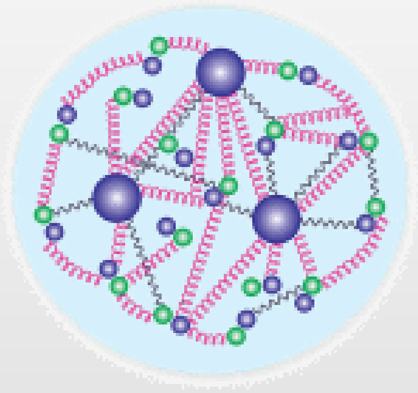
The Relation Between Fundamental Constants and Particle Physics Parameters



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The Presentation in a Nutshell

- Many new physics extensions of the standard model predict time variations of basic physics parameters such as the Quantum Chromodynamic Scale Λ_{QCD}, the Higgs Vacuum Expectation Value ν, and the Yukawa couplings h_i. *Hard to measure directly*.
- The proton to electron mass ratio μ and the fine structure constant α are two dimensionless fundamental constants that are functions of these parameters.
- Any change in the basic physics parameters induces changes in μ and α. *Fairly easy to measure directly*.
- Astronomical observations have established stringent limits on the time variation of μ and α at look back times on the order of the age of the universe. *Time base only available to astronomical observations*.
- These limits in turn establish limits on the time variation of Λ_{QCD}, ν and h. *Limits on the order of 10^{-5} or smaller at greater than half the age of the universe*.

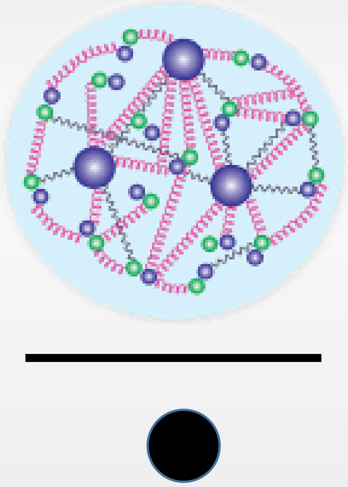
Outline of the talk

- What are the astronomical observational constraints on the time variation of the fundamental constants μ and α?
- What is the relationship between the variation of the physics parameters Λ_{QCD}, ν and h and the fundamental constants μ and α?
- What is the form of the derived constraints on the physics parameters?
- Use an example model to produce concrete limits?
- How do these limits evolve with time?
- Show the link between μ and α and the dark energy equation of state *w*.
- How does Λ_{QCD}, ν and h vary with time in a thawing and freezing quintessence cosmology?
- What is the predicted present day rate of change of Λ_{QCD}, ν and h for a quintessence cosmology?
- Make the time stability of fundamental constants a standard test for new physics and cosmologies.

Alpha and Mu

$$\alpha = \frac{2\pi e^2}{hc}$$

$$\mu = \frac{m_p}{m_e} =$$



As its name implies α is measured by the fine structure splitting in atomic spectra.

The proton to electron mass ratio is measured by its effect on the spectra of molecules (RIT 1975). An example is given in the following talk.

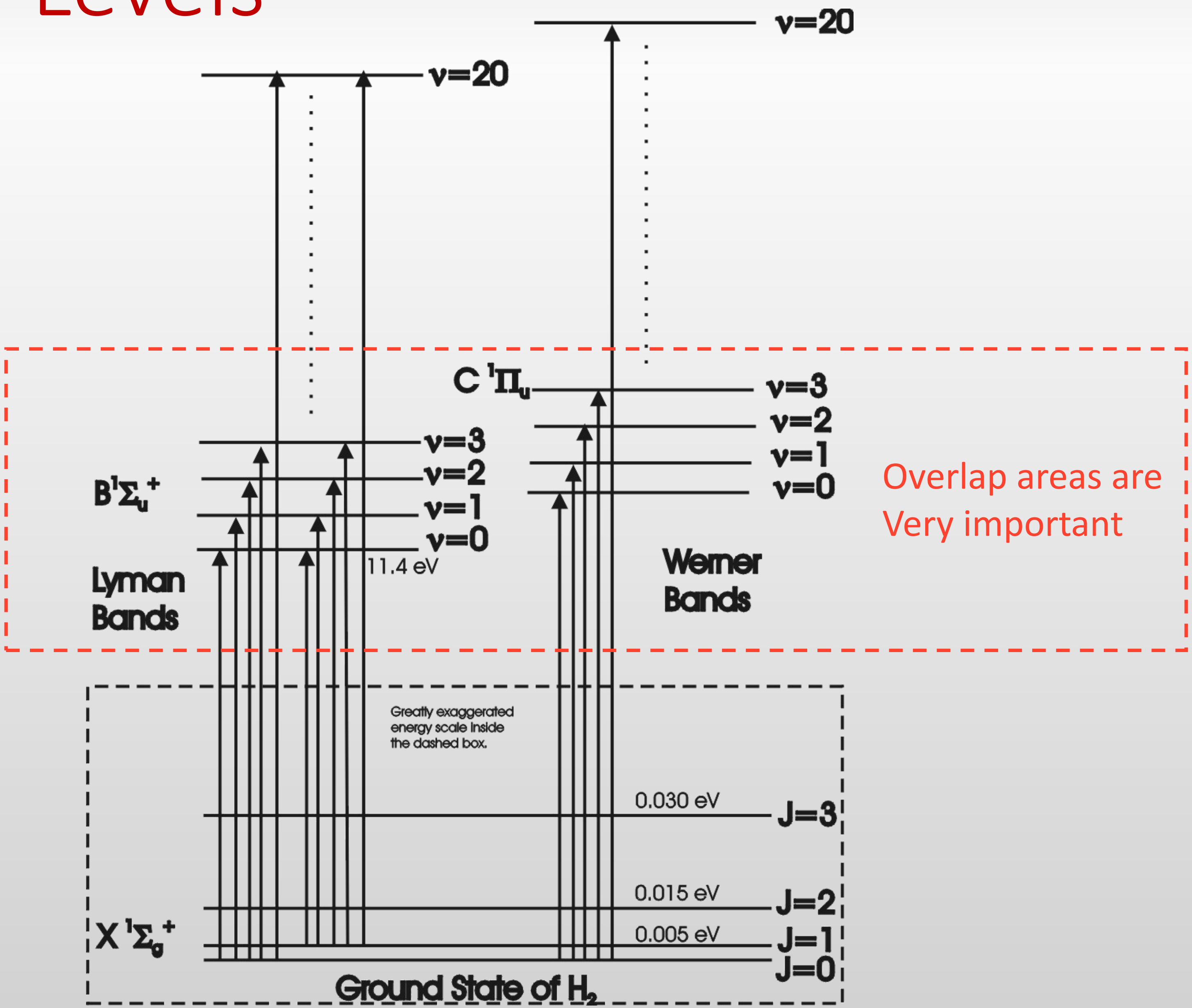
Observational Constraints on the Time Variation of μ and α

- Astronomical observations of atomic and molecular spectra in the early universe provide limits on the time variation of μ and α at lookback times on the order of the age of the universe.
- There is no established observed change in μ at this time.
- Although there are reports of both temporal and spatial variations in α these reports have not been confirmed and recent observations have established limits on time variations significantly below the reported changes. (See Plenary talks by Webb and Murphy on Thursday)
- For the purposes of this presentation the recent limits on the time variation of α are accepted and no time variation of α is assumed.

Proton to Electron Mass Ratio, μ, Constraints

- Optical observations of redshifted H_2 electronic transitions
  - Lyman and Werner bands
  - Absorption lines of cold H_2 from the ground electronic and vibrational states to a higher electronic state with varying vibrational and rotational states.
  - Redshifts from 2-4
  - Constraints on the order of $\frac{\Delta\mu}{\mu} \leq few\ x\ 10^{-6}$
  - **<u>Only 10 systems have been analyzed</u>**
- Radio observations of absorption lines in cold gas at moderate redshifts
  - Methanol and Ammonia absorption lines with high sensitivity to μ
  - Redshifts from 0.5-0.9 (greater than half the age of the universe)
  - Constraints on the order of $\frac{\Delta\mu}{\mu} \leq 10^{-7}$ (primary constraint)
  - Primary constraint $\frac{\Delta\mu}{\mu} \leq (0.29 \pm 1.0)x10^{-7}$ at a redshift of 0.88582 (Bagdonaite et al. 2012 and Kanekar et al. 2014)
  - **<u>Only two systems have been analyzed</u>**

H_2 Energy Levels

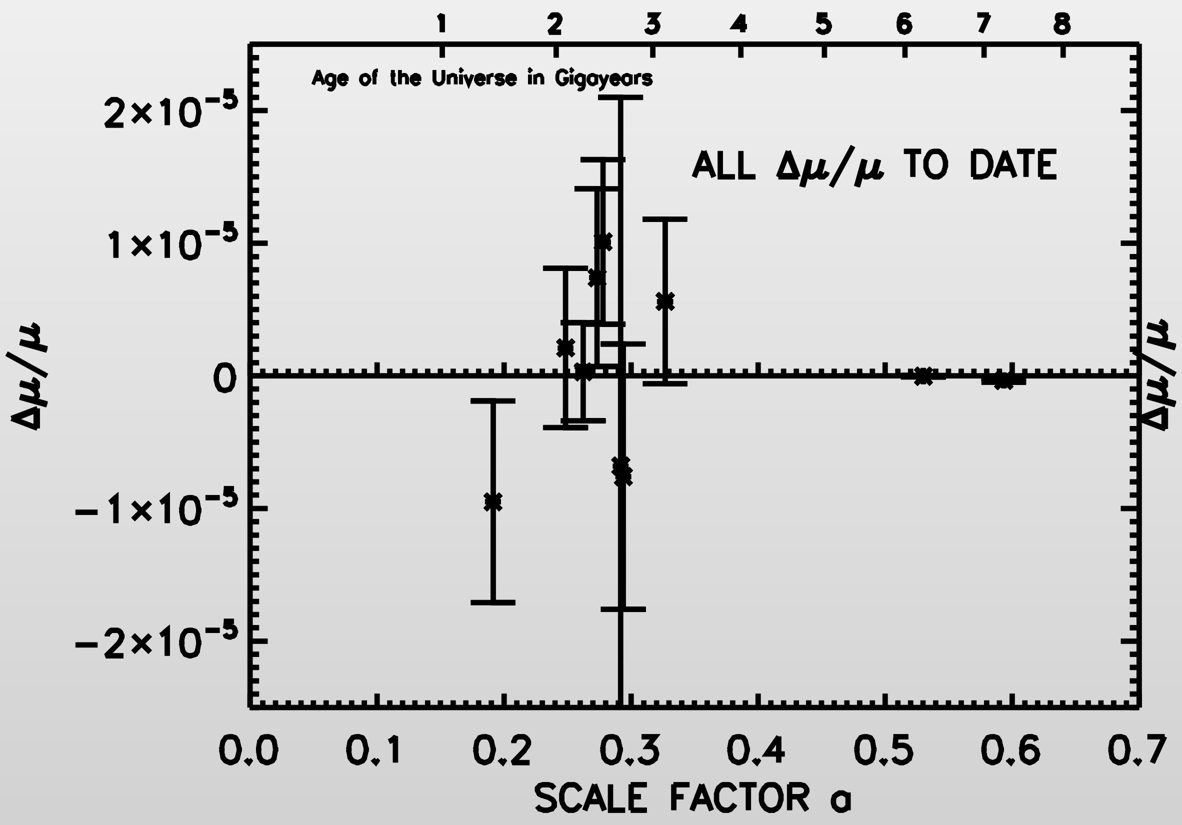


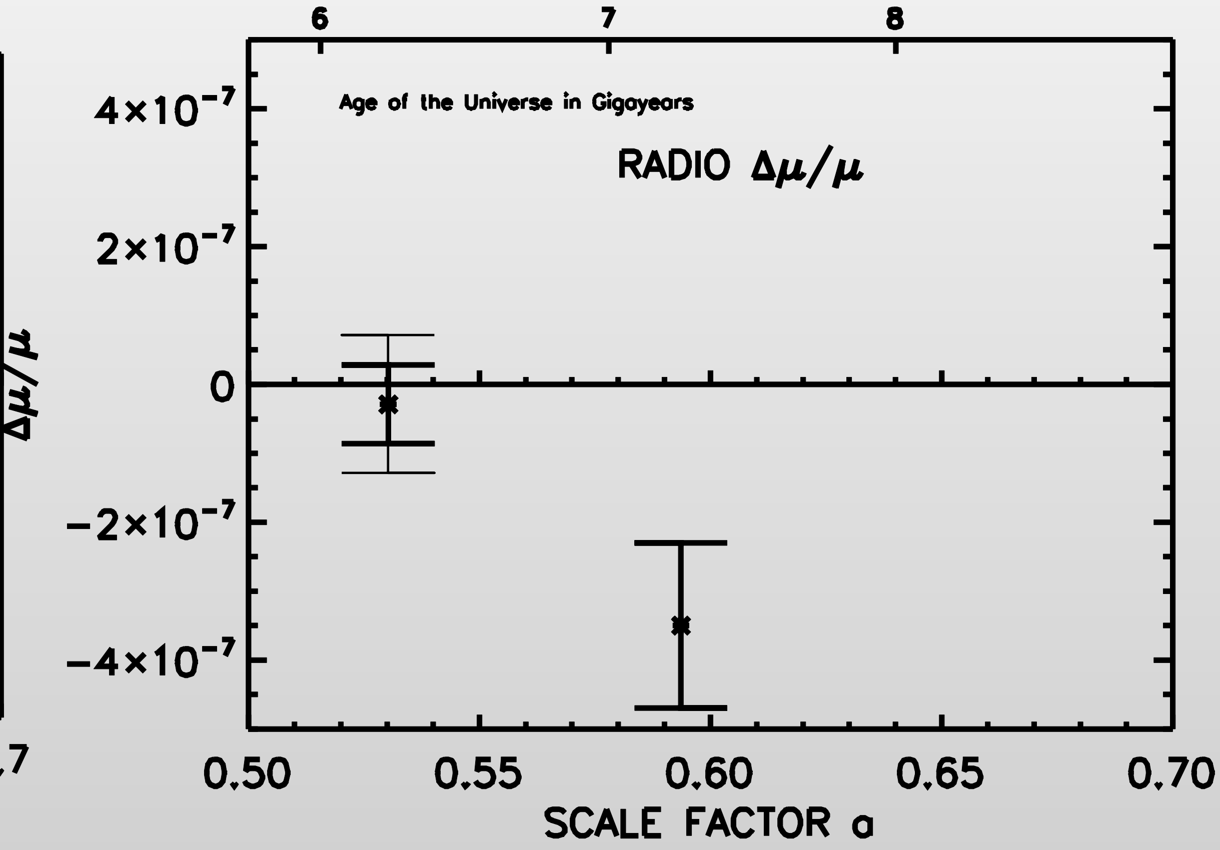
Overlap areas are
Very important

Observational Constraints on $\frac{\Delta\mu}{\mu}$

The primary constraint is $\frac{\Delta\mu}{\mu} \leq (2.9 \pm 5.7)x10^{-8}$ at z = 0.88582

Bagdonaite et al. 2013 and Kanekar et al. 2014 with methanol





Fine Structure Constant α Constraints

- Optical observations of multiple fine structure splittings in a large number of systems (many multiplet method)
  - Several thousand systems measured with many hundred at high accuracy but most spectra were taken for other reasons with UVES and HiRes.
  - Recently dedicated programs to measure a with high accuracy have been implemented.
  - The constraints on $\frac{\Delta\alpha}{\alpha}$ from Murphy, Malec and Prochaska 2016 are used in this study.
  - $\frac{\Delta\alpha}{\alpha} = (0.4 \pm 1.7)x10^{-6}$ at an average redshift of 1.54 (9.4 gigayear lookback)
  - The $\frac{\Delta\alpha}{\alpha}$ constraint is an order of magnitude looser than the radio $\frac{\Delta\mu}{\mu}$ constraint.

Constraints for this study

$\frac{\Delta\alpha}{\alpha} = (0.4 \pm 1.7)x10^{-6}$ at an average redshift of 1.54

$\frac{\Delta\mu}{\mu} = (0.29 \pm 1.0)x10^{-7}$ at a redshift of 0.88582

These limits are all 1σ limits and can be characterized as controversial, particularly the $\frac{\Delta\alpha}{\alpha}$ limit, but they serve as a basis for the method presented in the following.

Connecting $\frac{\Delta\mu}{\mu}$ and $\frac{\Delta\alpha}{\alpha}$ to $\frac{\Delta\Lambda_{QCD}}{\Lambda_{QCD}}$, $\frac{\Delta\nu}{\nu}$ and $\frac{\Delta h}{h}$

Λ_{QCD} = QCD Scale, ν = Higgs VEV and $h =$ Yukawa couplings

Although μ and α are fundamental constants their values depend on the values of the physics parameters Λ_{QCD}, ν and h.

We assume in the following that even though the parameters are allowed to vary with time the <u>Standard Model relations between the parameters and the constants still hold</u>.

The results are model dependent to some degree and as such they are tests of the models.

The relations are meant to be restrictions on the parameter space available to the models.

$\frac{d\mu}{\mu}$ as a Function of QCD, Higgs and Yukawa

Any change in the proton to electron mass ratio should depend on the basic physics parameter that determine the proton and electron mass.

$$\frac{d\mu}{\mu} = \frac{dm_P}{m_P} - \frac{dm_e}{m_e}$$

h_e is the electron Yukawa coupling
ν is the Higgs VEV and Λ_{QCD} is the QCD scale

The electron is easy $m_e = h_e \nu$, $\frac{dm_e}{m_e} = \frac{dh_e}{h_e} + \frac{d\nu}{\nu}$

For the proton $\frac{dm_P}{m_P} = a\frac{d\Lambda_{QCD}}{\Lambda_{QCD}} + b(\frac{dh^*}{h} + \frac{d\nu}{\nu})$ (Coc et al. 2007)

where a and b are scalars of order unity and a + b = 1.

$\therefore \frac{d\mu}{\mu} = a\frac{d\Lambda_{QCD}}{\Lambda_{QCD}} + (b-1)(\frac{dh}{h} + \frac{d\nu}{\nu}) = a[\frac{d\Lambda_{QCD}}{\Lambda_{QCD}} - (\frac{dh}{h} + \frac{d\nu}{\nu})]$

*Assumes that $\frac{dh_i}{h_i}$ is the same for all Yukawa couplings h_i

$\frac{d\alpha}{\alpha}$ as a Function of QCD, Higgs and Yukawa

The fine structure constant α has a different dependence on the particle physics parameters given by

$\frac{d\alpha}{\alpha} = \frac{1}{R}[\frac{d\Lambda_{QCD}}{\Lambda_{QCD}} - \frac{2}{9}(\frac{d\nu}{\nu} + \frac{dh}{h})]$ again from Coc et al. 2007.

R is a constant that is dependent on the particular GUT theory invoked as is the factor of $\frac{2}{9}$. R is a function of the beta function coefficients b_i which at the unification scale become unified to a single value b_U. At that scale $R = \frac{2\pi}{9\alpha}\frac{b_U+3}{\frac{8}{3}b_U-12}$. When the magnitude b_u becomes large R approaches the value 36 which will be used in the example presented later.

Solving for $\frac{d\Lambda_{QCD}}{\Lambda_{QCD}}$

Now have two equations in $\frac{d\Lambda_{QCD}}{\Lambda_{QCD}}$ and $\left(\frac{dh}{h}+\frac{d\nu}{\nu}\right)$

$$\frac{d\mu}{\mu} = a\left[\frac{d\Lambda_{QCD}}{\Lambda_{QCD}} - \left(\frac{dh}{h}+\frac{d\nu}{\nu}\right)\right]$$

$$\frac{d\alpha}{\alpha} = \frac{1}{R}\left[\frac{d\Lambda_{QCD}}{\Lambda_{QCD}} - \frac{2}{9}\left(\frac{d\nu}{\nu}+\frac{dh}{h}\right)\right]$$

Can solve for $\frac{d\Lambda_{QCD}}{\Lambda_{QCD}}$

$$\boldsymbol{\frac{d\Lambda_{QCD}}{\Lambda_{QCD}} = \frac{9R}{11}\frac{d\alpha}{\alpha} - \frac{2}{11a}\frac{d\mu}{\mu}}$$

Can Also Solve for $(\frac{dh}{h} + \frac{d\nu}{\nu})$

$$(\frac{dh}{h} + \frac{d\nu}{\nu}) = \frac{9}{11}(R\frac{d\alpha}{\alpha} - \frac{1}{a}\frac{d\mu}{\mu})$$

Without more information there are not solutions for $\frac{dh}{h}$ and $\frac{d\nu}{\nu}$ independently, however, ν and h have a model dependent relationship

$\nu = M_{Pl}\exp(-\frac{8\pi^2 c}{h_t^2})$ where M_{Pl} is the Planck mass h_t is the Yukawa coupling for the top quark and c is a constant of order unity (Coc et al. 07). This leads to

$$\frac{d\nu}{\nu} = \frac{158c}{h^2}\frac{dh}{h} \approx 160\frac{dh}{h} = S\frac{dh}{h}$$

Again assuming $\frac{dh_i}{h_i} = \frac{dh}{h}$ for all i and that S is a model dependent parameter.

Model Dependent Solution for $\frac{d\nu}{\nu}$

From the previous slide $\frac{dh}{h} = \frac{1}{S}\frac{d\nu}{\nu}$ therefore

$$(1+\frac{1}{S})\frac{d\nu}{\nu} = \frac{9}{11}(R\frac{d\alpha}{\alpha} - \frac{1}{a}\frac{d\mu}{\mu})$$

for a solution involving the model dependent parameters R, S and a.

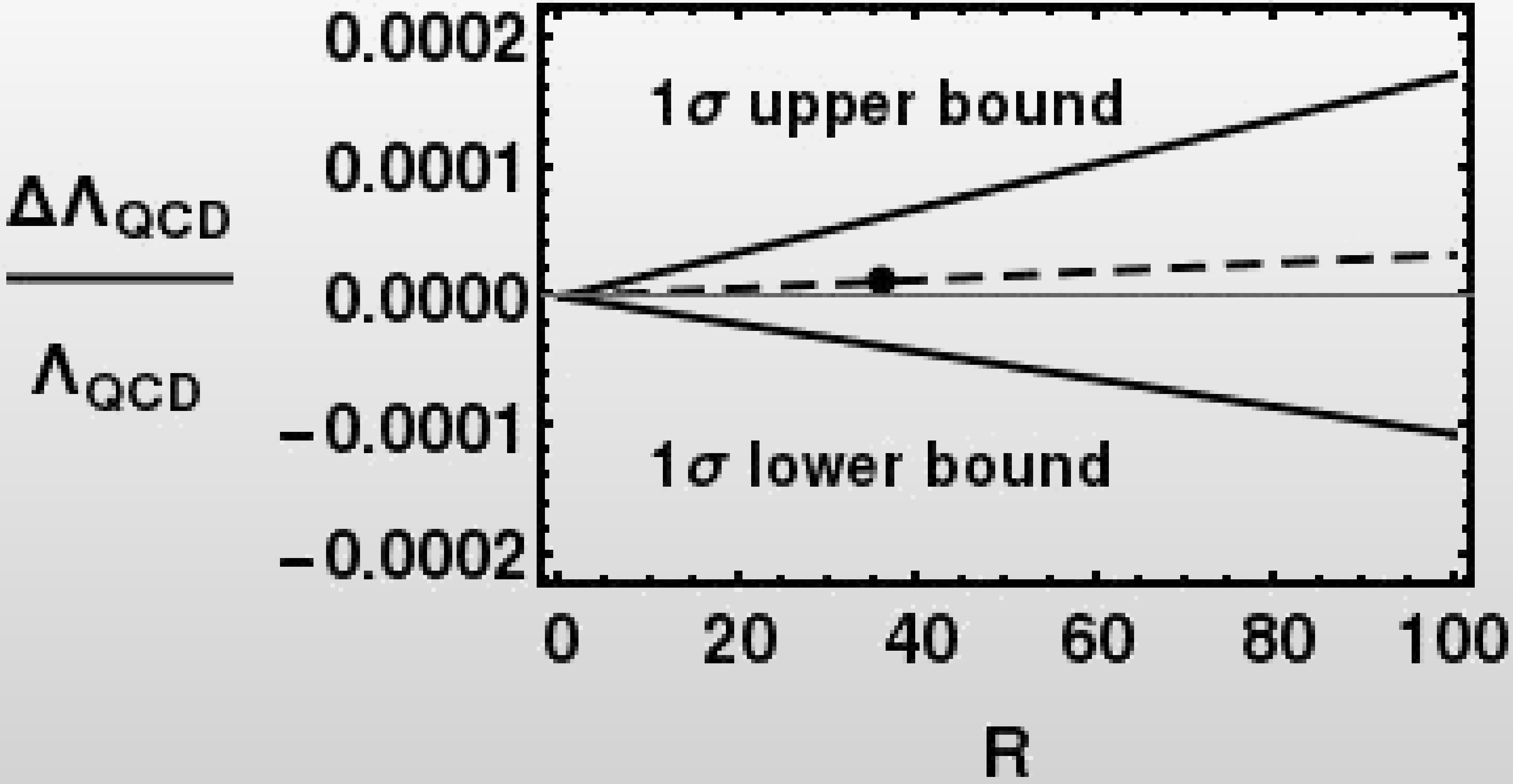
Observational Limits on the Time Variation of Λ_{QCD}

- $\frac{d\Lambda_{QCD}}{\Lambda_{QCD}} = \frac{9R}{11}\frac{d\alpha}{\alpha} - \frac{2}{11a}\frac{d\mu}{\mu}$
- $\frac{\Delta\alpha}{\alpha} = (0.4 \pm 1.7)x10^{-6}$
- $\frac{\Delta\mu}{\mu} = (0.29 \pm 1.0)x10^{-7}$
- $\frac{d\Lambda_{QCD}}{\Lambda_{QCD}} \leq (\pm 1.7x10^{-6})\frac{9R}{11} - (\pm 1.0x10^{-7})\frac{2}{11a}$
- The limit is model dependent in R and a.
- Need to examine a typical model

The Model of Coc et al. 2007

- The example is from: Coc, Nunes, Olive, Uzan & Vangioni 2007 Phys. Rev. D., 76, 023511
- The model parameters are R = 36, a = 0.76, b = 0.24, S = 160
- $\frac{d\Lambda_{QCD}}{\Lambda_{QCD}} \leq \left(\pm 1.7x10^{-6}\right)(29.5) - (\pm 1.0x10^{-7})(0.24) = \pm 5.0x10^{-5}$
- The look back time is $\approx$ 7 gigayears which gives a linear time evolution of $\frac{\dot{\Lambda}}{\Lambda} \leq 7x10^{-15}$ per year.
- The constraint is dominated by the loose limit on $\frac{\Delta\alpha}{\alpha}$.
- The parameter a is generally agreed to be close to 1 but different models have a wide range of R.

Limits on $\frac{\Delta\Lambda_{QCD}}{\Lambda_{QCD}}$ as a function of R



Observational Limit on the Higgs VEV (v) and Yukawa Coupling (h) Variation

$$\frac{d\nu}{\nu} = \left(\frac{S}{S+1}\right)\frac{9}{11}\left(R\frac{d\alpha}{\alpha} - \frac{1}{a}\frac{d\mu}{\mu}\right) = \left(\frac{160}{161}\right)\frac{9}{11}\left(36\frac{d\alpha}{\alpha} - \frac{1}{0.76}\frac{d\mu}{\mu}\right)$$

The result is essential independent of S for large S

$$\frac{d\nu}{\nu} = 29.3\frac{d\alpha}{\alpha} - 1.07\frac{d\mu}{\mu} = \pm 5.0x10^{-5}$$

$$\frac{dh}{h} = \frac{1}{S}\frac{d\nu}{\nu} = \pm 3.1x10^{-7}$$

Again for a look back time of 7 gigayears

Model Dependent α Limits

- The observational limits are essentially independent of the limits on $\frac{\Delta\mu}{\mu}$ due to the order of magnitude looser limit on $\frac{\Delta\alpha}{\alpha}$ and the order of magnitude larger coefficient of $\frac{\Delta\alpha}{\alpha}$.
- The model however predicts a smaller change in α than in μ by a factor of $\frac{1}{R}$.
- The model predicted change in α is $\frac{\Delta\alpha}{\alpha} = \frac{1}{R}\frac{\Delta\mu}{\mu} = \frac{1}{36}\frac{\Delta\mu}{\mu} = 2.8x10^{-9}$
- The constraint is of course consistent with the observational results but imposes a much stricter limit on $\frac{\Delta\alpha}{\alpha}$.

Constraints with the Model Dependent Limit on $\frac{\Delta\alpha}{\alpha}$

- Observational $\frac{\Delta\mu}{\mu} \leq \pm 1.0x10^{-7}$
- Modeled $\frac{\Delta\alpha}{\alpha} \leq \pm 2.9x10^{-9}$
- $\frac{\Delta\Lambda_{QCD}}{\Lambda_{QCD}} \leq \pm 6.2x10^{-8}$
- $\frac{\Delta\nu}{\nu} \leq \pm 2.5x10^{-8}$
- $\frac{\Delta h}{h} \leq \pm 1.6x10^{-10}$
- 7 gigayear look back time

Quick Derivation of μ and α Evolution in a Rolling Scalar Field

- For a linear (first term of Taylor series) coupling with a scalar field ϕ

$\frac{d\mu,\alpha}{\mu,\alpha} = \varsigma_{\mu,\alpha}\kappa(\phi - \phi_0),\ \kappa^2 = 8\pi {m_P}^{-2}$ and $\varsigma_{\mu,\alpha}$is the coupling constant

- The dark energy EoS w also is a function of ϕ
- $(w+1) = \frac{(\kappa\acute{\phi})^2}{3\Omega_\phi}$, $\acute{\phi}$ where the prime indicates $\frac{d}{d\,lna}$ Nunes & Lidsey* (2004)
- Expressing $\frac{d\mu,\alpha}{\mu,\alpha}$ in terms of $\frac{d}{d\,lna}$ we get $\frac{d\acute{\mu},\acute{\alpha}}{\mu,\alpha} = \varsigma_{\mu,\alpha}\sqrt{3\Omega_\phi(w+1)}$
- The evolution is then $\frac{\Delta\mu}{\mu} = \varsigma_\mu \int_1^a \sqrt{3\Omega(x)_\phi(w(x)+1)}x^{-1}dx$ between as scale factor of 1 (present time) and a, with a similar equation for α.
- The evolution of the dark energy EoS w and the ratio of the dark energy density to the critical density Ω_ϕ is determined by the chosen cosmology.
- In the following freezing and thawing quintessence is used as the example.

Quintessence Evolutionary Tracks of $\frac{\Delta\Lambda_{QCD}}{\Lambda_{QCD}}$ and $\frac{\Delta\nu}{\nu}$

- The evolutionary tracks of Λ_{QCD} and ν are functions of the evolution of α and μ and vice versa.
- For a given cosmology the evolution of α and μ can be calculated.
- As example a freezing and a thawing quintessence cosmology is examined.
  - In a freezing cosmology the dark energy equation of state w “freezes” toward minus one.
  - In a thawing cosmology the dark energy equation of state w “thaws” away from minus one.
- The evolution of the constants for this cosmology was considered previously (RIT12)

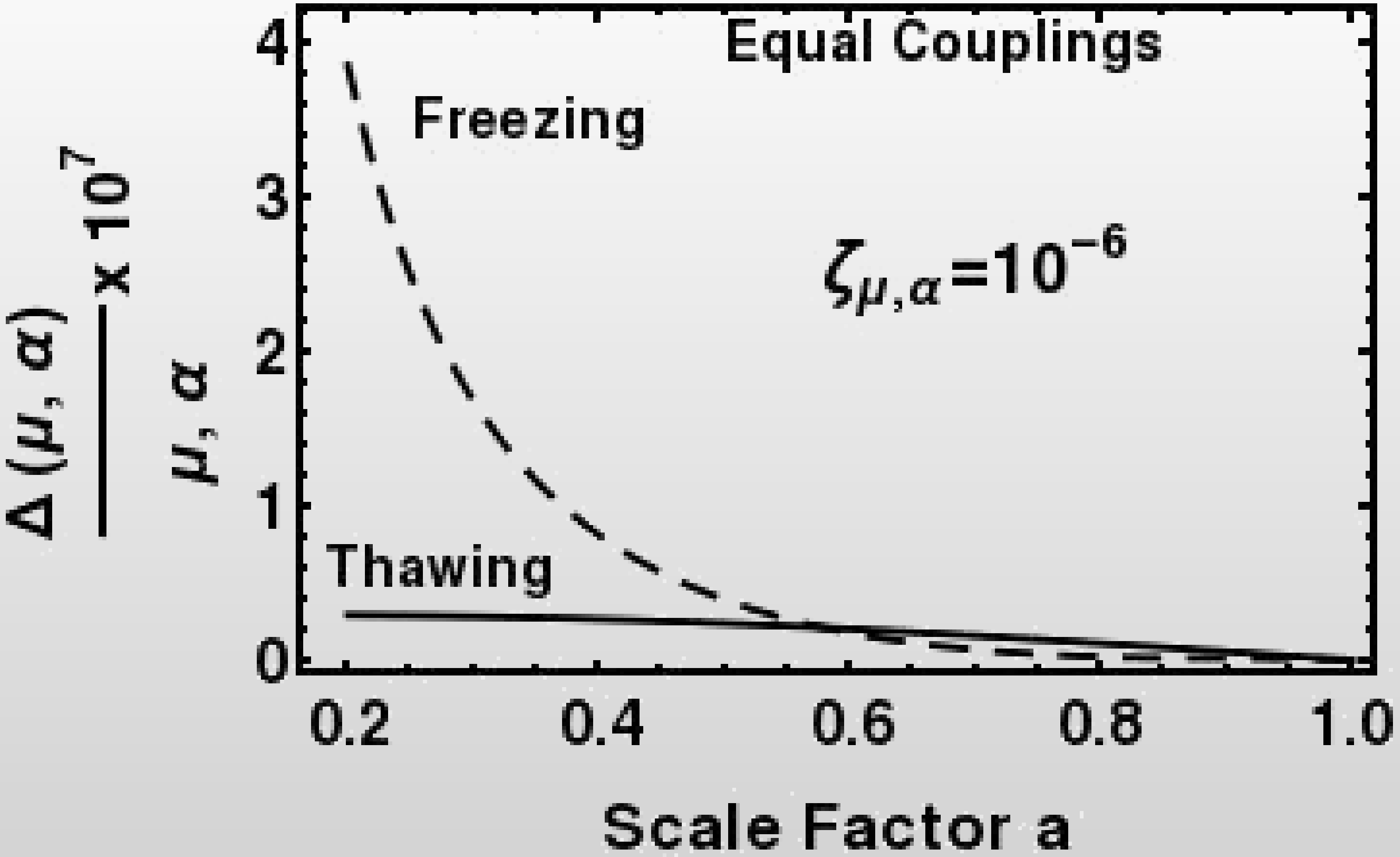
Basic Methods

- For the Quintessence cosmology $\Omega_\phi = [1 + \left(\Omega_{\phi_0}^{-1} - 1\right) a^{-3}]^{-1}$
- The equation of state is $(1+w) = \frac{1}{3}\lambda_0^2[\frac{1}{\sqrt{\Omega_\phi}} - (\frac{1}{\Omega_\phi} - 1)\left(tanh^{-1}\left(\sqrt{\Omega_\phi}\right) + C^2\right)]^2$ where λ_0 and C are based on slow roll and initial conditions. Here we set $\lambda_0 = 0.1$ for slow roll and $C = 1$ for the freezing cosmology and $C = 0$ for the thawing cosmology. The evolution of either μ or α is found by numerically integrating

$$\frac{\Delta\mu}{\mu} = \varsigma_\mu \int_1^a \sqrt{3\Omega_\phi(x)(w(x)+1)}x^{-1}dx$$

for a range of scale factors a and a similar equation for α.

Freezing and Thawing Evolution of μ and α

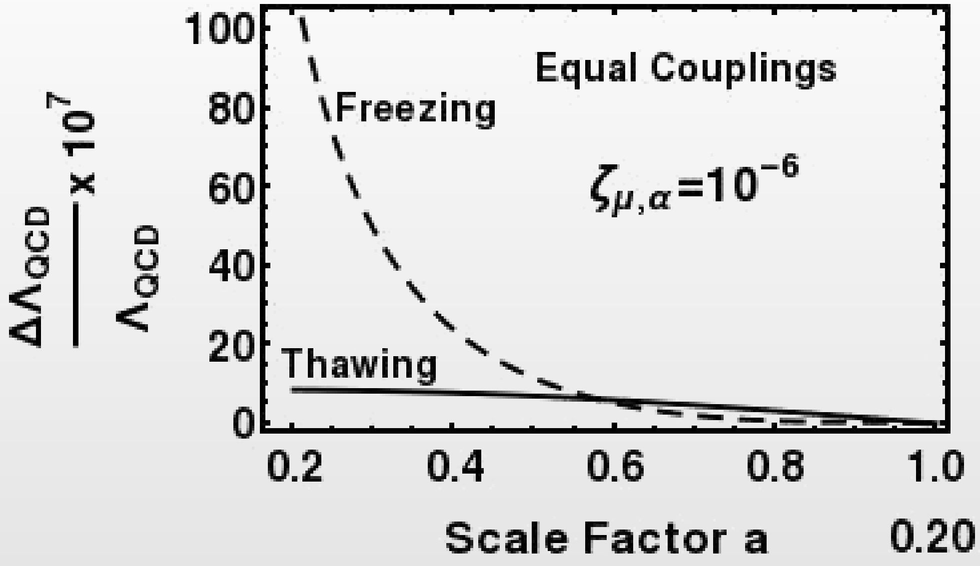


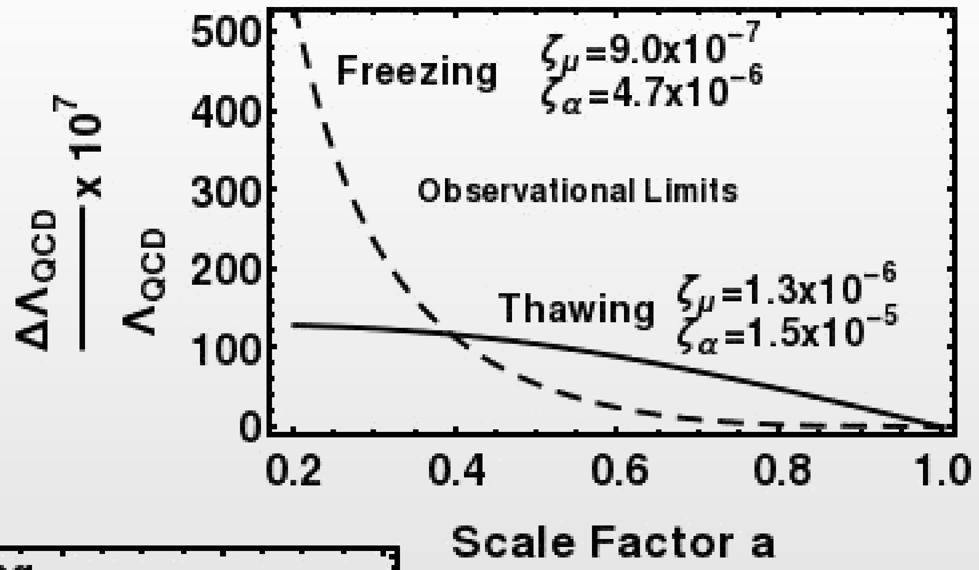
Example Cases and Cosmologies

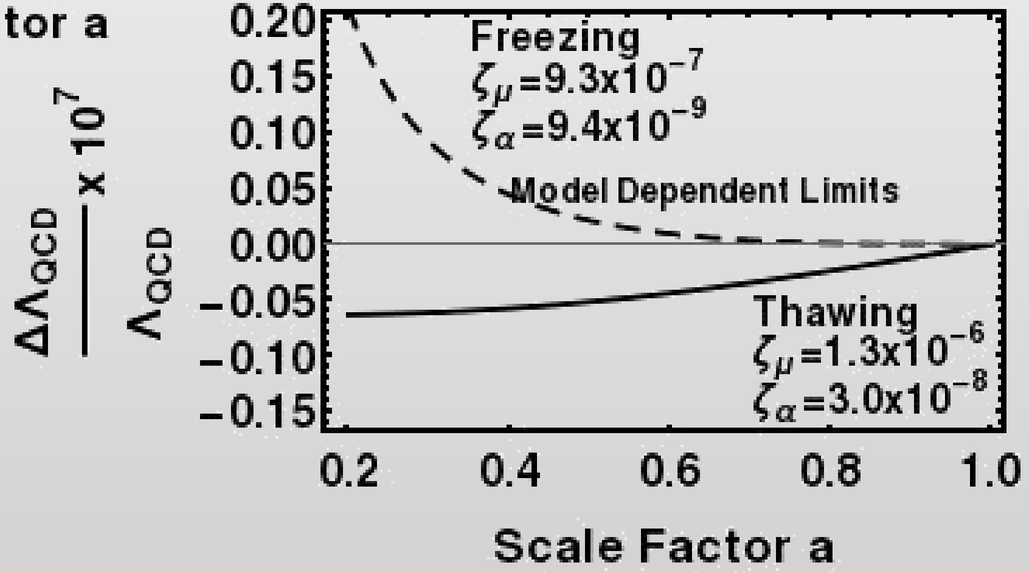
- Three cases are considered for both freezing and thawing quintessence:
  - A case where the coupling constants are equal $\varsigma_\mu = \varsigma_\alpha = 10^{-6}$
  - A case where the coupling constants are set to match the observational limits.
  - A case where the coupling constants are set to match the model dependent limits.

Case	**Cosmology**	$\Delta\alpha/\alpha$	$\Delta\mu/\mu$	ς_α	ς_μ
Equal ς	Freeze	NA	NA	10^{-6}	10^{-6}
Equal ς	Thaw	NA	NA	10^{-6}	10^{-6}
Obs. limits	Freeze	$4.0\text{x}10^{-7}$	$2.9\text{x}10^{-8}$	$4.7\text{x}10^{-6}$	$9.3\text{x}10^{-7}$
Obs. limits	Thaw	$4.0\text{x}10^{-7}$	$2.9\text{x}10^{-8}$	$1.5\text{x}10^{-5}$	$1.3\text{x}10^{-6}$
Model limits	Freeze	$8.0\text{x}10^{-10}$	$2.9\text{x}10^{-8}$	$9.4\text{x}10^{-9}$	$9.3\text{x}10^{-7}$
Model limits	Thaw	$8.0\text{x}10^{-10}$	$2.9\text{x}10^{-8}$	$3.0\text{x}10^{-8}$	$1.3\text{x}10^{-6}$

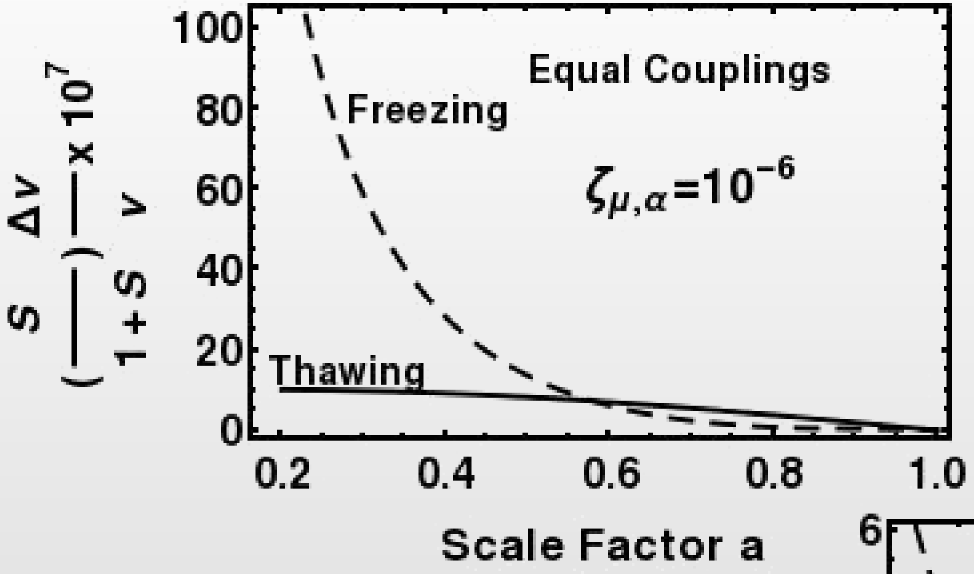
Evolution of $\frac{\Delta\Lambda_{QCD}}{\Lambda_{QCD}}$

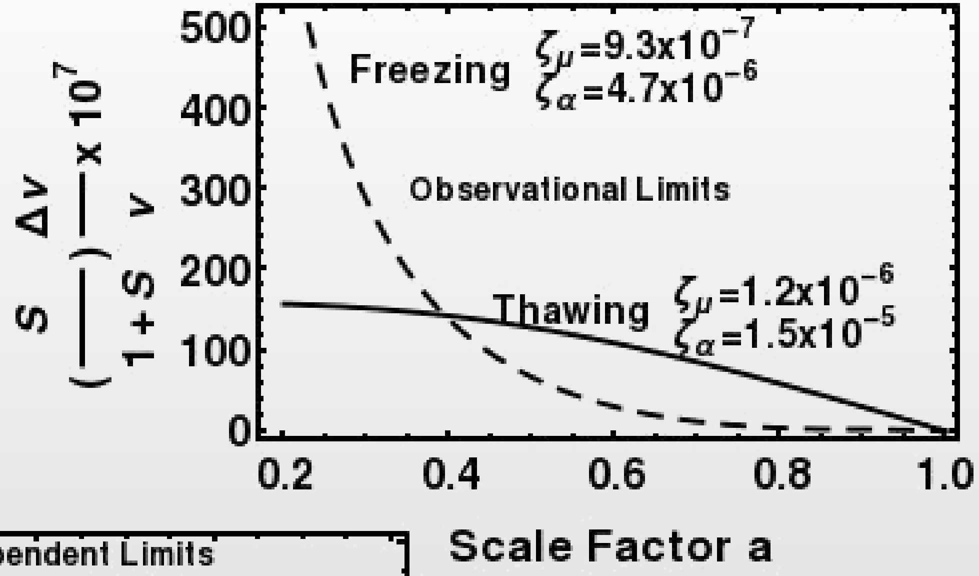


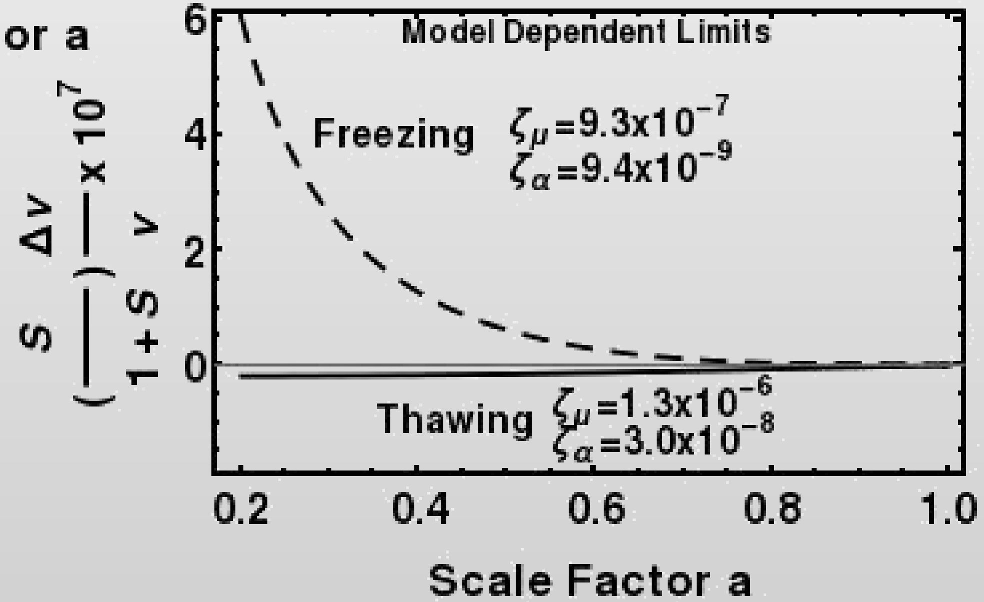




Evolution of $\frac{\Delta\nu}{\nu}$ $\quad \frac{\Delta h}{h} = \frac{1}{S}\frac{\Delta\nu}{\nu}$







Current Rates of Change of Λ_{QCD} and ν

- The current rate of change of μ or α per $\ln(a)$ is
  - $r_{\mu,\alpha} = \varsigma_{\mu,\alpha}\sqrt{3\Omega_{\phi}(x)(w(x)+1)}x^{-1}$ at $x = a = 1$
- The current rate of change of μ or α per year is $H_0 r_{\mu,\alpha}$ with H_0 in units of $years^{-1}$.
- Inserting those rates into the equations for $\frac{\Delta\Lambda_{QCD}}{\Lambda_{QCD}}$ and $\frac{\Delta\nu}{\nu}$ in terms of $\frac{\Delta\mu}{\mu}$ and $\frac{\Delta\alpha}{\alpha}$ gives the current rates of change $\frac{\dot{\Lambda}}{\Lambda}$ and $\frac{\dot{\nu}}{\nu}$ per year.

$\frac{\dot{\Lambda}}{\Lambda}$ and $\frac{\dot{\nu}}{\nu}$ per Year

Case	Cosmology	$\frac{\dot{\Lambda}}{\Lambda}$ *per year*	$\frac{\dot{\nu}}{\nu}$ per year
Equal ς	Freeze	-4.5×10^{-17}	7.7×10^{-19}
Equal ς	Thaw	-1.2×10^{-16}	2.1×10^{-18}
Obs. limits	Freeze	-2.3×10^{-16}	-4.0×10^{-18}
Obs. limits	Thaw	-1.8×10^{-15}	-4.4×10^{-17}
Model limits	Freeze	3.4×10^{-19}	1.9×10^{-18}
Model limits	Thaw	1.3×10^{-18}	7.1×10^{-18}

Summary

- Changes in the Quantum Chromodynamic Scale, the Higg VEV and the Yukawa couplings physics parameters produce changes in the fundamental constants μ and α that are observable in the early universe.
- The lack of any observable changes in μ and α put quantitative constraints on theories the predict time variation of the physics parameters.
- Although there are legitimate concerns about the validity of the Standard Model and ΛCDM cosmology the parameter space for alternative theories is significantly constrained.

Conclusion

- An important requirement for any new physics or cosmology is the prediction of the rate of variance of the fundamental constants and a comparison of these predictions with observational and laboratory limits.