Running of the Higgs quartic coupling, gravity and the stability of the Higgs effective potential

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Outline

1. Motivations
   - General & particular
2. Gravity and the Higgs potential
   - The Higgs, the mediator and the running
   - The one-loop effective potential

General motivations

Latest breakthroughs:

1. Discovery of the Higgs particle,
2. Discovery of gravitational waves.

Improvement in:

- Neutrino physics,
- Dark Matter sector,
- Cosmology.

Unification of gravity with SM?

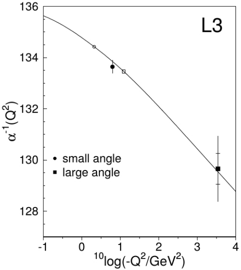
- What is the correct framework for it?
- Is it really possible?

Specific motivation – the stability of the SM

Are the coupling constants constant in quantum field theory?

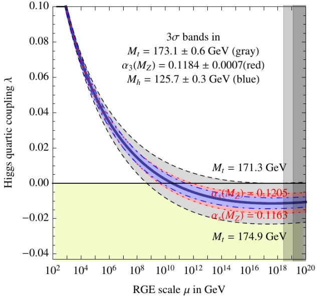
- To be meaningful quantum field theory requires renormalization.
- Renormalization introduces momentum/energy dependence to the renormalized constants.

The running of the fine structure constant



L3 Collaboration, Phys. Lett. B *476 (2000) 40*

The running of the Higgs quartic constant

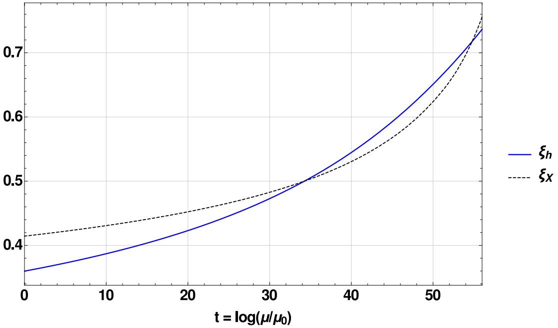


G. Degrassi et al. JHEP 08 (2012) 98

- Tree-level potential of scalars in the presence of gravity:

$$V_{HX} = \left(m_H^2 - \xi_h R\right)|H|^2 + \lambda_h |H|^4 + \left(m_X^2 - \xi_X R\right) X^2 + \lambda_X X^4 + \lambda_{hX}|H|^2 X^2.$$

- The running of the non-minimal coupling of scalars to gravity
  – the $\xi_h = \xi_X = 0.5$ case



The one-loop effective potential for the Higgs-top-mediator sector:

$$V^{(1)} = V_{HX} + \frac{1}{64\pi^2}\left\{\sum_i m_i^4\left[\alpha_i \ln\left(\frac{m_i^2}{\mu^2}\right) - b_i\right] + \frac{1}{3}y_t^2 h^2 \ln\left(\frac{b}{\mu^2}\right)R\right\} + \\ + \frac{\hbar}{64\pi^2}\left\{\frac{4}{180}\left(-R_{\alpha\beta}R^{\alpha\beta} + R_{\alpha\beta\mu\nu}R^{\alpha\beta\mu\nu}\right)\sum_i \alpha_i^{'} \ln\left(\frac{m_i^2}{\mu^2}\right) + \\ + \frac{4}{3}R_{\alpha\beta\mu\nu}R^{\alpha\beta\mu\nu}\ln\left(\frac{b}{\mu^2}\right)\right\}.$$

In the above equation

- $\alpha_i,\ \alpha_i^{'},\ b_i$ – numerical factors
- $m_i^2(h^2,\ X^2,\ R, p)$ – squares of the field dependent masses where p stands for parameters from the tree-level potential

Main differences between flat spacetime and gravity corrected one-loop effective potential

- tree-level non-minimal couplings,
- curvature dependent masses $m_i^2 \to m_i^2(R)$,
- terms of the $f(\mathcal{R}^2) \ln \left(m^2\right)$ type.

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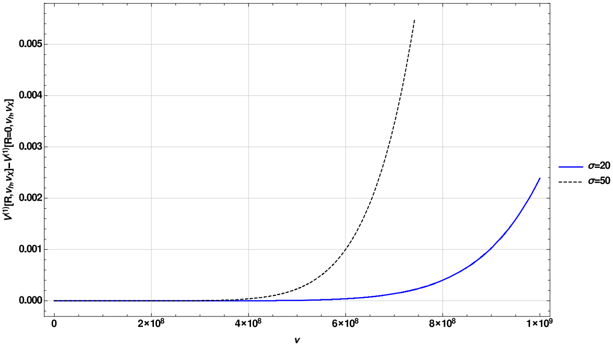
Other things that differ

- connection between running energy scale and field (for flat spacetime $\mu^2 \sim h^2$),
- meaning of the running energy scale (local and global aspects).

The influence of gravity in the small field region for the radiation dominated era



- Fixed fields values $X = v_X, \; h = v_h$,
- Fixed running energy scale $\mu = m_{top}$,
- Region of validity of the approximation used $\frac{\mathcal{R}^2}{h^4} << 1$.

What big curvature/strong gravity could do?

- Change the sign of the mass parameter.
- Change the critical temperature.
- To the running of the quartic coupling

$$V(h^4) = \frac{1}{4}\left[\lambda_{eff}(h) + \frac{4}{64\pi^2}\frac{4}{3}\frac{8}{3}\left(\bar{M}_P^{-2}\rho\right)^2\frac{\tilde{c}}{h^4}\right]_{|h=h_0} h^4 = \frac{1}{4}\bar{\lambda}_{eff}(h)h^4,$$

$$\rho = 4\pi h_0^2 \bar{M}_P^2\sqrt{\frac{9|\lambda_{eff}|}{32\tilde{c}}} \rightarrow \mu \sim 10^{13} \div 10^{14}\, GeV$$

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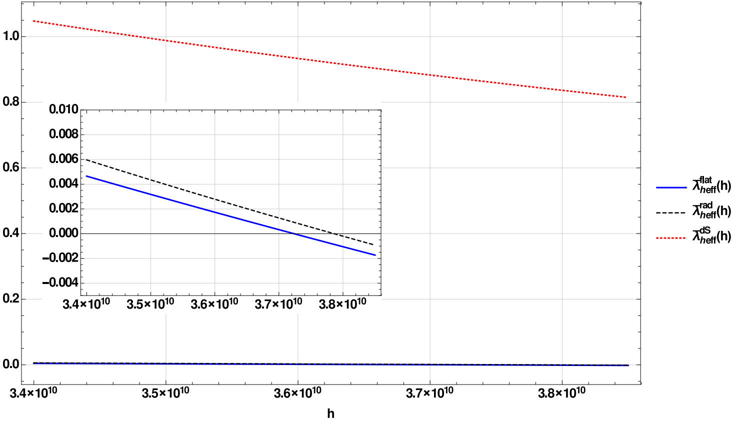
- To the running of the quartic coupling



Summary

- Using the flat spacetime method for obtaining the effective action above the energy scale of 10^{10}GeV may lead to inaccuracies.
- Classical gravity induces new terms in the effective action.
  - They introduce spacetime dependence to the Higgs vev.
  - They influence small field behavior of the theory (also the critical temperature).
- These new terms may have an impact on the problem of the stability of the Standard Model vacuum.

Thank you for your attention.

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Heat kernel and one-loop effective action

- Basic relations

$$S^{Tree} \to \int \sqrt{-g} d^4x \left\{ \hat{\Phi} \left[\frac{1}{2} \frac{\delta^2 S^{Tree}}{\delta\hat{\Phi}\delta\hat{\Phi}} \right] \hat{\Phi} \right\} = \int \sqrt{-g} d^4x \left\{ \hat{\Phi} D^2 \hat{\Phi} \right\},$$

$$D^2 = \Box + 2h^\mu d_\mu + Q,$$

$$\Gamma^{Tree+(1)} = S^{Tree} + \frac{i\hbar}{2} \ln\det\left(\frac{D^2}{\mu^2}\right)$$

- Use of the heat kernel

$$\frac{i\hbar}{2} \ln\det\left(\frac{D^2}{\mu^2}\right) = -\frac{i\hbar}{2} \int \sqrt{-g} d^4x Tr \left\{ \int_0^\infty \frac{ds}{s} K(x,x,s) \right\}$$

Heat kernel and one-loop effective action

- Equation for the heat kernel

$$i\frac{\partial}{\partial s}K(x,x^{'},s) = D^2 K(x,x^{'},s),$$
$$\lim_{s\to 0} K(x,x^{'},s) = \delta(x,x^{'})$$

- Schwinger-DeWitt representation for the heat kernel

$$K(x,x^{'},s) = i(4\pi i s)^{-n/2} exp\left[\frac{i\sigma(x,x^{'})}{2}\right]\Delta^{1/2}_{VM}(x,x^{'})F(x,x^{'},s),$$
$$\Delta^{1/2}_{VM}(x,x^{'}) = -|g(x)|^{-1/2}|g(x^{'})|^{-1/2}\det\left[\frac{-\partial^2\sigma(x,x^{'})}{\partial x^{\mu}\partial x^{'\nu}}\right],$$
$$F(x,x^{'},s) = \sum_{j=0}^{\infty}(is)^j a_j(x,x^{'})$$

Heat kernel and one-loop effective action

- Practical formula

$$\Gamma^{(1)} = \frac{\hbar}{64\pi^2}\int\sqrt{-g}d^4xTr\left\{\tilde{a}_0 M^4\left[\frac{2}{\varepsilon}-\ln\left(\frac{M^2}{\mu^2}\right)+\frac{3}{2}\right]+2\tilde{a}_2\left[\frac{2}{\varepsilon}-\ln\left(\frac{M^2}{\mu^2}\right)\right]\right\}$$

- Schwinger-DeWitt coefficients

$$\tilde{a}_0 = 1,$$
$$\tilde{a}_2 = \left\{-\frac{1}{180}R_{\mu\nu}R^{\mu\nu}+\frac{1}{180}R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}+\frac{1}{30}\Box R\right\}1+\frac{1}{6}\Box M^2+\frac{1}{12}W_{\mu\nu}W^{\mu\nu}$$

- $D^2 = \Box + 2h^\mu d_\mu + Q$

$$M^2 = Q + \frac{1}{6}R - d_\mu h^\mu - h_\mu h^\mu,$$
$$W_{\alpha\beta} = [d_\alpha, d_\beta]\,1 + 2d_{[\alpha}d_{\beta]} + 2h_{[\alpha}h_{\beta]}$$