On the precision of Bhabha scattering description

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OUTLINE

MOTIVATION

- Bhabha scattering is the basic QED process which is used for luminosity measurements at e^+e^- colliders (but no only)
- We get a record accuracy in small-angle Bhabha measurement and description at LEP. That was one of keystones for the high-precision verification of the Standard Model at LEP
- Since that time a considerable progress has been achieved:
  1) calculations of higher order radiative corrections
  2) computer tools, including adaptive Monte Carlo
  3) detector techniques, data analysis, etc.
- Still a lot has to be done to meet requirements of future experiments at e^+e^- linear collider(s)
- The aim of my talk is to overview the past achievements and to indicate and discuss the present problems (=tasks)

Statement of the problem

General task: we need a high-precision description of small-angle Bhabha scattering at high energies

General results: Monte Carlo generator(s), Monte Carlo integrator(s), and semi-analytic codes for cross-checks

Input: the beam energy distribution, hadronic vacuum polarization, analytic results from the literature

Output: various distributions and inclusive observables with estimates of uncertainties

PRELIMINARIES

Fortunately, in our case the general perturbation theory can be applied:

$$\frac{\alpha}{2\pi} \approx 1.2 \cdot 10^{-3}, \quad \left(\frac{\alpha}{2\pi}\right)^2 \approx 1.4 \cdot 10^{-6}$$

Moreover, other effects: hadronic vacuum polarization, (electro)weak contributions, hadronic pair emission, etc. are small and can be treated one-by-one separately

Nevertheless, there are some enhancement factors:

1) First of all, the large logarithm $L \equiv \ln \frac{\Lambda^2}{m_e^2}$ where $\Lambda^2 \sim Q^2$ is the momentum transferred squared, e.g., $L(\Lambda = 1\ \mathrm{GeV}^2 \approx 16$.

2) The energy region at the Z boson peak $(s \sim M_Z^2)$ requires a special treatment

THE BORN

Pay attention to the choice of the Born level definition

— Either pure QED or QED+EW

— Either the "standard" or improved Born approximation

— EW scheme choices: $\alpha(0)$, $\alpha(M_Z)$, G_{Fermi}, ...

The main point is to avoid double counting when Born is matched to higher order corrections

Another point is to be prepared for comparisons with other codes which might have a different choice

$\mathcal{O}(\alpha)$ CORRECTIONS

$\mathcal{O}(\alpha)$ corrections in QED and EW are known for years:
F.A. Redhead, Proc.Roy.Soc.'1953 R.V. Polovin, ZhETF'1956
F.A. Berends, K.J.F. Gaemers, R. Gastmans, Nucl.Phys.B '1974
M. Consoli, Nucl.Phys B '1979
M. Böhm, A. Denner, W. Hollik, Nucl.Phys B '1988

Remarks:

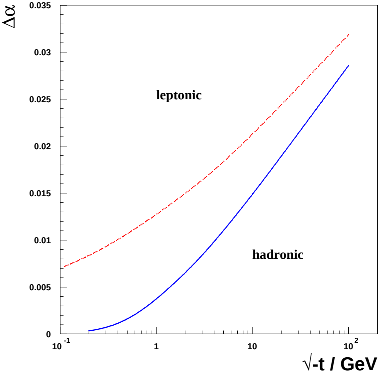
1) The t channel dominates everywhere even at very large scattering angles, except large angle scattering at narrow peak regions: Φ, J/Ψ, Z etc.
2) The pure QED Born contribution dominates everywhere below the Z peak, and also well above the peak for small scattering angles
3)Among radiative corrections, QED photonic contributions and vacuum polarization are the most important ones
4) Large logs $L = \ln(-t/m_e^2)$ provide the bulk of RC

Vacuum polarization

A.A. et al. EPJC' 2004

$\sqrt{s}$ (GeV)	91.187	91.2	189	206	500	1000	3000
	45 mrad $<\theta<$ 110 mrad						
$\langle\sqrt{-t}\rangle$ (GeV)	3.4	3.4	7.1	7.7	18.8	37.5	112.6
QED	51.428	51.413	11.971	10.077	1.7105	0.42763	0.047514
QED_t	51.484	51.469	11.984	10.088	1.7124	0.42809	0.047566
EW	51.436	51.413	11.965	10.072	1.7105	0.42871	0.049507
EW+VP_t	54.041	54.018	12.743	10.745	1.8590	0.47303	0.055748
EW+VP	54.036	54.013	12.742	10.744	1.8588	0.47296	0.055742
	5 mrad $<\theta<$ 50 mrad						
$\langle\sqrt{-t}\rangle$ (GeV)	1.1	1.1	2.2	2.4	5.8	11.6	34.8
QED	4963.4	4962.0	1155.4	972.54	165.08	41.271	4.5857
QED_t	4963.5	4962.1	1155.4	972.57	165.09	41.272	4.5858
EW	4963.4	4962.0	1155.4	972.53	165.08	41.272	4.5885
EW+VP_t	5075.0	5073.5	1190.6	1003.3	172.51	43.647	4.9603
EW+VP	5075.0	5073.5	1190.6	1003.3	172.51	43.646	4.9605

RUNNING OF α_{QED}



$$\alpha(q^2) = \frac{\alpha(0)}{1 - \Delta\alpha(q^2)}$$

See e.g. review S.Actis, A.A. et al., EPJC 66 (2010) 585

SECOND ORDER CORRECTIONS

The complete $\mathcal{O}\left(\alpha^2 L\right)$ analytic result was first received in A.A., V. Fadin, E. Kuraev, L. Lipatov, N. Merenkov, L. Trentadue [Nucl.Phys.B '1997]

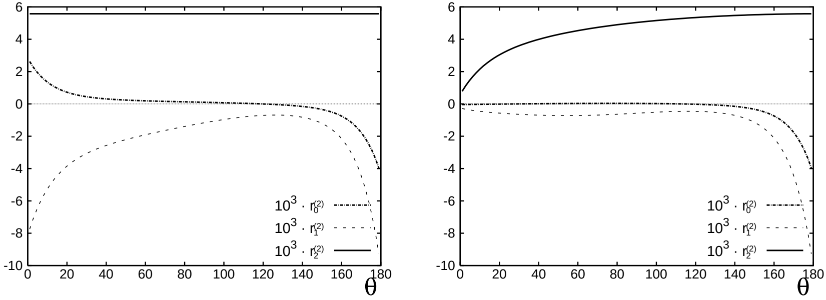
Two-loop virtual pure QED RC were computed by A. Penin [PRL'2005, NPB'2006]

Emission of one or two real photons was also added, see e.g. C. Carloni Calame, H. Czyz, J. Gluza, M. Gunia, G. Montagna, O. Nicrosini, F. Piccinini, T. Riemann, M. Worek *NNLO leptonic and hadronic corrections to Bhabha scattering and luminosity monitoring at meson factories* JHEP 1107 (2011) 126

A. A. Penin and G. Ryan, *Two-loop electroweak corrections to high energy large-angle Bhabha scattering*, JHEP'2011

SIZE OF SECOND ORDER RC

Let's look at soft + virtual $\mathcal{O}\left(\alpha^2\right)$ RC [A. Penin, PRL'2005, NPB'2006]: $\Delta = \sum_{n=0,1,2} C_n \ln^n \frac{M^2}{m_e^2} = \sum_{n=0,1,2} \eta_n$



Soft and virtual second order photonic relative radiative corrections in permil *versus* the scattering angle in degrees for $\Delta = 1$, $\sqrt{s}$=1 GeV; $M = \sqrt{s}$ on the left side and $M = \sqrt{-t}$ on the right side.

SOFT-HARD SEPARATOR

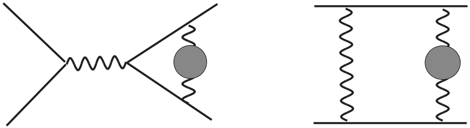
Beware of the proper separation of soft and hard radiation in higher orders: of photon energies or for each photon separately. Note that exponentiation of soft radiation assumes independent emission of each photon

See e.g. A.A., T.V. Kopylova, *On higher order radiative corrections to elastic electronproton scattering*, EPJC 2015

$\mathcal{O}\left(\alpha^2\right)$ HEAVY-FERMION AND HADRONIC CONTRIBUTIONS (I)

Virtual (loop) pair corrections

S. Actis, M. Czakon, J. Gluza and T. Riemann, PRD **78** (2008) 085019.



(c) (d)

$\mathcal{O}\left(\alpha^2\right)$ HEAVY-FERMION AND HADRONIC CONTRIBUTIONS (II)

Real pair emission corrections

Relevant semi-analytic results are known (should be updated), see LEPEWWG studies: M. Kobel et al. CERN Yellow Rep. '1999; A.A. JHEP '2001

Monte Carlo simulation for emission of real leptonic pairs is straightforward

Question: what are requirements for description of processes like $e^+e^- \to e^+e^- \; + \; \pi^+\pi^-$?

LEADING TWO-LOOP (ELECTRO)WEAK CORRECTIONS

In MCSANC v.1.20 we follow the recipe introduced by o J. Fleischer, O. Tarasov, and F. Jegerlehner in 1993 and implemented further in ZFITTER

The $\Delta\rho$ parameter as the ratio of the neutral current to charged current amplitudes at zero momentum transfer

$$\rho = \frac{G_{NC}(0)}{G_{CC}(0)} = \frac{1}{1-\Delta\rho}$$

The leading in $G_\mu m_t^2$ NLO EW contribution is

$$\Delta\rho^{(1)} = 3x_t = \frac{3\sqrt{2}G_\mu m_t^2}{16\pi^2}$$

At the two-loop level

$$\Delta\rho = N_c \frac{\sqrt{2}G_\mu m_t^2}{16\pi^2}\left[1 + \rho^{(2)}\left(M_H^2/m_t^2\right)\, x_t\right]\left[1 - \frac{2\alpha_s(M_Z^2)}{9\pi}(\pi^2+3)\right]$$

QED STRUCTURE FUNCTION APPROACH

The master formula for QED NLO corrections to Bhabha scattering reads

$$\mathrm{d}\sigma = \sum_{a,b,c,d=e,\bar{e},\gamma} \int_{\bar{z_1}}^{1} \mathrm{d}z_1 \int_{\bar{z_2}}^{1} \mathrm{d}z_2 \mathcal{D}^{\mathrm{str}}_{ae}(z_1) \mathcal{D}^{\mathrm{str}}_{b\bar{e}}(z_2) \Big[\mathrm{d}\sigma^{\mathrm{Born}}_{ab\to cd}(z_1,z_2)$$
$$+ \quad \mathrm{d}\bar{\sigma}^{(1)}_{ab\to cd}(z_1,z_2) + \mathcal{O}\left(\alpha^2 L^0\right) \Big] \int_{\bar{y_1}}^{1} \frac{\mathrm{d}y_1}{Y_1} \int_{\bar{y_2}}^{1} \frac{\mathrm{d}y_2}{Y_2} \mathcal{D}^{\mathrm{frg}}_{ec}\left(\frac{y_1}{Y_1}\right) \mathcal{D}^{\mathrm{frg}}_{\bar{e}d}\left(\frac{y_2}{Y_2}\right)$$

$\mathrm{d}\bar{\sigma}^{(1)}$ is the $\mathcal{O}(\alpha)$ correction to the massless Bhabha scattering in $\overline{\mathrm{MS}}$ scheme, $\mathcal{D}^{\mathrm{str}}_{ee}$ and $\mathcal{D}^{\mathrm{frg}}_{ee}$ are the electron structure and fragmentation functions

F.A.Berends, W.L.van Neerven, G.J.H.Burgers, NPB '1988
A.A., K.Melnikov, PRD '2002; A.A.,JHEP '2003
A.A., E.S. Scherbakova, JETP Lett. '2006

RC TO SMALL-ANGLE BHABHA SCATTERING AT LEP

A. Arbuzov *et al.* [LEPEWWG], *The Present theoretical error on the Bhabha scattering cross-section in the luminometry region at LEP*, PLB '1996

S. Jadach [hep-ph/0306083]:

Type of correction/error	LEPEWWG	hep-ph/9905235
Technical precision	—	— (0.03%)
Missing photonic $\mathcal{O}(\alpha^2 L)$	0.10%	0.027% (0.013%)
Missing photonic $\mathcal{O}(\alpha^3 L^3)$	0.015%	0.015% (0.006%)
Vacuum polarization	0.04%	0.040%
Light pairs	0.03%	0.010%
Z-exchange	0.015%	0.015%
Total	0.11%	0.054% (0.055%)

The global fit of LEP data would prefer to change the luminosity by 0.1%

OUTLOOK

- Precision theoretical description of small-angle Bhabha scattering is of ultimate importance for e^+e^- colliders
- Several effects of different nature should be taken into account
- Matching of those effects should be organized
- Common efforts of different group can give us reliable theoretical predictions
- Tuned comparisons should be performed
- Features of theoretical codes should meet experimental requirements
- The SANC team plans to contribute …