Varying Constants and Fundamental Cosmology
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Effective gravitational “constant” in scalar-(curvature)tensor and scalar-torsion gravities

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cosmology: 1003.1686, 1006.1246, 1112.5308, 1411.1947, 1511.03933;
parametrized post-Newtonian (Solar system): 1309.0031, 1607.02356

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Dark Universe

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Outline

1.1 Scalar-tensor gravity (STG)

STG action in the most general form for one scalar field Φ and no derivative couplings

$$S = \frac{1}{2\kappa^2}\int d^4x\sqrt{-g}\left\{\mathcal{A}(\Phi)R-\mathcal{B}(\Phi)g^{\mu\nu}\nabla_\mu\Phi\nabla_\nu\Phi-2\ell^{-2}\mathcal{V}(\Phi)\right\} + S_{matter}\left[e^{2\alpha(\Phi)}g_{\mu\nu},\chi\right]. \quad (1)$$

- Nonminimal coupling between a scalar field and curvature
  - higher dimensions, braneworlds, quantum corrections
  - inflation (Higgs, α-attractors), dark energy, dark matter
  - mathematically equivalent to $f(R)$, $f(R,\Phi)$, ...
- Two dimensionful constants κ^2, ℓ to make Φ dimensionless.
- Gravitational "constant" $G = \frac{\kappa^2}{8\pi\mathcal{A}(\Phi)}$

1.1 Transformations of STG

$$S = \frac{1}{2\kappa^2}\int d^4x\sqrt{-g}\left\{\mathcal{A}(\Phi)R - \mathcal{B}(\Phi)g^{\mu\nu}\nabla_\mu\Phi\nabla_\nu\Phi - 2\ell^{-2}\mathcal{V}(\Phi)\right\} + S_m\left[e^{2\alpha(\Phi)}g_{\mu\nu}, \chi\right]. \quad (1)$$

- The action is invariant (up to total divergence term) under
  - conformal rescaling of the metric (change of frame)

$$g_{\mu\nu} = e^{2\bar{\gamma}(\bar{\Phi})}\bar{g}_{\mu\nu}, \quad (2)$$

  - and reparametrization of the scalar field

$$\Phi = \bar{f}(\bar{\Phi}). \quad (3)$$

- In general four arbitrary functions $\mathcal{A}(\Phi)$, $\mathcal{B}(\Phi)$, $\mathcal{V}(\Phi)$, $e^{2\alpha(\Phi)}$.

1.1 Parametrizations of STG

By conformal rescaling and scalar field redefinition can fix two functions to get different parametrizations, e.g.

- Jordan frame Brans-Dicke-Bergmann-Wagoner (JF BDBW)

$$S = \frac{1}{2\kappa^2} \int d^4x \sqrt{-g} \left\{ \Psi R - \frac{\omega(\Psi)}{\Psi} g^{\mu\nu} \nabla_\mu \Psi \nabla_\nu \Psi - 2\ell^{-2} U(\Psi) \right\} + S_m \left[g_{\mu\nu}, \chi \right] . \tag{4}$$

- Jordan frame another parametrization

$$S = \frac{1}{2\kappa^2} \int d^4x \sqrt{-g} \left\{ \mathcal{F}(\phi) R - g^{\mu\nu} \nabla_\mu \phi \nabla_\nu \phi - 2\ell^{-2} \mathcal{U}(\phi) \right\} + S_m \left[g_{\mu\nu}, \chi \right] . \tag{5}$$

- Einstein frame canonical parametrization

$$S = \frac{1}{2\kappa^2} \int d^4x \sqrt{-g} \left\{ R - 2 g^{\mu\nu} \nabla_\mu \varphi \nabla_\nu \varphi - 2\ell^{-2} \mathcal{V}(\varphi) \right\} + S_m \left[e^{2\alpha(\varphi)} g_{\mu\nu}, \chi \right] . \tag{6}$$

NB! Physical observables independent of parametrization!

LJ, Kuusk, Saal, Vilson 1411.1947

1.2 Multiscalar-tensor gravity (MSTG)

MSTG action in the most general form for n scalar fields $\mathbf{\Phi} = \{\Phi^A\}$, no derivative couplings LJ, Kuusk, Vilson 1509.02903

$$S = \frac{1}{2\kappa^2}\int d^4x\sqrt{-g}\left\{\mathcal{A}(\mathbf{\Phi})R - \mathcal{B}_{AB}(\mathbf{\Phi})g^{\mu\nu}\nabla_\mu\Phi^A\nabla_\nu\Phi^B - 2\ell^{-2}\mathcal{V}(\mathbf{\Phi})\right\} + S_{matter}\left[e^{2\alpha(\mathbf{\Phi})}g_{\mu\nu},\chi\right]. \qquad (7)$$

- Nonminimal coupling between the scalar fields and curvature
  - higher dimensions, quantum corrections
  - inflation, dark energy (Higgs $SU(2)$ doublet), dark matter
  - mathematically equivalent to $f(R, \Box^i R, \nabla_\mu R \nabla^\mu R, \mathcal{G})$, hybrid metric-Palatini $f(R, \mathcal{R})$, nonlocal $f(R, \Box^{-i} R)$, ...
- Gravitational "constant" $G = \frac{\kappa^2}{8\pi\mathcal{A}(\mathbf{\Phi})}$

1.2 Multiscalar-tensor gravity (MSTG)

$$S = \frac{1}{2\kappa^2}\int d^4x\sqrt{-g}\left\{\mathcal{A}(\mathbf{\Phi})R - \mathcal{B}_{AB}(\mathbf{\Phi})g^{\mu\nu}\nabla_\mu\Phi^A\nabla_\nu\Phi^B - 2\ell^{-2}\mathcal{V}(\mathbf{\Phi})\right\} + S_{matter}\left[e^{2\alpha(\mathbf{\Phi})}g_{\mu\nu},\chi\right]. \qquad (8)$$

- The action is invariant (up to total divergence term) under
  - conformal rescaling of the metric (change of frame) and
  - reparametrization of the scalar fields.
- Nonminimally coupled generalization of nonlinear σ-model with metric on the space of fields,

$$\mathcal{F}_{AB} \equiv \frac{2\mathcal{A}\mathcal{B}_{AB} + 3\mathcal{A}_{,A}\mathcal{A}_{,B}}{4\mathcal{A}^2} \qquad (9)$$

Negative eigenvalues of this metric indicate a ghost among the scalars. Zero eigenvalue points to a nondynamical scalar.

LJ, Kuusk, Vilson 1509.02903

1.2 Multiscalar-tensor gravity (MSTG)

Two fields ϕ, Ψ, Jordan frame, BDBW-like parametrization

$$S = \frac{1}{2\kappa^2} \int d^4x \sqrt{-g} \Big(\Psi R - Z(\phi,\Psi) \partial_\rho \phi \partial^\rho \phi - \frac{\omega(\phi,\Psi)}{\Psi} \partial_\rho \Psi \partial^\rho \Psi - 2\kappa^2 U(\phi,\Psi) \Big) + S_m[g_{\mu\nu}, \chi_m] \,, \qquad (10)$$

- The metric on the space of scalar fields is already diagonal,

$$\mathcal{F}_{\alpha\gamma} = \begin{pmatrix} \frac{Z(\phi,\Psi)}{2\Psi} & 0 \\ 0 & \frac{2\omega(\phi,\Psi)+3}{4\Psi^2} \end{pmatrix} . \qquad (11)$$

- Can define the vector of nonminimal coupling,

$$\mathcal{K}_\alpha = \begin{pmatrix} 0 & -\frac{\kappa^2}{4\Psi^2} \end{pmatrix} . \qquad (12)$$

1.3 Teleparallel equivalent of general relativity (TEGR)

Based on the ideas of Einstein (1928), Møller (1961)

$$S = \int d^4x \, e \left[\frac{T}{2\,\kappa^2} + \mathcal{L}_m \right] . \tag{13}$$

- Geometrical variables are tetrad fields, $g_{\mu\nu} = \eta_{ab} \, e^a{}_\mu \, e^b{}_\nu$.
- Instead of Levi-Civita connection use Weitzenböck connection $\overset{\mathbf{w}}{\Gamma}{}^\lambda{}_{\nu\mu} \equiv e_a^\lambda \, \partial_\mu e_\nu^a$, which yields zero curvature but nonzero torsion

$$T^\lambda{}_{\mu\nu} = \overset{\mathbf{w}}{\Gamma}{}^\lambda{}_{\nu\mu} - \overset{\mathbf{w}}{\Gamma}{}^\lambda{}_{\mu\nu} = e_a^\lambda \left(\partial_\mu e_\nu^a - \partial_\nu e_\mu^a \right) , \tag{14}$$

$$T = \frac{1}{4} T^{\rho\mu\nu} T_{\rho\mu\nu} + \frac{1}{2} T^{\rho\mu\nu} T_{\nu\mu\rho} - T_{\rho\mu}{}^{\rho} T^{\nu\mu}{}_{\nu} . \tag{15}$$

- Equation of motion equivalent to general relativity, $T = -R - 2\nabla^\mu T^\lambda{}_{\mu\lambda}$.
- What if we modify TEGR instead of GR?

1.3 Scalar field nonminimally coupled to torsion

Scalar-torsion gravity Geng, Lee, Saridakis, Wu 1109.1092

$$S = \int d^4x\, e \left[(1+\kappa^2 f(\phi)) \frac{T}{2\,\kappa^2} + \frac{1}{2}\,\partial_\mu\phi\,\partial^\mu\phi - V(\phi) + \mathcal{L}_m \right] . \qquad (16)$$

- Equations of motion different from scalar-(curvature) tensor gravity.
- Gravitational "constant" depends on the value of the scalar field, $G = \frac{\kappa^2}{8\pi(1+\kappa^2 f(\phi))}$
- Equations of motion must be invariant under local Lorentz transformations $e'^a{}_\mu = \Lambda^a{}_b(x) e^b{}_\mu$, otherwise acausality, etc.

  They will be if one also includes purely inertial spin connection $\omega'^a{}_{b\mu} = \Lambda^a{}_c \partial_\mu \Lambda_b{}^c$ as an independent variable (still leading to zero curvature), to delineate the effects of inertia and gravitation.

  Saridakis, Krššák 1510.08432, Krššák 1510.06676

2.1 STG cosmology

BDBW parametrization, flat ($k = 0$) FLRW, barotropic fluid

$$H^2 = +\frac{\kappa^2}{\Psi}\frac{\rho}{3} + \frac{\ell^{-2}}{\Psi}\frac{U(\Psi)}{3} - H\frac{\dot{\Psi}}{\Psi} + \frac{1}{6}\frac{\dot{\Psi}^2}{\Psi^2}\,\omega(\Psi)\,, \tag{17}$$

$$2\dot{H} + 3H^2 = -\frac{\kappa^2}{\Psi}\mathrm{w}\rho + \frac{\ell^{-2}}{\Psi}\,U(\Psi) - 2H\frac{\dot{\Psi}}{\Psi} - \frac{1}{2}\frac{\dot{\Psi}^2}{\Psi^2}\,\omega(\Psi) - \frac{\ddot{\Psi}}{\Psi}\,, \tag{18}$$

$$\ddot{\Psi} = -3H\dot{\Psi} - \frac{1}{2\omega(\Psi)+3}\,\frac{d\omega(\Psi)}{d\Psi}\,\dot{\Psi}^2 + \frac{\kappa^2}{2\omega(\Psi)+3}\,(1-3\mathrm{w})\,\rho + \frac{2\ell^{-2}}{2\omega(\Psi)+3}\left[2U(\Psi) - \Psi\,\frac{dU(\Psi)}{d\Psi}\right], \tag{19}$$

In the general relativity limit

$$\dot{\Psi}_\star = 0\,, \qquad \frac{1}{2\omega(\Psi_\star)+3} \equiv 0 \tag{20}$$

get ΛCDM with $G = \frac{\kappa^2}{8\pi\Psi_\star}$, $\Lambda = \ell^{-2}U(\Psi_\star)$.

2.1 Approximation near the GR limit

Consider small deviations $x \sim \dot{x} \sim h$

$$\begin{aligned} \Psi(t) &= \Psi_\star + x(t)\,, && (21) \\ \dot{\Psi}(t) &= \dot{x}(t)\,, && (22) \\ H(t) &= H_\star(t) + h(t)\,, && (23) \end{aligned}$$

Expand in series, keeping the leading terms, obtain approximate equations which depend on $\omega(\Psi_\star)$, $U(\Psi_\star)$, $\frac{d\omega}{d\Psi}|_{\Psi_\star}$, $\frac{dU}{d\Psi}|_{\Psi_\star}$, etc.

LJ, Kuusk, Saal 1003.1686

Background

$$\begin{aligned} H_\star(t) &= \frac{2}{3(t-t_s)} \quad , \quad \text{if} \quad \rho \gg U(\Psi)\,, \mathrm{w} = 0 \\ H_\star(t) &= \sqrt{\frac{3\ell^{-2}U(\Psi_\star)}{\Psi_\star}} \quad , \quad \text{if} \quad U(\Psi) \gg \rho\,, \end{aligned} \qquad (24)$$

Equations for $x(t)$ and $h(t)$ are nonlinear and depend implicitly on time, but can be solved analytically.

2.1 Solutions to the approximate equations

In the dust matter dominated regime LJ, Kuusk, Saal 1112.5308

$$\Psi(t) = \Psi_\star \pm \begin{cases} \frac{1}{t}\left[M_1 t^{\frac{\sqrt{D}}{2}} - M_2 t^{-\frac{\sqrt{D}}{2}}\right]^2 , & \text{if} \quad D > 0 \, , \\ \frac{1}{t}\left[M_1 \ln t - M_2\right]^2 , & \text{if} \quad D = 0 \, , \\ \frac{1}{t}\left[M_1 \sin\left(\frac{\sqrt{|D|}}{2} \ln t\right) - M_2 \cos\left(\frac{\sqrt{|D|}}{2} \ln t\right)\right]^2 , & \text{if} \quad D < 0 \, , \end{cases}$$

and in the potential dominated regime LJ, Kuusk, Saal 1006.1246

$$\Psi(t) = \Psi_\star \pm \begin{cases} e^{-C_1 t}\left[M_1 e^{\frac{1}{2} t \sqrt{C}} - M_2 e^{-\frac{1}{2} t \sqrt{C}}\right]^2 , & \text{if} \quad C > 0 \, , \\ e^{-C_1 t}\left[M_1 t - M_2\right]^2 , & \text{if} \quad C = 0 \, , \\ e^{-C_1 t}\left[M_1 \sin(\tfrac{1}{2} t \sqrt{|C|}) - M_2 \cos(\tfrac{1}{2} t \sqrt{|C|})\right]^2 , & \text{if} \quad C < 0 \, . \end{cases}$$

Here M_1, M_2 are constants of integration (determined by initial conditions) and

$$A_\star \equiv \frac{d}{d\Psi}\left(\frac{1}{2\omega(\Psi) + 3}\right)\Big|_{\Psi_\star} , \qquad C_1 \equiv \sqrt{\frac{3\ell^{-2} U(\Psi_\star)}{\Psi_\star}} , \qquad C_2 \equiv 2\ell^{-2} A_\star \left(2U(\Psi) - \frac{dU(\Psi)}{d\Psi}\Psi\right)\Big|_{\Psi_\star}$$

$$D \equiv 1 + \frac{8}{3}\Psi_\star A_\star \, , \qquad C = C_1^2 + 2C_2 \, .$$

2.1 GR as an attractor to STG and MSTG

- STG solutions dynamically converge towards the GR regime
  - GR limit exists, if $\exists \Psi_\star$, such that

$$\frac{1}{2\omega(\Psi_\star)+3} = 0\,.$$

  - Attractor in the dust matter dominated era

$$\frac{d}{d\Psi}\left(\frac{1}{2\omega(\Psi)+3}\right)\Bigg|_{\Psi_\star} \Psi_\star < 0\,.$$

  - Attractor in the potential dominated era

$$U(\Psi_\star) > 0\,, \qquad \left[\frac{\Psi}{2U(\Psi)}\frac{dU(\Psi)}{d\Psi}\right]_{\Psi_\star} < 1\,.$$

- Can get analytical solutions in cosmological time for H, $\mathrm{w}_{\mathrm{eff}}$, $\frac{\dot{G}}{G}$, etc.
- Analogously for multiscalar-tensor gravity LJ, Kuusk, Randla forthcoming

2.3 Scalar-torsion cosmology

Flat FLRW geometry, $e^a_\mu = \mathrm{diag}(1, a(t), a(t), a(t))$, perfect fluid

$$3H^2 = \frac{\kappa^2}{1+\kappa^2 f(\phi)} \left[\rho_m + V(\phi) + \frac{\dot{\phi}^2}{2} \right], \tag{25}$$

$$2\dot{H} = -\frac{\kappa^2}{1+\kappa^2 f(\phi)} \left[(1+\mathrm{w}_m)\rho_m + \dot{\phi}^2 + 2Hf'(\phi)\dot{\phi} \right], \tag{26}$$

$$\ddot{\phi} = -3H\dot{\phi} - 3H^2 f'(\phi) - V'(\phi). \tag{27}$$

- Equations different from scalar-(curvature)tensor cosmology! E.g. the scalar field is not sourced by matter energy-momentum.
- In the general relativity limit

$$\dot{\phi} = 0\,, \qquad f'(\phi_\star) = f'_\star = 0\,, \qquad V'(\phi_\star) = V'_\star = 0\,, \tag{28}$$

  get ΛCDM with $G = \frac{\kappa^2}{1+\kappa^2 f(\phi_\star)}$, $\Lambda = V(\phi_\star)$.

- Does scalar-torsion cosmology have GR attractor?

2.3. Approximate solutions in scalar-torsion gravity

Expand around the GR limit $\phi(t) = \phi_\star + x(t)$, $H(t) = H_\star(t) + h(t)$, can solve analytically LJ, Toporensky 1511.03933

If matter dominates $\rho_m \gg V$ and $V''_\star > 0$

$$x(t) = t^{-\frac{1}{2}\left(\frac{1-\mathrm{w}_m}{1+\mathrm{w}_m}\right)} \left(c_1 \, J_\nu(\sqrt{V''_\star} t) + c_2 \, Y_\nu(\sqrt{V''_\star} t) \right) \, , \tag{29}$$

If potential dominates $V \gg \rho_m$

$$x(t) = e^{-\frac{3H_0 t}{2}} \left(c_1 e^{\frac{3H_0 v_0 t}{2}} + c_2 e^{-\frac{3H_0 v_0 t}{2}} \right) \tag{30}$$

Here

$$\nu = \sqrt{\frac{1}{4}\left(\frac{1-\mathrm{w}_m}{1+\mathrm{w}_m}\right)^2 - \frac{4}{3}\frac{f''_\star}{(1-\mathrm{w}_m)^2}} \, , \quad v = \sqrt{1 - \frac{4f''_0}{3} - \frac{4}{3}\frac{(1+\kappa^2 f_0)V''_0}{\kappa^2 V_0} - \frac{8}{3}\frac{f'_0 V'_0}{V_0}} \, . \tag{31}$$

Can deduce when the GR limit is an attractor.

2. Recap

- Determined the conditions on a generic scalar-(curvature)tensor gravity for its solutions to dynamically converge to the general relativity limit (cf Damour, Nordtvedt 1994)
- Found the general analytic form of solutions near the GR limit, i.e. $\Psi(t)$, $H(t)$, Can also compute $\mathrm{w}_{\mathit{eff}}(t)$, $\frac{\dot{G}}{G}$, etc. Complete classification. LJ, Kuusk, Saal, 1006.1246, 1112.5308
- Physical observables (periods of oscillation, $\mathrm{w}_{\mathit{eff}}$, G) independent of frame and scalar field reparametrization LJ, Kuusk, Saal, Vilson 1411.1947, 1504.02686
- The same method works on multiscalar-(curvature)tensor gravity LJ, Kuusk, Randla forthcoming
- As well as for scalar-torsion gravity LJ, Toporensky 1511.03933.

3. Parametrized post-Newtonian formalism (PPN)

PPN: weak quasi-static field, sourced by a perfect fluid, expansion orders of magnitude relative to velocity $v^i = \frac{u^i}{u^0}$.

- Point source at the origin (no pressure or internal energy), Newtonian potential $\mathrm{U}_N = \frac{M_0}{r}$.

$$
\begin{aligned}
g_{00} &= -1 \quad +2G_{\mathrm{eff}}\mathrm{U_N} \quad -2G_{\mathrm{eff}}^2\beta\mathrm{U_N^2} + \dots \quad \mathcal{O}(6) \\
g_{0j} &= \quad \mathcal{O}(5) \\
g_{ij} &= \delta_{ij} \quad +2G_{\mathrm{eff}}\gamma\mathrm{U_N}\delta_{ij} \quad +\mathcal{O}(4) \\
\Phi &= \Phi_0 \quad +\overset{(2)}{\Phi} \quad +\overset{(4)}{\Phi} \quad \mathcal{O}(6)
\end{aligned}
\tag{32}
$$

Asymptotically Minkowski, $\mathcal{U}(\Phi_0) = 0$, $\frac{\partial\mathcal{U}}{\partial\Phi}|_{\Phi_0} = 0$, $\overset{(2)}{\Phi}|_{r\to\infty} = 0$, $\overset{(4)}{\Phi}|_{r\to\infty} = 0$.

Experimentally $\gamma - 1 = (2.1 \pm 2.3) \cdot 10^{-5}$ Cassini,
$4\beta - \gamma - 3 = (0.6 \pm 5.2) \cdot 10^{-4}$ lunar laser ranging.

3.1 PPN parameters in scalar-(curvature)tensor gravity

BDBW parametrization. In terms of the mass of the scalar

$$m_\Psi \equiv \frac{1}{\ell}\sqrt{\frac{2\Psi_0}{2\omega_0+3}\frac{d^2U}{d\Psi^2}|_0} . \qquad (33)$$

the parameters are given by Hohmann, LJ, Kuusk, Randla 1309.0031

$$G_{\text{eff}} = \frac{\kappa^2}{8\pi\Psi_0}\left(1+\frac{e^{-m_\Psi r}}{2\omega_0+3}\right) , \qquad (34)$$

$$\gamma - 1 = -\frac{\kappa^2}{8\pi\Psi_0 G_{\text{eff}}}\frac{2e^{-m_\Psi r}}{2\omega_0+3} , \qquad (35)$$

$$\beta - 1 = \frac{\kappa^4}{64\pi^2\Psi_0^2 G_{\text{eff}}^2}\left(\frac{\Psi_0 \frac{d\omega}{d\Psi}|_0 e^{-2m_\Psi r}}{(2\omega_0+3)^3} - \frac{m_\Psi r}{(2\omega_0+3)}\beta(r)\right) , \qquad (36)$$

where $\beta(r)$ involves exponential integrals $\mathrm{Ei}(-m_\psi r) = -\int_{m_\psi r}^{\infty} \frac{e^{-t}}{t} dt$.

- For massive scalar the results depend on the distance from the source r.

3.2 PPN parameters in multiscalar-(curvature)tensor grav.

For two fields, in BDBW parametrization Hohmann, LJ, Kuusk, Randla, Vilson 1607.02356

$$G_{\text{eff}} = \frac{\kappa^2}{8\pi\Psi_0}\left(1 + \frac{\cos^2\vartheta_+ \, e^{-m_+ r} + \cos^2\vartheta_- \, e^{-m_- r}}{2\omega_0 + 3}\right) \tag{37}$$

and

$$\gamma - 1 = -2\frac{\cos^2\vartheta_+ \, e^{-m_+ r} - \cos^2\vartheta_- \, e^{-m_- r}}{2\omega_0 + 3 + \cos^2\vartheta_+ \, e^{-m_+ r} + \cos^2\vartheta_- \, e^{-m_- r}} \, . \tag{38}$$

Here the mass eigenvalues are

$$m_\pm^2 = \frac{\kappa^2}{2Z_0(2\omega_0 + 3)}\left((2\omega_0 + 3)\left.\frac{\partial^2\mathcal{U}}{\partial\phi^2}\right|_0 + 2\Psi_0 Z_0 \left.\frac{\partial^2\mathcal{U}}{\partial\Psi^2}\right|_0 \pm B\right) , \tag{39}$$

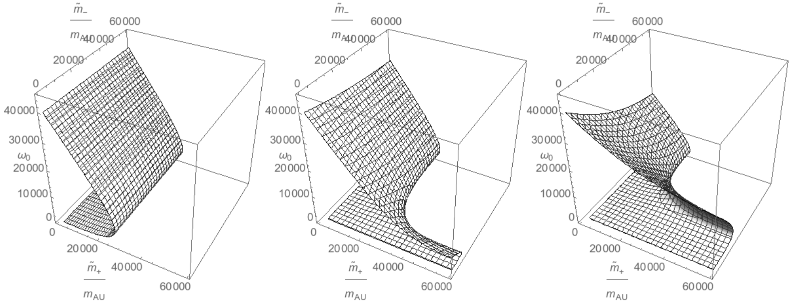
while the mass eigenvectors are orthogonal to each other with

$$\cos^2\vartheta_+ = \frac{1}{2}\left(1 + \frac{A}{B}\right) , \qquad \cos^2\vartheta_- = \frac{1}{2}\left(1 - \frac{A}{B}\right) , \qquad \cos^2\vartheta_+ + \cos^2\vartheta_- = 1 , \tag{40}$$

where

$$B = \sqrt{A^2 + 8(2\omega_0 + 3)Z_0\Psi_0\left(\left.\frac{\partial^2\mathcal{U}}{\partial\phi\,\partial\Psi}\right|_0\right)^2} , \qquad A = 2\Psi_0 Z_0 \left.\frac{\partial^2\mathcal{U}}{\partial\Psi^2}\right|_0 - (2\omega_0 + 3)\left.\frac{\partial^2\mathcal{U}}{\partial\phi^2}\right|_0 . \tag{41}$$

3.2 PPN parameters in multiscalar-(curvature)tensor grav.



Constraints at 2σ from the Cassini measurement of the PPN parameter γ on the rescaled masses of the two scalar fields and the parameter ω_0 for $\vartheta_+ = 0$ (left), $\vartheta_+ = \frac{\pi}{8}$ (middle), and $\vartheta_+ = \frac{\pi}{4}$ (right).
The allowed region of the parameter space is to the right of the plotted surface.

Here $\tilde{m}_\pm = \sqrt{2\omega_0 + 3}\, m_\pm$ normalized by the mass corresponding to the astronomical unit.

Radio signals passing by the Sun at a distance $r \approx 7.44 \cdot 10^{-3} AU$.

Hohmann, LJ, Kuusk, Randla, Vilson 1607.02356

3. Recap

- Massive nonminimal scalar modifies G_{eff} and PPN γ, β by a correction which falls off exponentially in distance.
- Coupling a massless scalar to a massive scalar can greatly reduce the bounds on Brans-Dicke ω.
- Have a general formula for G_{eff} and PPN γ for arbitrary number scalar fields nonminimally coupled to curvature. It depends on the masses as well as on the alignment of the fields in the space of fields.
  Hohmann, LJ, Kuusk, Randla, Vilson 1607.02356
- Again, the observables are independent of frame and scalar field reparametrization. LJ, Kuusk, Vilson 1509.02903, forthcoming
- The results apply to many models that reduce to multiscalar-tensor gravity.
- In scalar-torsion gravity the PPN parameters coincide with the values from general relativity Chen, Wu, Wei 1410.7715