Measurement of the *Moli‘ere radius* from the 2014 TB data

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motivation

- A major design aim of LumiCal is it's compact structure.
- One aspect is to limit the development of the electromagnetic showers in the transverse direction.
- It allow better separation and identification of the electromagnetic element.

Moli'ere radius definition

The transverse development of electromagnetic showers in different materials scales fairly accurately with the *Moli'ere radius* $R_{\mathcal{M}}$, given by

$$R_{\mathcal{M}} = X_0 \frac{E_s}{E_c} \tag{1}$$

where $E_s \approx 21$ MeV, and E_c is the critical energy. In a compound the *Moli'ere radius* is given by

$$\frac{1}{R_{\mathcal{M}}} = \frac{1}{E_s} \sum \frac{w_j E_{cj}}{X_{0j}} = \sum \frac{w_j}{R_{\mathcal{M}}} \tag{2}$$

On the average,only 10% of the energy lies outside the cylinder with radius of 1 *Moli'ere radius*. The distributions are characterized by a narrow core, and broaden as the shower develops, often represented as the sum of two Gaussians.

density considerations

- In order to take in to effect the density the $R_{\mathcal{M}}$ units are $[gr/cm^2]$.
- During discussion on a structures and compounds the $R_{\mathcal{M}}$ is corrected for the density, ρ like

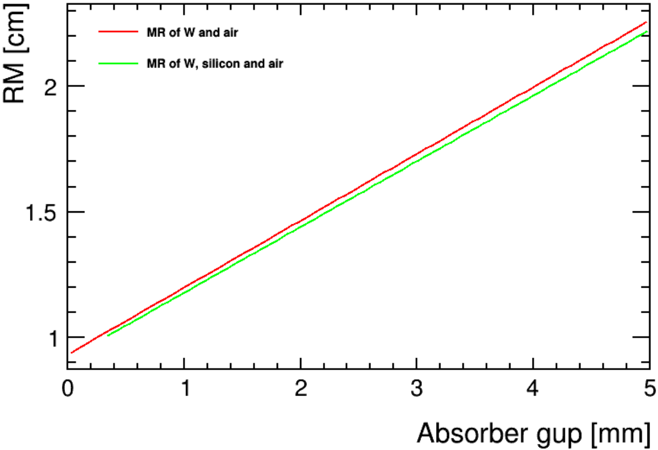
$$R_{\mathcal{M}}[cm] = \frac{R_{\mathcal{M}}[gr/cm^2]}{\rho[gr/cm^3]} \tag{3}$$

- We can calculate the $R_{\mathcal{M}}$ of the stack in different configuration using :

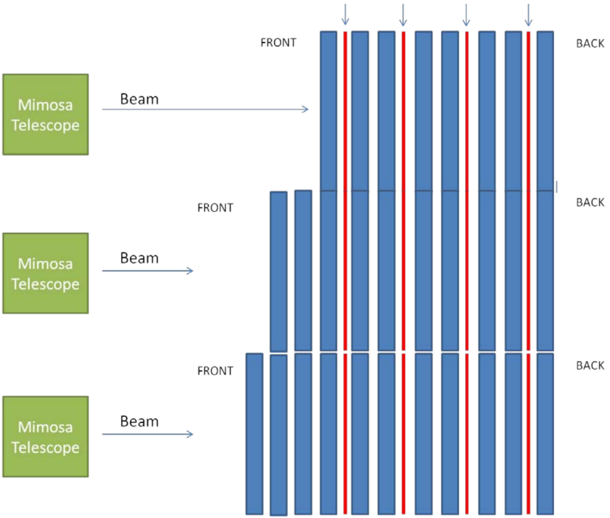
$$\frac{1}{R_{\mathcal{M}}[cm]} = \frac{\rho[gr/cm^3]}{R_{\mathcal{M}}[gr/cm^2]} = \rho \sum \frac{w_j}{R_{\mathcal{M}}} = \frac{W}{V} \sum \frac{W_j}{WR_{\mathcal{M}}} = \sum \frac{\rho_j \frac{Z_j}{Z}}{R_{\mathcal{M}}} \tag{4}$$

air gap

We can see the importance of the gap between absorbers in the calorimeter design on the the *Moli'ere radius* from the calculation :



2014 configuration



Moli'ere radius of 2014 configuration

Summery of all the material in our setup

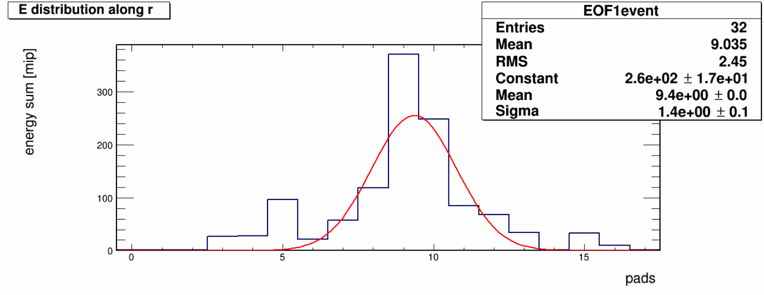
material	W	Cu	Ni	PL-95%	MGS-93%	air	Si	PCB
density	19.3	8.96	8.9	18.0*	17.8*	0.0012	2.33	1.7
$R_{\mathcal{M}}[gr/cm^2]$	18.0	14.0	13.4	17.7**	17.6**	8.8	11.5	10.3
$R_{\mathcal{M}}[cm]$	0.93	1.57	1.51	0.98	0.99	7330	4.94	6.06

Summery of calculated *Moli'ere radius*

	PL-95%	MGS-93%	air	Si	PCB	total	$R_{\mathcal{M}}[cm]$
general 93	0	0.7	0.37	0.032	0.25	1.35	1.79
general 95	0.7	0	0.37	0.032	0.25	1.35	1.78
CONF 1	1.05	1.75	1.57	0.128	1.0	5.5	1.81
CONF 2	1.75	1.75	1.77	0.128	1.0	6.4	1.71
CONF 3	2.1	1.75	1.87	0.128	1.0	6.85	1.67

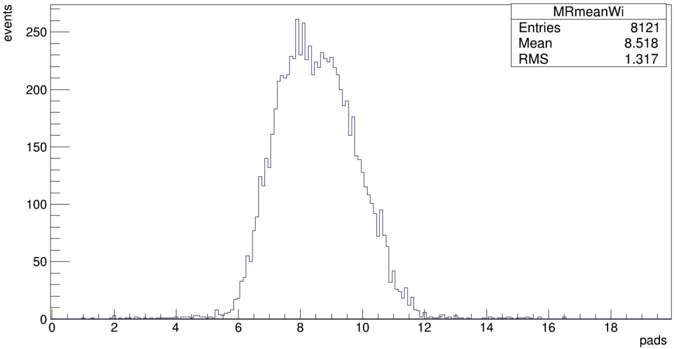
1 event

- For each event from the data or simulation, we can look on the sum of the energy deposit, along the radial direction.
- The sum of energy deposit include both instrumented sectors in all 4 sensor layers.
- The hit position can be estimate By fitting or by calculating the center of gravity.



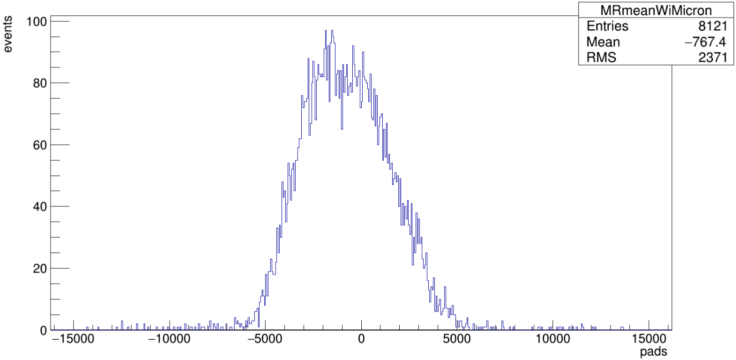
hit position

and for all events :



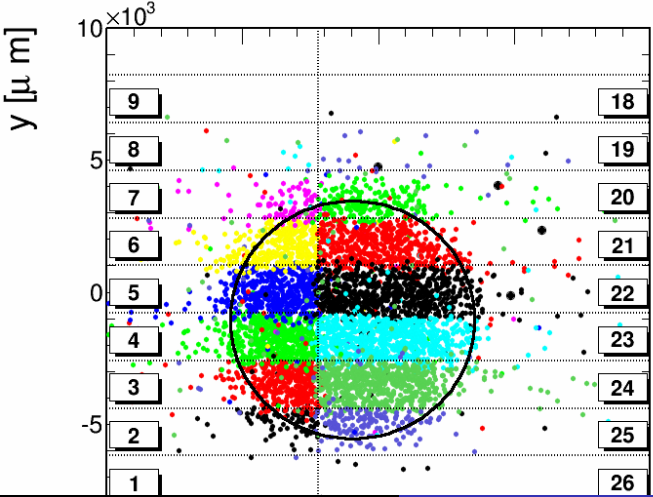
hit position

and for all events :



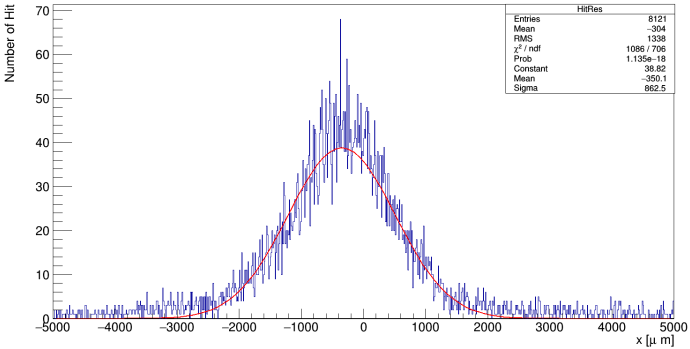
hit position

we also so :



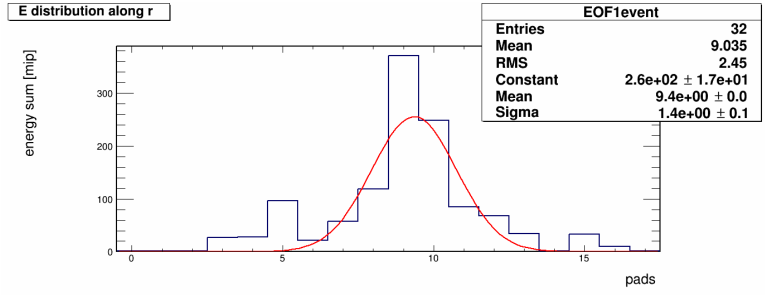
hit position resolution

We can compare between the LumiCal reconstructed hit position and the extrapolated hit position from the beam Telescope to the LumiCal first layer



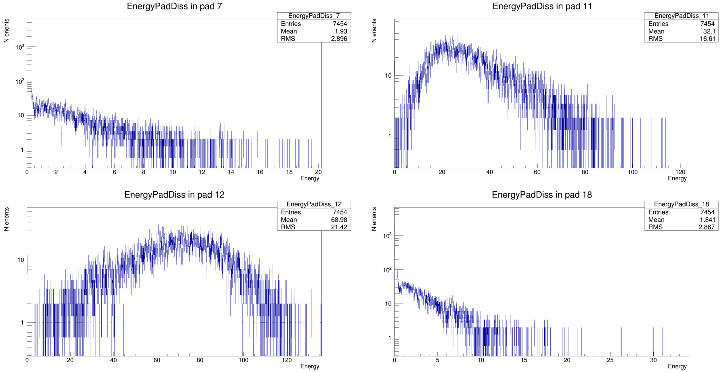
energy deposit

- By folding all events to start in a central pad the beam profile is canceled out.
- the mean radial energy deposit distribution can extracted from the single pad energydistribution



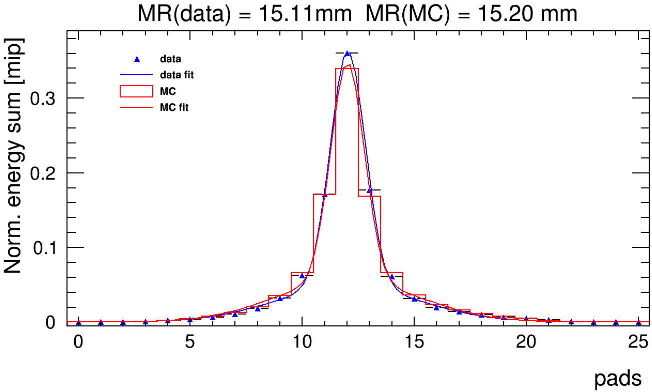
energy deposit

The pad energy deposit distribution (pad numbers are shifted):



Radial energy distribution

From the radial energy deposit distribution the *Moli'ere radius* can by estimate by the fit of the sum of 2 Gaussians.



calculation

- The result is not direct from the fit.
- LumiCal pads are long (strip like).

$$0.9 = \frac{\oint_0^R \oint_0^{2\pi} F(E)}{\oint_0^\infty \oint_0^{2\pi} F(E)} \tag{5}$$

- LumiCal pads are long (strip like) and acts like 1 dimension integration.
- by setting

$$\oint F(E)dA = \oint_{-\infty}^{\infty} \oint_{-\infty}^{\infty} Ae^{-(\frac{x^2+y^2}{B^2})} + Ce^{-(\frac{x^2+y^2}{D^2})} dxdy \tag{6}$$
$$= A\pi B^2 + C\pi D^2 \equiv 1$$

calculation

- By extracting the Gaussians constant the *Moli'ere radius* can by calculated by solving :

$$0.1 = \pi AB^2 e^{-(\frac{R}{B})^2} + \pi CD^2 e^{-(\frac{R}{D})^2} \tag{7}$$

Summary

- *Moli'ere radius* of 15 16 mm can by calculated from the 2014 TB data.
- MC simulation (Lucas) is giving similar results.
- *Moli'ere radius* error can by calculated from psodo experiment.
- error calculation is not complete and has big efect on measurement.