From Quantum Regime to Cosmology via forcing and exotic 4-smoothness

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Motivations

Cosmological models and evolution of the Universe are based on the constant (not changing during the evolution) real line $\mathbb{R}$ (Diff invariance of GR requires $\mathbb{R}$ to be complete)

On every energy scale and stage of the evolution one deals with the same (up to diffeomorphisms) 'absolute' $\mathbb{R}$.

We address the following issues in building the cosmological models:

- Do we really have tools to manage the variation of $\mathbb{R}$ (and $\mathbb{N}$)?
- Does real numbers line on cosmological scale emerge somehow from pure quantum regime?
- Are smooth structures on 4-spacetimes always fixed and standard?
- Is the steadiness of $\mathbb{R}$ and 4-smoothnes any physical law? Or rather Can the change of $\mathbb{R}$ and the smoothness help explaining some problems in cosmology?

The tools: Extending the real line $\mathbb{R}$ by adding new reals

I. Robinson's nonstandard models of arithmetic and analysis, ${}^*R$. They contain infinitely big, $r^* > r, \forall r \in \mathbb{R}$, and infinitely small, $|r_*| < \frac{1}{n}, \forall n \in \mathbb{N}$, real numbers.

II. Forcing in set theory: $\mathbb{R}$ in a model M of ZFC (Zermelo-Fraenkel set theory) is extended to $R[G]$ via forcing adding reals ($M[G]$ is the extended model of ZFC).

III. Real line R_T in T a smooth topos. R_T contains nilpotent infinitesimals, $r \in R_T$, i.e. $r^2 = 0$, and Robinson's non-standard numbers. The logic can not be classical. It is intuitionistic one. Natural numbers has to vary which shows geometric impact in dimension 4 (M.Heller,JK,2016).

QM to GR and the modification of $\mathbb{R}$

We focus on the program:

Start with QM at micro-scale (the lattice of projections); recognize the real numbers line R; reach the cosmological scale (GR) with its $\mathbb{R}$; compare R and $\mathbb{R}$; draw the physical conclusions.

Completing the program gives rise to:

$$\text{(primordial) QM regime} \xrightarrow[R\to R[G]]{\text{Forcing(ZFC)}} \text{large scale GR} \tag{1}$$

but also

$$\text{QM regime} \xrightarrow[\text{flat } \mathbb{R}^4 \to \text{curved } R^4]{\mathbb{R}^4 \to \text{exotic } R^4} \text{large scale GR} + \text{4d curvature} \tag{2}$$

Moreover,

(3) The 4d curvature appearing in (2) plays the role of the cosmological constant in the model.

Forcing and Exotic R^4 from $\mathbb{L}$

Let us start with QM lattice of projections $\mathbb{L}$

(a) The local descriptions of $\mathbb{L}$ (Boolean contexts) are given by forcing models of ZFC (sheaves on the measure algebras); thus the real line is enlarged by the random forcing (based on the measure Boolean algebra of the spectrum of quantum operators) [G.Takeuti,1975;M.Ozawa,1988;JK,et al.,2015-2016]

The quantum real line R corresponds to the continuous spectrum of the position operator Q. Then the 'classical' $\mathbb{R}$ has to be enlarged by the random forcing

(b) Since $\mathbb{L}$ cannot be globally Boolean (none of local Boolean contexts can be global),

then

a smooth structure on $\mathbb{R}^4$ derived from quantum $\mathbb{L}$, cannot be standard (every local open cover of $\mathbb{R}^4$ is **not** reduced to the single-patch $\mathbb{R}^4$)

QM, 4-smoothness and the real line R

Forcing on $\mathbb{R}$ and exotic R^4 are both derived from the quantum lattice $\mathbb{L}$!

Physics: forcing and zero-modes

The presence of forcing in passing from micro to macro scale cancels the zero-modes of quantum fields

$$\frac{E}{V} = \int_{\mathbb{R}^3_M} \frac{\mathrm{d}^3 k}{(2\pi)^3} \frac{\sqrt{\mathbf{k}^2 + m^2}}{2}, \; m \in \mathbb{R}_{M[G]}, \; \mathbf{k} \in \mathbb{R}^3_M,$$

which all vanish, since we integrate over the 'meager' (null) set $\mathbb{R}^3_M \subset \mathbb{R}^3_{M[G]}$.

Thus $\mathrm{CC} = 0$

Physics: exotic R^4 and CC

Physically, the 4-curvature of exotic smooth R^4 is the non-vanishing value of the cosmological constant

$$0 \neq \mathrm{CC} \sim \mathrm{curv}(R^4)$$

when R^4 is embedded in $\mathbb{R}^4$ [T.Asselmeyer-Maluga,JK,2014]
More precisely

- Exotic smooth R^4 cannot be flat, so that $R_{\mu\nu\rho\sigma} \neq 0$ (CC)
- R^4 determines the sequence of hyperbolic 3-manifolds; then the constancy of CC follows from the Mostow rigidity (curvature and volume are topological invariants) [T.Asselmeyer-Maluga,JK,2014;2016]

Cosmology from exotic 4-smoothness on $\mathbb{R}^4$

Given exotic R^4 and removing the singular point $\mathrm{pt.} \in R^4$ we have exotic $S^3 \times_\Sigma \mathbb{R}$

Non-canceling smoothly pairs of $1-2$ handles in R^4 lead to necessarily exponential potential for the inflation field with the scalar field of the Starobinsky model

The change of the spatial (3D) topology in $S^3 \times_\Sigma \mathbb{R}$ is described by the Morse function (of handles) and it is the scalar field ϕ

Let us see how it works ...

The example: topology change from S^3 to a homology 3-sphere Σ driven by exotic R^4

Spacetime out of singular point is modeled by exotic $S^3 \times_\Sigma \mathbb{R}$
compact 3-manifold Σ is homology 3-sphere (Brieskorn sphere) given by the solution set

$$x, y, z \in \mathbb{C} : \quad x^2 + y^5 + z^7 = 0 \quad |x|^2 + |y|^2 + |z|^2 = 1$$

The change $S^3 \to \Sigma$ is given by scalar field model (ϕ scalar field)

$$\mathcal{L} = R + \partial^\mu \phi \partial_\mu \phi + \frac{1}{8\alpha}(1 - \exp(-\phi))^2$$

leading to the Starobinsky model

$$\mathcal{L} = R + \alpha \cdot R^2$$

Topology change inside exotic R^4: wild $S^3 \to \Sigma \to P\#P$ (TAM, JK 2014, 2016(forthcoming))

Exponential growth of the universe (inflation)

Expansion scale factor ϑ is completely determined by the topology of the change $S^3 \to \Sigma$ (smooth cancelation of 1-/2-handles), namely
the radial coordinate $a(t)$ (FRW metric) scales after inflation (using Witten + infinite Casson handles) like

$$a = a_0 \cdot \exp\left(\frac{\vartheta}{2}\right)$$
$$\vartheta = \frac{3}{CS(\Sigma)}$$

where a_0 is the radius of S^3

The value of the Chern-Simons invariant of the Brieskorn sphere and the corresponding value of expansion factor, read

$$CS(\Sigma) = \frac{9}{280}$$
$$\vartheta = \frac{280}{3} \approx 93.33333...$$

Using the Planck scale at beginning $(a_0 = L_P)$ we have:

$$a = L_P \cdot \exp\left(\frac{3}{2 \cdot CS(\Sigma)}\right) \approx L_P \cdot 1.8 \cdot 10^{20} \approx 7.5 \cdot 10^{-15} m$$

Not enough inflation; BUT exotic smoothness enforces another topology change!

The second topology change: $\Sigma \to P\#P$ (sum of two Poincare spheres) with the expansion factor:

$$CS(P\#P) = \frac{1}{60}$$
$$\vartheta = 180$$

Using Planck scale at beginning ($a_0 = L_P$):

$$a = L_P \cdot \exp\left(\frac{3}{2 \cdot CS(\Sigma)} + \frac{3}{2 \cdot CS(P\#P)}\right) \approx L_P \cdot 2.2 \cdot 10^{59}$$
$$\approx 10^9 \text{Light years}$$

What is α? α represents the energy scale when the first transition starts (= canceling of the first handles pair)

$$\frac{1}{\alpha} = 1 + \frac{3}{2 \cdot CS(\Sigma)} + \frac{1}{2}\left(\frac{3}{2 \cdot CS(\Sigma)}\right)^2 + \frac{1}{6}\left(\frac{3}{2 \cdot CS(\Sigma)}\right)^3$$
$$\alpha \approx 5.5 \cdot 10^{-5}$$

Good agreement with measurement!!

What about CC?

CC is the curvature of the exotic R^4 (inside standard $\mathbb{R}^4$)

$$CC = \frac{1}{a^2} = (L_P)^{-2} \exp\left(-\frac{3}{2 \cdot CS(\Sigma)} - \frac{3}{2 \cdot CS(P\#P)}\right)$$

Taking recent value $(H_0)_{Planck} = 68 \, \frac{km}{s \cdot Mpc}$ we obtain

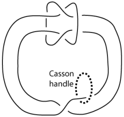
$$\Omega_\Lambda(CC) = 0.6836$$

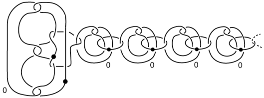
Very good agreement with Planck measurement $\Omega_\Lambda = 0.683$

exotic R^4, $S^3 \times_\Theta \mathbb{R}$ and Casson handles

Exotic smoothness is carried by infinite geometrical constructions coded by Casson handles and 'gropes'

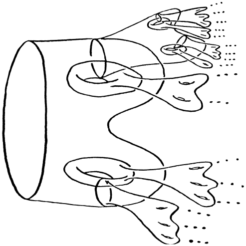
The simplest Casson handle and exotic R^4





More complicated structures are the gropes (capped)

These infinite constructions make the inflaton potential to be exponential rather than polynomial. If one deals with homeomorphic objects all the constructions collapse to the finite 2-handles (and the smoothness to the standard one)



- We presented the model where quantum lattice of projections determined forcing and exotic smooth $S^3 \times_\Sigma \mathbb{R} \subset \text{exotic}\, R^4 \subset \mathbb{R}^4$
- Forcing cancels the QFT zero-modes while R^4 reintroduces its small value (good agreement)
- Exotic smoothness reintroduces non-vanishing CC and exponential inflation potential via purely topological invariants (good agreement)
- Whether it is fully realistic model requires further analysis, especially experimental verification (CMB should have coded the forcing era of the expansion)

Thank You!