Constraining discrete flavour symmetries with neutrino oscillation experiments

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Outline of talk

1. What do we know about leptonic flavour?
2. Parameter correlations from discrete symmetries
   - Semi-direct A_5 with generalised CP
3. Conclusions

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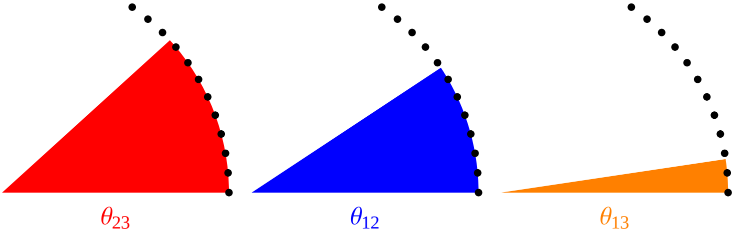
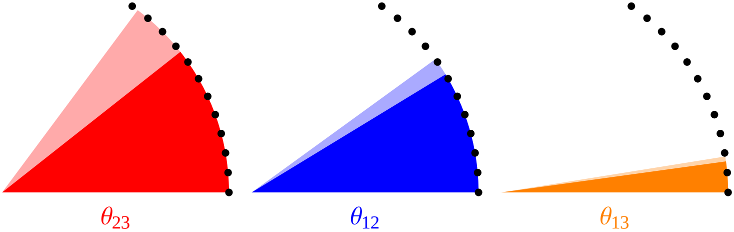
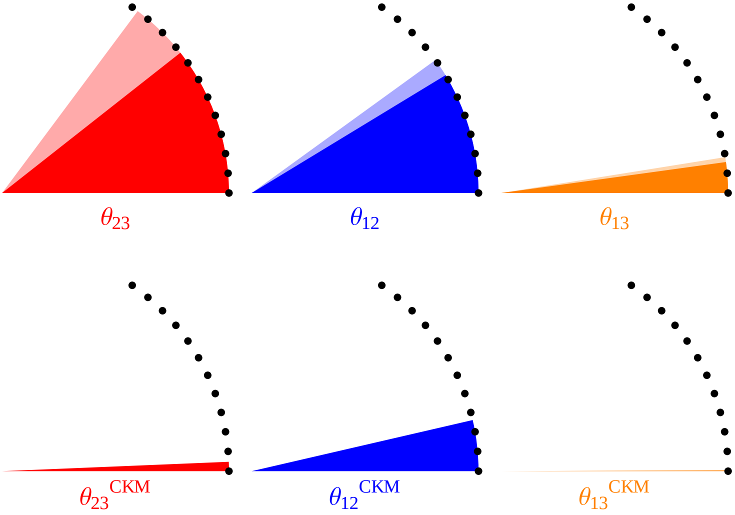
Leptonic flavour



Leptonic flavour: current knowledge



Leptonic flavour: some explanation?



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Leptonic flavour symmetries

- Extend the SM symmetry group by a discrete non-Abelian factor. Assign fields to representations of this group.

$$G_{\mathrm{SM}} \to G_{\mathrm{SM}} \times G_{\mathrm{F}} \qquad \text{and} \qquad \begin{pmatrix} L_e \\ L_\mu \\ L_\tau \end{pmatrix} \sim \underline{\mathbf{3}}$$

- The symmetry restricts the most general mass terms of the theory:

$$T^\dagger \left(m_e^\dagger m_e \right) T = m_e^\dagger m_e \qquad \text{and} \qquad S^\mathsf{T} m_\nu S = m_\nu .$$

- The full symmetry cannot be exact at low-energies whilst maintaining distinct neutrino masses. However, a set of residual symmetries may enforce a given mixing pattern or, most generally, specific correlations between observables.

Parameter correlations

In some cases, the symmetry alone does not predict the full PMNS matrix. Instead only predicting a constrained form.

For example...

If $G_\nu = \mathbb{Z}_2$ we see that only a single column of the PMNS matrix can be specified:

$$e.g. \qquad U_{\text{PMNS}} = \begin{pmatrix} \sqrt{\frac{2}{3}} & ? & ? \\ \frac{1}{\sqrt{6}} & ? & ? \\ \frac{1}{\sqrt{6}} & ? & ? \end{pmatrix}$$

which leads to correlations between mixing parameters:

$$s = \sqrt{1 - \frac{2r^2}{2 - r^2}} \qquad \text{and} \qquad a \approx r \cos\delta,$$

s.t $\sin\theta_{23} \equiv \frac{1+a}{\sqrt{2}}$, $\sin\theta_{12} = \frac{1+s}{\sqrt{3}}$, and $\sin\theta_{13} \equiv \frac{r}{\sqrt{2}}$, (see King *0710.0530*).

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Generalised CP symmetries

- With three flavours, a generalised CP symmetry combines a flavour transformation with a CP transformation:

$$\Psi_\alpha \to X_{\alpha\beta}\Psi_\beta^{\mathsf{CP}}$$

- The theoretical consistency of this picture is constrained and the form of X must be related to an *automorphism of the flavour group*. See Holthausen, Lindner & Schmidt *1211.6953*, Chen *et al.* *1402.0507*.
- This creates novel constraints on the mass matrix, specifically regarding its complex phases,

$$X m_\nu X = m_\nu^*$$

- Taking this symmetry to be a residual symmetry of the lepton mass matrix can lead to predictions of the PMNS matrix including complex phases: δ, α_{21} and α_{31}. See *e.g.* Feruglio, Hagedorn & Ziegler *1211.5560*.

A_5 with GCP

- In 1503.07543, we[†] considered the group A_5 and systematically computed all of the predictions from different combinations of residual symmetries with GCP.

$$A_5 = \langle S, T \,|\, S^2 = T^5 = (ST)^3 \rangle$$

such that,

$$S = \begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad \text{and} \quad T = \frac{1}{2}\begin{pmatrix} 1 & -\varphi & -\varphi_g \\ \varphi & -\varphi_g & -1 \\ -\varphi_g & 1 & \varphi \end{pmatrix}$$

- We focus on the case when $G_\nu = \mathbb{Z}_2$, which produces parameter correlations similar to the atmospheric sum rules mentioned previously; however, the phases are more constrained.

† PB, Pascoli & Turner.

A_5 with GCP: predictions

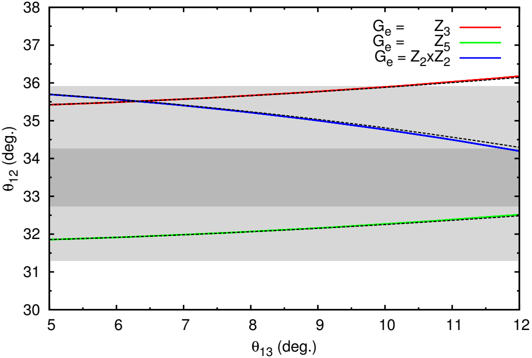
- Taking $G = A_5$ with GCP, breaking into $G_\nu = \mathbb{Z}_2$ with GCP and a cyclic group for G_e, we find 10 distinct sets of observables.

G_e	θ_{12}	θ_{23}	$\sin \alpha_{ji}$	δ
$\mathbb{Z}_3$	$35.27^\circ + 10.13^\circ\, r^2$	45°	0	90°
				270°
$\mathbb{Z}_5$	$31.72^\circ + 8.85^\circ\, r^2$	$45^\circ \pm 25.04^\circ\, r$	0	0°
				180°
		45°	0	90°
				270°
$\mathbb{Z}_2 \times \mathbb{Z}_2$	$36.00^\circ - 34.78^\circ\, r^2$	$31.72^\circ + 55.76^\circ\, r$	0	0°
				180°
		$58.28^\circ - 55.76^\circ\, r$	0	0°
				180°

From PB, Pascoli & Turner *1503.07543*. See also Li & Ding *1503.03711* and De Iura, Hagedorn & Meloni *1503.04140*.

A_5 with GCP: θ_{12}

- Three distinct predictions for θ_{12} in terms of θ_{13}.



- Change in θ_{12} over 3σ range of r only 0.07°, 0.08° and 0.13° for $\mathbb{Z}_3$, $\mathbb{Z}_5$ and $\mathbb{Z}_2 \times \mathbb{Z}_2$. Testing these relationships in themselves will be beyond the capabilities of upcoming reactor experiments.

A_5 with GCP: θ_{23} and δ

- We find an interesting relationship between θ_{23} and δ.

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A_5 with GCP: θ_{23} and δ

- Taking an order-5 generator in the charged-lepton sector, we find two choices depending on X:

$$\begin{pmatrix} \frac{\varphi}{\sqrt{2+\varphi}} & \frac{1}{\sqrt{2+\varphi}} & 0 \\ -\frac{1}{\sqrt{4+2\varphi}} & \frac{\varphi}{\sqrt{4+2\varphi}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{4+2\varphi}} & -\frac{\varphi}{\sqrt{4+2\varphi}} & \frac{1}{\sqrt{2}} \end{pmatrix} R_{13}(\theta)$$

$$\sin^2\theta_{23} = \frac{1}{2}\frac{\left(\sin\theta + \sqrt{1+\varphi^2}\cos\theta\right)^2}{1+\varphi^2\cos^2\theta}$$

$$\delta = \begin{cases} 0 & \theta \in (0, \frac{\pi}{2}) \\ \pi & \theta \in (\frac{\pi}{2}, \pi) \end{cases}$$

$$\begin{pmatrix} \frac{\varphi}{\sqrt{2+\varphi}} & -\frac{i}{\sqrt{2+\varphi}} & 0 \\ -\frac{i}{\sqrt{4+2\varphi}} & \frac{\varphi}{\sqrt{4+2\varphi}} & \frac{1}{\sqrt{2}} \\ \frac{i}{\sqrt{4+2\varphi}} & -\frac{\varphi}{\sqrt{4+2\varphi}} & \frac{1}{\sqrt{2}} \end{pmatrix} R_{13}(\theta)$$

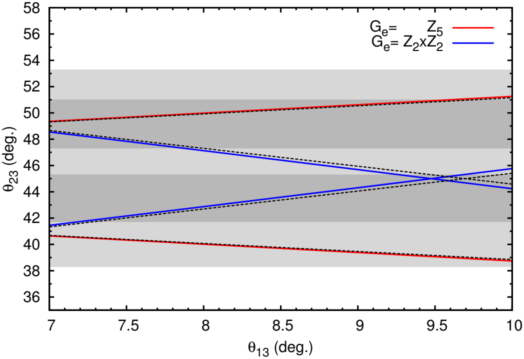
$$\sin^2\theta_{23} = \frac{1}{2} \qquad \forall\theta$$

$$\delta = \begin{cases} \frac{3\pi}{2} & \theta \in (0, \frac{\pi}{2}) \\ \frac{\pi}{2} & \theta \in (\frac{\pi}{2}, \pi) \end{cases}$$

PB, Pascoli, Turner *1503.07543*.

Non-maximal θ_{23} & Dirac CP conservation

- If δ is unobserved at T2K and NOνA, then we can look for information in the (possible) deviation from $\theta_{23} = 45°$.



- For $\mathbb{Z}_2 \times \mathbb{Z}_2$, the deviation from maximality is around $1°$. However, a greater than 2σ exclusion of maximality would be possible for $\mathbb{Z}_5$ with T2K and NOνA.

In conclusion...

- Parameter correlations, predicted in some models of discrete symmetries, are attractive (and accessible) signatures of fairly generic model-building ideas.

- I discussed a recent study on A_5 with a generalised CP. We found 10 distinct sets of parameter correlations.

- High-precision θ_{12} measurements (JUNO, RENO-50) will be able to distinguish between these candidates; although, the correlations only weakly depend on θ_{13}.

- θ_{23}–δ correlations are interesting in these models:
  - maximal-maximal predictions for θ_{23} and δ (T2K, NOνA)
  - CP conservation with θ_{23}–θ_{13} measurements (T2HK, DUNE).

Thank you.

backup slides

Solar sum rules

- The PMNS matrix measures the discrepancy between charged lepton and neutrino mass bases w.r.t a common flavour basis.

$$U_{\mathrm{PMNS}} = U_e^\dagger U_\nu$$

such that

$$U_e^\dagger m_e^\dagger m_e U_e = \mathrm{Diag}(m_e^2, m_\mu^2, m_\tau^2) \qquad U_\nu^T m_\nu U_\nu = \mathrm{Diag}\left(m_1, m_2, m_3\right).$$

- Leading-order mass matrices may be predominately symmetric but with higher-order symmetry violating terms. In the lepton sector, this would lead to

$$U_e = U_e^{\mathrm{Symmetric}} V^{\mathrm{Correction}} \qquad \Longrightarrow \qquad U_{\mathrm{PMNS}} = V^{\mathrm{Correction}} U_0$$

Solar sum rules

- We considered PMNS matrices of the form

$$U_{\text{PMNS}} = U^{e\dagger}_{23} U^{e\dagger}_{12} R^{\nu}_{23} R^{\nu}_{12} P$$

where R_{ij} denotes a real rotation from the neutrino sector, U_{ij} a unitary transformation in the ij-plane, and P a diagonal matrix of phases.

- There is a simple derivation for a prediction of $\cos\delta$. Multiplying out the PMNS matrix above,

$$|U_{\tau 1}/U_{\tau 2}| = \tan\theta^{\nu}_{12}$$

which, in terms of observable mixing parameters, can be expressed as

$$\cos\delta = \frac{t_{23}s^2_{12} + s^2_{13}c^2_{12}/t_{23} - s^{\nu 2}_{12}(t_{23} + s^2_{13}/t_{23})}{\sin 2\theta_{12}s_{13}}.$$

Solar sum rules: leading-order patterns

Survey of possibilities finds 4, all with $\theta_{13}^{\nu} = 0$ and $\theta_{23}^{\nu} = \frac{\pi}{4}$:

$$\theta_{12}^{\nu} = 45^{\circ} \quad \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ -\frac{1}{2} & \frac{1}{2} & \frac{1}{\sqrt{2}} \\ \frac{1}{2} & -\frac{1}{2} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

$$\theta_{12}^{\nu} = 35.3^{\circ} \quad \begin{pmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

$$\theta_{12}^{\nu} = 31.7^{\circ} \quad \begin{pmatrix} \frac{\varphi}{\sqrt{2+\varphi}} & \frac{1}{\sqrt{2+\varphi}} & 0 \\ -\frac{1}{\sqrt{4+2\varphi}} & \frac{\varphi}{\sqrt{4+2\varphi}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{4+2\varphi}} & -\frac{\varphi}{\sqrt{4+2\varphi}} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

$$\theta_{12}^{\nu} = 20.9^{\circ} \quad \begin{pmatrix} \frac{\varphi}{\sqrt{3}} & \frac{\varphi_g}{\sqrt{3}} & 0 \\ -\frac{\varphi_g}{\sqrt{6}} & \frac{\varphi}{\sqrt{6}} & \frac{1}{\sqrt{2}} \\ \frac{\varphi_g}{\sqrt{6}} & -\frac{\varphi}{\sqrt{6}} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

Solar sum rules: predictions for δ

- We can use global data to make statistical predictions for $\cos\delta$.

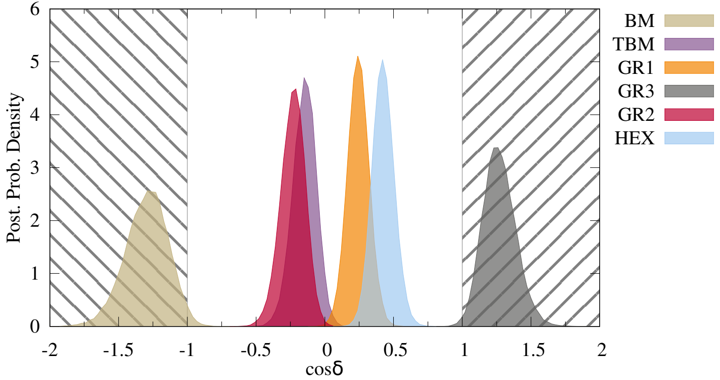


Figure from PB, King, Luhn, Pascoli, Schmidt *1410.7573*.