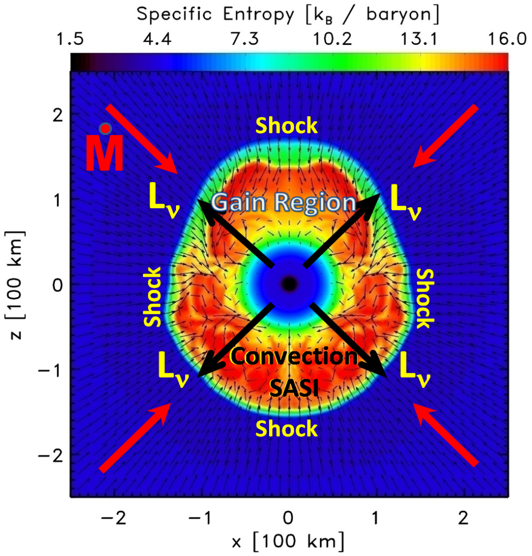
- ~10^{51} erg = 1 B kinetic and internal energy of the ejecta.
  (Extreme cases: 10 B; "hypernova")

- 99% of the energy is radiated in neutrinos on $\mathcal{O}(10)$s
  -> Strong evidence from SN 1987A neutrino observations.

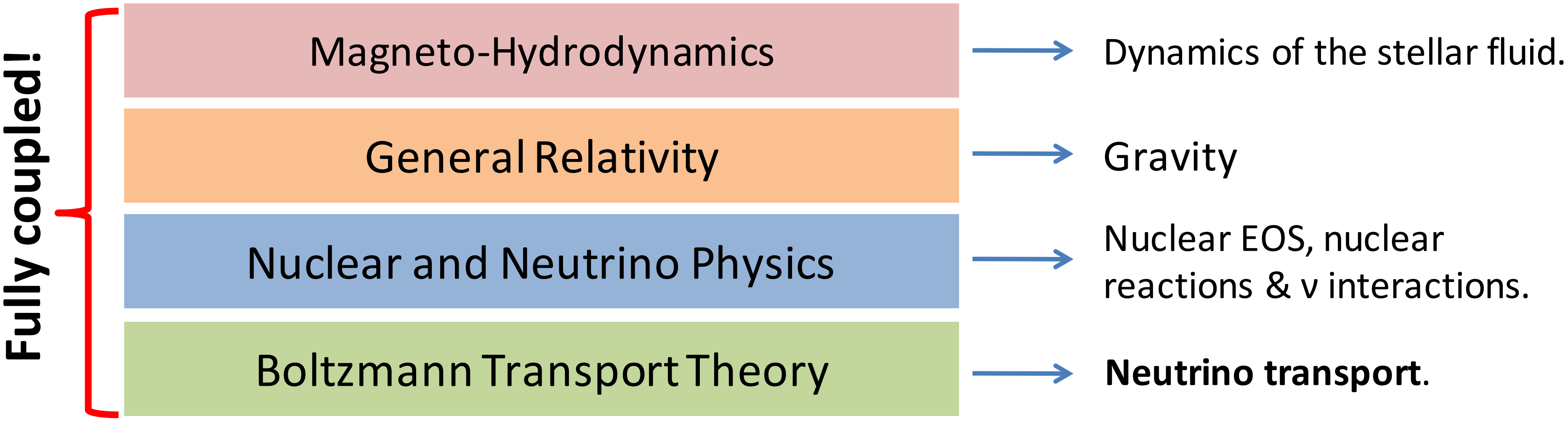
Example 3: Core-Collapse Supernovae

- Explosions of massive stars: **Gravity bombs**.
- Multi-dimensional, multi-physics, **multi-scale** problem.
- What is the detailed explosion mechanism?
- Sources of GW bursts -> GWs carry information on multi-D dynamics and explosion mechanism (Ott 09).
- Multi-Messenger Astronomy -> neutrinos, GWs, photons!





Detailed CCSN Simulations: Ingredients



Fully coupled!

Magneto-Hydrodynamics → Dynamics of the stellar fluid.

General Relativity → Gravity

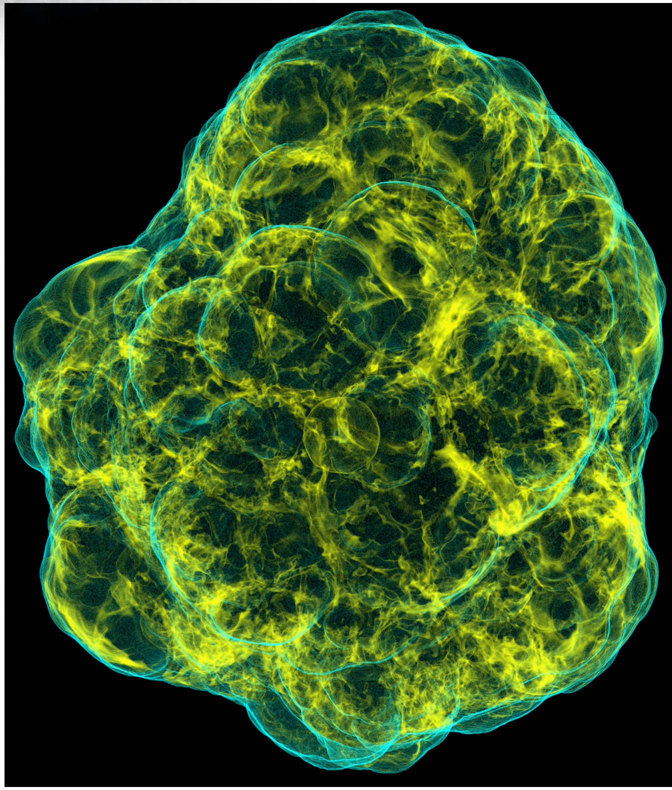
Nuclear and Neutrino Physics → Nuclear EOS, nuclear reactions & ν interactions.

Boltzmann Transport Theory → **Neutrino transport**.

- Additional Complication: Core-Collapse Supernovae are 3D
  - Rotation, **fluid instabilities**, **magnetic fields**, multi-D stellar structure from convective burning, etc.
- Full problem: 3D space, 3D momentum space + time

The 3D Frontier – Petascale Computing!

- Modeling: only for photons (light curve, spectra).
- **Simulation required for everything else.**
- Some early work: Fryer & Warren 02, 04
- **Loads of new work since ~2010**:
  Fernandez 10, Nordhaus+10, Takiwaki+11,13, Burrows+12, Murphy+13, Dolence+13, Hanke+12,13, Kuroda+12, Ott+13, Couch 13, Takiwaki+13, Couch & Ott 13, 15, Abdikamalov+15, Couch & O'Connor 14, Lentz+15, Melson+15ab, Cardall&Budiardja 15, Radice+15, Summa+15



Ott+2013

Ott+2013
Caltech,
full GR,
parameterized
neutrino heating

-6.18 ms

Caltech

Gravitational-Waves from Core-Collapse Supernovae

Reviews: Ott 09, Kotake 11, Fryer & New 11

Need:

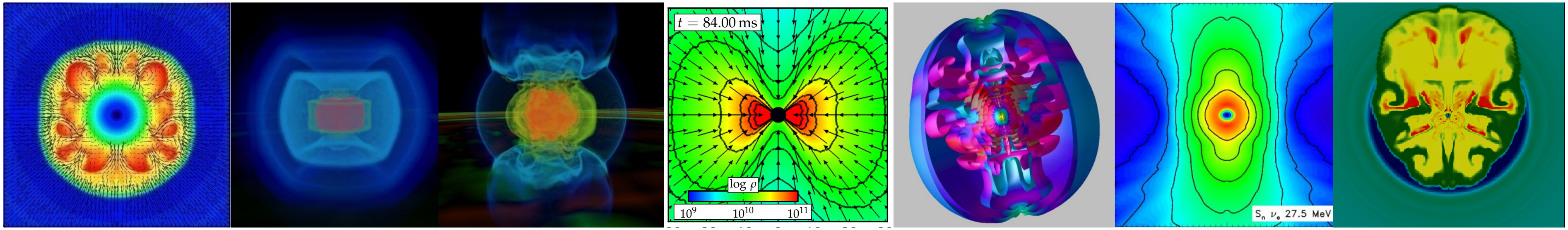
$$h^{TT}_{jk}(t,\vec{x}) = \left[\frac{2}{c^4}\frac{G}{|\vec{x}|}\ddot{I}_{jk}\left(t-\frac{|\vec{x}|}{c}\right)\right]^{TT} \rightarrow$$

accelerated aspherical (quadrupole) mass-energy motions

Candidate Emission Processes:

- Turbulent convection & shock instability (SASI)
- Rotating collapse & bounce
- 3D rotational instabilities
- Aspherical mass-energy outflows:
  -> aspherical neutrino emission
  -> aspherical explosion

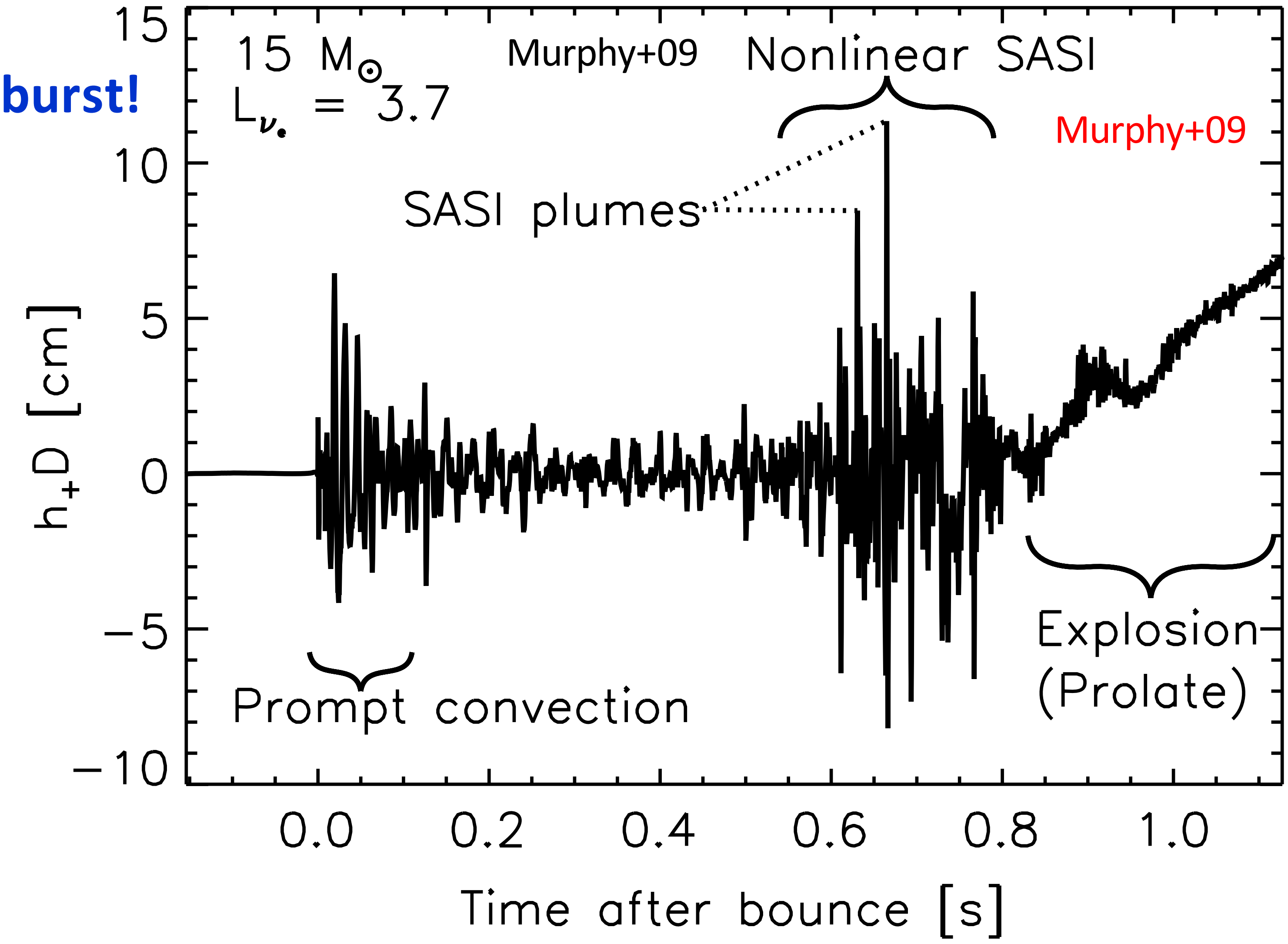
GW emission weak – detectable only for galactic CCSN



GWs from Convection & Standing Accretion Shock Instability

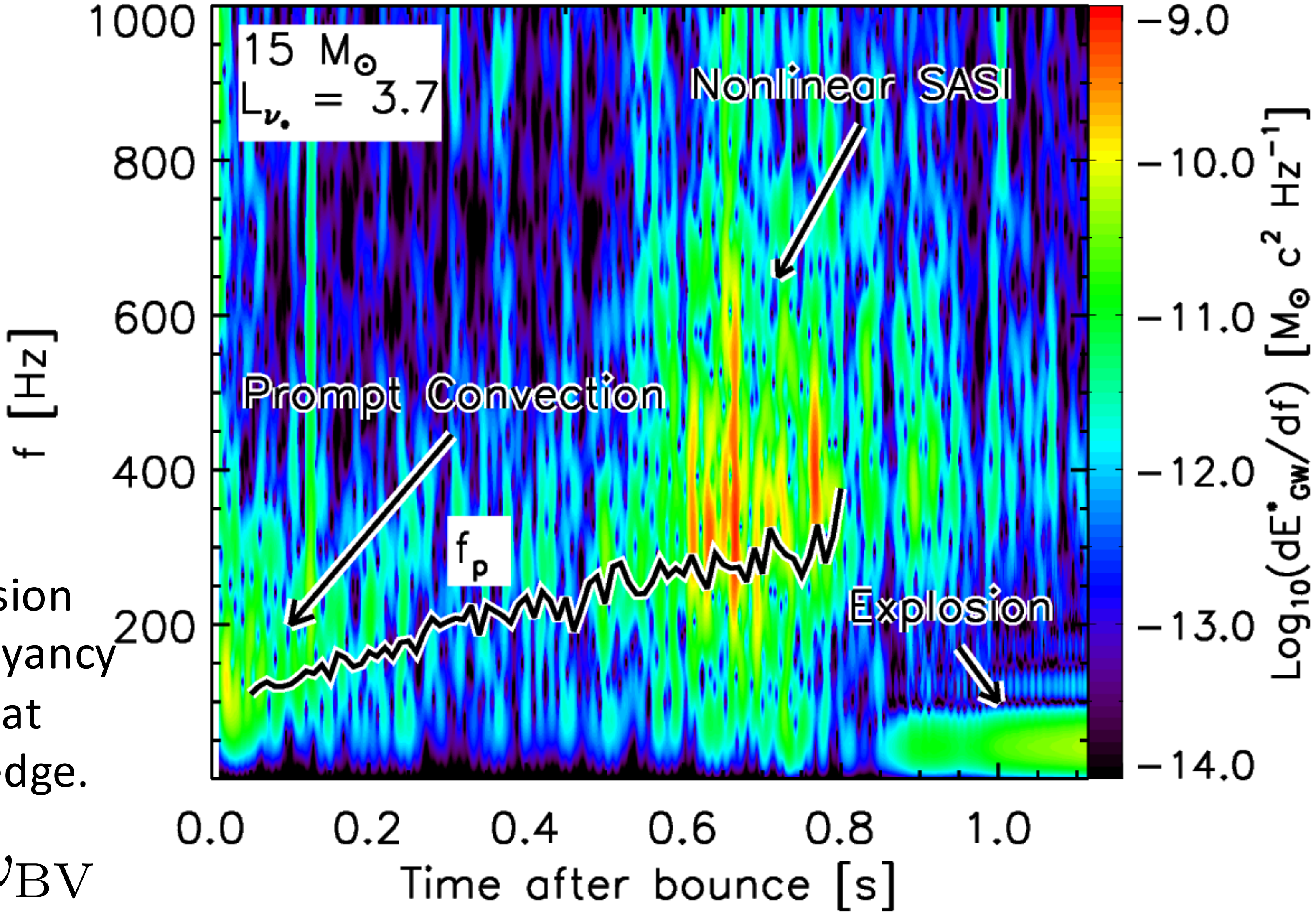
Recent work: Murphy+09, Kotake+09, 11, Yakunin+10, E. Müller+12, B.Müller+13

GW burst!



Time-Frequency Analysis of GWs

Murphy, Ott, Burrows 09, see also B. Müller+13



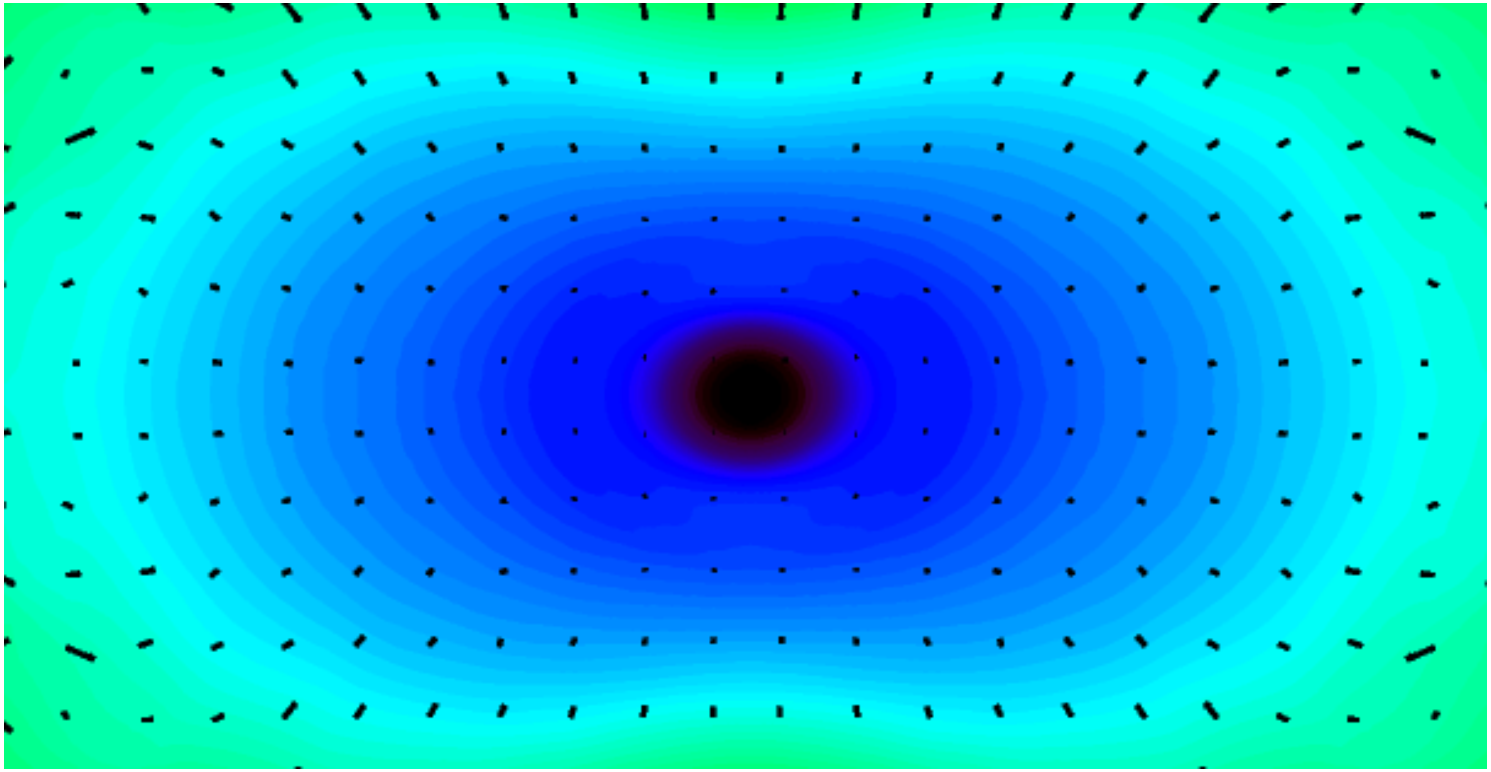
Peak emission traces buoyancy frequency at proto-NS edge.

$$f_p \sim \frac{\omega_{\mathrm{BV}}}{2\pi}$$

(buoyancy frequency)

GWs from Rotating Collapse & Bounce

Recent work: Dimmelmeier+08, Scheidegger+10, Ott+12, Abdikamalov+14

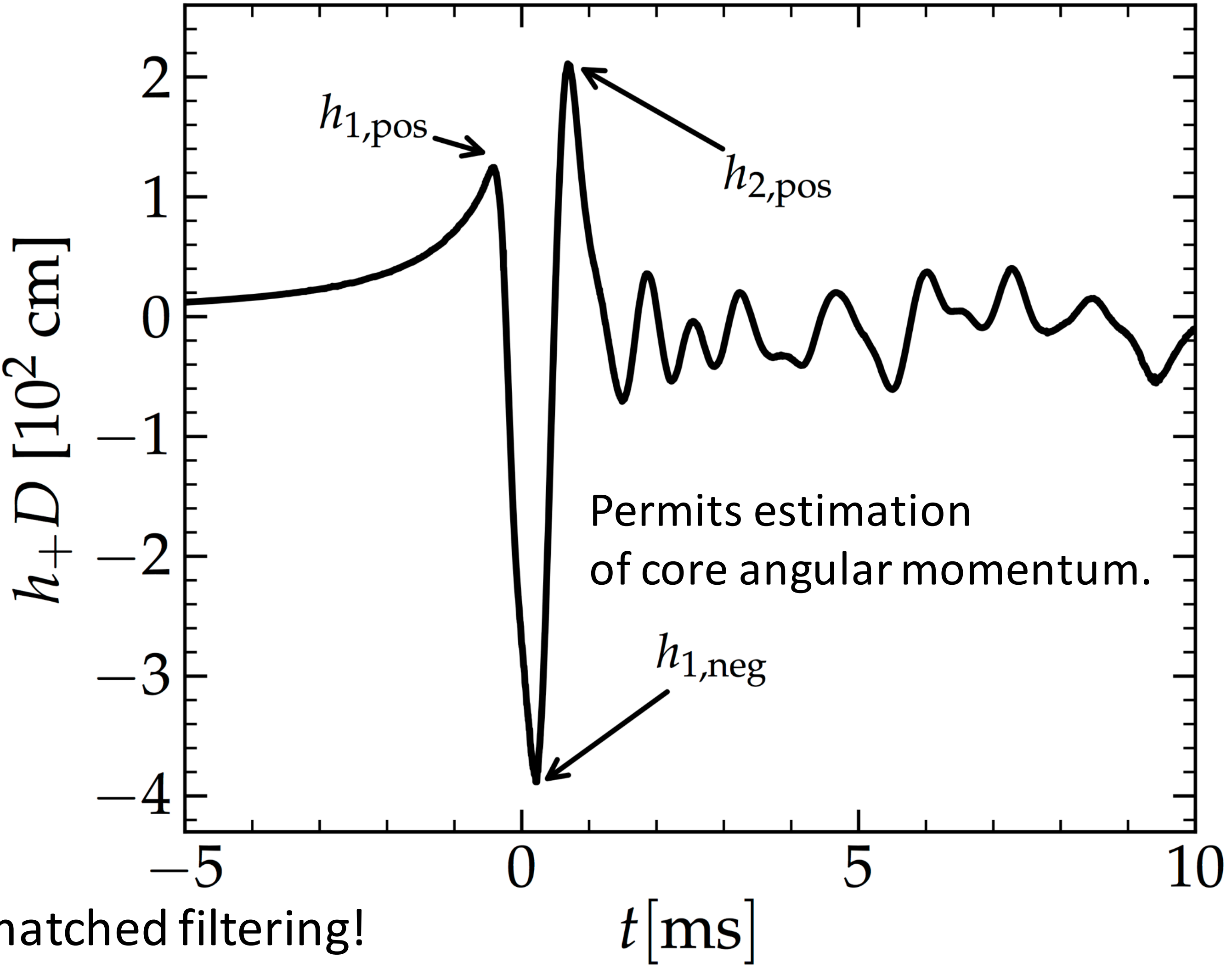


- **Axisymmetric: ONLY h_+**
- Simplest GW emission process: **Rotation** + mass of inner core + **gravity** + **stiffening of nuclear EOS**
- Strong signals for rapid rotation (-> millisecond proto-NS).

GWs from Rotating Collapse & Bounce

Recent work: Dimmelmeier+08, Scheidegger+10, Ott+12, Abdikamalov+14

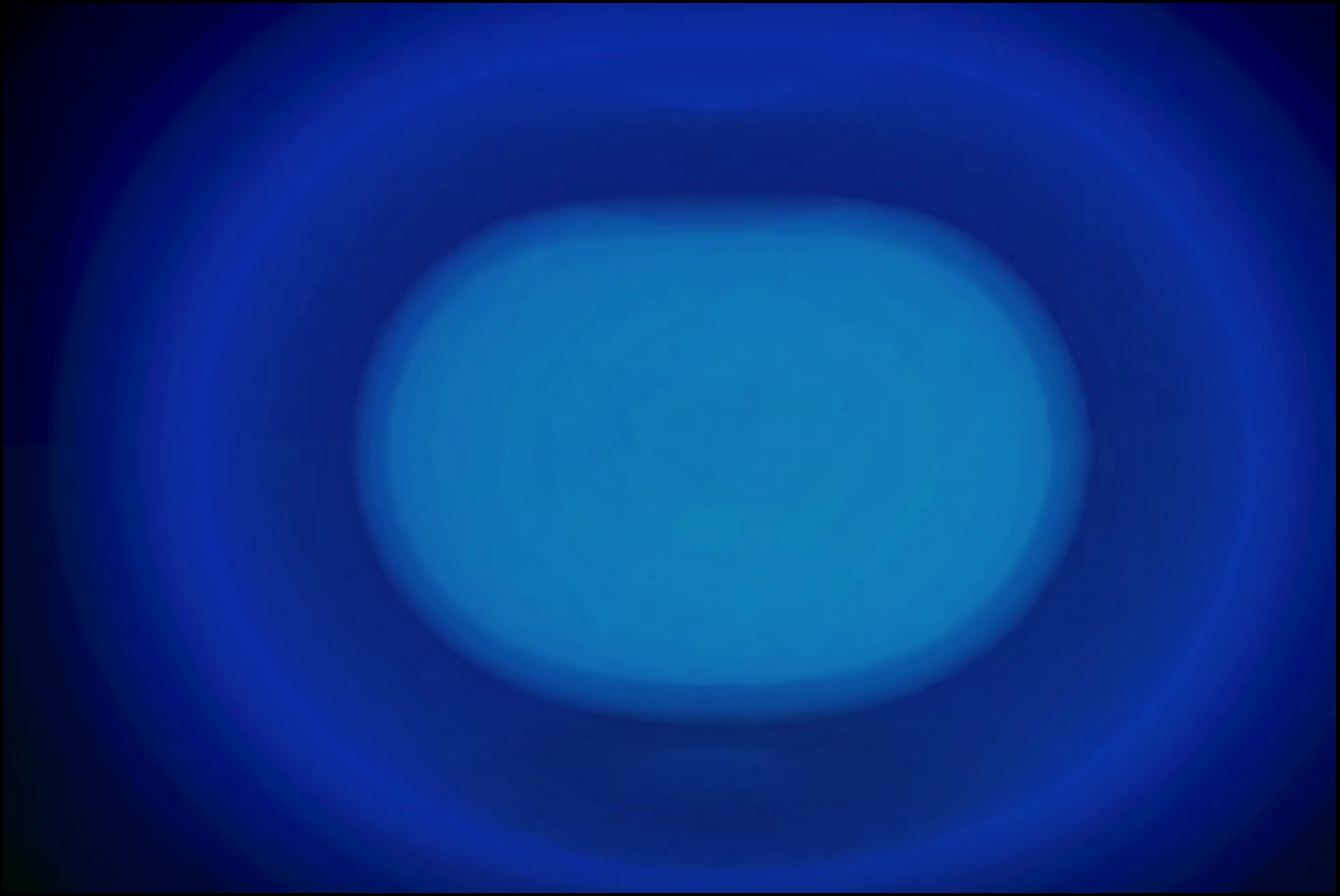
Simple signal features: Axisymmetric rotating collapse



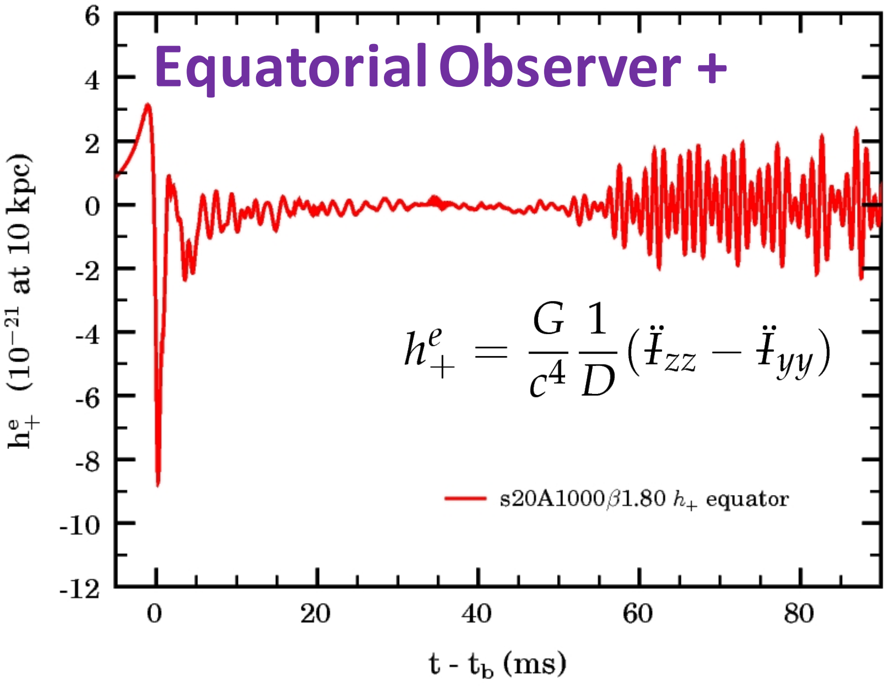
Permits estimation
of core angular momentum.

Can (almost)
be used for matched filtering!

3D Rotational Instabilities

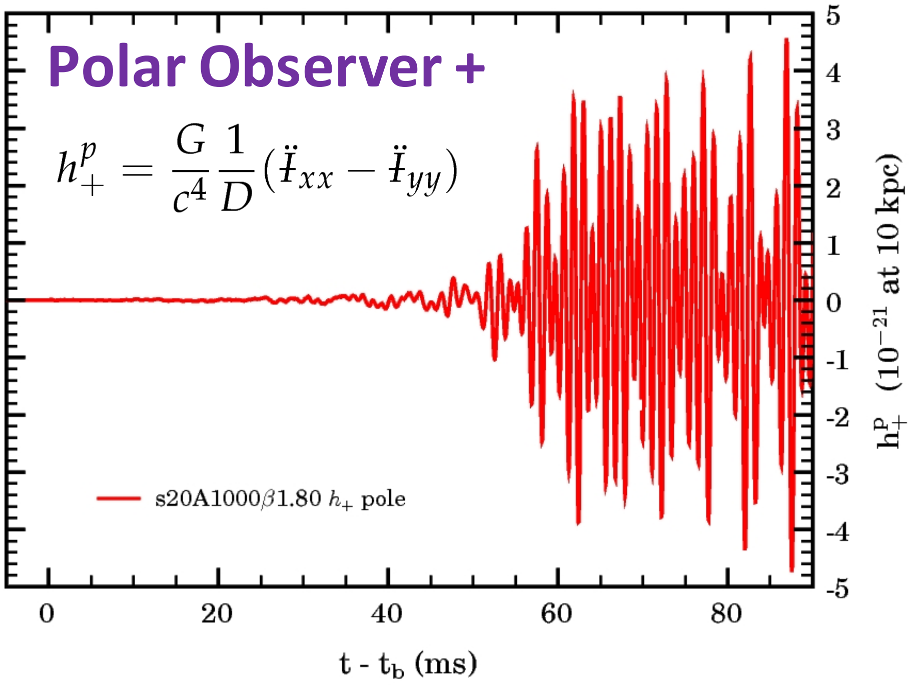


Simulation: C. D. Ott, Visualization: R. Kaehler



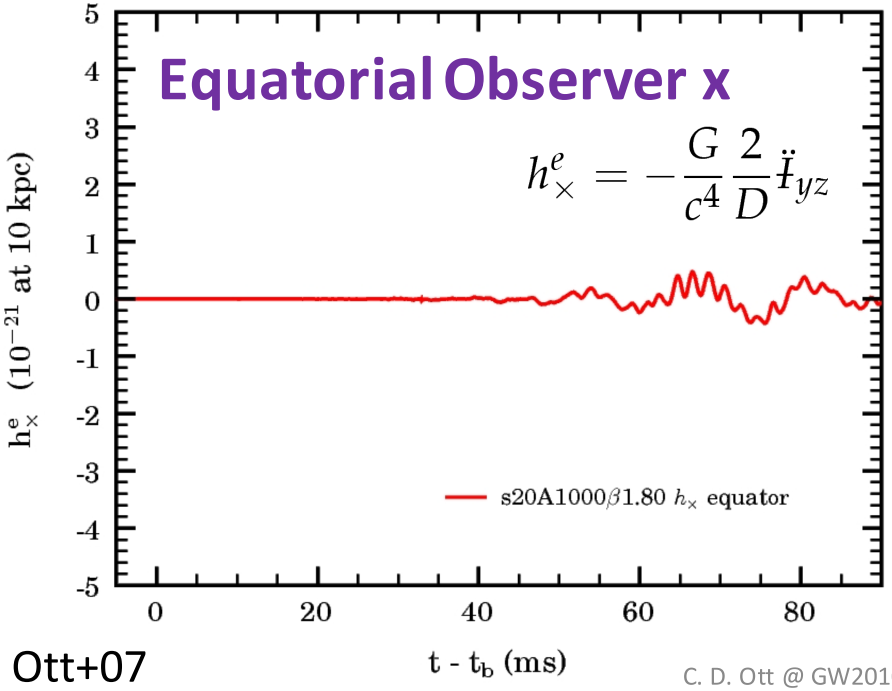
Equatorial Observer +

$$h_+^e = \frac{G}{c^4}\frac{1}{D}(\ddot{I}_{zz} - \ddot{I}_{yy})$$



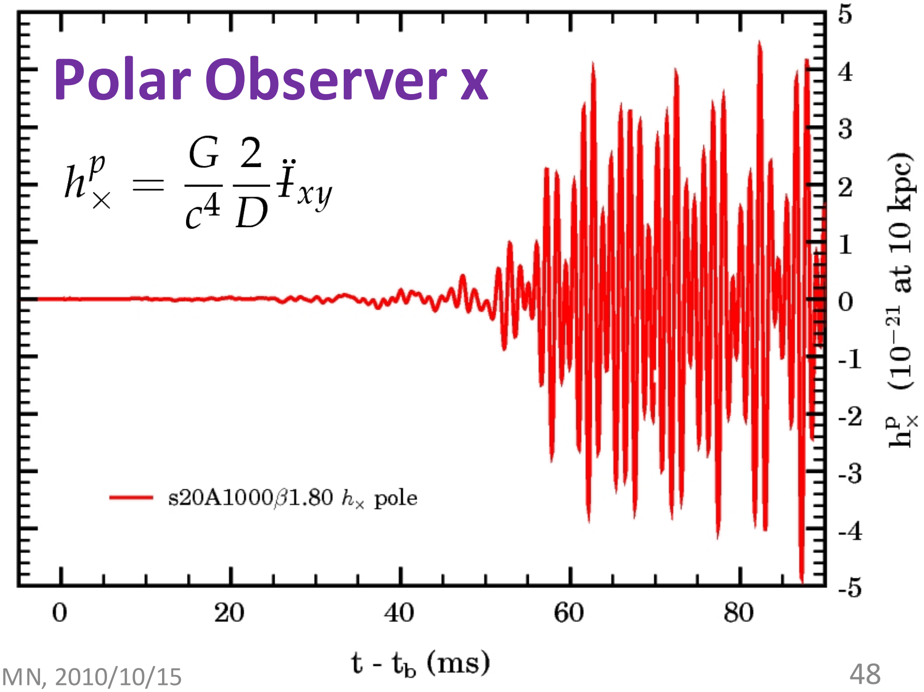
Polar Observer +

$$h_+^p = \frac{G}{c^4}\frac{1}{D}(\ddot{I}_{xx} - \ddot{I}_{yy})$$



Equatorial Observer x

$$h_\times^e = -\frac{G}{c^4}\frac{2}{D}\ddot{I}_{yz}$$



Polar Observer x

$$h_\times^p = \frac{G}{c^4}\frac{2}{D}\ddot{I}_{xy}$$

Ott+07

GWs from Asymmetric Neutrino Emission

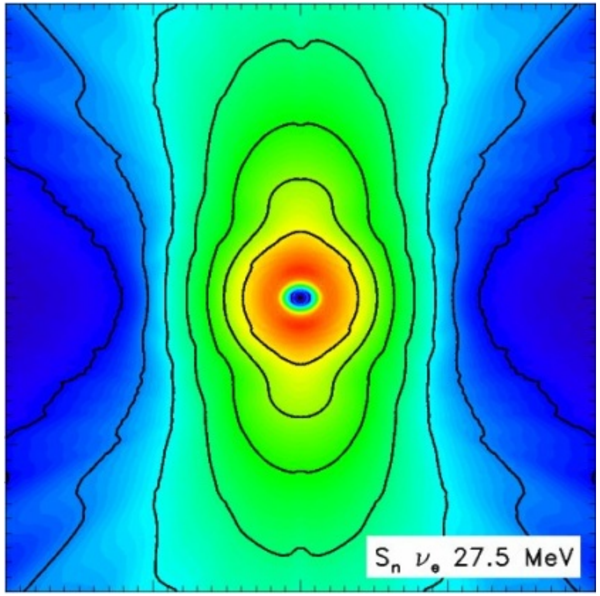
[Epstein 1978, Burrows & Hayes 1996, Janka & Müller 1997, Müller et al. 2004, Dessart et al. 2006, Ott 2009]

- Any accelerated mass-energy quadrupole will emit GWs. Asymmetric neutrino radiation:

$$h^{TT}_{+,e}(t) = \frac{2G}{c^4 D} \int_{-\infty}^{t-D/c} \alpha(t') L_\nu(t') dt'$$

$$\alpha(t) = \frac{1}{L_\nu(t)} \int_{4\pi} \Psi(\vartheta', \varphi') \frac{dL_\nu(\vec{\Omega}', t)}{d\Omega'} d\Omega'$$

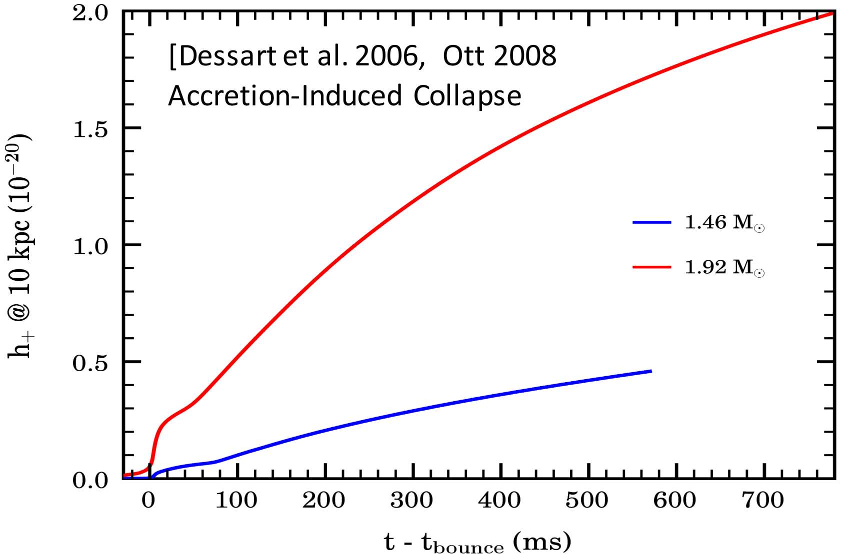
GW "Memory"



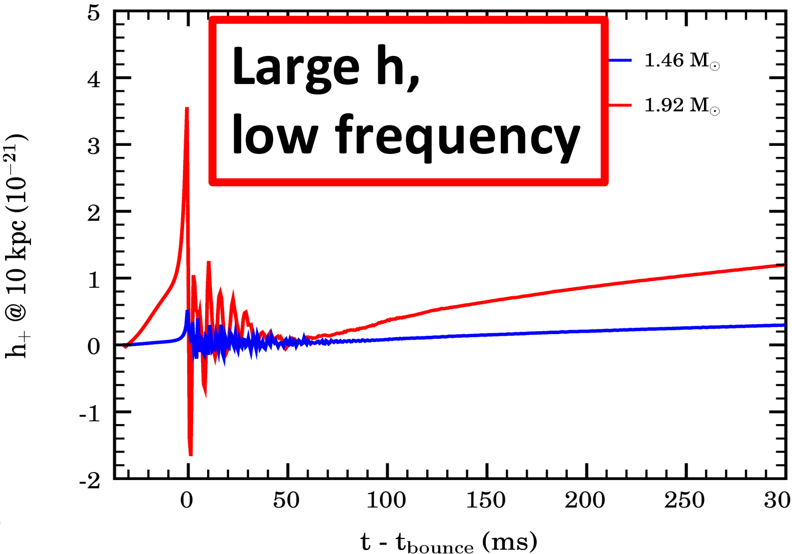
[Ott et al. 2008]

Asymmetric neutrino emission in core-collapse SNe:

- **Convection**: small-scale variations.
- **Rapid rotation**: large-scale asymmetry.
- **Large-scale asymmetries**: large-scale asymmetry.



[Dessart et al. 2006, Ott 2008
Accretion-Induced Collapse



Large h, low frequency

The Einstein Toolkit Project

http://einsteintoolkit.org

Mösta+14
Löffler+12

The Einstein Toolkit

http://einsteintoolkit.org

- Collection of open-source software components for the simulation and analysis of general-relativistic astrophysical systems.

Mösta+14
Löffler+12

The Einstein Toolkit

http://einsteintoolkit.org

Mösta+14
Löffler+12

- Collection of open-source software components for the simulation and analysis of general-relativistic astrophysical systems.
- Supported by NSF via collaborative grant to Georgia Tech, LSU, RIT, and Caltech.
- ~110 users, 53 groups; ~10 active maintainers.
- Goals:
  - Reproducibility.
  - Build a community codebase for numerical relativity and computational relativistic astrophysics.
  - Enable new science by lowering technological hurdles for researchers with new ideas. Enable code verification/validation, physics benchmarking, regression testing.
  - Make it easy for users to take advantage of new technologies.
  - Provide cyberinfrastructure tools for code and data management.

The Einstein Toolkit

- Regular releases of stable code versions.
  Most recent: “Somerville” release, November 2015
- Support via mailing list and weekly open conference calls.
- Working examples for BH mergers, NS mergers, isolated NSs, rotating, magnetized core collapse.

Mösta+14
Löffler+12

Available Components:

- Cactus (framework), Carpet (adaptive mesh refinement)
- GRHydro – GRMHD solver
- McLachlan – BSSN/Z4c spacetime solver
  (code auto-generated based on Mathematica script, GPU-enabled)
- Initial data solvers / importers
- Analysis tools (wave extraction, horizon finders, etc.)
- Visualization via VisIt (http://visit.llnl.gov)

The Dawn of Gravitational Wave Astronomy

Betelgeuse, D~200 pc

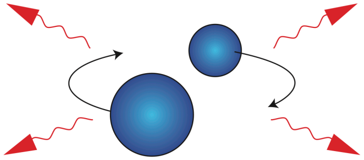
Stay Tuned…

Supplemental Slides

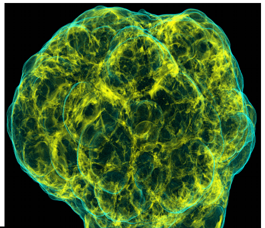
(Expected) Astrophysical Sources of GWs

-> Anything that has a large time-changing quadrupole moment!

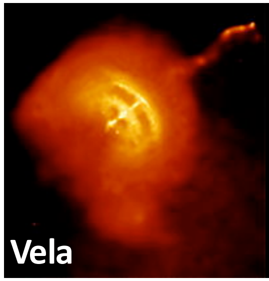
Coalescing binaries of compact stars

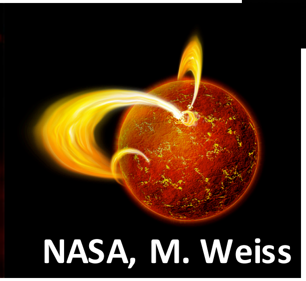


Stellar collapse & core-collapse supernovae



Galactic neutron stars:
mountains, glitches, quakes

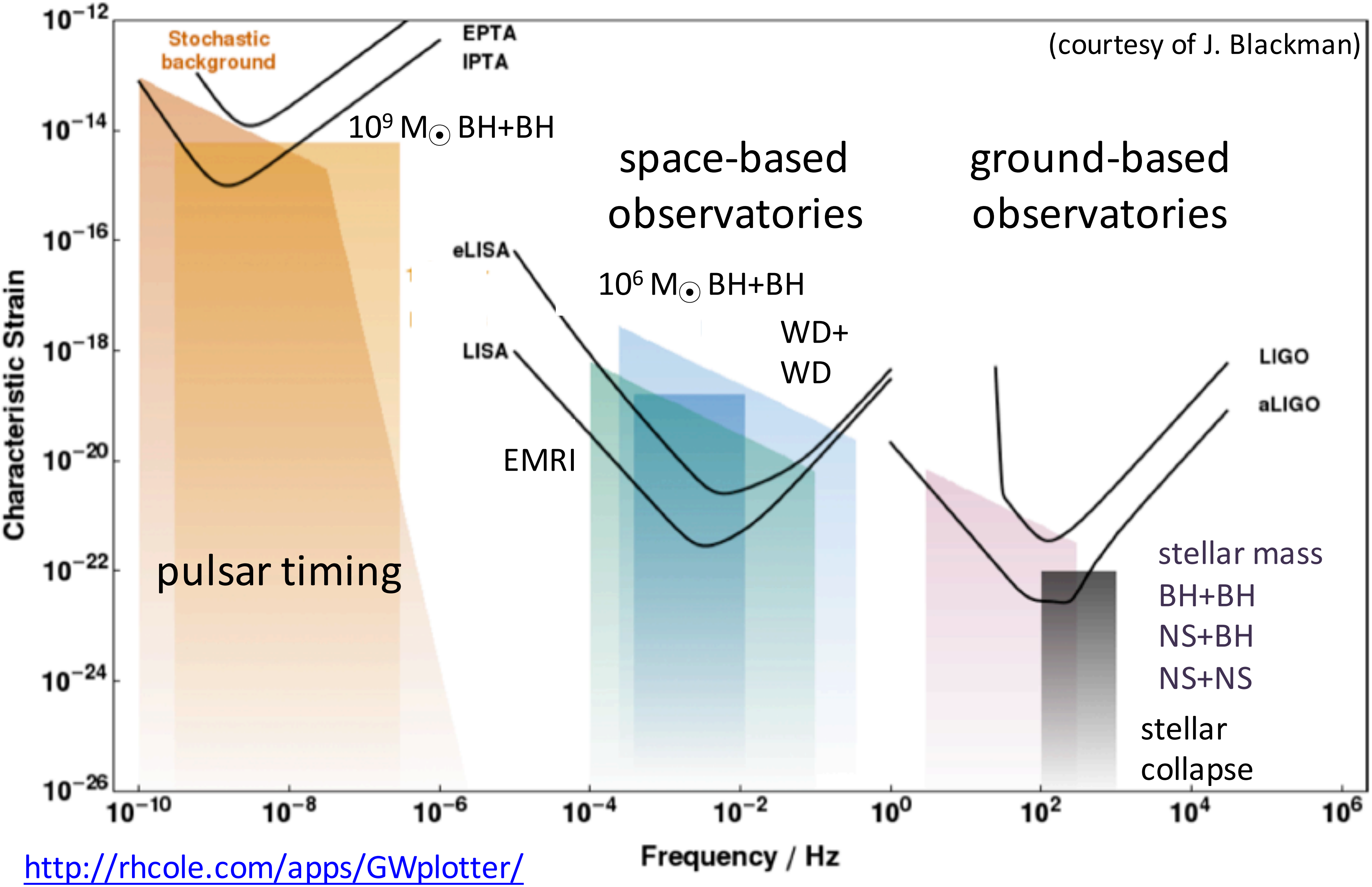




Cosmological and astrophysical stochastic backgrounds

Cosmic string cusps, fast radio bursts,
+ your favorite hypothetical source

GW Frequency Windows



(courtesy of J. Blackman)

$10^9\,M_\odot$ BH+BH

space-based observatories

ground-based observatories

$10^6\,M_\odot$ BH+BH

WD+ WD

EMRI

pulsar timing

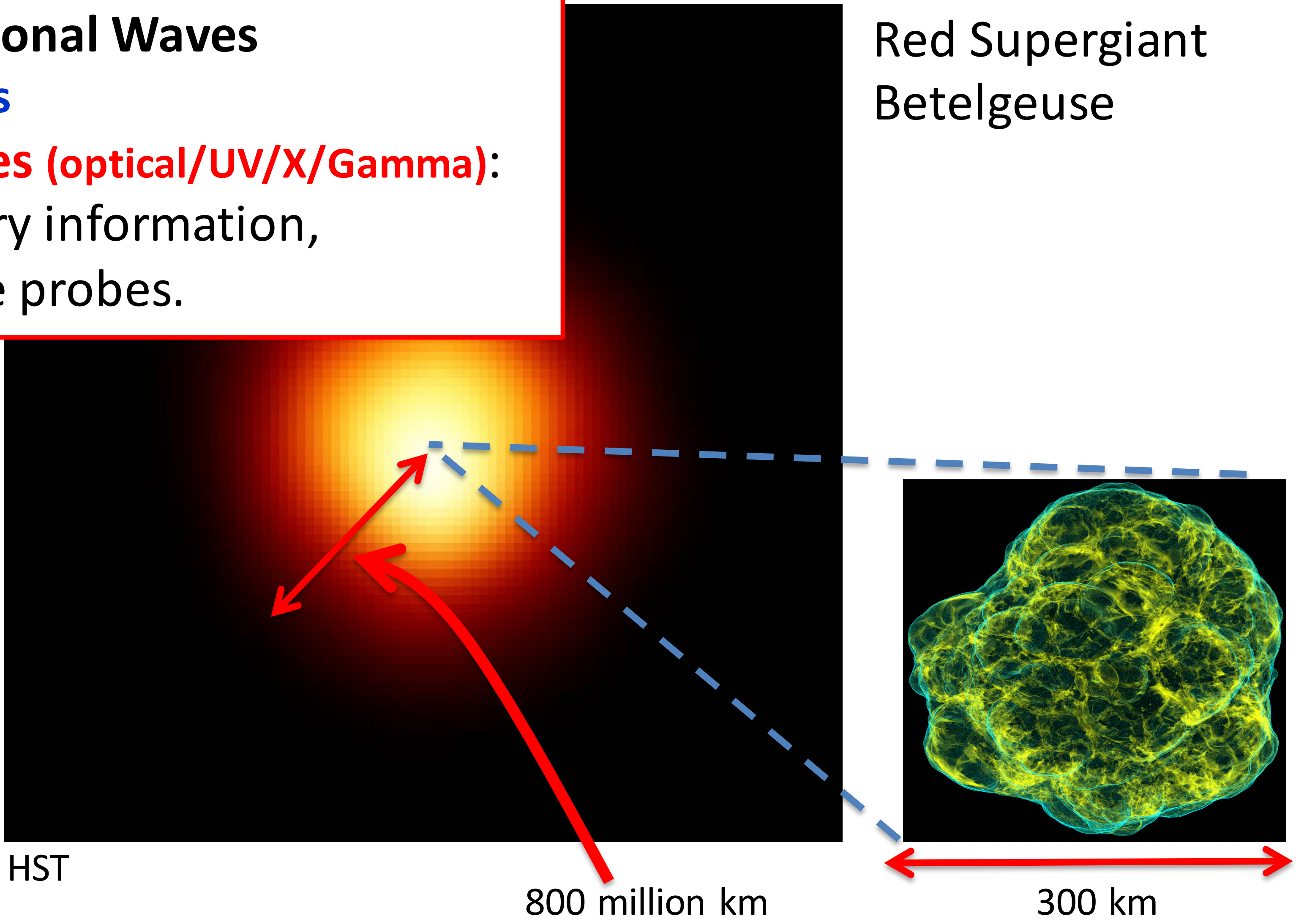
stellar mass BH+BH NS+BH NS+NS

stellar collapse

http://rhcole.com/apps/GWplotter/

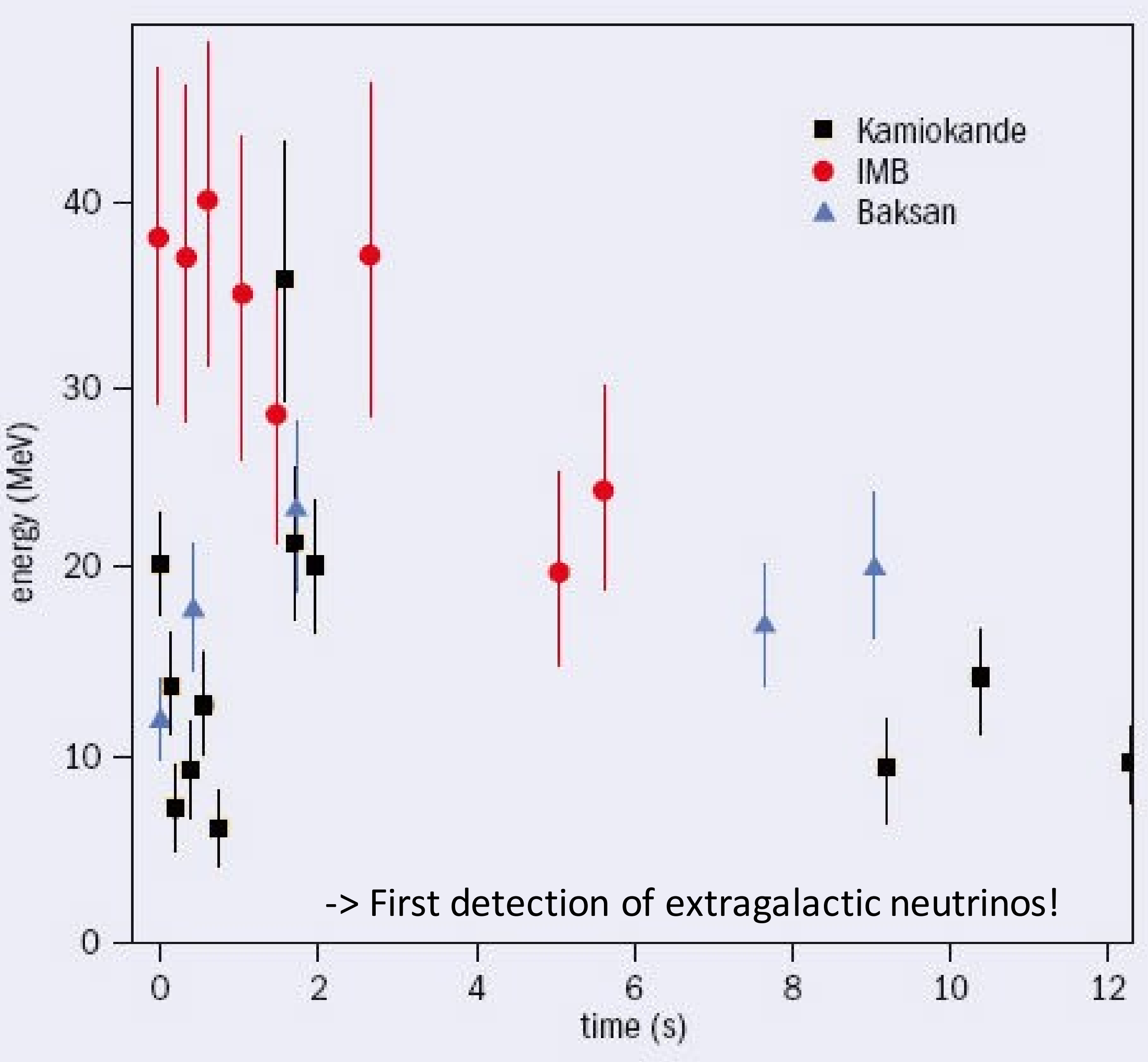
Observing the Heart of a Supernova

Probes of Supernova Physics:
- **Gravitational Waves**
- **Neutrinos**
- **EM waves (optical/UV/X/Gamma)**: secondary information, late-time probes.



SN 1987A: Neutrino Detection

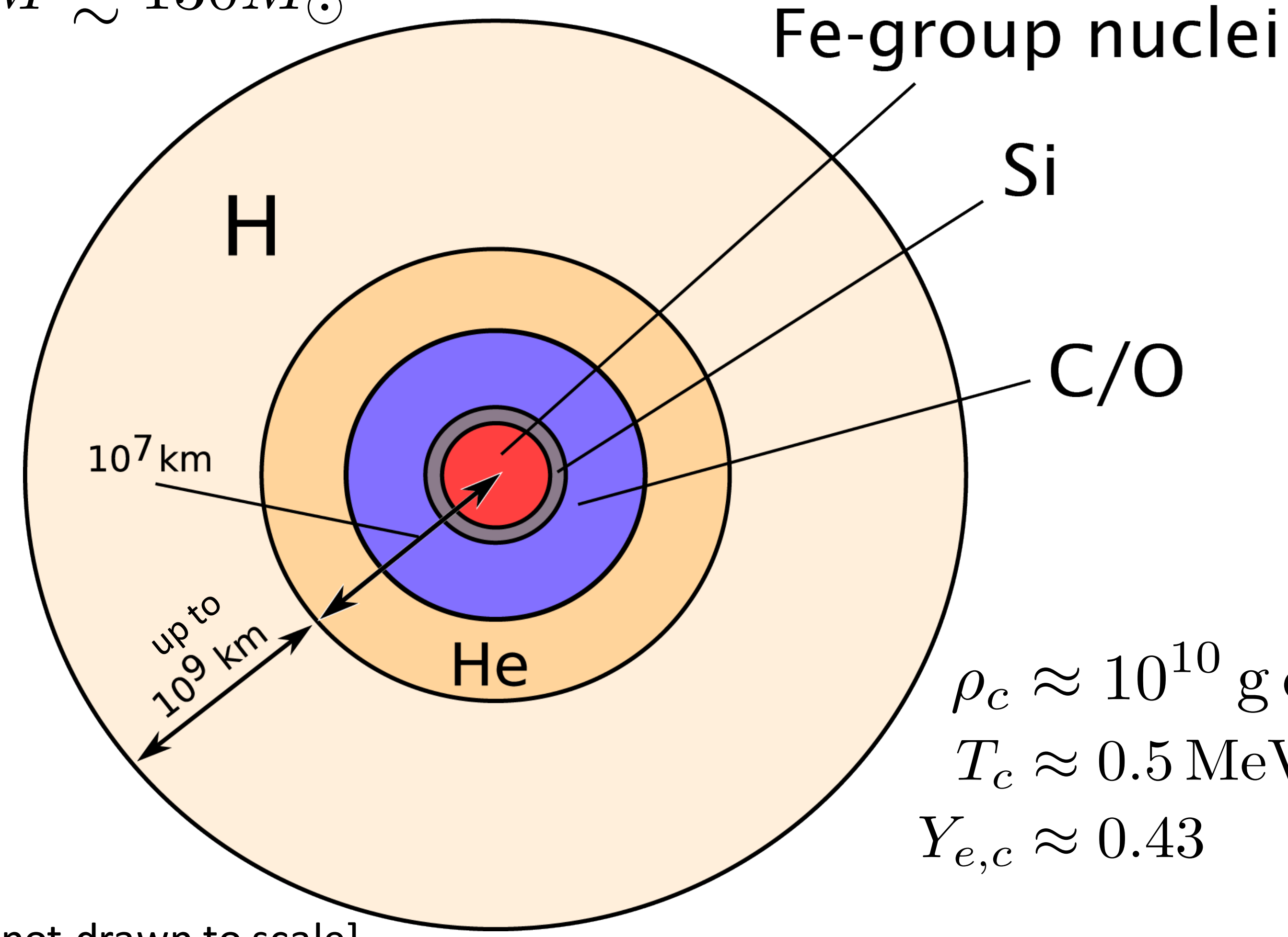
Hirata+87
Bionta+87



-> First detection of extragalactic neutrinos!

The Basic Theory of Core Collapse

$8M_\odot \lesssim M \lesssim 130M_\odot$

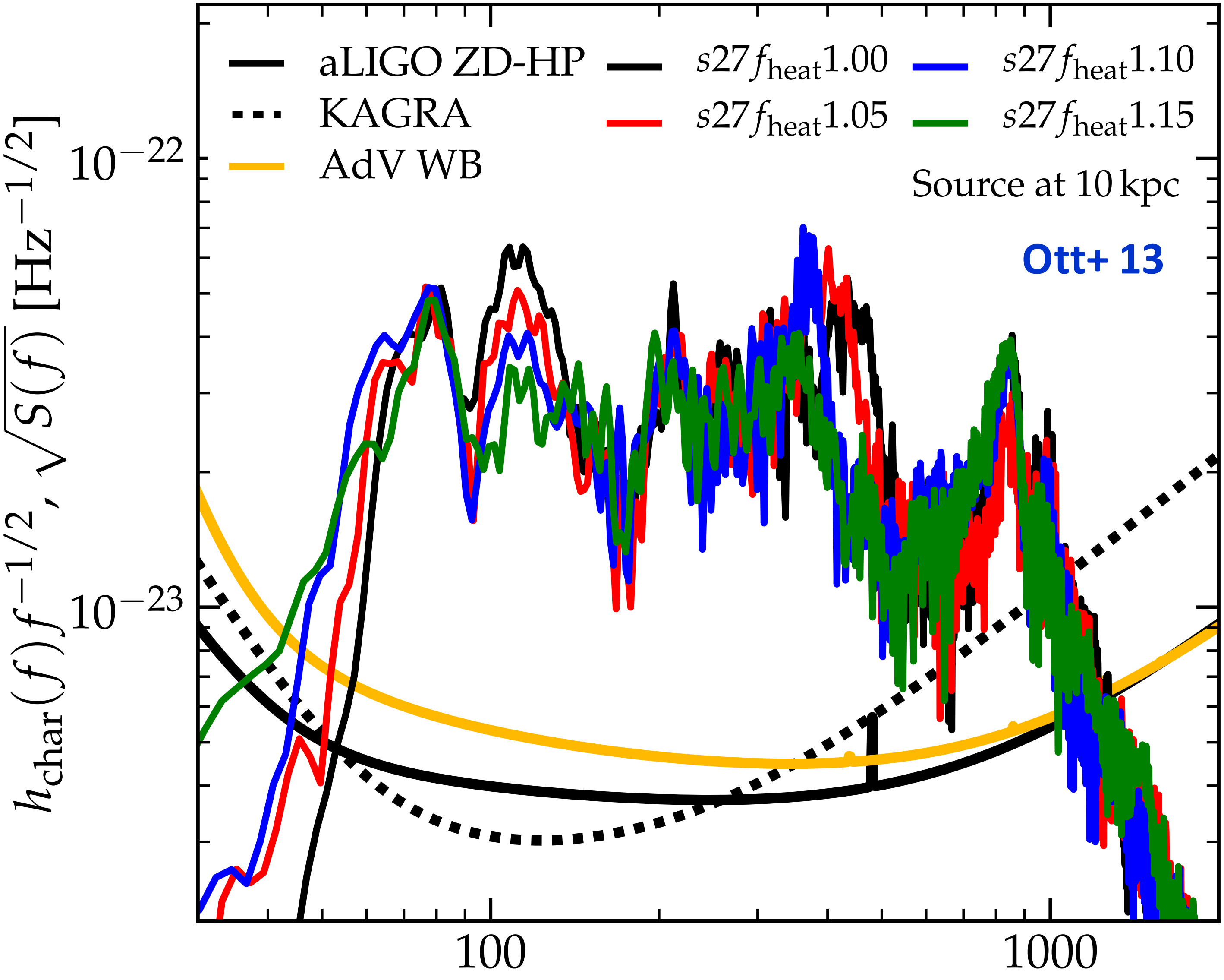


[not drawn to scale]

$$\rho_c \approx 10^{10}\,\mathrm{g\,cm^{-3}}$$
$$T_c \approx 0.5\,\mathrm{MeV}$$
$$Y_{e,c} \approx 0.43$$

$$M_{\mathrm{Ch}} \approx 1.44 \left(\frac{Y_e}{0.5}\right)^2 M_\odot$$

Detectability? -> Milky Way

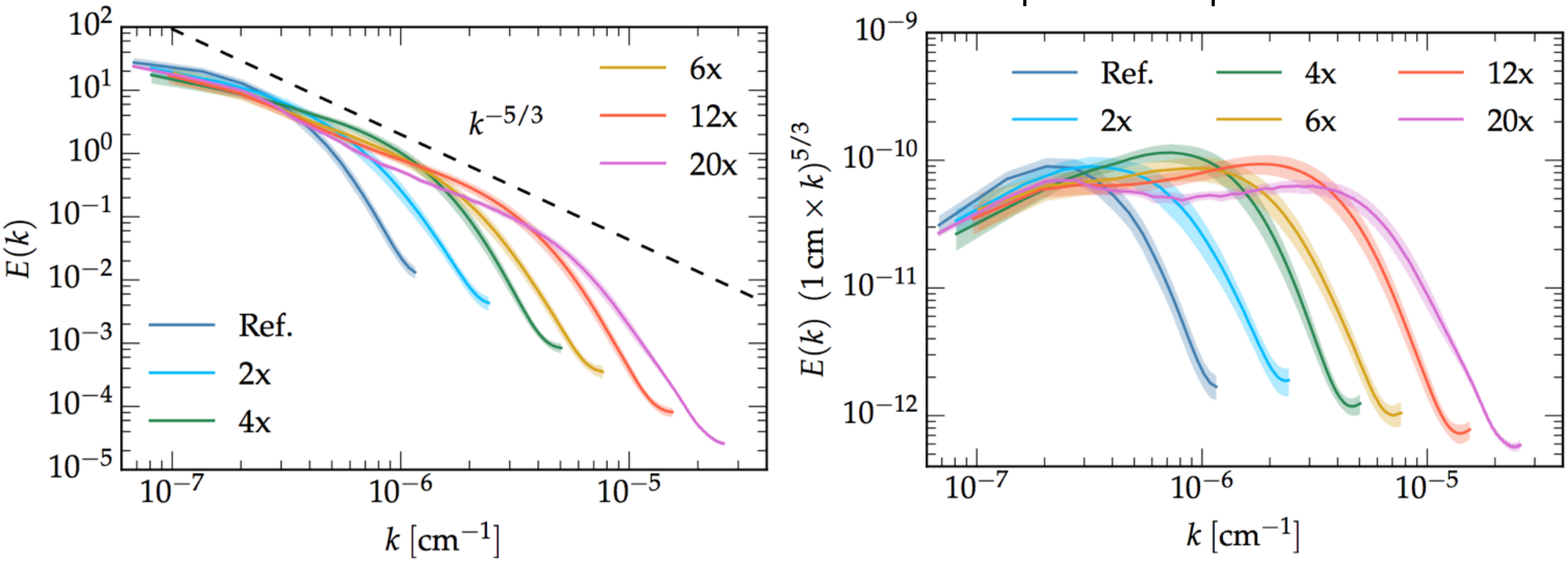


Frequency [Hz]

$$h_{\mathrm{char}}(f) = \sqrt{\frac{2}{\pi^2} \frac{G}{c^3} \frac{1}{D^2} \frac{dE_{\mathrm{GW}}(f)}{df}},$$

Neutrino-Driven Turbulent Convection

(Radice+15b)



“compensated” spectrum

Neutrino absorption drives anisotropic turbulent convection.

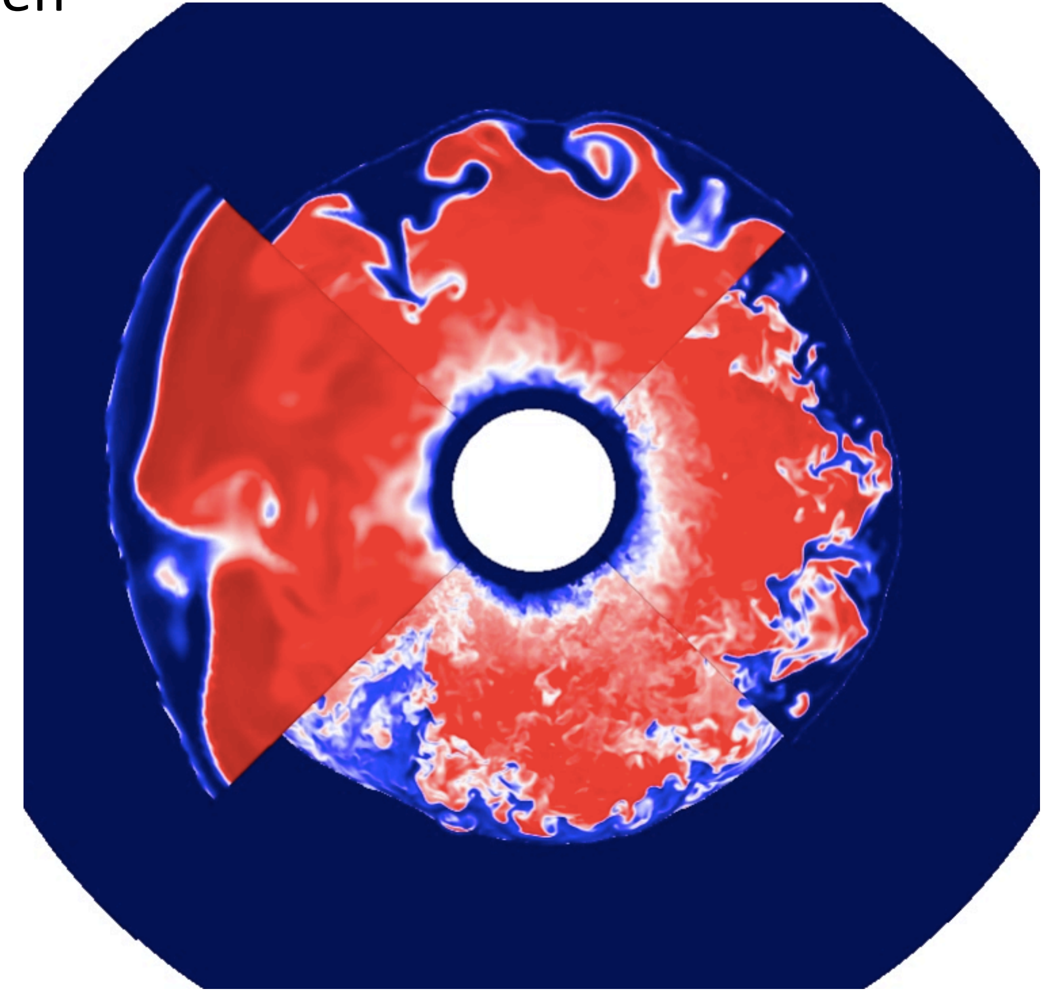
Core-collapse supernova turbulence obeys Kolmogorov scaling!

But: Can’t afford to run global simulations at necessary resolution to resolve inertial range!

Resolution Comparison (Radice+15b)

- Semi-global simulations of neutrino-driven turbulence; simplified physics.

dθ,dϕ = 0.9°
dr = 1.9 km
2 x



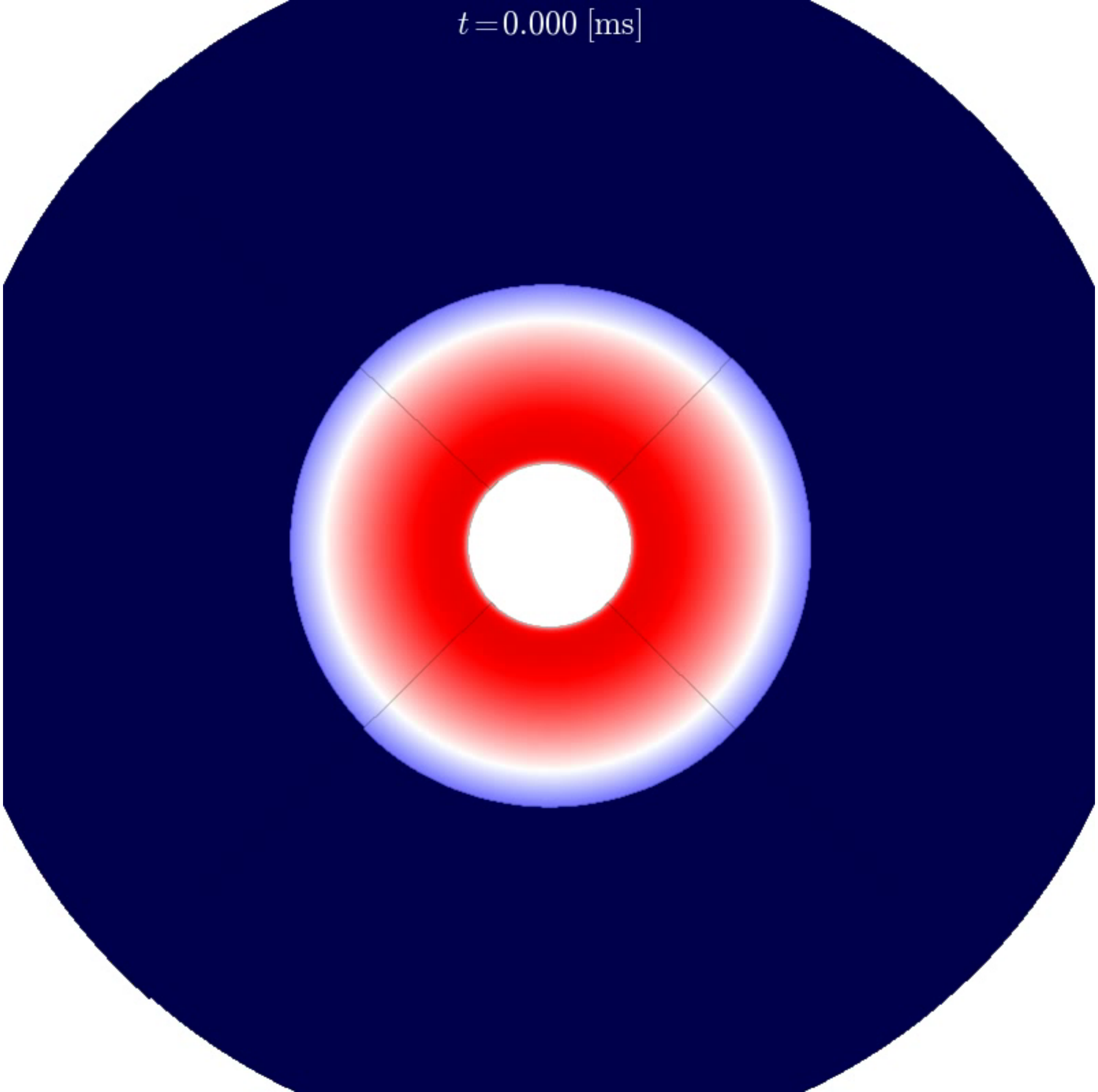
dθ,dϕ = 1.8°
dr = 3.8 km

(typical resolution of 3D rad-hydro sims)

4 x

dθ,dϕ = 0.45°
dr = 0.9 km

dθ,dϕ = 0.15°
dr = 0.32 km
12 x


$t = 0.000$ [ms]