# Universe's memory and spontaneous coherence in loop quantum cosmology

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# Why bother?

- Recent series of very interesting results in perturbative regime of loop quantum cosmology
  (Ashtekar & Agullo, previous talks of this session, ...)
  effective classical dynamics, modified classical constraint algebra, ...
    - Origin from quantum theory of LQC
    - Implicit assumption of well behaving and large semiclasical sector
- Nonperturbative character of LQG/LQC
    - Existence and properties of semiclassical sector – highly nontrivial
    - Crucial properties of the dynamics are of genuine quantum nature.
- Questions highly nontrivial even for isotropic background of perturbations.
- Effective techniques require a guaranteed & well behaving semiclassical sector.

# Outline

Quantum mechanics under the hood.

- Basic features of loop theories
- Quantum dynamics of loop quantum cosmology
- Techniques of probing semiclassical sector
  - Ultra-short review
- deSitter universe with matter:
  - Peculiar features of quantum dynamics
  - decoherence & spontaneous coherence

# Key technique of LQG/LQC

Polymer quantization: Example of 1D free particle.

- Standard (Schrödinger) quantization:
  canonical variables $(x, p)$ promoted to operators $\hat{x}, \hat{p}$.
- Alternative Weyl quantization:
  - Instead promote Weyl algebra elements $e^{i(\lambda x + \sigma p)}$ to operators.
  - In standard topology $(\mathbb{R}, \mathrm{d}x)$: operators $\hat{x}, \hat{p}$ defined as derivatives
    - Condition: weak continuity of $\widehat{e^{i\lambda x}}, \widehat{e^{i\sigma p}}$.
- Change of topology:
  - Points: almost periodic functions

$$(f, g) = \lim_{L\to\infty} \tfrac{1}{2L} \int_{-L}^{L} \mathrm{d}y f^\star(y) g(y),$$

    - Uncountable set of discrete points: $(e^{ikx}, e^{ik'x}) = \delta_{k,k'}$
  - Continuity broken: only one (in LQG none) of $\hat{x}, \hat{p}$ exists.
  - Crucially changes predictions!

# Loop gravity & cosmology

- Loop quantum gravity:
  - Orthonormal triads of vectors instead of metrics.
  - Holonomy-flux algebra of parallel transports of (su(2) valued) connections and fluxes of triads.
  - Quantization: Holonomies represented by cyllindrical functions.
  - Wave function support: graphs embedded in 3D mainfold
  - Properties:
    - UV regular.
    - Background independent.
    - Geometric observables discrete
    - Very exotic vacuum, smooth spacetime nontrivially emergent
- Loop quantum cosmology:
  - Technique applied to symmetric spacetimes
  - Quantum Mechanical systems

# Isotropic sector of LQC

- Basic canonical variables $(v, b)$: rescaled volume & Hubble parameter.
- Polymer quantization: $v$ - almost periodic functions
  - $\mathbb{R}$ with discrete measure. Hilbert space basis: $\langle v|v'\rangle = \delta_{v,v'}$.
- Quantum deparametrization: matter (massless scalar, dust, Maxwell) used as clocks
- "Hamiltonian": 2nd order difference operator ($\hat{N}|v\rangle = |v+1\rangle$):

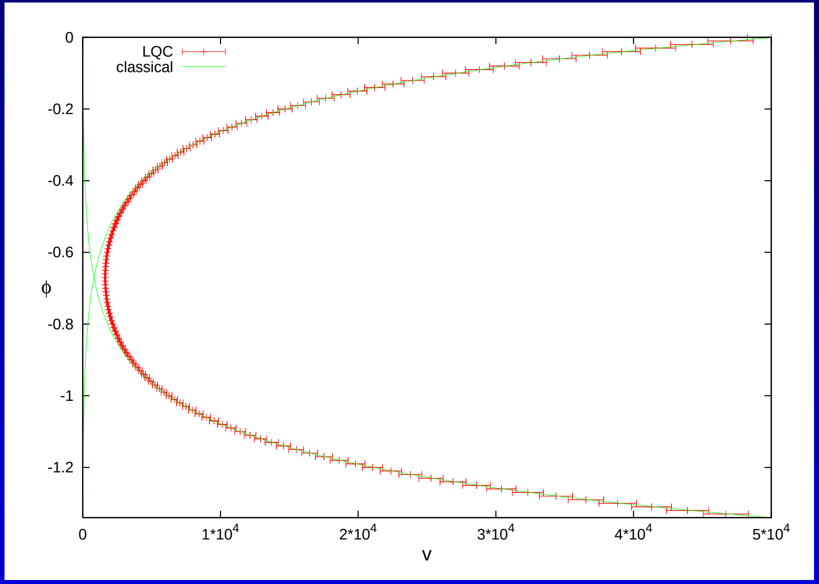
$$-\partial_T^2 \Psi(v,T) = \hat{\Theta}\Psi(v,T), \quad \hat{\Theta} = f_+(v)\hat{N}^4 + f_o(v)\mathbb{I} + f_-(v)\hat{N}^{-4}.$$

- Dynamical results:
  - Upper energy density bound (Planck scale).
  - Conformation to GR in low energies
  - Effective "Anti-gravity" force causing bounce at Planck energy scale
  - Bounce deterministically connects several (semiclassical?) low energy epochs

# LQC states

Decomposition in "energy" eigenstates:

- $\Psi(v,T) = \int_0^\infty \mathrm{d}k \tilde{\Psi}(k) e_k(v) e^{i\omega(k)T}$
  - $\mathrm{d}k$ can be discrete,
  - $e_k$: $\hat{\Theta} e_k = \omega^2(k) e_k$ determined numerically
- Observables defined for slices of constant $T$.




- Semiclassicality properties differ between pre and post-bounce epochs.
  - The beahavior/magnitude of these differences had to be tested.

# Testing the semiclassicality

Just $\Lambda = 0$ and massless scalar or radiation

- Genuine quantum dynamics numerical tests:
  Ashtekar, TP, Singh 2006; Mena-Marugan, Olmedo, TP 2010; Corichi, Montoya 2011; Diener, Gupt, Medevand, Singh 2014
  - Pros: Precise results throughout the evolution
  - Cons: Finite examples, very few shapes of $\tilde{\Psi}(k)$.
- Analytical evaluations: Corichi, Singh, 2007
  - Simplest models can be solved analytically (tailoring ambiguities).
  - Pros: Cover considerable region of phase space.
  - Cons: Just simplest model (one specific field as matter content)
- Scattering picture: Kamiński, TP 2010
  - Pros: General results, extendability.
  - Cons: Only probing asymptotic future/past.

# Scattering picture

- Wheeler-DeWitt analog
  - Model can be quantized in Schrödinger rep. (geometrodynamics).
  - Result: Klein-Gordon equation $[\partial_\phi^2 - 12\pi G[v\partial_v]^2]\Psi(v,\phi) = 0$.
    - Two classes: ever expanding and contracting
- WDW limit: in large $v$ limit $e_k$ approach standing WDW waves.
  - Can define future/past WDW limit wave packets.
- Scattering: global evolution described as transition between past and future WDW states.
- For $\langle \Delta \hat{p}_\phi \rangle < \infty$ the values $\langle \hat{X}_\phi \rangle$, $\langle \Delta \hat{X}_\phi \rangle$ of $X_\phi := \ln |\hat{v}|_\phi$ agree in $\phi \to \pm\infty$ limit.
- Numerical analysis of the scattering matrix: global triangle inequality

$$\lim_{\phi\to\infty} \langle \Delta \hat{X}_\phi \rangle < \lim_{\phi\to\infty} \langle \Delta \hat{X}_\phi \rangle + \langle \Delta \ln |\hat{p}_\phi| \rangle$$

- Results of other treatments similar. Analytical: weaker bounds at final $\phi$.

# deSitter model

$\Lambda > 0$ and massless scalar field as sole matter content. Ashtekar TP 2012

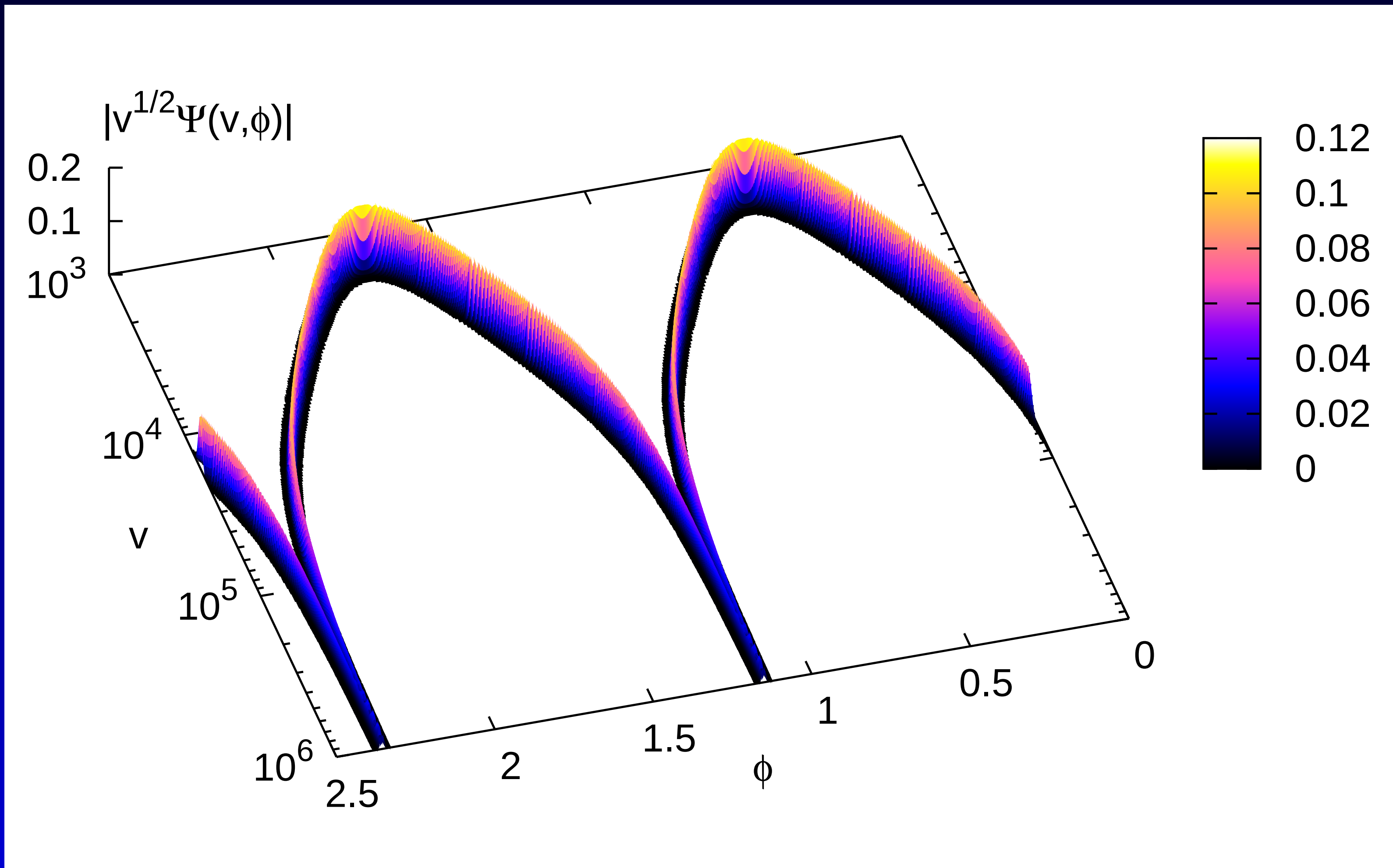
- $U(1)$ family of inequivalent unitary evolutions (extensions) (infinite size reached at final value of the clock field)
- Each extension generated by a discrete Hamiltonian $\omega(k) = C(\Lambda)k$

$$\Psi(v,\phi) = \sum_{n=0}^{\infty} \tilde{\Psi}_n e_n^{\beta}(v) e^{i\omega(k_{\beta,n})\phi},$$

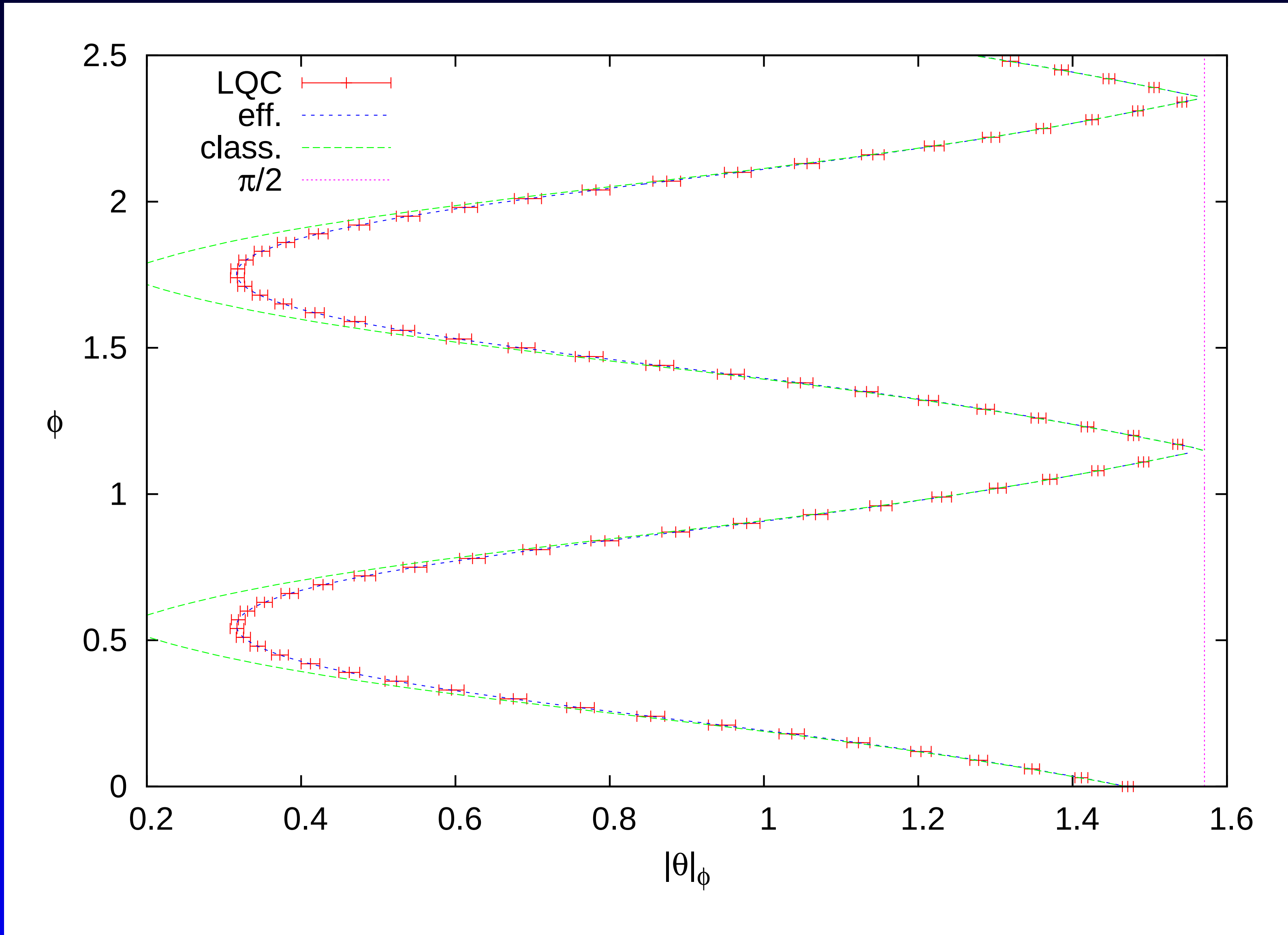
- Wave numbers: $\tan(k_{\beta,n} y_o) + \tanh[k_{\beta,n}(\pi - y_o)] \tan(\beta) = 0$, $[0,\pi) \ni y_o = y_o(\Lambda)$, $[0,\pi) \ni y = y(b,\Lambda)$ - through elliptic integrals.
- The spectrum asymptotics $k_n = (n\pi - \beta)/y_o + O(e^{-2n\pi(\pi - y_o)/y_o})$
- The dynamics: Infinite chain of bounces and reflections from SCRI.
  - Interesting expansion of Penrose "rings on the sky" proposal.
- Compactified size observable ($K$ - arbitrary constant)

$$\hat{\theta}_K \Psi(v,\phi) := \arctan(|v|/K).$$

# The wave packets



# The trajectory



# The decoherence

Have the spectrum of $\sqrt{\hat{\Theta}_\Lambda}$ been uniform the evolution would be cyclic.

This is not the case!

- We expect a nontrivial decoherence over time. How fast?
- Within obe cycle expected siilar behavior to $\Lambda = 0$. Between cycles?
- We compare the spreads near SCRI reflection points.
- Asymptotics of the energy eigenstates:

$$e^\beta_n(v) = N_n[e^{i\alpha(\beta,n)}e^\beta_{+n} + e^{-i\alpha(\beta,n)}e^\beta_{-n} + O(v^{-3})$$

$$e^\beta_{\pm n} = |v|^{-1}e^{\pm i\Omega(\Lambda)|v|}e^{\pm i[A(\Lambda)\omega_n^2 + B(\Lambda)]|v|^{-1}}$$

- Cast from discrete to continuous system: “instantiations”: interpolation of amplitudes and phases.
  - Reproduce to leading order $\langle \hat{p}_\phi \rangle$, $\langle \hat{\theta}_K(\phi) \rangle$ and dispersions.
- Estimate spread-out over one cycle from asymptotics of $\omega_n$.

$$\Delta\langle\Delta\hat{\theta}_K(\phi)\rangle \approx C_o \exp[-2(1 - \langle\Delta\hat{p}_\phi\rangle/\langle\hat{p}_\phi\rangle)\pi\langle n\rangle(\pi - y_o)/y_o], \quad C_o = O(1)$$

# Spontaneous coherence

After many sycles the coherent state will eventually spread-out. But under what conditions a generic state can cohere?

- $\hat{p}_\phi$ – constant of motion, thus necessarily $\langle\Delta\hat{p}_\phi\rangle/\langle\hat{p}_\phi\rangle \ll 1$.
- Sharply peaked state – can cut off tails.
  - Problem of linear flow on $N$-torus.
- Trajectory dense on $N$-torus if $\omega_n$ not-rationally related, however:
  - some parameters determined numerically (lack of data)
- Luckily $\omega_n$ uniform up to exponentially decaying correction!
  - Via number theory possible to determine upper bound on distances between sections of trajectory.
- Result: for $\langle\Delta\hat{p}_\phi\rangle/\langle\hat{p}_\phi\rangle \ll 1$ there exists an epoch where $\Delta\hat{p}_\phi\Delta\theta_K$ is of the order of mimimal value allowed by Heisenberg uncerainty.
- Conclusion: Provided a deSitter universe is semiclassical in constant of motion it will admit a semiclassical epoch.

# Thank you for your attention!

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