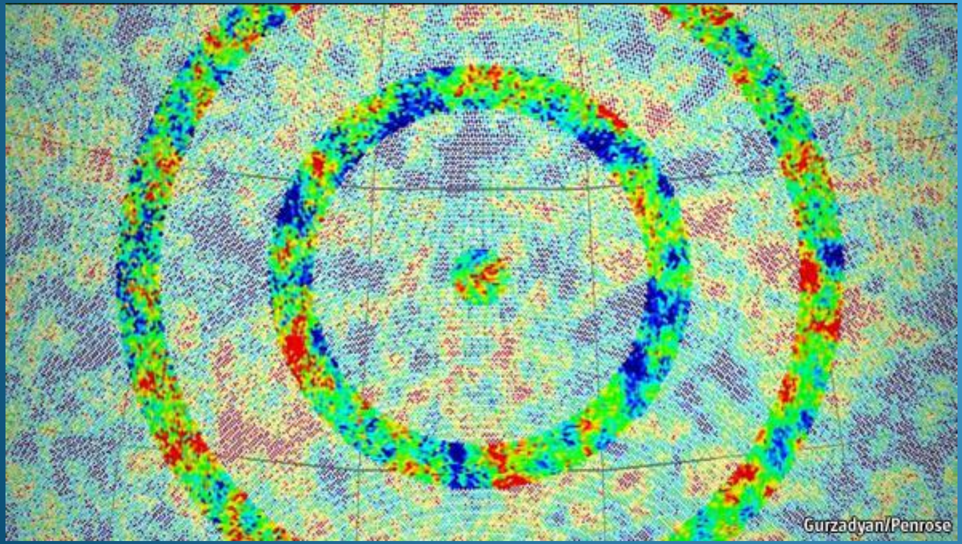
Varying constants and Cyclic Universes



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Multiverse in varying G

Field equation with varying „G" and „c"

$$\varrho(t) = \frac{3}{8\pi G(t)} \left(\frac{\dot{a}^2}{a^2} + \frac{kc^2(t)}{a^2} \right) ,$$
$$p(t) = -\frac{c^2(t)}{8\pi G(t)} \left(2\frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2} + \frac{kc^2(t)}{a^2} \right)$$

And conservation equation:

$$\dot{\varrho}(t) + 3\frac{\dot{a}}{a} \left(\varrho(t) + \frac{p(t)}{c^2(t)} \right) = -\varrho(t)\frac{\dot{G}(t)}{G(t)} + 3\frac{kc(t)\dot{c}(t)}{4\pi G a^2}$$

Multiverse in varying G

First thermodynamics law:

$$d\rho + \frac{dV}{V}\left(\rho + \frac{p}{c^2}\right) - \frac{T}{Vc^2}dS = 0$$

And conservation equation:

$$d\rho + \frac{dV}{V}\left(\rho + \frac{p}{c^2}\right) = -\rho\frac{dG}{G}$$

If we combine this equations we will get:

$$-\frac{T}{Vc^2}dS = \rho\frac{dG}{G}$$

Multiverse in varying G

Which give:

$$dS = -\frac{\rho V c^2}{T}\frac{dG}{G}$$

Assuming that the factor is constant (Clapeyron equation):

$$\frac{\rho V c^2}{T} = \text{const.} = N k_B$$

We can express entropy density by:

$$S(t) = N k_B \ln\left[\frac{A_0}{G(t)}\right],$$

Multiverse with constant entropy

To avoid the problem of decreasing entopy in cyclic universe (2nd law of thermodynamics) we may assume that the entropy of the multiverse being a sum of entropies of individual universes is constant:

$$S = S_1 + S_2 + S_3 + ... + S_n = const.$$

Multiverse in varying G

Let us consider a two universe in with in one of universe $G_1\left(t\right) = \frac{G_A}{a^n}$

$$S = S_1 + S_2$$

$$S_1 = \frac{\rho_1 V_1 c^2}{T_1} \ln\left(\frac{A_1}{G_1(t)}\right)$$

$$S_2 = \frac{\rho_2 V_2 c^2}{T_2} \ln\left(\frac{A_2}{G_2(t)}\right)$$

$$S = \frac{\rho_1 V_1 c^2}{T_1} \ln\left(\frac{A_1}{G_1(t)}\right) + \frac{\rho_2 V_2 c^2}{T_2} \ln\left(\frac{A_2}{G_2(t)}\right)$$

To keep entropy constants we can assume that:

$$\frac{\rho_1 V_1 c^2}{T_1} \widetilde{A_1} = \frac{\rho_2 V_2 c^2}{T_2} \widetilde{A_2}$$

where $\widetilde{A_1} = \ln\left(\frac{A_1}{G_A}\right)$

Multiverse in varying G

Then gravitational constant in second universe will be:

$$G_2(t) = G_B a^n$$

If $n \geq 2$ then singularities in density and pressure in first universe is removed.

Multiverse in varying G

If we use a folowing sale factor:

$$a(t) = a_0 \left|\sin\left(\pi \frac{t}{t_c}\right)\right|$$

Then Energy density and pressure take the form(for n=2):

$$\rho(t) = \frac{3}{8\pi G_0}\left[\frac{\pi^2 a_0^2 \cos^2\left(\pi \frac{t}{t_c}\right)}{t_c^2} + c^2\right],$$

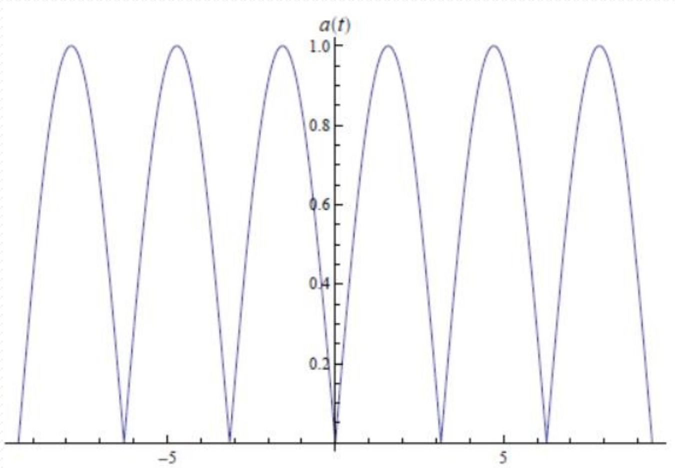
$$p(t) = -\frac{c^2}{8\pi G_0}\left[\frac{3\pi^2 a_0^2 \cos^2\left(\pi \frac{t}{t_c}\right)}{t_c^2} + c^2 - 2\frac{\pi^2 a_0^2}{t_c^2}\right] .$$

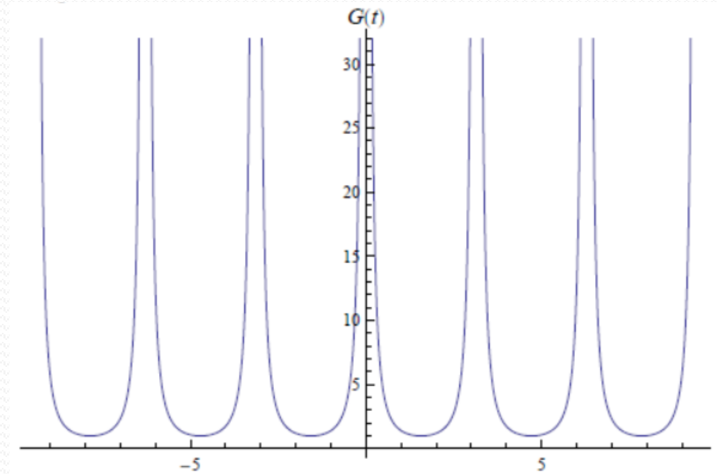
Equation of state:

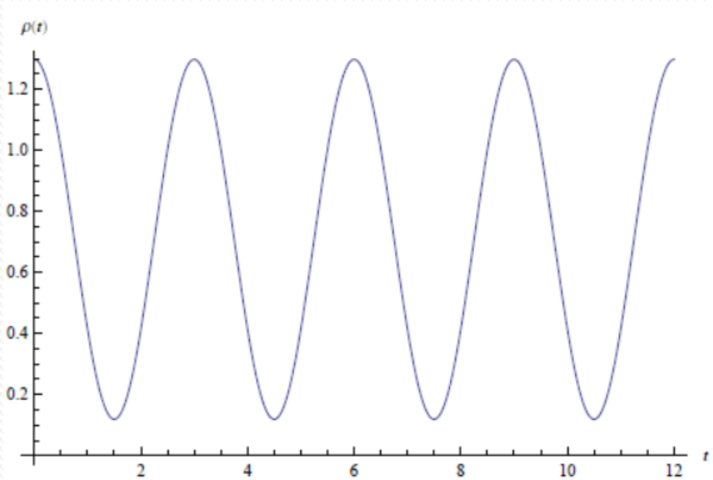
$$p(t) = -\rho(t)c^2 + p_0,$$

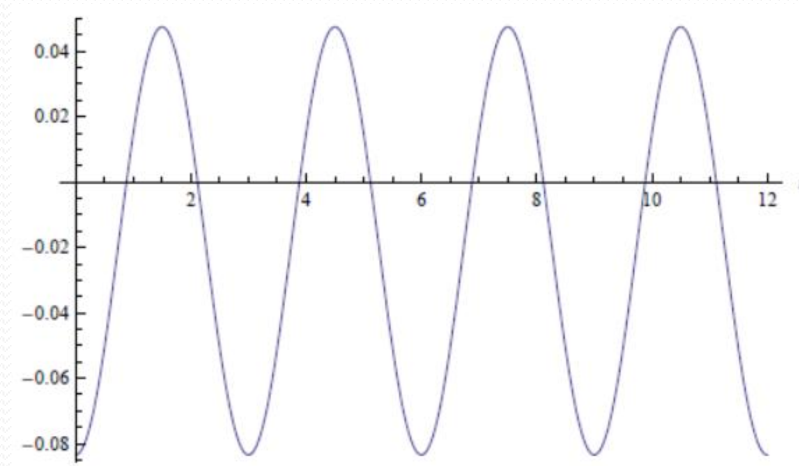
Where: $p_0 = \frac{c^2}{4\pi G_0}\left(\frac{\pi^2 a_0^2}{t_c^2} + c^2\right).$

Multiverse in varying G



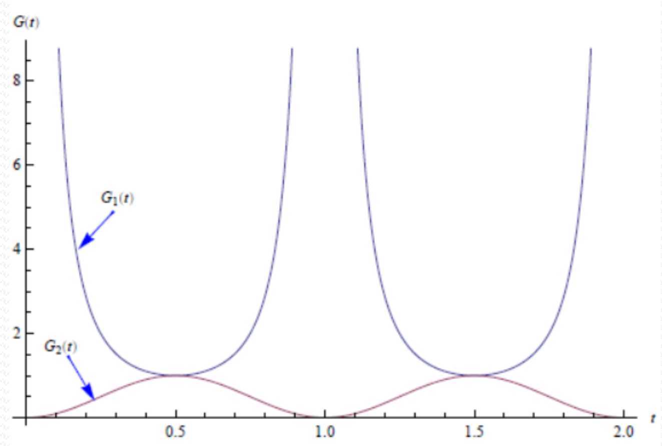




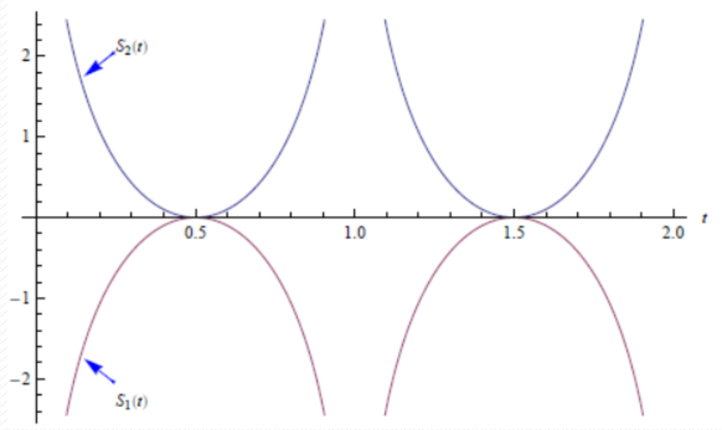


Multiverse in varying G

Gravitational in both universes



Entropy in both universes



Varying speed of light theory

Action for VSL theory where $\psi = c^4$:

$$S = \int d^4x \left[\sqrt{-g} \left\{ \frac{c^4}{16\pi G} (\mathcal{R} - 2\Lambda) + \mathcal{L} \right\} + \mathcal{L}_c \right]$$

when we use Robertson-Walker metric then we will get Friedmann equation with varying speed of light:

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho + \frac{c^2}{3}\Lambda - \frac{kc^2}{a^2},$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\left(\rho + 3\frac{p}{c^2}\right) + \frac{c^2}{3}\Lambda,$$

and conservation equation:

$$\dot{\rho} + 3\frac{\dot{a}}{a}\left(\rho + \frac{p}{c^2}\right) = \frac{3kc\dot{c}}{4\pi G a^2} - \frac{c\dot{c}}{4\pi G}\Lambda$$

Barrow JD, Magueijo J, 1999, Solutions to the quasi-flatness and quasi-lambda problems, Physics Letters B, Vol:447
Albrecht A, Magueijo J, 1999, Time varying speed of light as a solution to cosmological puzzles, Physical Review D, Vol:59

Thermodynamics in VSL Theory

First thermodynamics law:

$$\dot{\rho} + \frac{\dot{V}}{V}\left(\rho + \frac{p}{c^2}\right) + 2\rho\frac{\dot{c}}{c} = \frac{T}{Vc^2}\dot{s}$$

And conservation equation:

$$\dot{\rho} + 3\frac{\dot{a}}{a}\left(\rho + \frac{p}{c^2}\right) = \frac{3kc\dot{c}}{4\pi G a^2}$$

If we combine these equations we will get:

$$2\rho\frac{\dot{c}}{c} + \frac{3kc\dot{c}}{4\pi G a^2} = \frac{T}{Vc^2}\dot{s}$$

Thermodynamics in VSL Theory

Which give:

$$\dot{s} = 2\frac{\breve{\rho} V c^2}{T}\frac{\dot{c}}{c}$$

Where:

$$\breve{\rho} = \frac{3}{8\pi G}\left[\left(\frac{\dot{a}}{a}\right)^2 + 2\frac{kc^2}{a^2}\right]$$

Assuming that the factor is constant (Clapeyron equation):

$$\frac{\breve{\rho} V c^2}{T} = const.$$

We can express entropy density by:

$$S = 2\frac{\breve{\rho} V c^2}{T} ln(A_0 c)$$

Multiverse with constant entropy

For example let us consider a double-universe where their entropies are:

$$S_1 = 2\frac{\breve{\rho}_1 V_1 c_1^2}{T_1} ln(A_1 c_1) = \frac{2}{\tilde{w}} N_1 k_B \ln[c_1(t)],$$

$$S_2 = 2\frac{\breve{\rho}_2 V_2 c_2^2}{T_2} ln(A_2 c_2) = \frac{2}{\tilde{w}} N_2 k_B \ln[c_2(t)].$$

Multiverse with constant entropy

Let us assume that:

$$c_1(t) = e^{\lambda_1 \phi_1(t)} \quad , \quad c_2(t) = e^{\lambda_2 \phi_2(t)} \quad .$$

Where ϕ_1 , ϕ_2 are arbitrary functions of time.

Then entropy of our Multiverse is given by:

$$S = 2\frac{\breve{\rho}_1 V_1 c_1^2}{T_1}[ln(A_1) + \lambda_1\phi_1] + 2\frac{\breve{\rho}_2 V_2 c_2^2}{T_2}[ln(A_2) + \lambda_2\phi_2]$$

To keep entropy constant one should assume:

$$\frac{\breve{\rho}_1 V_1 c_1^2}{T_1}\lambda_1 = \frac{\breve{\rho}_2 V_2 c_2^2}{T_2}\lambda_2$$

Multiverse with constant entropy

where $\phi_1(t)$ and $\phi_2(t)$ can be in following form:

$$\phi_1(t) = \sin^2(t)$$

$$\phi_2(t) = \cos^2(t)$$

Entropies:

$$S_1 = \frac{2}{\tilde{w}} N_1 k_B \lambda_1 \sin^2\left(\pi \frac{t}{t_s}\right),$$

$$S_2 = \frac{2}{\tilde{w}} N_2 k_B \lambda_2 \cos^2\left(\pi \frac{t}{t_s}\right).$$

Multiverse with constant entropy

Assuming that:

$$\frac{\tilde{p}(t)}{c^2(t)} \sim \tilde{\rho}(t),$$

we can write:

$$\frac{\tilde{p}(t)}{c^2(t)} = -\frac{1}{3}\tilde{\rho}(t) + \frac{p_0(t)}{c^2(t)},$$

where:

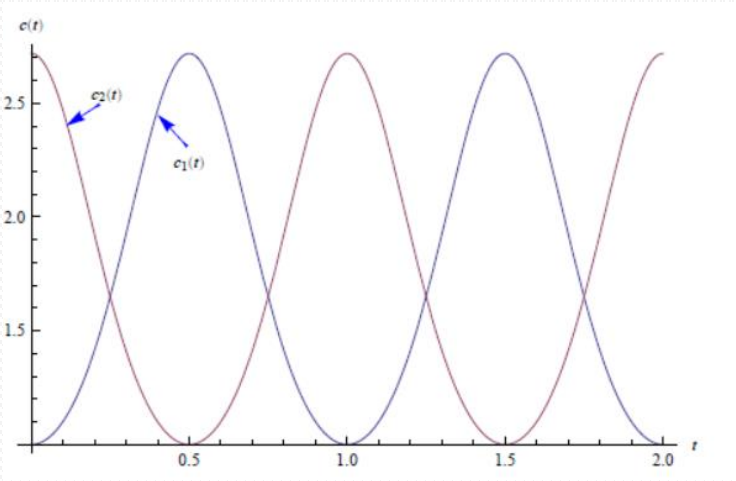
$$\frac{p_0(t)}{c^2(t)} = -\frac{1}{4\pi G}\frac{\ddot{a}}{a},$$

Then

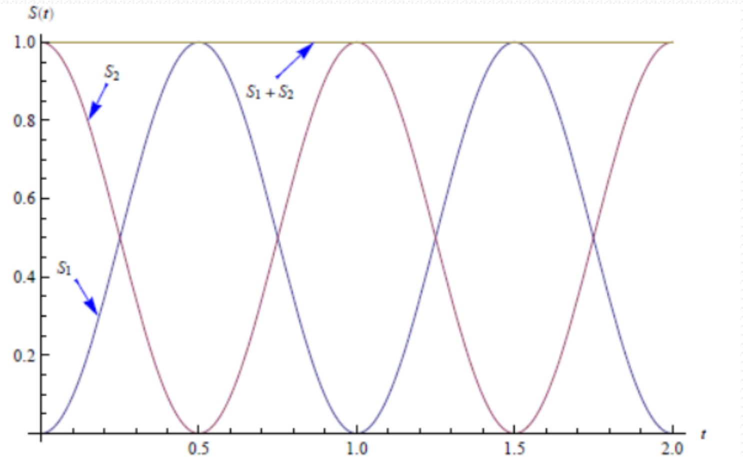
$$\frac{(\tilde{p} - p_0)\,V}{T} = const.$$

Multiverse with constant entropy

Speed of light:



Entropy:



Cyclic Brans-Dicke Universe

Action in Brans-Dicke Theory:

$$S = \int d^4x\sqrt{-g}\left(\frac{\phi R - \omega\frac{\partial_a\phi\partial^a\phi}{\phi}}{16\pi} + \mathcal{L}_M\right)$$

Field Equations:

$$\rho(t) = \frac{3\phi(t)\left[\dot{a}^2(t) + kc^2\right]}{8\pi a^2(t)} + \frac{3\dot{a}(t)\dot{\phi}(t)}{8\pi a(t)} - \frac{\omega\dot{\phi}^2(t)}{16\pi\phi(t)},$$

$$p(t) = -\frac{c^2}{8\pi}\left[\frac{\phi(t)\left[\dot{a}^2(t) + kc^2\right]}{a^2(t)} - \frac{2\dot{a}(t)\dot{\phi}(t)}{a(t)} - \frac{\omega\dot{\phi}^2(t)}{2\pi\phi(t)} - \frac{2\phi(t)\ddot{a}(t)}{a(t)} - \ddot{\phi}(t)\right],$$

$$\Box\phi = \frac{8\pi}{3+2\omega}T,$$

Cyclic Brans-Dicke Universe

Continuity equation with constant entropy:

$$\dot{\rho} + \frac{\dot{V}}{V}\left(\rho + \frac{p}{c^2}\right) = 0,$$

When we assume that entropy isn't constant we can write:

$$\dot{\rho} + \frac{\dot{V}}{V}\left(\rho + \frac{p}{c^2}\right) - 16\pi \frac{T\dot{S}}{V}\frac{\phi}{\dot{\phi}} = 0.$$

This leads to modification of of the field function:

$$\Box\phi = \frac{8\pi}{3 + 2\omega} T + N(t),$$

And the entropy in this model is:

$$\dot{S} = \frac{N(t)\, V}{16\pi T}\frac{\dot{\phi}}{\phi}.$$

Cyclic Brans-Dicke Universe

We assume that:

$$N(t) = \frac{8\pi\alpha\rho(t)}{3+2\omega} + \frac{8\pi\beta p(t)}{3+2\omega},$$

The entropy express by:

$$S = \frac{[\alpha\rho(t) + \beta p(t)]\,V}{2T\,(3+2\omega)} \ln\phi,$$

Cyclic Brans-Dicke Universe

For: $\omega = -1$ and $k = 1$, we can take the acsale factor for first uniwerse

$$a_1(t) = a_A \left|\sin\left(\pi \frac{t}{t_s}\right)\right|,$$

And scalar field

$$\phi_1(t) \sim a_1^2(t) = \phi_A \sin^2\left(\pi \frac{t}{t_s}\right),$$

For this model $\alpha_1 = 4/5,\ \beta_1 = -32/15$ and $c = (a_A \pi)/(t_s)$ necessarily to fulfill the field equation.

Cyclic Brans-Dicke Universe

The field equation are:

$$\rho_1(t) = \frac{\phi_A \pi}{8{t_s}^2}\left[3 + 11\cos^2\left(\pi\frac{t}{t_s}\right)\right],$$

$$p_1(t) = \frac{c^2\phi_A \pi}{8{t_s}^2}\left[3 - 9\cos^2\left(\pi\frac{t}{t_s}\right)\right],$$

And equation of state:

$$\frac{p_1(t)}{c^2} = -\frac{9}{11}\rho_1(t) + \frac{15\phi_A\pi}{22{t_s}^2}$$

Cyclic Brans-Dicke Universe

In second universe:

Scale factor: $a_2(t) = a_B \left|\sin\left(\pi \frac{t}{t_s}\right)\right|,$ Scalar field: $\phi_2(t) \sim \frac{1}{a_2^2(t)} = \frac{\phi_B}{\sin^2\left(\pi \frac{t}{t_s}\right)},$

$\alpha_2 = 0$, $\beta_2 = -4$, $c = (a_B\pi)/t_s$

Field equations

$$\rho_2(t) = \frac{\phi_2\pi}{8t_s{}^2}\left[\frac{2+\sin^2\left(\pi\frac{t}{t_s}\right)}{\sin^4\left(\pi\frac{t}{t_s}\right)}\right],$$

$$p_2(t) = -\frac{c^2\phi_2\pi}{8t_s{}^2}\left[\frac{2-\sin^2\left(\pi\frac{t}{t_s}\right)}{\sin^4\left(\pi\frac{t}{t_s}\right)}\right],$$

To keep entropy constant following condition must be conserved

$$\frac{[\alpha_1\rho_1(t)+\beta_1p_1(t)]\,V_1}{T_1} = \frac{[\alpha_2\rho_2(t)+\beta_2p_2(t)]\,V_2}{T_2},$$

Conclusions

To avoid a decrease of entropy in the universe by variation of constants can be solved by consider the Multiverse model where an entropy can decrease in one of universe and increase in another.

By variation of constants it is also possible to remove an all of singularities in density and pressure.