3-FORM COSMOLOGY:
phantom behavior, singularities and interactions

"Varying Constants and Fundamental Cosmology" - VARCOSMOFUN'16

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This talk is based on the work

JM, M. Bouhmadi-López, K. Sravan Kumar, J. Marto, Y. Tavakoli

Distinguishing interactions in 3-form dark energy models

[arXiv:1608.01679]

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Reviewing the 3-form field $A_{\mu\nu\rho}$

The 3-form action

A 3-form is a totally anti-symmetric covariant tensor, $A_{\mu\nu\rho} = A_{[\mu\nu\sigma]}$.
We will consider the following action for a massive 3-form minimally coupled to gravity[1,2]

$$S^A = \int \mathrm{d}^4\mathbf{x}\sqrt{|\det g_{\mu\nu}|}\left[-\frac{1}{48}F^{\mu\nu\rho\sigma}F_{\mu\nu\rho\sigma} - V\left(A^{\mu\nu\rho}A_{\mu\nu\rho}\right)\right]. \quad (1)$$

The strength tensor, a 4-form, is defined through the exterior derivative:

$$F_{\mu\nu\rho\sigma} \equiv 4\nabla_{[\mu}A_{\nu\rho\sigma]}. \quad (2)$$

[1] C. Germani and A. Kehagias, J. Cosmol. Astropart. Phys. 2009, 28 (2009)
[2] T. S. Koivisto, D. F. Mota, and C. Pitrou, J. High Energy Phys. 2009, 92 (2009)

The 3-form equation of motion

The equation of motion, obtained from variation of S^A, is[1,2]

$$\nabla_\sigma F^\sigma{}_{\mu\nu\rho} - 12\frac{\partial V}{\partial\left(A^2\right)}A_{\mu\nu\rho} = 0\,. \tag{3}$$

$\Rightarrow$ a massless 3-form is equivalent to a cosmological constant[3]!

[1] C. Germani and A. Kehagias, J. Cosmol. Astropart. Phys. 2009, 28 (2009)
[2] T. S. Koivisto, D. F. Mota, and C. Pitrou, J. High Energy Phys. 2009, 92 (2009)
[3] M. Duff and P. Van Nieuwenhuizen, Phys. Lett. B 94, 179 (1980)

3-form Cosmology

We consider homogeneous and isotropic universe described by the Friedmann-Lemaître-Robertson-Walker line element

$$ds^2 = -dt^2 + a^2(t)\gamma_{ij}dx^i dx^j \,. \tag{4}$$

t - cosmic time, $\dot{\{\}} = d\{\}/dt$
a - scale factor
x^i - comoving spatial coordinates (roman indices run from 1 to 3).

Only the purely spatial components of the 3-form are dynamical[2]:

$$A_{0ij} = 0\,, \qquad A_{ijk} = a^3(t)\chi(t)\epsilon_{ijk}\,. \tag{5}$$

[2] T. S. Koivisto, D. F. Mota, and C. Pitrou, J. High Energy Phys. 2009, 92 (2009)

3-form Cosmology: background equations

⇒ Friedmann Equation

$$3H^2 = \kappa^2 \rho_\chi = \kappa^2 \left[\frac{1}{2} \left(\dot{\chi} + 3H\chi \right)^2 + V(\chi^2) \right] . \tag{6}$$

⇒ Raychaudhuri equation

$$\dot{H} = -\frac{\kappa^2}{2} \left(\rho_\chi + P_\chi \right) = -\kappa^2 \chi^2 \frac{\partial V}{\partial \chi^2} . \tag{7}$$

A 3-form can show phantom-like behavior if $\partial V / \partial \chi^2 < 0$.

⇒ Equation of motion

$$\ddot{\chi} + 3H\dot{\chi} + 3\dot{H}\chi + 2\chi \frac{\partial V}{\partial \chi^2} = 0 . \tag{8}$$

3-form Cosmology: evolution for large χ

For non-negative potentials, the Friedmann equation imposes

$$-\sqrt{\frac{2}{3\kappa^2}} \leq \frac{\dot{\chi} + 3H\chi}{3H} \leq \sqrt{\frac{2}{3\kappa^2}} \tag{9}$$

Equality is achieved when $V/H^2 = 0$.

$\Rightarrow$ If $H > 0$ the 3-form decays as $\chi \sim a^{-3}$ for $|\chi| \gg \chi_c = \sqrt{2/(3\kappa^2)}$.

$\Rightarrow$ Once inside the interval $[-\chi_c, \chi_c]$, the field χ cannot escape from it.

$\Rightarrow$ If $\chi = \pm\chi_c$, $\dot{\chi} = 0$ and $V(\pm\chi_c) \neq 0$, then $H \to \infty$.

This result only depends on the sign of V, not on its shape!

3-form Cosmology: evolution for small χ

Combining the Raychaudhuri equation and the equation of motion for χ:

$$\ddot{\chi} + 3H\dot{\chi} + 2\chi\left(1 - \frac{\chi^2}{\chi_c^2}\right)\frac{\partial V}{\partial \chi^2} = 0\,. \quad (10)$$

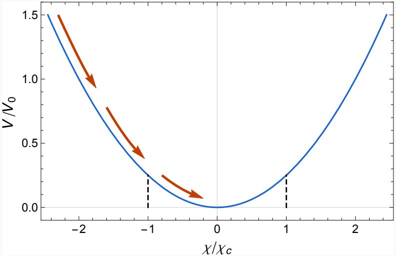
The static solutions are:

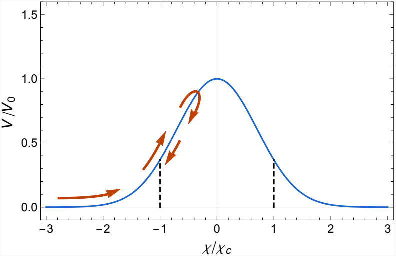
- the origin: $\chi = 0$,
- the critical points of the potential: $\dfrac{\partial V}{\partial \chi^2} = 0$,
- the limiting points: $\chi = \pm\chi_c$.

Once inside the interval $[-\chi_c, \chi_c]$, the field χ evolves towards a local minimum of V. However...

3-form Cosmology: evolution for small χ

...since $[-\chi_c, \chi_c]$ is a closed interval, the limiting points $\chi = \pm\chi_c$ act as local extrema:





local maxima if $\dfrac{\partial V}{\partial \chi^2} > 0$

local minima if $\dfrac{\partial V}{\partial \chi^2} < 0$

Little Sibling of the Big Rip

The Little Sibling of the Big Rip

The Little Sibling of the Big Rip (LSBR)[4] is a cosmological event that happens at infinite time and is characterized by

- $a\,(t \to \infty) \to +\infty$,
- $H\,(t \to \infty) \to \infty$,
- $\dot{H}\,(t \to \infty) \to \mathrm{constant}$.

In general, this can be obtained with an equation of state:

$$p = -\rho - A \qquad (A > 0)\,. \tag{11}$$

Solving the conservation equation we find $H^2 \propto \log(a)$ and $\dot{H} = (\kappa^2/2)A$.

[4] M. Bouhmadi-López, A. Errahmani, P. Martín-Moruno, T. Ouali, and Y. Tavakoli, Int. J. Mod. Phys. D 24, 1550078 (2015)

The LSBR and the 3-form

When $\chi = \pm\chi_c$ acts as a minimum of the 3-form potential, V, and if at the minimum $V(\pm\chi_c) \neq 0$, we find that

- $H \to +\infty$ $\quad (\chi = \pm\chi_c\,,\ \dot{\chi} = 0$ and $V(\pm\chi_c) \neq 0)$
- $\dot{H} \to -\kappa^2\chi_c^2 \left.\dfrac{\partial V}{\partial \chi^2}\right|_{|\chi|=\chi_c} > 0$ $\quad \left(\dfrac{\partial V}{\partial \chi^2} < 0\right)$

$\Rightarrow$ The 3-form leads the universe to a LSBR!

Avoiding the LSBR

Can the presence of dark matter (DM) avoid this behavior?

- Non-interacting DM decays too fast

$$\ddot{\chi} + 3H\dot{\chi} + 2\chi\left(1 - \frac{\chi^2}{\chi_c^2}\right)\frac{\partial V}{\partial \chi^2} = -\frac{3\kappa^2}{2}\chi\,\rho_m\,, \quad (\rho_m \sim a^{-3}) \tag{12}$$

and does not change the end state of the universe:

- *What about interacting DM? Can the LSBR be avoided through a suitable DM/3-form interaction?*

Interacting DM and 3-form

Interacting DM and 3-form

Let us consider a phenomenological model of interacting DM/3-form, where the 3-form plays the role of dark energy (DE).

The evolution of the matter fluids is given by

$$\dot{\rho}_m + 3H\rho_m = -Q\,, \qquad \ddot{\chi} + 3H\dot{\chi} + 3\dot{H}\chi + 2\chi\frac{\partial V}{\partial \chi^2} = \frac{Q}{\dot{\chi} + 3H\chi}\,. \quad (13)$$

The interaction is controlled by the term Q:

- $Q > 0$ energy transfer from DM to DE,
- $Q < 0$ energy transfer from DE to DM.

Dynamical System representation

To find which interactions remove the LSBR we employ a dynamical system approach.

The individual terms of the Friedmann equation are mapped to the compact variables[5] s, y and z:

$$s \equiv \sqrt{\frac{\kappa^2 \rho_m}{3H^2}} \,, \quad y \equiv \frac{\dot{\chi} + 3H\chi}{3H\chi_c} \,, \quad z \equiv \sqrt{\frac{\kappa^2 V}{3H^2}} \,. \tag{14}$$

The field χ is mapped to the compact variable[6] u

$$u \equiv \frac{2}{\pi} \arctan\left(\frac{\chi}{\chi_c}\right) \,. \tag{15}$$

[5] T. S. Koivisto and N. J. Nunes, Phys. Rev. D 80, 103509 (2009)
[6] C. G. Boehmer, N. Chan, and R. Lazkoz, Phys. Lett. B 714, 11 (2011)

Evolution equations

The Friedmann equation, $y^2 + z^2 + s^2 = 1$, allows us to eliminate one variable. A 3-dimensional dynamical system $(u,\, y,\, z)$ can be obtained

$$u' = \frac{6}{\pi}\cos^2\left(\frac{\pi u}{2}\right)\left[y - \tan\left(\frac{\pi u}{2}\right)\right]\,,$$
$$y' = \frac{3}{2}\left\{\left[1 - y^2 - z^2\right]y + \frac{\lambda\,(u)}{3}z^2\left[1 - \tan\left(\frac{\pi u}{2}\right)y\right]\right\} + \frac{1}{6y}\frac{\kappa^2 Q}{H^3}\,,$$
$$z' = \frac{3}{2}z\left\{\left[1 - y^2 - z^2\right] - \frac{\lambda\,(u)}{3}\left[y - \tan\left(\frac{\pi u}{2}\right)\left(1 - z^2\right)\right]\right\}\,. \qquad (16)$$

Here $\lambda(u) = -3\chi_c V^{-1}\partial V/\partial\chi$.

The system (16) is closed if $\kappa^2 Q/H^3$ is a function of $(u,\, y,\, z)$.

Fixed Points: categories

We identify 3 different categories of fixed points (FP):

- **Type I:** $z_{fp} = 0$ and $u_{fp} \neq \pm 1$. The existence of FP-I does not depend on the type of potential (though the stability does). The LSBR event, corresponding to FP $(\pm 1/2, \pm 1, 0)$ is in this category.
- **Type II:** $z_{fp} \neq 0$ and $u_{fp} \neq \pm 1$. In the absence of interaction, the FP-II correspond to the critical points of the potential. Both the existence and stability depend on V.
- **Type III:** $u_{fp} = \pm 1$, i.e. states at χ-infinity. The FP-III were identified for the first time in this work. They depend strongly on the potential and need to be analysed with care. If physically viable, they correspond to the asymptotic past of the system.

Choosing the potential

We choose to work with the Gaussian potential

$$V = V_0 e^{-\frac{\xi}{9}\frac{\chi^2}{\chi_c^2}} \, . \qquad (17)$$

$\Rightarrow$ the 3-form behaves in the past as a cosmological constant and in the future drives the universe to a LSBR event.

$\Rightarrow$ At late-time, the 3-form has a phantom-like behavior (slightly favored by observations[7]) and the squared speed of sound is positive (no DE instabilities).

$\Rightarrow$ In terms of the dynamical variables we can write $\lambda\left(u\right) = \frac{2\xi}{3}\tan\left(\frac{\pi}{2}u\right)$.

[7] [Planck Collaboration], P. A. R. Ade et al., arXiv 1502.01589 (2015).

Choosing the interaction

We consider a phenomenological class of interactions

$$Q = 3H\left(\rho_m + \rho_\chi\right)\sum_{i=0}^{2}\alpha_i\left(\frac{\rho_\chi}{\rho_m + \rho_\chi}\right)^i . \tag{18}$$

This includes several examples which are abundant in the literature: $Q = 3H\alpha_m\rho_m$, $Q = 3H\alpha_{DE}\rho_{DE}$, $Q = 3H\alpha\,\rho_m\rho_{DE}/(\rho_m + \rho_{DE})$, ...

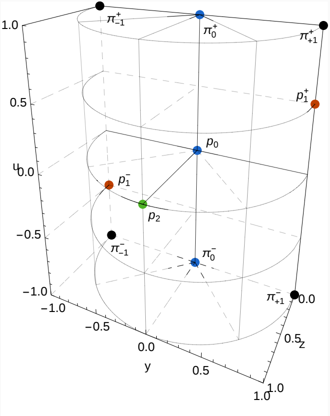
Most works set constraints on the maximum value of $|\alpha_i|$ on the order of[8,9] $10^{-2} \sim 10^{-1}$. We take these values as guidelines.

The dynamical system can be closed as $\kappa^2 Q/H^3 = 9\sum_{i=0}^{2}\alpha_i\left(y^2 + z^2\right)^i$.

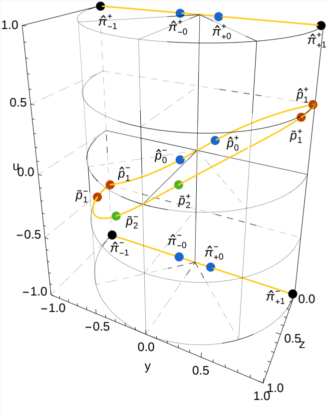
[8] A. A. Costa, L. C. Olivari, and E. Abdalla, Phys. Rev. D 92, 103501 (2015)
[9] C. Feng, B. Wang, E. Abdalla, and R. Su, Phys. Lett. B 665, 111 (2008)

Fixed Points: position

No interaction



Quadratic interaction



Removing the LSBR

In the absence of interaction we find that the system:

- has a matter dominated era in the past: $\pi_0^{\pm} = (\pm 1, 0, 0)$,
- evolves towards a future LSBR event: $p_1^{\pm} = (\pm 1/2, \pm 1, 0)$.

When the interaction is turned on, the position and stability of FP are altered. In particular the LSBR event is removed if and only if

$$\alpha_0 + \alpha_1 + \alpha_2 \neq 0 \tag{19}$$

If this condition is met, the universe evolves towards a de Sitter expansion with a DE-dominated scaling solution ($\rho_{DE}/\rho_{DM} = const. \ll 1$).

Distinguishing interactions

Distinguishing interactions

How can we distinguish different interactions? Do they leave imprints on cosmological observables?

We take two test interactions that remove the LSBR:

- $Q = 3H\alpha_\chi \rho_\chi$ (interaction I)
- $Q = 3H\alpha_{\chi\chi} \rho_\chi^2/(\rho_m + \rho_\chi)$ (interaction II)

and look for characteristic differences in their evolution.

The non-interacting model, calibrated to fit the current observations, was used to set the initial values of the (u, y, z) at *Redshift* ~ 6.

Both interaction parameters were fixed at $\alpha_\chi = \alpha_{\chi\chi} = -0.03$, which are well inside the observational constraints[8,9].

[8] A. A. Costa, L. C. Olivari, and E. Abdalla, Phys. Rev. D 92, 103501 (2015)
[9] C. Feng, B. Wang, E. Abdalla, and R. Su, Phys. Lett. B 665, 111 (2008)

Statefinder Hierarchy

The first method consist in analysing the evolution of the statefinder parameters[10] $S_n^{(1)}$:

$$S_3^{(1)} = A_3\,, \tag{20}$$

$$S_4^{(1)} = A_4 + 3\left(1 - A_2\right)\,, \tag{21}$$

$$S_5^{(1)} = A_5 - 2\left(4 - 3A_2\right)\left(1 - A_2\right)\,, \tag{22}$$

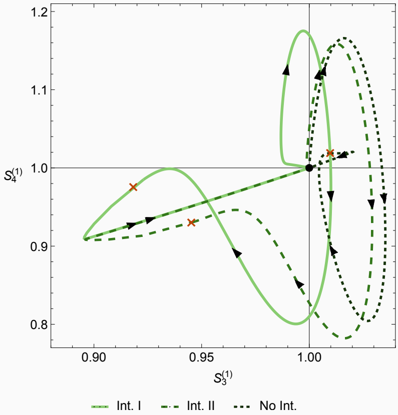
where $A_n = (aH^n)^{-1}(\partial^n a/\partial t^n)$.

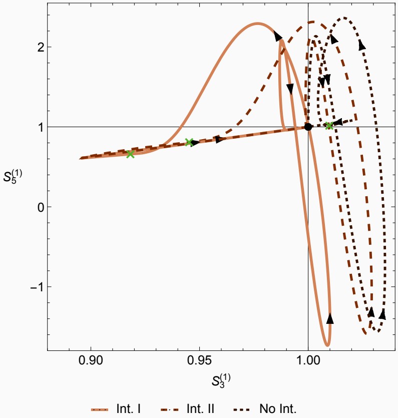
The statefinders are defined so that for ΛCDM, $S_n^{(1)} = 1$ at all orders.

By plotting the graphs $\{S_3^{(1)}, S_4^{(1)}\}$ and $\{S_3^{(1)}, S_5^{(1)}\}$, we can observe how each model deviates from ΛCDM, indicated by the point $\{1, 1\}$.

[10] V. Sahni, T. D. Saini, A. A. Starobinsky, and U. Alam, J. Exp. Theor. Phys. Lett. 77, 201 (2003)

Statefinder Hierarchy results





Composite Null Diagnostic

The second method[11] combines the statefinder parameter $S_3^{(1)}$ and a preliminary analysis of the growth factor $\epsilon(a)$

$$\epsilon(a) \equiv \frac{f(a)}{f_{\Lambda\mathrm{CDM}}(a)}\,, \qquad f(a) \equiv \frac{a}{\delta_m}\frac{\partial \delta_m}{\partial a}\,. \tag{23}$$

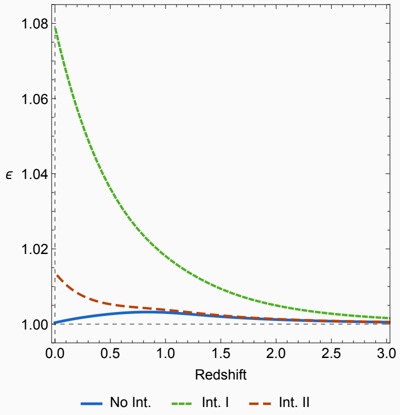
where δ_m is the fractional energy density perturbation of DM.

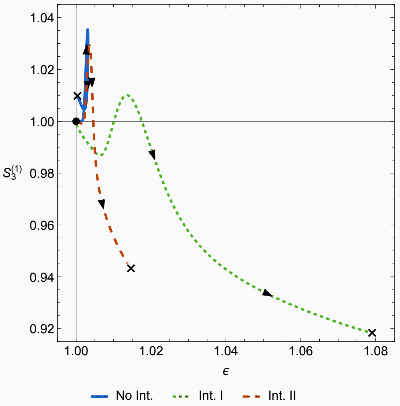
For each model, the evolution of $f(x)$ was numerically computed in the approximation of a smooth DE fluid.

By plotting the graph $\{\epsilon,\, S_3^{(1)}\}$ we can observe how each model fares against ΛCDM, which is indicated by the point $\{1,\, 1\}$.

[11] J.-L. Cui, L. Yin, L.-F. X. and Y. Wang, Y.-H. Li, and X. Zhang, JCAP, 1009 (2015)

Composite Null Diagnostic results





Conclusions

Conclusions

The 3-form, although an attractive alternative to standard scalar fields, can lead the Universe to a late-time LSBR event.

Through a dynamical system approach we were able to show that, for a fairly general class of phenomenological interactions, this late-time event can be eliminated and replaced by a final de Sitter expansionary phase.

In order to understand the dynamics of the system in the asymptotic past, the FP of the system at χ-infinity need to be analysed. In this work, these FP were identified and their stability studied for the first time.

By combining different diagnostic tools like the Statefinder Hierarchy $\{S_3^{(1)}, S_4^{(1)}\}$, $\{S_3^{(1)}, S_5^{(1)}\}$, and the Composite Null Diagnostic $\{\epsilon, S_3^{(1)}\}$ we can discriminate different interactions and identify which ones fit better the observations.

Thank you for your attention