Current status of dark energy singularities: A Quantum analysis

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Outline

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1. Introduction
2. Cosmological singularities related to dark energy within GR
3. The quantum fate of singularities in a dark-energy dominated universe
4. DE singularities and the EiBI theory: a quantum approach
5. Conclusions

Introduction-1-: A brief sketch of the universe

- The universe is homogeneous and isotropic on large scales (cosmological principle).
- The matter content of the universe:
  - Standard matter
  - Dark matter
  - Dark energy as a mean to accelerate the late universe.
- The acceleration of the universe is backed by several measurments: $H(z)$, SneIa, BAO, CMB, LSS (matter power spectrum, growth function)...

Introduction-2-

- The effective equation of state of dark energy is roughly -1. For a wCDM model with w constant and $k = 0$, Planck results implies $w = -1.006 \pm 0.045$.
- Therefore, there could be room for dark energy with $w_0 < -1 \Longrightarrow$ phantom energy
- In phantom energy models
  - Null energy condition is not satisfied
  - Energy density is a growing function of the scale factor (in an expanding Universe like ours)
  - May be there is some future singularity/abrupt event.

Starobinsky 00, Caldwell 02, Caldwell, Kamionkowski and Weinberg 03

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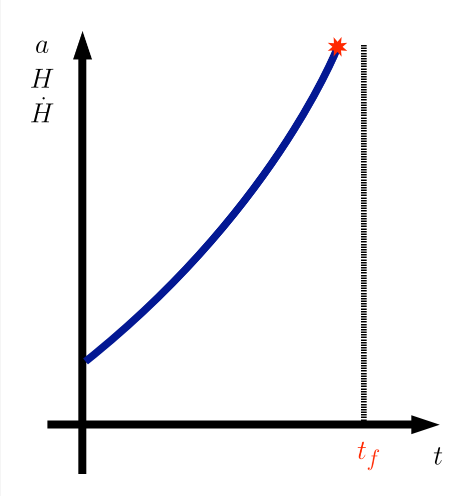
Cosmological singularities related to dark energy

- Classification of the cosmological singularities related to dark energy
  - Big rip singularity
  - Sudden singularity, big brake singularity, big démarrage singularity
  - Big freeze singularity
  - Type IV singularity
  - w-singularity
  - Little rip event
  - Little sibling of the big rip

Kamenshchik 13 (review mainly on type II sing.)

Big rip singularity

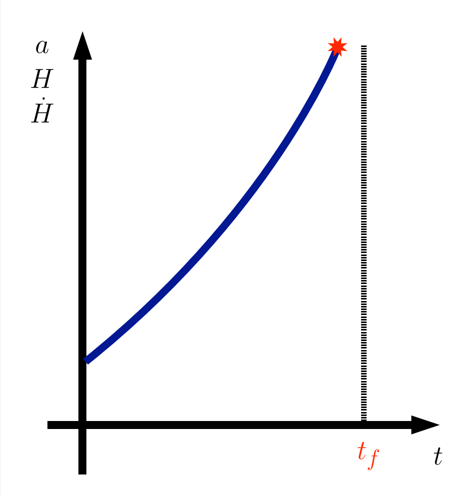
- For this singularity the null energy condition is violated. The scale factor diverges in a finite time. It is accompanied with a divergence of the Hubble rate and the cosmic derivative of the Hubble rate.
- For example this singularity might appear in a hologrphic Ricci dark energy model (C.Gao et al 09).



Starobinsky 00, Caldwell 02, Caldwell, Kamionkowski and Weinberg 03

Little rip abrupt event-1-

- For this abrupt event, the null energy condition is violated. The scale factor diverges in an infinite time ($t_f \to \infty$). It is accompanied with a divergence of the Hubble rate and the cosmic derivative of the Hubble rate.



Little rip abrupt event-2-

- The name of little rip was introduced by Frampton, Ludwick and Scherrer '11
- This kind of singularity corresponds to a big rip sent towards an infinite cosmic time
- Examples:
  - This kind of singularity can happens in a FLRW universe filled with a perfect fluid $p = -\rho - A\rho^{1/2}$
    (Nojiri, Odintsov and Tsujikawa 05', Stefançic 05')
  - Also presents in some dilatonic brane-world models (BL 05').
  - First example was found by Ruzmaikina and Ruzmaiki back in 1970 corresponding to a past little rip

Little sibling of the big rip

- This event is much smoother than the big rip singularity. When the little sibling of the big rip is reached, the Hubble rate and the scale factor blow up but the cosmic time derivative of the Hubble rate does not. This abrupt event takes place at an infinite cosmic time where the scalar curvature explodes.
- It turns out that eventhough the event seems to be harmless as it takes place in the infinite future, the bound stuctrure in the universe would be unavoidably destroyed in a finite cosmic time from now.

BL, Errahmani, Martín-Moruno, Ouali, Tavakoli (2015)

Outline

On the quantum fate of singularities in a dark-energy dominated universe

- There is no successful quantum gravity theory so far that would lead to THE theory of quantum cosmology
- There are, however, several approaches in this direction. Here we will follow the most conservative one which corresponds to the Wheeler deWitt approach.

The holographic dark energy scenario

- Bekenstein proposal: The entropy of a system of volume L^3 is bounded by a quantity proportional to its area L^2 (Bekenstein 73, 81)
- It was proposed as well a bound on the energy density of such a system (Cohen et al 99)
- This idea was applied to cosmology given rise to the holographic dark energy model (Li 04, Hsu 04)

$$\rho_{\mathrm{H}} = \frac{3c^2}{\kappa_4^2 L^2} \tag{1}$$

- There are several choices for the scale: the Hubble scale, the event horizon, the particle horizon, a scale related to the curvature ...
- Here, we will consider an holographic Ricci dark energy model (Gao, Wu, Chen, Shen '09).

The holographic Ricci dark energy model

- After using the Friedmann equation for a universe filled with rad. and matter, the energy density of HRDE reads

$$\rho_R = \frac{3H_0^2}{8\pi G}\left[\left(\frac{\beta}{2-\beta}\right)\Omega_{m0}\left(\frac{a}{a_0}\right)^{-3} + \Omega_{p0}\left(\frac{a}{a_0}\right)^{-2\left(2-\frac{1}{\beta}\right)}\right]$$

where Ω_{p0} is an integration constant that quantifies the effective amount of DE in the HRDE.

1. If $1 < \beta$, the cosmic acceleration is negative.
2. If $1/2 < \beta < 1$, the Universe enters in an accelerating state when the HRDE dominates. The Universe is asymptotically flat in the future.
3. If $\beta = 1/2$, the model is asymptotically de Sitter.
4. If $0 < \beta < 1/2$, the Universe not only enters in an accelerated state, but also super accelerates ($\dot{H} > 0$) in the future hitting a Big Rip. The universe hits a singularity at a finite cosmic time.

- Obsevertional constraints on the HRDE favours the case $0 < \beta < 1/2$ (Xu and Wang 2010 and Suwa, Kobayashi, Oshima 2014).

- We start assuming a FLRW universe, then the gravitational Lagrangian reads

$$L = N\left[\frac{3\pi}{4G}\left(-\frac{a\dot{a}^2}{N^2} + ka - \Lambda(a)\frac{a^3}{3}\right)\right]$$

where $\Lambda(a)$ encodes the matter content of the universe

- We next obtain the Hamiltonian

$$\mathcal{H} = N\left[-\frac{G}{3\pi}\frac{p_a^2}{a} + \frac{\pi}{4G}\Lambda\left(a\right)a^3\right]$$

where p_a is the canonical momentum of a.

- So far everything is classical

Example on how to obtain the Wheeler deWitt-2-

- In the quantum framework, the term p_a^2/a generates the operator

$$\frac{p_a^2}{a} = -\hbar^2 \left[a^{-\frac{1}{2}}\partial_a\right]\left[a^{-\frac{1}{2}}\partial_a\right]$$

- Then, the quantum Hamiltonian operator can be written as
$$\hat{\mathcal{H}} = N\left\{\frac{3G\hbar^2}{4\pi a_0^3}\partial_x^2 + \frac{3\pi H_0^2 a_0^3}{4G}\left[\Omega_{r0}x^{-\frac{2}{3}} + \left(\frac{2}{2-\beta}\right)\Omega_{m0} + \Omega_{p0}x^{-\frac{2}{3}\left(1-\frac{2}{\beta}\right)}\right]\right\}$$
where $x = \left(\frac{a}{a_0}\right)^{\frac{3}{2}}$.
- Finally, the WDW equation comes from the variation of the Hamiltonian with respect to the lapse function N which produces the Hamiltonian constraint

$$\hat{\mathcal{H}}\Psi(x) = 0.$$

Quantisation of the holographic Ricci dark energy model

- We devided the expanssion of the universe in 3 different regions.
  - A purely initial radiation domintes epoch
  - An epoch where the main components are radiation and matter
  - A final epoch where matter and DE on the form of the HRDE are present
- We solved the Wheeler DeWitt equation on the different regions and connected them smoothly
- It is possible to impose the DeWitt argument for small and very large factors which can be seen as a quantum avoidance of the big bang in the past and big rip in the future

Albarran, BL 2015

More on the quantum fate of singularities in a dark-energy dominated universe

Within the framework of quantum geometrodynamics and a Born Oppenheimer approximation

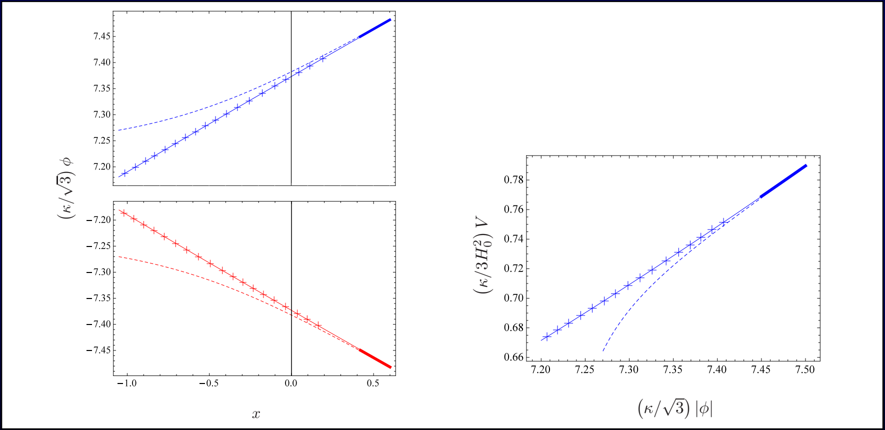
- It was shown by Dabrowski, Kiefer and Sandhöfer 06' that the big rip can be removed.
- It was shown by Kamenshchik, Kiefer and Sandhöfer 07' the avoidance of a big brake singularity.
- It was shown by BL, Kiefer, Sandhöfer and Vargas Moniz 09' the avoidance of a big démarrage singularity and a big freeze.
- Type IV singularity is removed (BL, Krämer and Kiefer 2014).

The LR driven by a phantom scalar field

- Phantom scalar field ϕ: $\rho_\phi = -\frac{1}{2}\dot{\phi}^2 + V(\phi)$, $\quad p_\phi = -\frac{1}{2}\dot{\phi}^2 - V(\phi)$

$$\ddot{\phi} + 3H\dot{\phi} - V'(\phi) = 0, \quad V(\phi) \propto \phi^4 \text{ (asymp.)}$$

- $\rho_\phi = \rho$, $p_\phi = P$ with $P = -\rho - A\sqrt{\rho}$, $0 < A$



Albarran, BL, Kiefer, Morais, Vargas Moniz, 16

The Wheeler-DeWitt equation

- Quantisation of the classical scenario in the quantum geometrodynamical framework
- The Wheeler-DeWitt equation in quantum cosmology is the analogous to Schrödinger equation in quantum mechanics.
- The Wheeler-DeWitt equation for the space variables (a,ϕ)

$$\left\{ \frac{\hbar^2}{4\pi^2} \left[\frac{\kappa^2}{6} \partial_x^2 + \partial_\phi^2 \right] + \sigma e^{6x} \phi^4 \right\} \Psi(x,\phi) = 0,$$

where σ is a constant and $x = \ln(a)$

- Notice that in the quantum case ϕ is no longer a function of a
- General remark: the Wheeler-DeWitt equation does not depend on time (!)

Decomposing the Wheeler-DeWitt equation

- We make a change of variables: $\phi = r(z)\,\varphi, \quad x = z,$
- We apply some approximations (which works better the closer is our model from ΛCDM which is indeed the case)
- We conclude that the wave functions can be written as

$$\Psi(z,\varphi) = \sum_k U_k(\varphi)\, C_k(z)\, q_k,$$

where q_k is the amplitude for each solution and the functions $C_k(z)$ and $U_k(\varphi)$ are the solutions of the following differential equations

$$\left\{\frac{\hbar^2\kappa^2}{24\pi^2}\partial_z^2 + ke^{2z}\right\} C_k(z) = 0, \quad \left\{\frac{\hbar^2}{4\pi^2}\partial_\varphi^2 + \sigma\varphi^4 - k\right\} U_k(\varphi) = 0.$$

- It can be shown that there is a set whole of solutions that decays assymptotically for large x (or a) and φ. Therefore, the wave function vanishes when approaching LR leading to the fullfilment of the DeWitt argument.
- The same conclusion can be reached using a WKB approximation and/or a perfect fluid approach for the matter content.

Little sibling of the big rip

- This event is much smoother than the big rip singularity. When the little sibling of the big rip is reached, the Hubble rate and the scale factor blow up but the cosmic time derivative of the Hubble rate does not. This abrupt event takes place at an infinite cosmic time where the scalar curvature explodes.
- This kind of event can happen for a FLRW universe filled (for example) with a perfect fluid with the following equation of state:

$$p = -\rho - \frac{A}{3}, A > 0$$

- The energy density:

$$\rho(a) = \Lambda + A \ln\left(\frac{a}{a_0}\right)$$

- Can also be present in 3-form models (J. Morais talk tomorrow)

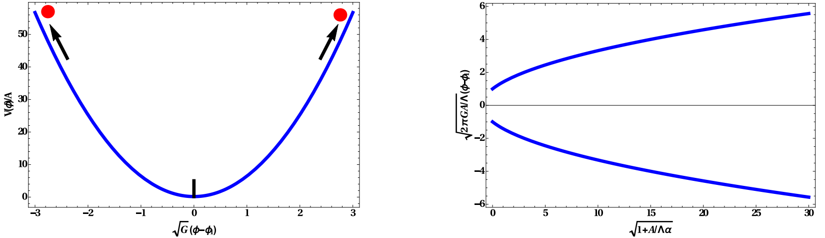
BL, Errahmani, Martín-Moruno, Ouali and Tavakoli '15.

LSBR driven by a standard scalar field

- Phantom scalar field ϕ: $\rho_\phi = -\frac{1}{2}\dot{\phi}^2 + V(\phi)$, $p_\phi = -\frac{1}{2}\dot{\phi}^2 - V(\phi)$

$$\ddot{\phi} + 3H\dot{\phi} - V'(\phi) = 0, \quad V(\phi) = \frac{A}{6} + 2\pi AG\,(\phi - \phi_1)^2$$

$\alpha = \ln(a/a_0)$



Albarran, BL, Cabral, Martín-Moruno, 15

LSBR: Quantisation with a scalar field

- The Wheeler-DeWitt equation:

$$\frac{\hbar^2}{2}\left[\frac{\kappa^2}{6}\frac{\partial^2}{\partial\alpha^2}+\frac{\partial^2}{\partial\phi^2}\right]\psi(\alpha,\phi)+a_0^6e^{6\alpha}V(\phi)\psi(\alpha,\phi)=0$$

- Can be solved within the BO:
  - The gravitational part are oscilatory or exponential functions.
  - The matter part can be written as parabolic cylinder functions that decay to zero at large value of the scale factor.
- Consequently, the DeWitt condition is again fullfilled as long as the gravitational part is finite; i.e. as long as the increasing exponential solution is dropped.

Outline

EiBI theory-1-

- There have been many proposals for alternative theories of GR as old as the theory itself
- One of the oldest proposal was due to Eddington
- In Eddington proposal, the connection rather than the metric plays the fundamental role of the theory
- It is equivalent to GR in vacuum
- BUT does not incorporate matter
- An Eddington-inspired-Born-Infeld theory has been proposed by Bañados and Ferreira

Bañados and Ferreira (2010)

EiBI theory-2-

$$\mathcal{S}_{\rm EiBI}(g,\Gamma,\Psi)=\frac{2}{\kappa}\int d^4x\left[\sqrt{|g_{\mu\nu}+\kappa R_{\mu\nu}(\Gamma)|}-\lambda\sqrt{|g|}\right]+\mathcal{S}_{\rm m}(g,\Gamma,\Psi)$$

- We consider the action under the Palatini formalism, i.e., the connection $\Gamma^{\alpha}_{\mu\nu}$ is *not* the Levi-Civita connection of the metric $g_{\mu\nu}$
- This Lagrangian has two well defined limits: (i) when $|\kappa R|$ is very large, we recover Eddington's theory and (ii) when $|\kappa R|$ is small, we obtain the Hilbert-Einstein action with an effective cosmological constant $\Lambda=(\lambda-1)/\kappa$
- A solution of the above action can be characterized by two different Ricci tensors: $R_{\mu\nu}(\Gamma)$ as presented on the action and $R_{\mu\nu}(g)$ constructed from the metric g
- There are in addition three ways of defining the scalar curvature. These are: $g^{\mu\nu}R_{\mu\nu}(g)$, $g^{\mu\nu}R_{\mu\nu}(\Gamma)$ and $R(\Gamma)$. The third one is derived from the contraction between $R_{\mu\nu}(\Gamma)$ and the metric compatible with the connection Γ
- Therefore whenever one refers to singularity avoidance, one must specify the specific scalar curvature(s). This will affect the way we apply the DeWitt criterium

EiBI theory-3-

- Gravitational action:

$$\mathcal{S}_{\rm EiBI}(g,\Gamma,\Psi) = \frac{2}{\kappa}\int d^4x \left[\sqrt{|g_{\mu\nu}+\kappa R_{\mu\nu}(\Gamma)|} - \lambda\sqrt{|g|}\right] + \mathcal{S}_{\rm m}(g,\Gamma,\Psi)$$

- The parameter κ has been constrained using observationally for example from BBN (Casanellas et al 2012, Avelino 2012).
- The model can avoid the Big Bang singularity, for example, in a radiation dominated universe (Bañados and Ferreira 2012).
- Has been proposed as an alternative scenario to the inflationary paradigm (Avelino 2012)
- It was shown that if the null energy condition is fullfilled then the *apparent* null energy condition is also fullfilled (Deslate and Steinhoff 2012).
- Can this theory avoid the big rip singularity? Answer No (BL, Che-Yu Chen, Pisin Chen '14,'15)

Towards EiBI quantization-1-

- The equation of motion of EiBI theory can be equally obtain from the action (Delsate and Steinhoff '12)

$$S_a = \lambda \int d^4x\sqrt{-q}\Big[R(\Gamma) - \frac{2\lambda}{\kappa} + \frac{1}{\kappa}(q^{\alpha\beta}g_{\alpha\beta} - 2\tau)\Big] + S_M(g),$$

- Unlike the EiBI action, this action is linear on $R(\Gamma)$. This makes the quantization much “simpler”
- The starting point is the homogeneous and isotropic ansatz of the Universe:

$$\begin{aligned} g_{\mu\nu}dx^{\mu}dx^{\nu} &= -N(t)^2dt^2 + a(t)^2d\vec{x}^2, \\ q_{\mu\nu}dx^{\mu}dx^{\nu} &= -M(t)^2dt^2 + b(t)^2d\vec{x}^2. \end{aligned}$$

where $N(t)$ and $M(t)$ are the lapse functions of $g_{\mu\nu}$ and $q_{\mu\nu}$, respectively.

Towards EiBI quantization-2-

- The Lagrangian reads

$$\mathcal{L} = \lambda M b^3 \left[-\frac{6\dot{b}^2}{M^2 b^2} - \frac{2\lambda}{\kappa} + \frac{1}{\kappa}\left(\frac{N^2}{M^2} + 3\frac{a^2}{b^2} - 2\frac{Na^3}{Mb^3} \right) \right] - 2\rho(a) N a^3,$$

- The Hamiltonian reads (as there is no singularity at $b = 0$ for the model, we can safely rescale the Hamiltonian as)

$$b^3 \mathcal{H} = M \left[-\frac{b^2 p_b^2}{24\lambda} + \frac{2\lambda^2}{\kappa} b^6 + \frac{1}{\kappa\lambda}(\lambda + \kappa\rho(a))^2 a^6 - \frac{3\lambda}{\kappa} a^2 b^4 \right] = 0$$

- Then, we can write down the WDW equation by choosing the following factor ordering:

$$b^2 p_b^2 = -\hbar^2 \left(b\frac{\partial}{\partial b} \right)\left(b\frac{\partial}{\partial b} \right) = -\hbar^2 \left(\frac{\partial}{\partial x} \right)\left(\frac{\partial}{\partial x} \right)$$

where $x = \ln(\sqrt{\lambda} b)$.

Towards EiBI quantization-3-

- Therefore, the WDW equation reads:

$$\left[\frac{\partial^2}{\partial x^2} + V_1(a,x)\right]\Psi(a,x) = 0,$$

where

$$V_1(a,x) = \frac{24}{\kappa\hbar^2}\left[2\mathrm{e}^{6x} - 3a^2\mathrm{e}^{4x} + (\lambda + \kappa\rho(a))^2 a^6\right].$$

- It can be shown that the wave function decays and vanishes when approaching $a \to 0$.
- The DeWitt criterium is therefore fullfilled!!!

BL, Che-Yu Chen '16

Outline

Conclusions

- In this talk, we have reviewed the cosmological singularities that have appeared on the literature over the last few years, motivated (initially) from the possible presence of an exotic dark energy component
- Then we have shown how these singularities could be appeased or removed within a quantum approach