# ENTROPY/INFORMATION FLUX IN HAWKING RADIATION


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## INTRODUCTION

- It is well known that there exists an entropy/information “puzzle” associated with the black hole evaporation process
  - Assuming unitarity, we can consider that the information comes out with the radiation (Page calculations $\longrightarrow$ firewalls)
- We think that a key point for understanding the problem is to start looking at the entropy/information budget in a pure statistical mechanics process where all the physics is under control
- The next step is to apply these considerations to the Hawking evaporation process in black hole physics
- For black hole systems, we calculate the classical and quantum entropies to show that the Hawking evaporation is relatively benign, no worst than the previous case

## ON BURNING A LUMP OF COAL

- It is a unitary process $\longrightarrow$ strict conservation of the von Neumann entropy

- Standard statistical mechanics applied to a furnace with a small hole $\longrightarrow$ blackbody radiation

  - Planck spectrum: implies some coarse graining

  - Every photon transfers thermodynamic entropy to the radiation field (Clausius entropy)

$$S = \frac{E}{T} = \frac{\hbar\omega}{T}$$

- Now consider the effect of coarse graining the (von Neumann) entropy

$$S_{coarse\ grained} = S_{before\ coarse\ graining} + S_{correlations}$$

$$\Downarrow$$

$$S_{before\ coarse\ graining} = S_{coarse\ grained} + I_{correlations}$$

Information as negative entropy

- On our single-photon definition of entropy, it is often convenient to measure the entropy in "natural units of information" (nat)

$$\hat{S} = \frac{S}{k_B} \quad \Rightarrow \quad \text{In an equivalent number of bits: } \hat{S}_2 = \frac{S}{k_B \ln(2)}$$

- The average energy per photon in blackbody radiation

$$\langle E \rangle = \hbar \langle \omega \rangle = \hbar \frac{\int \omega f(\omega) d\omega}{\int f(\omega) d\omega} = \frac{\pi^4}{30\zeta(3)} k_B T$$

- Consequently, the average entropy: $\langle \hat{S} \rangle = \dfrac{\langle E \rangle}{k_B T \ln(2)} \approx 3.897$ bits/photon

- It is easy to calculate also the standard deviation

$$\sigma_{\hat{S}_2} = \frac{\sqrt{\langle E^2 \rangle - \langle E \rangle^2}}{k_B T} \approx 2.521 \quad \text{bits/photon}$$

- So, the main result we obtained is that the entropy per photon in blackbody radiation is

$$3.897 \pm 2.521 \quad \text{bits/photon}$$

  - Relevant for a photon for which the only thing you know is that it was emitted as part of some blackbody spectrum

- Since we know the underlying physics is unitary $\longrightarrow$ this entropy is compensated by an equal "hidden information"

  - Hidden in the correlations we choose to ignore in our coarse graining procedure

- The blackbody photons arising from burning a lump of coal carry both thermodynamic entropy and "hidden information"

## HAWKING EVAPORATION OF A BLACK HOLE

- Using the previous results as a starting point to understand what happens in a general relativistic black hole system
  - We have to deal with the existence of horizons ⟶ apparent/trapping horizons, so the information could escape
  - Complete evaporation process
  - We assume then that the evaporation has to be unitary

➡ Does the information comes out throughout the Hawking emission? How is the process?

- Page in 1993 developed a similar idea that led to a paradoxical behaviour of the black holes
- We consider a different model
  - Compare and contrast the behaviour of the classical thermodynamic entropy with the quantum entanglement entropy

## THERMODYNAMIC ENTROPY IN THE HAWKING FLUX

- Loss of Bekenstein entropy of a Schwarzschild black hole

$$\frac{dS}{dN}=\frac{dS/dt}{dN/dt}=\frac{d\left(4\pi k_B GM^2/\hbar c\right)/dt}{dN/dt}=\frac{k_B\pi^4}{30\zeta(3)}$$

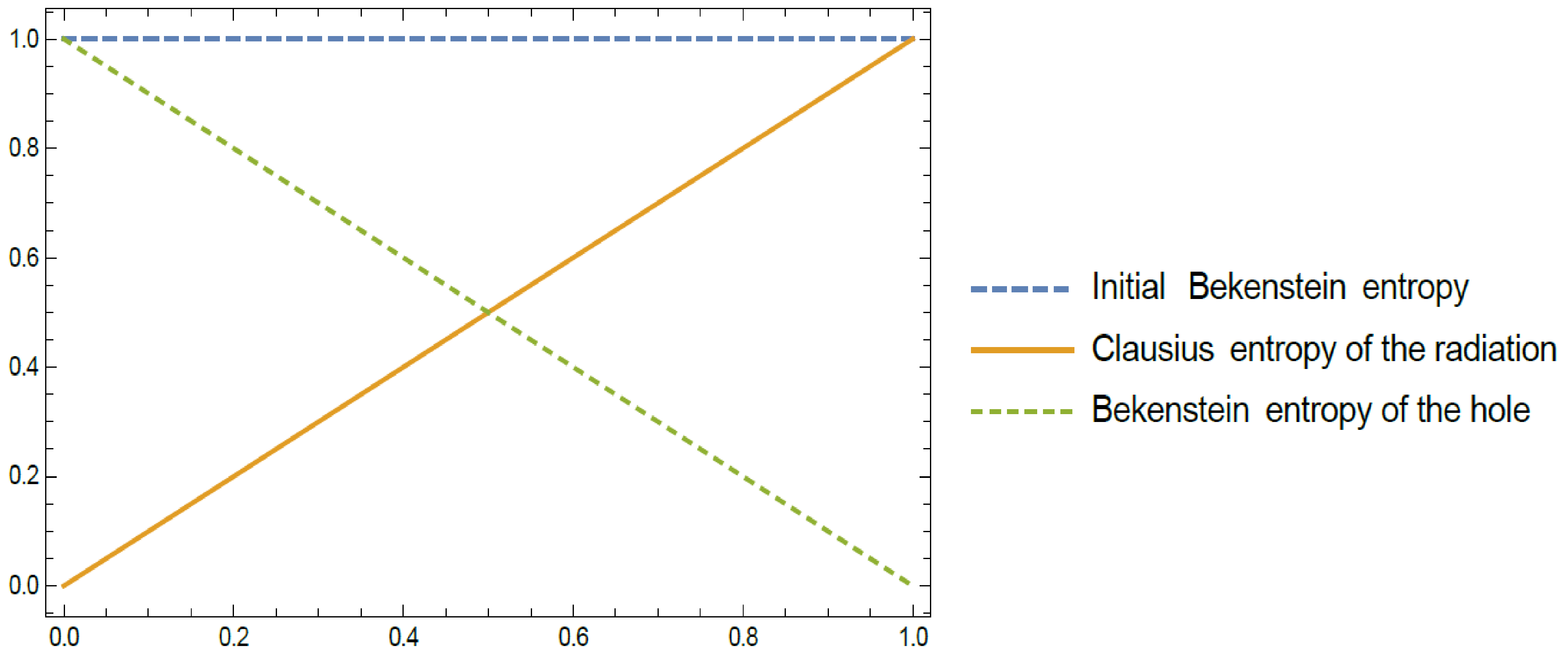
- Thermodynamic entropy gain (Clausius entropy gain) of the external radiation field per emitted quanta $\longrightarrow$ entropy gain of the Hawking flux

$$\frac{dS}{dN}=\frac{dE/dT_H}{dN}=\frac{\hbar\langle\omega\rangle dN}{T_H dN}=\frac{k_B\pi^4}{30\zeta(3)}$$

- The Hawking radiation is essentially (adiabatically) transferring Bekenstein entropy from the hole into Clausius entropy of the radiation field

- If we measure the entropy in bits: $\dfrac{d\hat{S}_2}{dN}\approx 3.897$ bits/quanta

- Throughout the evaporation process we have $S_{Bekenstein}(t)+S_{Clausius}(t)=S_{Bekenstein,0}$




- So, semiclassically everything holds together very well

## ENTANGLEMENT ENTROPY IN THE HAWKING FLUX

- Average subsystem entropies (Page, 1993)
- Consider a Hilbert space that factorizes $H_{AB} = H_A \otimes H_B$

  Pure state $\rho_{AB} = |\psi\rangle\langle\psi|$ $\longrightarrow$ Subsystem density matrices $\rho_A = tr_B(|\psi\rangle\langle\psi|)$

  Subsystem von Neumann entanglement entropy $\hat{S}_A = -tr(\rho_A \ln \rho_A)$

- Uniform average over all pure states, taking:

  $n_1 = dim(H_A)$ , $n_2 = dim(H_B)$ and $m = min[n_1, n_2]$

  The central result $\hat{S}_{n_1, n_2} = \langle \hat{S}_A \rangle = \langle \hat{S}_B \rangle \leqslant \ln m$

- Average subsystem entropy is very close to its maximum possible value

  Strict bound (combined with our results): $\hat{S}_{n_1, n_2} = \langle \hat{S}_A \rangle = \langle \hat{S}_B \rangle \in \left(\ln m - \frac{1}{2}, \ln m\right)$

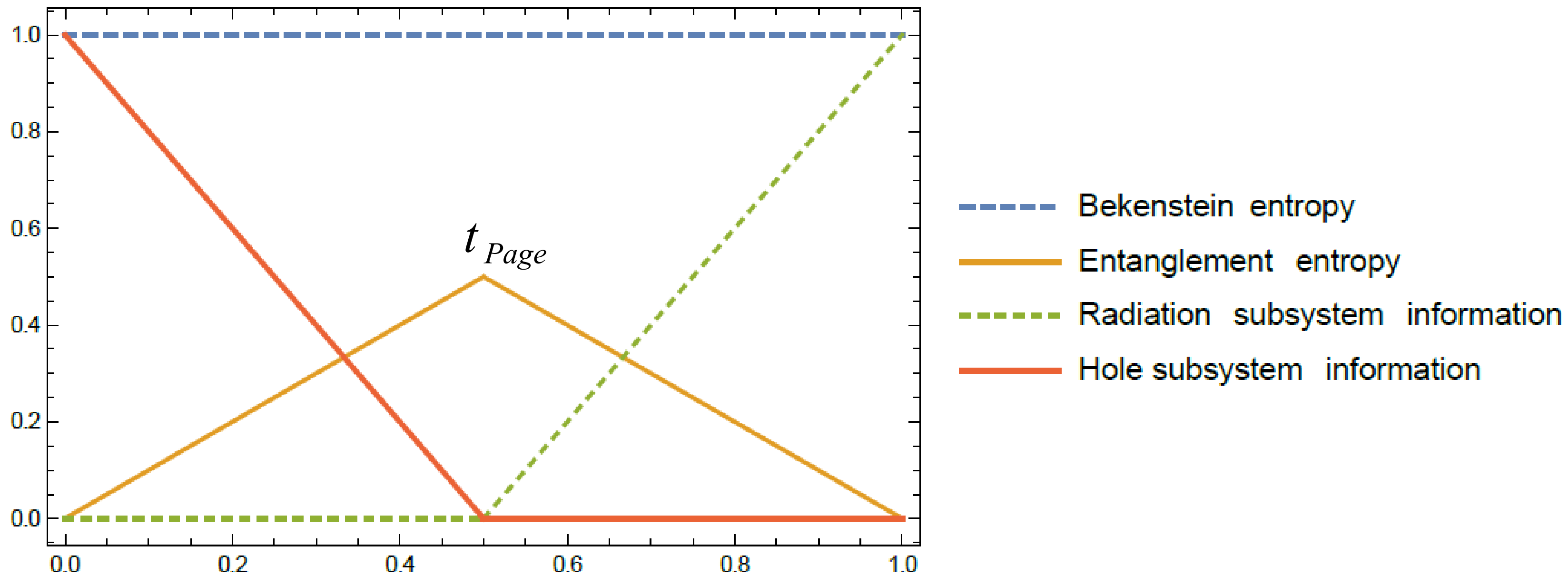
## Bipartite entanglement: black hole + Hawking radiation (Page, 1993)

- "Closed box" argument
- Initially there is no yet any Hawking ratiation $\begin{cases} H_R & \text{trivial} \\ H_H & \text{enormous} \end{cases} \longrightarrow (\hat{S}_{n_H, n_R})_0 = 0$
- After the black hole has completely evaporated: $H_H$ is trivial $\longrightarrow (\hat{S}_{n_H, n_R})_\infty = 0$
- At intermediate times both dimensionalities are nontrivial $\longrightarrow (\hat{S}_{n_H, n_R})_t \neq 0$
  - Since the evolution is assumed unitary: total Hilbert space is constant

$$n_H(t)\, n_R(t) = n_{H_0} = n_{R_\infty} \longrightarrow (\hat{S}_{n_H, n_R})_t = \ln\left\{ n_H(t), \frac{n_{H_0}}{n_H(t)} \right\}$$

Maximized when $n_H(t) \approx \sqrt{n_{H_0}} \longrightarrow \hat{S}_{n_H, n_R}(t = t_{Page}) \approx \frac{1}{2} \ln n_{H_0}$

➢ Page curve:




- It is the shape of this curve that underlies much of the modern discussion surrounding the "information puzzle"
- Subsystem entropy is initially zero $\longrightarrow$ tension with Bekenstein entropy
- If we entangle the black hole with the environment, then the total state is not pure

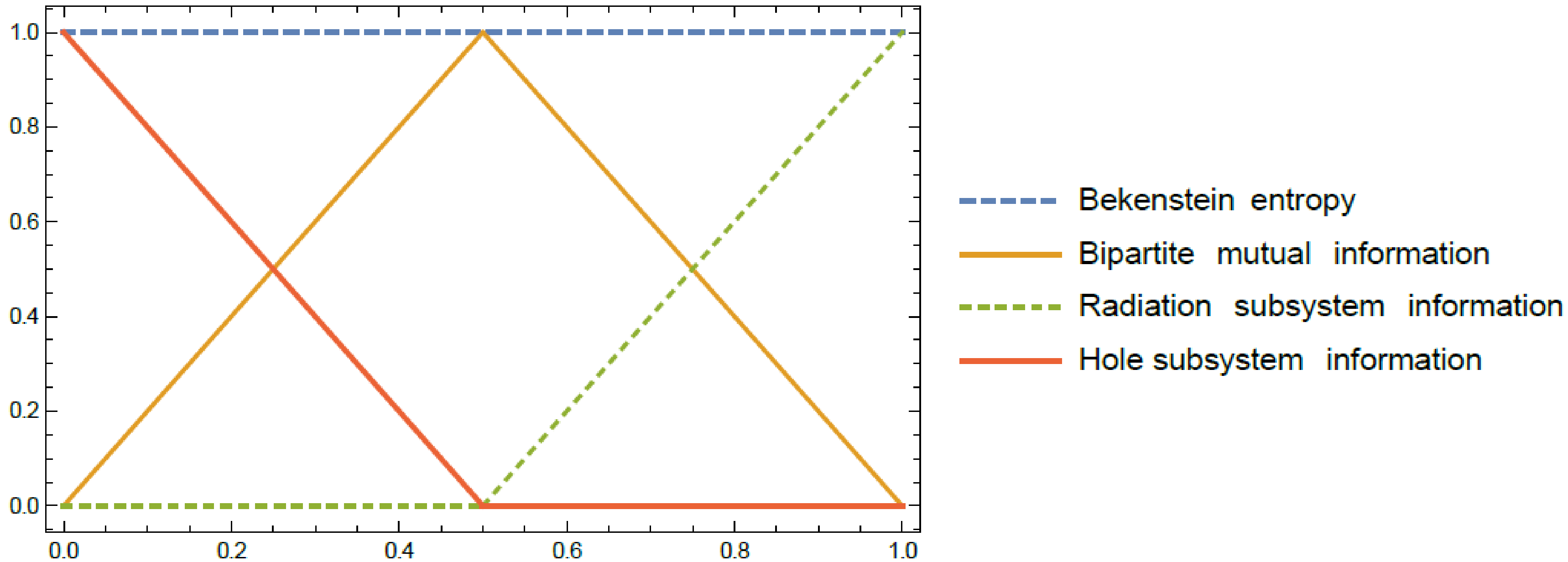
➢ Pages defines a novel asymmetric version of the subsystem information

$$\tilde{I}_{n_1,n_2} = \ln n_1 - \hat{S}_{n_1,n_2}$$

## Mutual information

- In general: $I_{A:B} = S_A + S_B - S_{AB}$
- Bipartite entanglement: $I_{H:R} = 2\mathrm{S}_H = 2\mathrm{S}_R$
- Applying the "average subsystem" argument and combined with the Page's asymmetric subsystem information, we found:

$$\langle \tilde{I}_{H,R} \rangle + \langle \tilde{I}_{R,H} \rangle + \langle \hat{I}_{H:R} \rangle = \ln(n_H n_R) \approx \hat{S}_{Bekenstein,0}$$



- What happen with these results?

  - The late radiation is maximally entangled with the early radiation and with the black hole subsystems

$\Downarrow$

Monogamy of entanglement

$\Downarrow$

Firewalls

- We though that the point is the considered “closed box” system

  - May be it is more appropiate to consider a tripartite system, including the interaction with the environment

## Tripartite entanglement: bh + Hawking radiation + rest of the universe

- The Hilbert space is now $H_{HRE} = H_H \otimes H_R \otimes H_E$
- Take the entire universe be in a pure state $S_{HRE}(t) = 0$

  And now the subsystem entropies $S_H(t) = S_{ER}(t)$, $S_R(t) = S_{HE}(t)$, $S_E(t) = S_{HR}(t)$

- Initially $S_{H_0} = S_{E_0}$ , $S_{R_0} = 0 = S_{HE_0}$
- Once the black hole has completely evaporated $S_{H_\infty} = 0 = S_{ER_\infty}$, $S_{R_\infty} = S_{E_\infty}$
- The evolution is assumed unitary, with the unitary time evolution operator factorized as

$$U_{HRE} = U_{HR}(t) \otimes U_E(t)$$

Therefore
$$\begin{cases} n_{E_0} = n_{E_\infty} \equiv n_E \\ n_H(t)\, n_R(t) = n_{H_0} = n_{R_\infty} \end{cases}$$

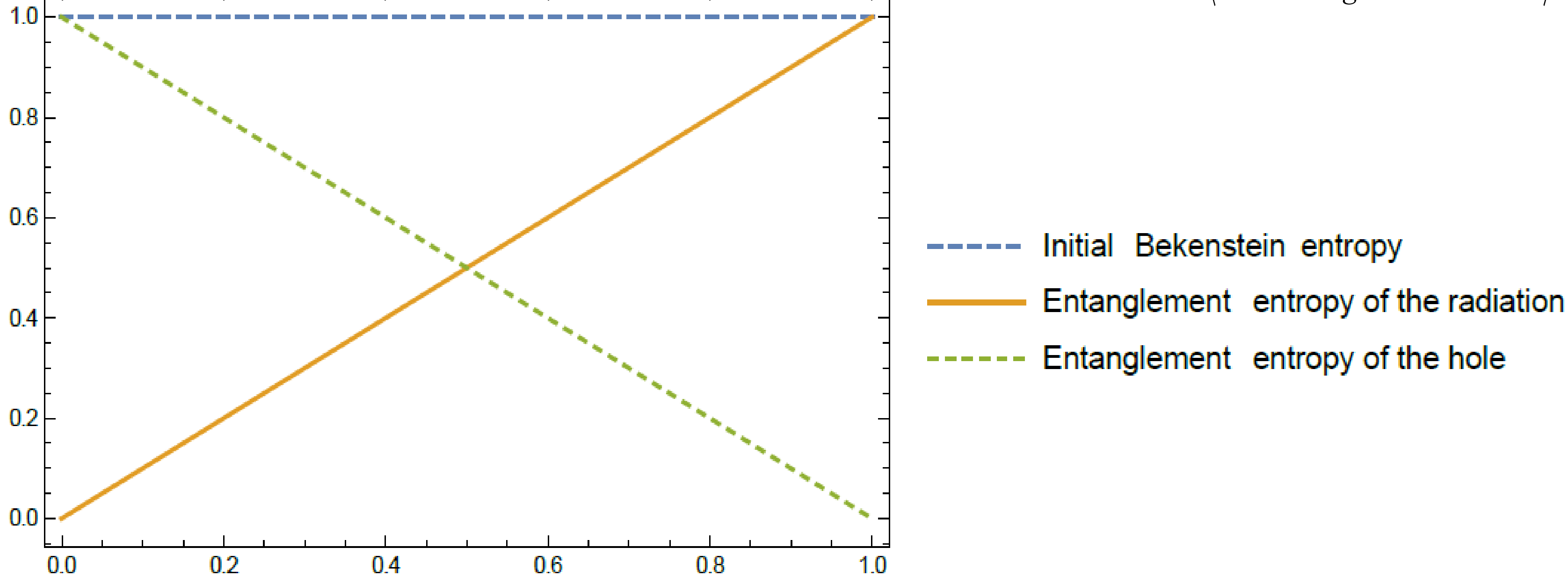
- We make an additional assumption: That the Bekenstein entropy can be identified with the average entanglement entropy

- Then, the entropies:

  - $\left\langle \hat{S}_H(t) \right\rangle \approx \ln min[n_H(t), n_R(t) n_E] \approx \ln n_H(t)$
  - $\left\langle \hat{S}_R(t) \right\rangle \approx \ln min[n_R(t), n_H(t) n_E] \approx \ln n_R(t)$

$$\left\langle \hat{S}_H(t) \right\rangle + \left\langle \hat{S}_R(t) \right\rangle \approx \ln[n_H(t) n_R(t)] = \ln n_{H_0}$$

$$\hat{S}_{Bekenstein}(t) + \left\langle \hat{S}_{Hawking\ radiation}(t) \right\rangle \approx \hat{S}_{Bekenstein,0}$$




- The "rest of the universe environment": the extent to which the subsystem is entangled

$$\left\langle \hat{S}_E(t) \right\rangle \approx \ln min[n_E(t), n_H(t) n_R] \approx \ln n_{H_0} \approx \hat{S}_{Bekenstein,0}$$

## Mutual information

- For the tripartite system: $I_{H:R} = S_H + S_R - S_{HR}$ $\quad S_E$
- Averaging over the pure states in the total system we obtain

$$\left\langle \hat{I}_{H:R} \right\rangle \leqslant \frac{n_{H_0}}{2\mathrm{n}_E} \leqslant \frac{1}{2}$$

➡ So the average mutual information never exceeds ½ nat throughout the entire evaporation process

## Infinite-dimensional environment

- For the tripartite system: environment is used to initially entangle the black hole with the rest of the universe
  - There is no loss of generality in taking the limit $n_E \to \infty$
- In this limit we therefore have the equality

$$\lim_{n_E \to \infty} \left( \left\langle S_H \right\rangle + \left\langle S_R \right\rangle \right) = \lim_{n_E \to \infty} \left\langle S_E \right\rangle \quad \Longrightarrow \quad \lim_{n_E \to \infty} \left\langle I_{H:R} \right\rangle = 0$$

## DISCUSSION

- There is no “information puzzle” in burning a lump of coal
  - Entropy per photon in blackbody radiation is compensated (since the process is unitary) by an equal “hidden information” in the correlations
- We calculated the classical thermodynamics entropy and the Bekenstein entropy and they compensate perfectly
- We calculate the quantum (entanglement) entropy by means of a tripartite system, which results that is completely agree with the classical expected results, to within 1 nat
- When we restrict attention to a particular subsystem we perceive an amount of entanglement entropy (a loss of information), there exists a complementary amount of entropy that is codified in the correlations between the subsystems
- Assuming the unitarity of the evolution of the (black hole) + (Hawking radiation) subsystem, we showed that there are no weird physical effects in the evaporation of a black hole

## Thank you for your attention!

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Work in progress.