Observational consequences of an interacting multiverse

Salvador Robles-Pérez et al.

IFF - CSIC / EEBM

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Varcosmofun'16 - Szczecin

Outline

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The quantum multiverse

- Multiverse: classically disconnected regions of the space-time (singularities, multiply connected topology,...)
- The wave function of each single universe is given by the Wheeler-deWitt equation, which in the case of a homogeneous and isotropic space-time endorsed with a scalar field, φ, can be written as:

$$\ddot{\phi} + \frac{1}{a}\dot{\phi} - \frac{1}{a^2}\phi'' + \omega^2(a,\varphi)\phi = 0$$

  - $\phi \equiv \phi(a,\varphi)$ is the wave function of the universe,
  - $\dot{\phi} \equiv \frac{\partial \phi}{\partial a}$, $\phi' \equiv \frac{\partial \phi}{\partial \varphi}$,
  - $\omega^2(a,\varphi) = \sigma^2 \left(H^2 a^4 - a^2\right)$, with $\sigma^2 \equiv \frac{3\pi M_P^2}{2}$, and, $H^2 = \frac{8\pi}{3M_P^2}V(\varphi)$.

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Third quantization

- QFT of the wave function $\phi(a, \varphi)$, which can be seen as a *field* that propagates in the mini-superspace spanned by the scale factor, a, and the scalar(s) field(s), φ $(\vec{\varphi})$, with **metric element** given by

$$d\mathfrak{s}^2 = G_{MN} dq^N dq^M = -a da^2 + a^3 d\varphi^2,$$

$$G_{MN} = \begin{pmatrix} -a & 0 \\ 0 & a^3 \end{pmatrix}, \; q^N = \{a, \varphi\}$$

- We can use the customary machinery of a quantum field theory: **action, Lagrangian density, Hamiltonian density, ...**

$$S = \int da d\varphi \, \mathcal{L}(\phi, \dot{\phi}, \phi'; a), \text{ where}$$

$$\mathcal{L}(\phi, \dot{\phi}, \phi'; a) = \frac{1}{2}\left(-a\dot{\phi}^2 + \frac{1}{a}\phi'^2\right) + \frac{a\,\omega^2}{2}\phi^2, \text{ and}$$

$$\mathcal{H} = -\frac{1}{2}\left(\frac{1}{a}P_\phi^2 + \frac{1}{a}\phi'^2 + a\omega^2\phi^2\right), \text{ with } P_\phi \equiv \frac{\delta\mathcal{L}}{\delta\dot{\phi}} = -a\dot{\phi}.$$

- **h.o. with mass,** $M(a) = a$**, and frequency,** $\omega(a, \varphi) = \sigma\sqrt{H^2 a^4 - a^2}$

Interaction scheme in the multiverse

- We can now pose an interaction scheme between N universes with a total Hamiltonian given by:

$$\boxed{\mathcal{H} = \sum_{n=1}^{N} \mathcal{H}_n^0 + \mathcal{H}_n^I}, \text{ where}$$

$$\mathcal{H}^0 = -\frac{1}{2}\left(\frac{1}{a}P_\phi^2 + \frac{1}{a}\phi'^2 + a\omega^2\phi^2\right), \text{ and}$$

$$\mathcal{H}_n^I = \frac{a\lambda^2(a)}{8}\left(\phi_{n+1} - \phi_n\right)^2, \text{ with } \phi_{N+1} \equiv \phi_1$$

- With the following Fourier transformation

$$\tilde{\phi}_k = \frac{1}{\sqrt{N}}\sum_n e^{-\frac{2\pi i k n}{N}}\phi_n,\ \tilde{P}_k = \frac{1}{\sqrt{N}}\sum_n e^{\frac{2\pi i k n}{N}}P_n,$$

The interaction depends on the representation

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The effective value of the Wheeler-deWitt equation

- ϕ representation (interacting universes → *external* observer):

$$\ddot{\phi} + \frac{1}{a}\dot{\phi} - \frac{1}{a^2}\phi'' + \sigma^2 \left(H^2 a^4 - a^2\right)\phi = 0, \text{ with}$$

$$H^2 = \frac{8\pi}{3M_P^2} V(\varphi).$$

- $\tilde{\phi}$ representation (non-interacting universes → p.o.v. real observer),

$$\ddot{\tilde{\phi}}_k + \frac{1}{a}\dot{\tilde{\phi}}_k - \frac{1}{a^2}\tilde{\phi}_k'' + \omega_k^2(a,\varphi)\tilde{\phi}_k = 0, \text{ with}$$

$$\omega_k^2(a,\varphi) = \sigma^2(\tilde{H}_k^2 a^4 - a^2),$$

where,

$$\tilde{H}_k^2 = \frac{8\pi}{3M_P^2}\tilde{V}_k(a,\varphi), \text{ with } \boxed{\tilde{V}_k(a,\varphi) = V(\varphi) + \frac{\lambda^2(a)}{4\pi^2 a^4}\sin^2\frac{\pi k}{N}}$$

Semiclassical regime

Modified value of the effective potential

The potential of the scalar field is effectively modified by the interaction with other universes. However, the classical field equations are not modified.

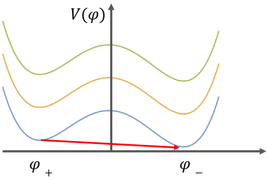
$$\boxed{\tilde{V}_k(a,\varphi) = V(\varphi) + \frac{\lambda^2(a)}{4\pi^2 a^4}\sin^2\frac{\pi k}{N}}$$

- The classical field equations are not modified (here, $\dot{\varphi} \equiv \frac{\partial\varphi}{\partial t}$):

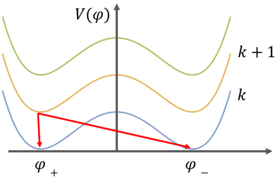
$$\ddot{\varphi} + \frac{3\dot{a}}{a}\dot{\varphi} + \frac{d\tilde{V}_k}{d\varphi} = \ddot{\varphi} + \frac{3\dot{a}}{a}\dot{\varphi} + \frac{dV_k}{d\varphi} = 0,$$

Vacuum decay

- The interactions induce a landscape structure of different false vacua and two true vacua



Coleman-DeLucia potential



Quartic potential .

- Decaying rate per unit volume: $\frac{\Gamma}{V} = A e^{-\frac{B}{\hbar}}$, with (SRP et *al.*, PLB 759 (2016) 328)

$$B = \frac{10\pi^2 m^{12}}{3\lambda_\varphi^8} \frac{1}{\left[\sin^2 \frac{\pi k}{N} - \sin^2 \frac{\pi(k-1)}{N}\right]^3} \left(\frac{4\pi^2 a^4}{\lambda^2}\right)^3$$

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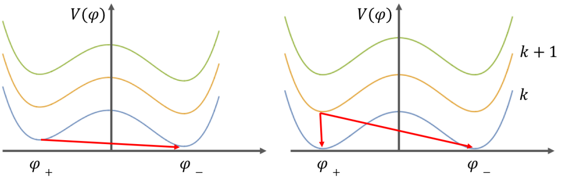

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Case $\lambda(a) \propto a^2$

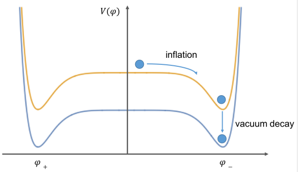
- The potential term:

$$\tilde{V}_k(a,\varphi) = V(\varphi) + \frac{\lambda^2(a)}{4\pi^2 a^4} \sin^2 \frac{\pi k}{N}$$

with the value, $\lambda^2(a) = \frac{9\pi M_P^2}{2} \Lambda a^4$. It turns out to modify the effective value of the cosmological constant, $\Lambda^{\rm eff}$, as

$$\boxed{\Lambda_k^{\rm eff} = \Lambda_0 + \Lambda \sin^2 \frac{\pi k}{N}}$$

- Discretized value of the c.c., with $\Lambda_k^{\rm eff} \in [\Lambda_0, \Lambda_0 + \Lambda]$, where $\Lambda_0 \ll M_P^4$ and $\Lambda \sim M_P^4$.



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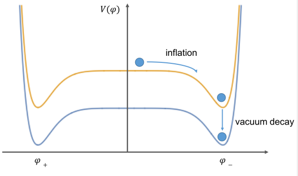

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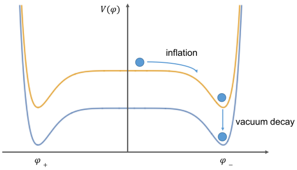

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Case λ = constant.

- The Wheeler-deWitt equation:

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where,

$$E_k = E_0 \sin^2 \frac{\pi k}{N}$$

- The radiation like terms induces a pre-inflationary stage in the evolution of the universe. For the flat branch

$$\frac{da}{dt} = \frac{\omega}{a} = \frac{\sigma}{a}\sqrt{H^2 a^4 + E_k}$$

$$a(t) = a_0 \sinh^{\frac{1}{2}}(2Ht + \theta_0) \sim e^{Ht}$$

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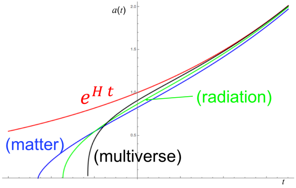

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Case $\lambda \propto a^{-1}$.

- The frequency is now given by

$$\omega^2(a) = \sigma^2 \left(H^2 a^4 - a^2 + \frac{c_0^2}{a^2} \sin^2 \frac{\pi k}{N} \right).$$

- On the other hand, during the earliest stage of the evolution, $\dot{\varphi} \approx 0$, and we can decompose the wave function of the universe in partial waves:

$$\phi(a,\varphi) = \int \frac{dk}{\sqrt{2\pi}} e^{ik\varphi} \phi_k(a) + e^{-ik\varphi} \phi_k^*(a) \ .$$

which satisfy a Wheeler-deWitt equation with the frequency:

$$\omega^2(a) = \sigma^2 \left(H^2 a^4 - a^2 + \frac{k^2}{a^2} \right).$$

- The partial wave decomposition is equivalent to an interaction scheme with $\lambda \propto a^{-1}$. They both can be seen as a quantum effect having no classical analogue (classically $k = 0$, quantum mechanically $k \gtrsim \hbar \neq 0$)

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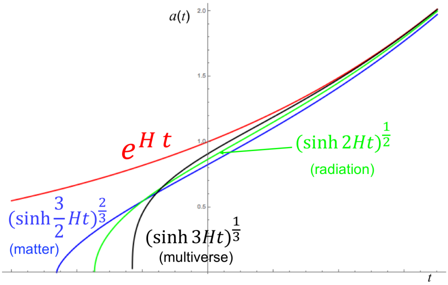
Case $\lambda \propto a^{-1}$.

- The term $\lambda \propto a^{-1}$ induces a pre-inflationary stage of the universe that is more abrupt (a^{-6}) than those induced by a matter (a^{-3}) or a radiation (a^{-4}) content in the early universe. E.g., for the flat branch

$$\frac{1}{a}\frac{da}{dt} = \sigma\sqrt{\Lambda + \frac{c_k^2}{a^6}} \rightarrow \boxed{a(t) = a_0 \sinh^{\frac{1}{3}}(3Ht+\theta_0)}.$$



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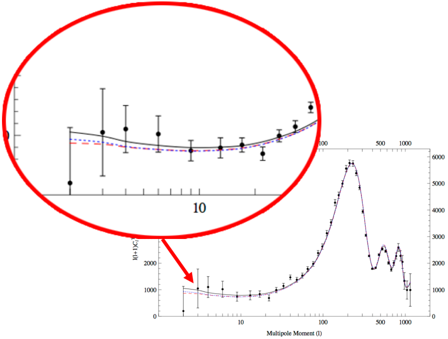
Observational imprints in the power spectrum of the CMB

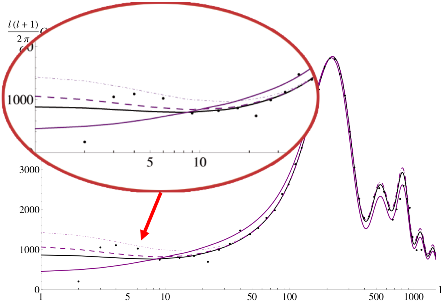
- A pre-inflationary stage in the evolution of the early universe would produce a suppression of the lowest modes of the power spectrum of the CMB (Bouhmadi-López et *al.*, PRD87:103513,2013)



(dotted : NFDW ($\rho \propto a^{-1}$); dashed: NFCS ($\rho \propto a^{-2}$))

Observational imprints in the power spectrum of the CMB

- A matter or radiation dominated pre-inflationary stage of the universe would also produce a suppression of the lowest modes of the power spectrum of the CMB (Scardigli et *al.*, Phys.Rev.D83:063507,2011)



(full : matter ($\rho \propto a^{-3}$); dashed-dotted: radiation ($\rho \propto a^{-4}$))

Observational imprints in the power spectrum of the CMB

- The computations show a suppression of the lowest modes del CMB ($l \sim 2$) that is compatible with the astronomical data (they are not conclusive, though).
- It would seem that the observational fit is better for a radiation like term (a^{-4}) than for a matter like term (a^{-3}), and a stronger effect might be needed to produce the best fit with observations.
- The term induced by the interacting multiverse (a^{-6}) is expected to produce a greater suppression of the lowest modes and a better fit therefore (SRP et *al*., In preparation).
- We need to analyze other terms that might come up from the underlying theories: the string theories and the quantum theory of gravity.
- More observable effects: initial conditions for inflation to occur (e.g. plateau models), inter-universal entanglement,....

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2. They not only leave observable and distinguishable imprints in the properties of the CMB but they might even produce a better fit.

3. It brings the interacting multiverse to **the same footing of testability as any other scientific theory** (at least as any other cosmological one).

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