Varying Constants and Fundamental Cosmology

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Varying constants quantum cosmology

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Introduction

- **Classical cosmology**- based on classical general relativity and particle physics

- **Confirmed by observations:** expansion (Hubble law, supernovae), CMB

- **Planck epoch:** $t_p \sim 10^{-44}$ s, $l_p \sim 10^{-35}$ m, $\rho_p \sim 10^{96}$ kg/m^3

- **Quantum gravity is required**

- Canonical quantum gravity applied to cosmology →

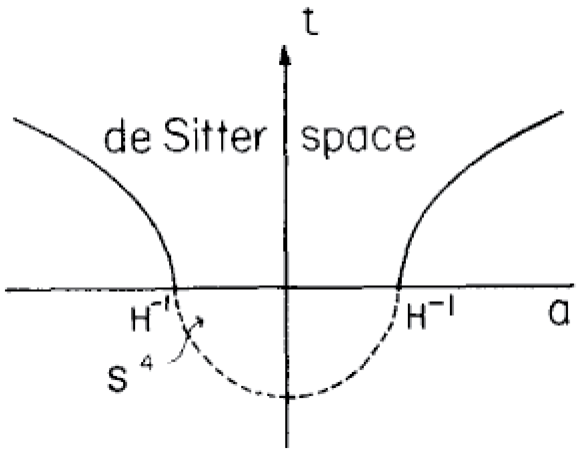
QUANTUM COSMOLOGY

Introduction

[1] A. Vilenkin, Phys. Lett. **117**, 25 (1982)
[2] A. Vilenkin, Phys. Rev. D **27**, 2848 (1983)

- The way how to avoid the initial singularity [1][2]:

The Universe spontaneously created from „nothing”- no space, no time.



- **No change** in any fundamental equations of physics required.
- **No singularity** in a quantum discreption of the beginning of the Universe problem

Closed universe (k=+1)

$$ds^2 = c^2 dt^2 - a^2(t)[dr^2(1-r^2) + r^2 d\,\Omega]$$

FLRW metric

$$\dot{a}^2 + 1 = \frac{8\pi G}{3} \varrho_v a^2$$

$$t = -i\tau$$

$$a(t) = H^{-1}\cosh(Ht)$$

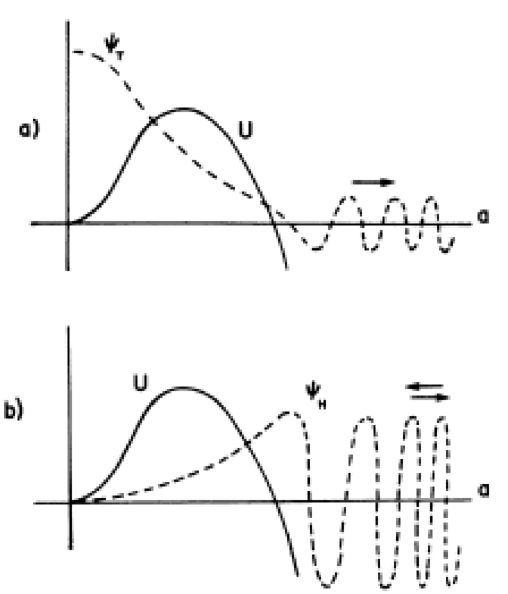
$$a(\tau) = H^{-1}\cos(H\tau)$$

The wave function of the Universe

- **The Wheeler- DeWitt equation**- the fundamental equation of canonical quantum gravity; time plays no role in here [3]

$$\hat{H}\,\psi\left(h_{\mu\nu},\varphi\right)=o$$

- $|\psi|^2$ - gives **probability of creation** of a universe with a specific geometry and matter fields



Tunneling wave function

„At the boundaries of superspace, the wave function of the Universe includes only outgoing modes (carrying flux out of superspace) [4]"

$$\psi_T(h,\varphi)=\int_{\emptyset}^{(h,\varphi)} dg\, d\varphi\, e^{iS(g,\varphi)}$$

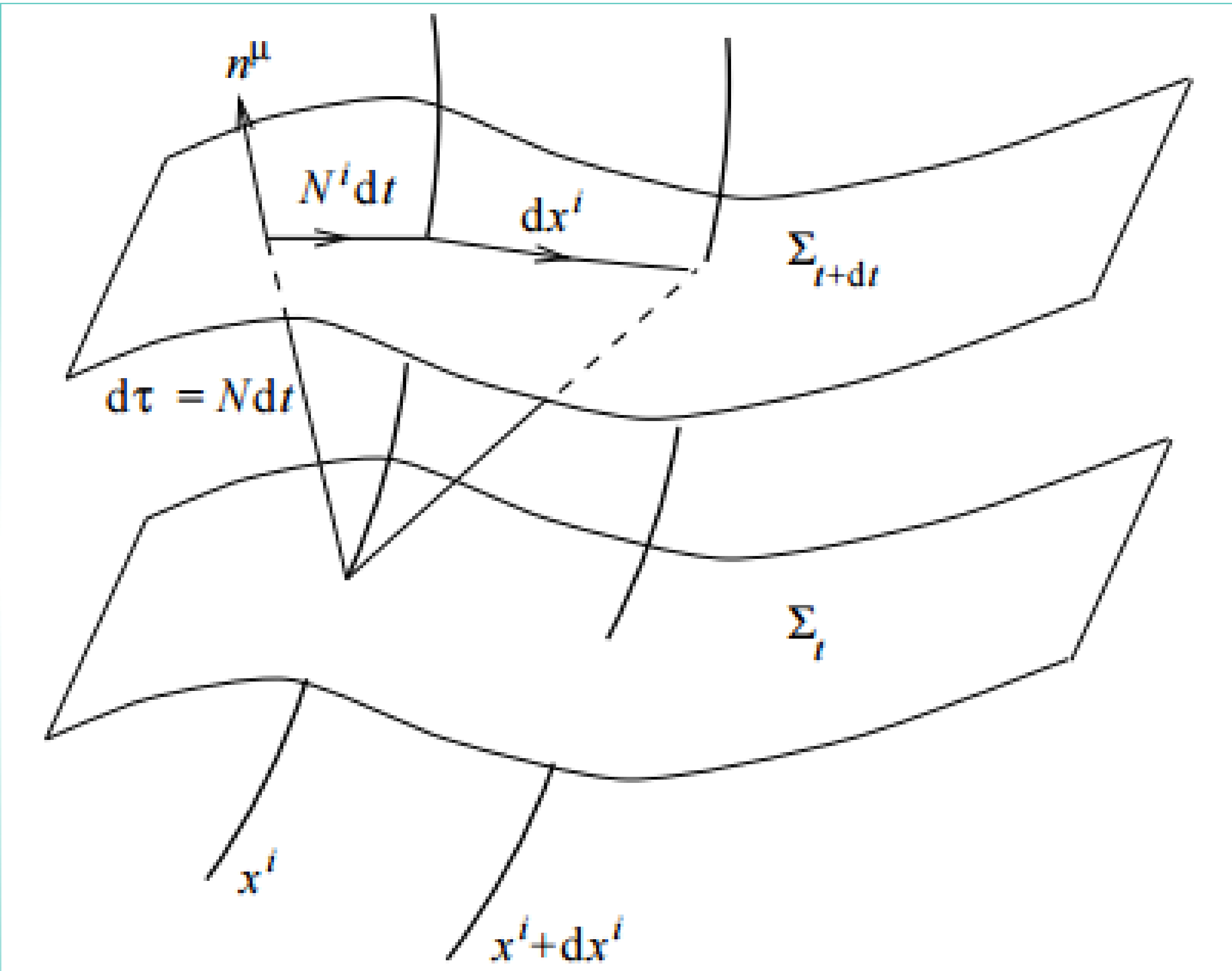
Hartle-Hawking wave function

„The boundary condition of the universe is that it has no boundary [5]."

$$\psi_{HH}(h,\varphi)=\int^{(h,\varphi)} dg\, d\varphi\, e^{-S_E(g,\varphi)}$$

[3] B. DeWitt, Phys. Rev. D **160**, 1113 (1967), [4] A. Vilenkin, Phys. Rev. D **33**, 3560 (1986)
[5] B. Hartle, S. Hawking, Phys. Rev. D **28**, 2960 (1983)

The 3+1 decomposition



In order to study **the hamiltonian formulation of General Relativity** one has to slice the four-dimensional spacetime by three-dimensional surfaces (hypersurfaces)[6].

- **N(t, x^k)- lapse function**
- **N^i(t, x^k)- shift vector**

Rys. The 3+1 decomposition of the manifold (D.L. Wiltshire, An introduction to quantum cosmology, arXiv:gr-qc/0101003)

[6] R. Arnowitt, S. Deser, C. Misner, Phys. Rev. **D 116**, 1322 (1959)

Wheeler-DeWitt minisuperspace equations

The Einstein-Hilbert action:

$$S_g = \int_M d^4x \sqrt{-g}\, {}^{(4)}R \frac{c^3}{16\pi G}$$

The Gibbons- Hawking boundary term:

$$S_{GH} = \int_{\partial M} \sqrt{h} K \frac{c^3}{8\pi G} d^3x$$

The matter action:

$$S_m = \int_M L_m \sqrt{-g}\, d^4x = -\int_M \rho c \sqrt{-g}\, d^4x$$

The Λ- term:

$$S_\Lambda = -\int_M \frac{2\Lambda c^3}{16\pi G} \sqrt{-g}\, d^4x$$

The total action: $S_{tot} = S_g + S_{GH} + S_m + S_\Lambda$

Wheeler-DeWitt minisuperspace equations

- In our approach we take the Friedman metric:

$$ds^2 = -\left(dx^0\right)^2 + a^2\left(x^0\right)\left[d\chi^2 + S^2(\chi)\, d\Omega^2\right]$$

where:

$$S(\chi) = \begin{cases} \sin\chi\,, & k=+1 \\ \chi\,, & k=0 \\ sh\,\chi\,, & k=-1 \end{cases}$$

$$x^\mu = \left(x^0, x^1, x^2, x^3\right)$$
$$x^0 = ct$$

- For the isotropic and homogeneous Friedman geometry we have:

$$\sqrt{-g} = a^3\left(x^0\right) S^2(\chi) \sin\theta$$

$$\sqrt{h} = a^3\left(x^0\right) S^2(\chi) \sin\theta$$

(4)

$$R = \frac{6}{a^2}\left(k + a^2{}_{,0} + aa_{,00}\right)$$

and so:

$$\int_M d^4x \sqrt{-g} = \int a^3\left(x^0\right) dx^0 \int S^2(\chi) \sin\theta \, d\chi \, d\varphi$$
$$= V_3 \int a^3\left(x^0\right) dx^0$$

Wheeler-DeWitt minisuperspace equations

The gravitational action

$$S_g = \frac{V_3}{16\pi}\int dx^0 a^3(x^0)\frac{c^3}{G}\left[\frac{6}{a^2(x^0)}\left(k + a^2{}_{,0} + aa_{,00}\right)\right]$$

The boundary term

$$S_{GH} = -\frac{3}{8\pi}\int_M \frac{\partial}{\partial x^0}\left(a_{,0}\, a^2\right)\frac{c^3}{G}S(\chi)\sin\theta\, d^4x$$

The matter action

$$S_m = -V_3\int \varrho(x^0)c\, a^3(x^0)dx$$

The Λ- term

$$S_\Lambda = -\frac{3V_3}{8\pi}\int \frac{\Lambda c^3}{3G}a^3(x^0)dx$$

TOTAL ACTION+ POSSIBLE VARIATION OF c and G

$$S = S_g + S_{GH} + S_\Lambda + S_m = \frac{3V_3}{8\pi}\int dx^0\frac{c^3(x^0)}{G(x^0)}\left[ka - a_{,0}^2\, a - \frac{\Lambda}{3}a^3 - \frac{8\pi G(x^0)}{3c^2}\varrho a^3\right]$$

Varying constants minisuperspace equations

According to $S = \int L(a, \dot{a}, t)\, dt$ the Lagrangian reads as:

$$L = \frac{3V_3 c^3(x^0)}{8\pi G(x^0)} \left(ka - a_{,0}^2\, a - \frac{\Lambda}{3} a^3 - \frac{8\pi G(x^0)}{3c^2} \varrho a^3 \right)$$

To **quantize the model** we need to find the canonical momenta [3]:

$$p_a = \frac{\partial L}{\partial a_{,0}} = -\frac{3V_3 c^3}{4\pi G} a a_{,0}$$

$$p_a \longrightarrow i\hbar \frac{\partial}{\partial a}$$

and the Hamiltonian:

$$H = p_a a_{,0} - L = -\frac{2\pi G(x^0)}{3V_3 c^3(x^0) a} p_a^2 - \frac{3V_3 c^3(x^0)}{8\pi G(x^0)} ka + \frac{V_3 c^3(x^0)}{8\pi G(x^0)} \Lambda a^3 + V_3 \varrho c(x^0) a^3$$

The Wheeler-DeWitt equation

$$\left[\hbar^2 \frac{\partial^2}{\partial a^2} - U(a) \right] \Psi(a) = 0$$

Varying constants minisuperspace equations

The Wheeler- DeWitt equation: $\left[\hbar^2 \frac{\partial^2}{\partial a^2} - U(a)\right]\Psi(a) = 0,$

NO CLASSICAL TIME!

where

$$U(a) = \left(\frac{3V_3 c^2(a) a}{4\pi G(a)}\right)^2 \left(kc^2(a) - \frac{\Lambda}{3} a^2 c^2(a) - \frac{8\pi G(a)}{3} \varrho(a) a^2\right).$$

$$c = c\left(x^0\right) \rightarrow c = c(a)$$
$$G = G\left(x^0\right) \rightarrow G = G(a)$$

After imposing the barotropic equation of state $p = w \varrho c^2$

and using the ansatze:

$$c(a) = c_0 a^n$$
$$G(a) = G_0 a^q,$$

$n, q = const.$

we have:

$$U(a) = \left(\frac{3V_3 c_0{}^2 a^{2n+1-q}}{4\pi G_0}\right)^2 \left(k c_0{}^2 a^{2n} - \frac{\Lambda}{3} c_0{}^2 a^{2n+2} - \frac{8\pi G_0}{3} \varrho(a) a^{2+q}\right).$$

The conservation equation: $\dot{\varrho} + 3\frac{\dot{a}}{a}\left(\varrho + \frac{p}{c^2}\right) = -\varrho \frac{\dot{G}}{G} + 3\frac{k\ c\dot{c}}{4\pi G a^2}$

can be solved by:

$$\varrho(a) = \frac{C}{a^{3(w+1)+q}} + \frac{3c_0{}^2 n}{4\pi G_0}\left(\frac{k}{2n+3w+1} - \frac{\Lambda a^2}{3(2n+3w+3)}\right) a^{2(n-1)-q}$$

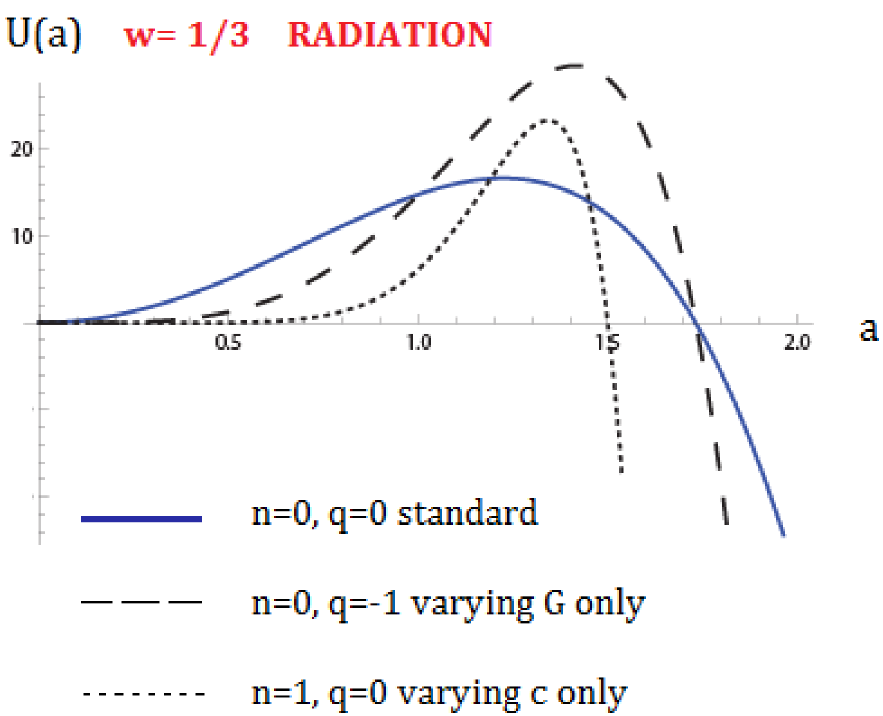
Varying constants minisuperspace equations

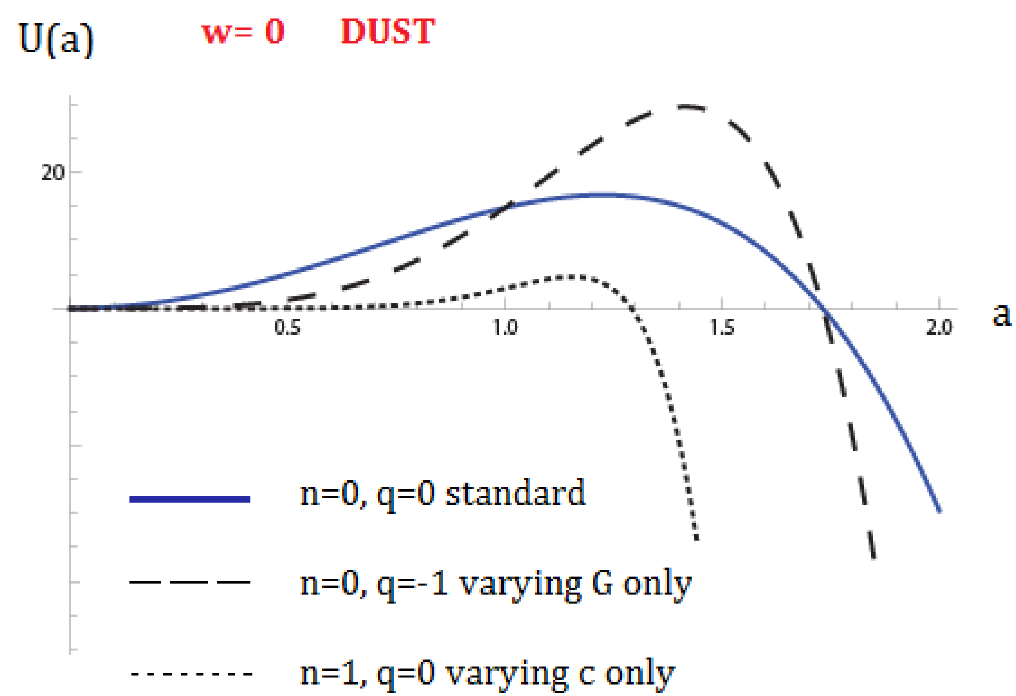
The minisuperspace potential:

$$U(a)=K_0^2 a^{2(3n+1-q)}\left(\frac{3w+1}{2n+3w+1}k-\frac{\Lambda(w+1)}{2n+3(w+1)}a^2-\frac{8\pi G_0 C}{3c_0^2 a^{3w+1+2n}}\right),\qquad K_0=\frac{3V_3 c_0^3}{4\pi G_0}$$

The simplest case C=0:

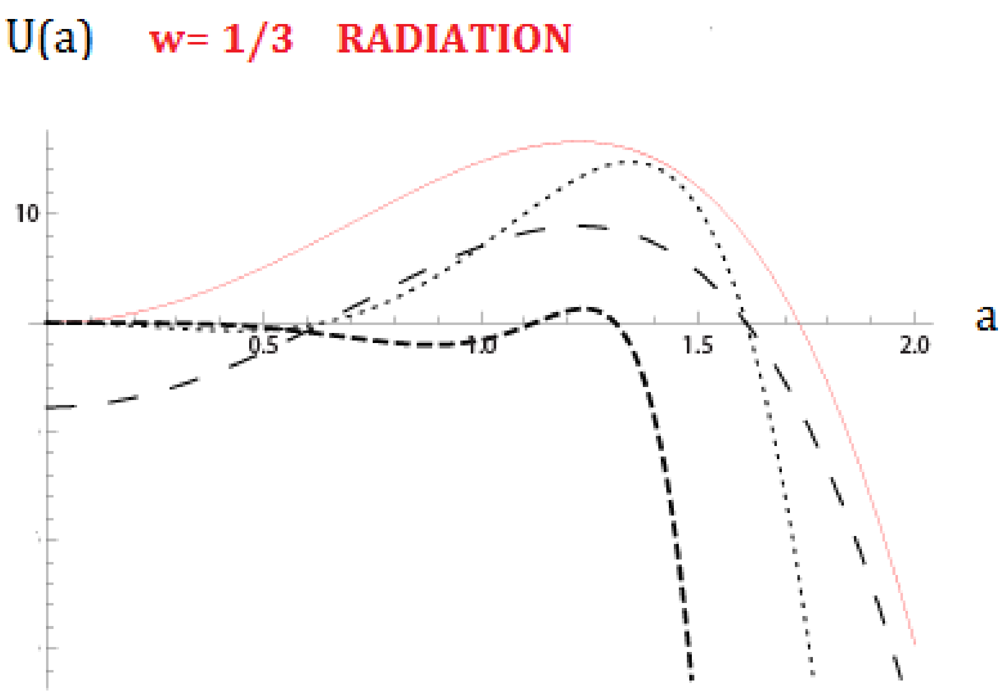
$$U(a)=K_0^2 a^{2(3n+1-q)}\left(\frac{3w+1}{2n+3w+1}k-\frac{\Lambda(w+1)}{2n+3(w+1)}a^2\right),\quad U(a_t)=0\ ,\quad a_t=\sqrt{\frac{k(3w+1)(2n+3(w+1))}{\Lambda(w+1)(2n+3w+1)}}$$





The potentials for Λ= 1. It allowes the tunneling of the Universe from a=0 to a= a_0

The minisuperspace potential C≠ 0

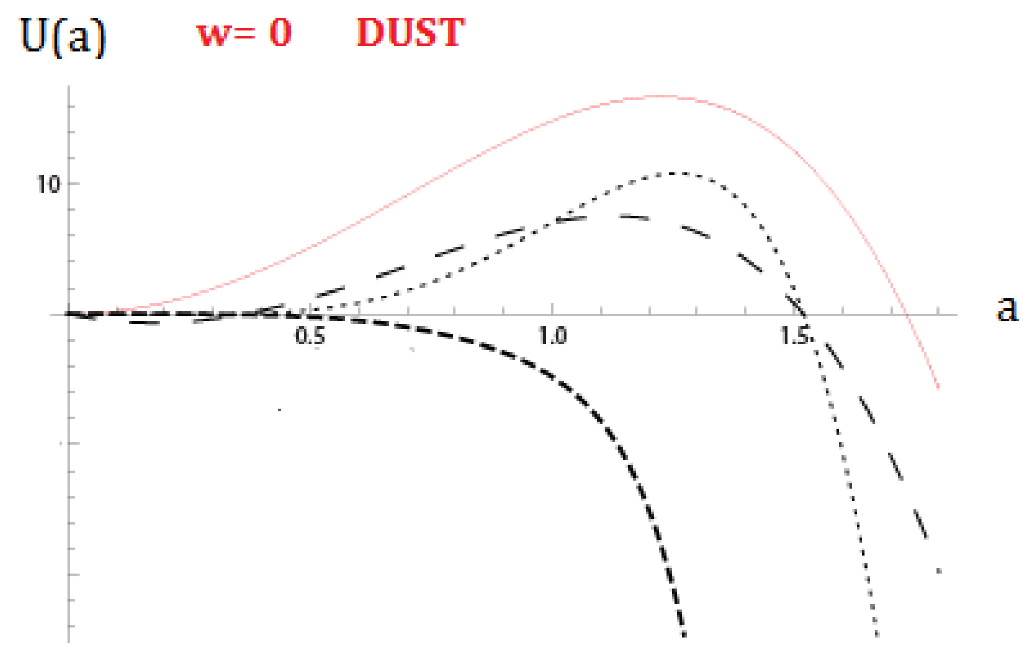


n=0, q=0 standard, C=0

n=0, q=0 standard, C= 0.35

n=1, q=0 varying c only, C= 0.35

n=0, q=-1 varying G only, C= 0.35



n=0, q=0 standard, C=0

n=0, q=0 standard, C= 0.35

n=1, q=0 varying c only, C= 0.35

n=0, q=-1 varying G only, C= 0.35

The potentials for $\Lambda = 1$.

Quantum tunneling in varying constants cosmology

- Now, we use the **WKB method [4][7]** to calculate **the probability** of tunneling of the Universe **„from nothing"** (a=0) to the Friedmann geometry with a= a_t which reads as:

$$P \simeq \exp\left[-\frac{2}{\hbar}\int_0^{a_t}\sqrt{2(E-U(a))}\,da\right]$$

$$= \exp\left[-\frac{2K_0}{\hbar}\int_0^{a_t} a^{3n+1-q}\left(\frac{\Lambda(w+1)}{2n+3(w+1)}a^2-\frac{3w+1}{2n+3w+1}k\right)^{1/2} da\right]$$

- Using the definition of Beta and Gamma finctions the integral can be solved under an additional condition (3n+2>q):

$$P \simeq \exp\left[-\frac{K_0}{\hbar}\sqrt{\left|\frac{3w+1}{2n+3w+1}k\right|}\,\tilde{x}_0^{\frac{1}{2}(3n-q+2)}\, B\left(\frac{1}{2}(3n+2-q),\frac{3}{2}\right)\right]$$

$$\simeq \exp\left[-\frac{K_0}{\hbar}\sqrt{\left|\frac{3w+1}{2n+3w+1}k\right|}\,\tilde{x}_0^{\frac{1}{2}(3n-q+2)}\,\frac{\sqrt{\pi}}{2}\frac{\Gamma\left(\frac{1}{2}(3n+2-q)\right)}{\Gamma\left(\frac{1}{2}(3n+5-q)\right)}\right]$$

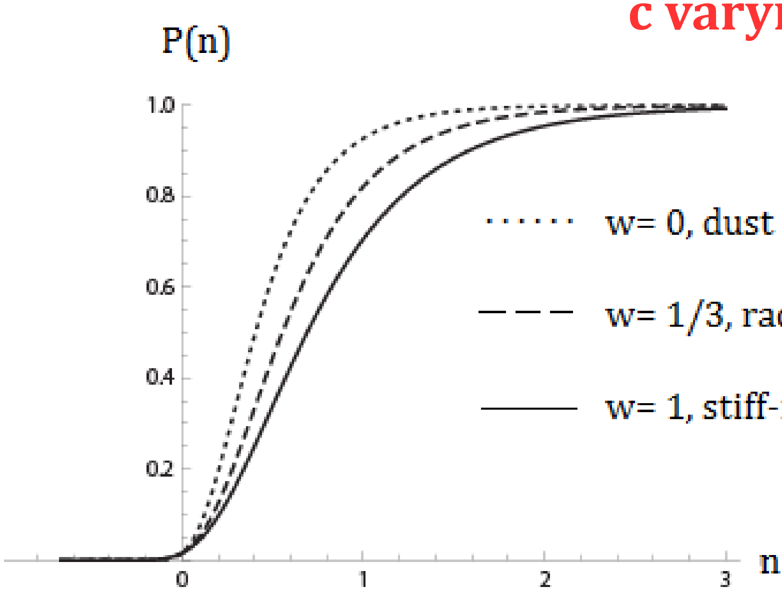
Tunneling probability

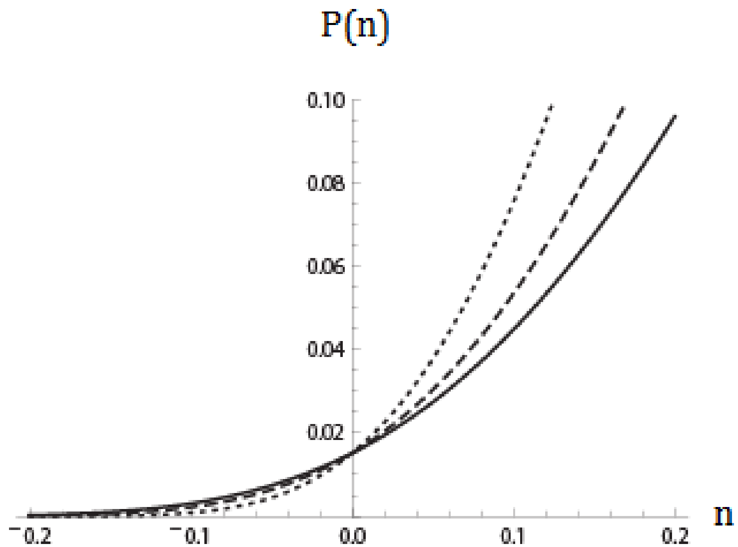
$$\tilde{x}_0 = a_t^2$$

[7] D. Atkatz, Am. J. Phys. **62**, 619 (1994)

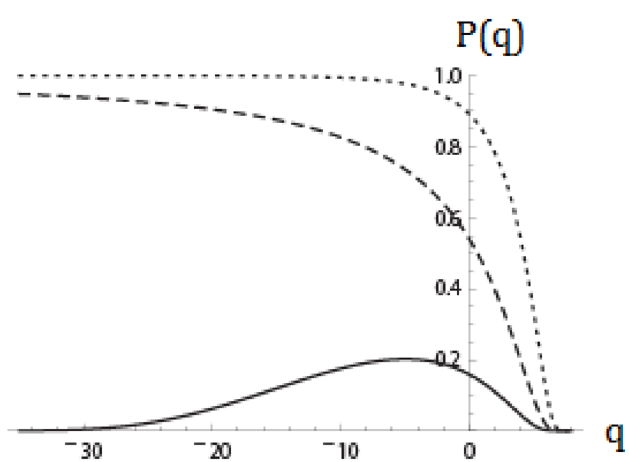
The probability of the tunneling

c varying only





The probability of tunneling for varying c minisuperspace models with k=+1, q=0, Λ=5



The probability of tunneling for varying G minisuperspace models with k=+1, n= 2, Λ= 5

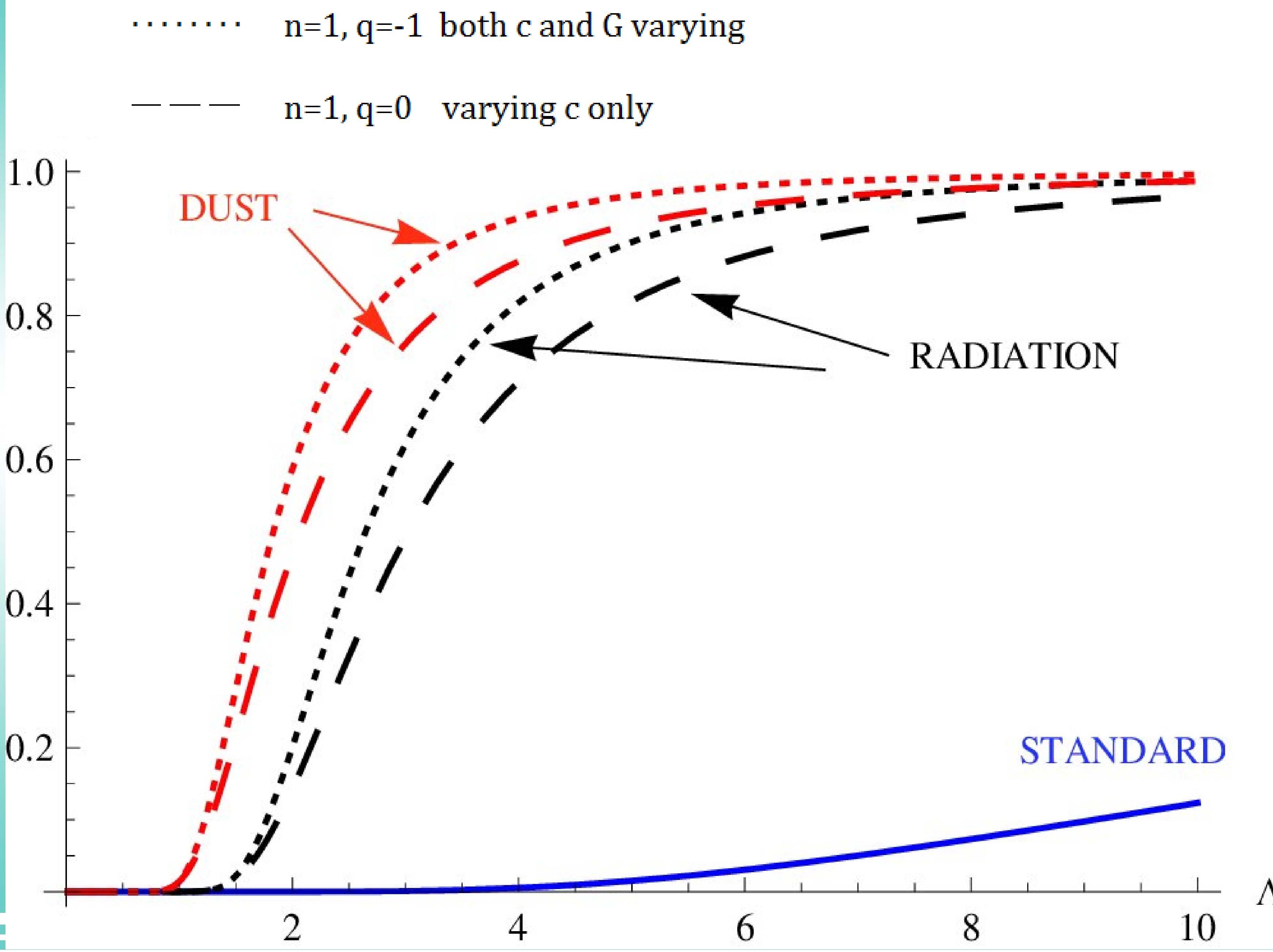
⋯⋯ w= 0, dust

— — — w= 1/3, radiation

——— w= 1, stiff-fluid

G varying,
c varying: n=2

The probability of the tunneling



Conclusions

- Most of the varying c and G minisuperspace potentials are of the tunneling type. This allowe us to use WKB approximation to calculate the probabilty of tunneling of the universe „from" nothing to the Friedman geometry with some fixed value of the scale factor $a_{t.}$
- The probability strongly depends on both c and G being not constant: it is large for growing c models and is strongly suppressed for diminishing c models;
  it is large for gravitational constant decreasing, while small for G increasing.
- The variability of c and G influences the probability of tunneling as compared with the standard matter content universe models.
- The probability is larger for larger value of the cosmological constant.

Thank you for your attention!