Lepton mixing predictions from (generalised) CP and discrete flavour symmetry

Thomas Neder

School of Physics and Astronomy, University of Southampton

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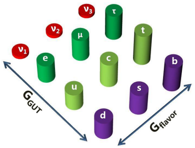
What is the question, that is the question

Why are there 3 generations?
Why do fermions mix as they do?
What is the difference between quarks and leptons?
What is the origin of CP violation?
Is there CP violation in the lepton sector?
Are neutrinos Dirac or Majorana fermions?
Why are the masses of fermions what they are?
Which mechanism generates neutrino masses?

Flavour symmetries

- Flavour symmetries extend the symmetry group of the Standard Model by an additional ("horizontal") symmetry that connects different flavours of particles: $G_{\text{gauge}} \times G_{\text{Flavour}}$.
- In more detail this means that the (lepton) Lagrangian is invariant under the following transformations:

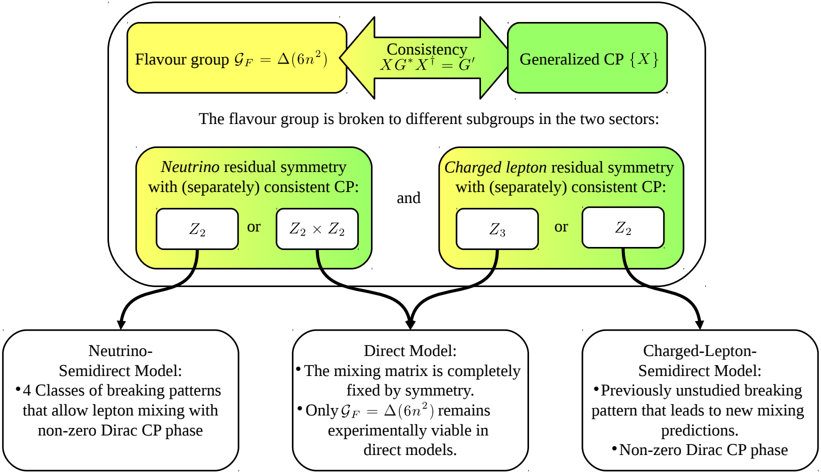


Picture from

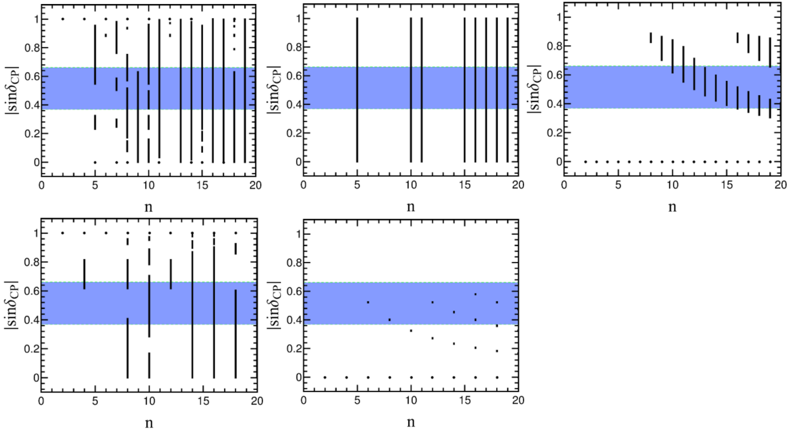
http://theophys.kth.se/tepp/Flavor.jpg

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} \mapsto G_\nu \begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} \quad \text{and} \quad \begin{pmatrix} e \\ \mu \\ \tau \end{pmatrix} \mapsto G_l \begin{pmatrix} e \\ \mu \\ \tau \end{pmatrix}$$

with $G_\nu, G_l \in G_{\text{Flavour}}$.


Flavour group $\mathcal{G}_F = \Delta(6n^2)$
Consistency
$XG^*X^\dagger = G'$
Generalized CP $\{X\}$
The flavour group is broken to different subgroups in the two sectors:
Neutrino residual symmetry with (separately) consistent CP:
Z_2
or
$Z_2 \times Z_2$
and
Charged lepton residual symmetry with (separately) consistent CP:
Z_3
or
Z_2
Neutrino-Semidirect Model:
• 4 Classes of breaking patterns that allow lepton mixing with non-zero Dirac CP phase
Direct Model:
• The mixing matrix is completely fixed by symmetry.
• Only $\mathcal{G}_F = \Delta(6n^2)$ remains experimentally viable in direct models.
Charged-Lepton-Semidirect Model:
• Previously unstudied breaking pattern that leads to new mixing predictions.
• Non-zero Dirac CP phase

$\Delta(6n^2)$ semidirect mixing: Predictions for δ_{CP}



Summary and outlook (1)

- Mass terms of fermions have a residual symmetry.
- We examine lepton mixing patterns in direct models with $\Delta(6n^2)$ groups and consistent generalised CP.
- In direct flavour models all mixing angles and the Dirac CP phase are predicted purely from symmetry
- Inclusion of a CP symmetry predicts Majorana phases
- In direct flavour models only $\Delta(6n^2)$ groups remain viable and can be analysed simultaneously in n.

Summary and outlook (2)

- We analyse the mixing predictions arising in neutrino-semidirect models and in charged-lepton-semidirect models
- Despite an additional free parameter, different groups predict only small ranges of mixing parameters and can be distinguished by future measurements
- Future Work:
  - Extend analysis to other interesting groups
  - Construct $\Delta(6n^2)$ flavour model(s) for promising n
  - Understand better the (spontaneous) breaking of flavour and CP symmetries in explicit potentials

BACKUP

Residual symmetries or why flavour symmetries?

- Imagine a mass term for several generations of Majorana fermions:

$$\nu^T M^\nu \nu$$

- M^ν diagonalised by $\nu \to U^\nu \nu$
- Define

$$G_1 = +u_1 u_1^\dagger - u_2 u_2^\dagger - u_3 u_3^\dagger$$

$$G_2 = -u_1 u_1^\dagger + u_2 u_2^\dagger - u_3 u_3^\dagger$$

$$G_3 = -u_1 u_1^\dagger - u_2 u_2^\dagger + u_3 u_3^\dagger$$

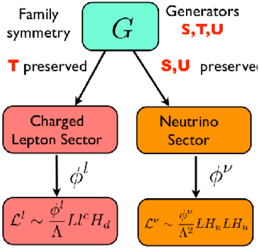
- $\Rightarrow Z_2 \times Z_2$ "residual" symmetry (Reason to go discrete)
- For Dirac fermions: $U(1) \times U(1)$

- Models are classified by how much of the neutrino residual symmetry is identified with a subgroup of G_{Flavour}:
  - $Z_2 \times Z_2 \in G_F$ "Direct model"
  - Z_2 "Semidirect model"
  - Or "Indirect model"
- Every conserved generator (for left-handed fields) produces a constraint on mass matrix:

$$G^T M G = M \text{ (Majorana)}$$

$$G^\dagger M^\dagger M G = M^\dagger M \text{ (Dirac)}$$

Direct models (1)



(From Steve F. King, Christoph Luhn, Neutrino Mass and Mixing with Discrete Symmetry, 1303.6180)

- In direct models, the residual symmetry is fully contained in the flavour group:

$$G^e \simeq Z_n \text{ with } n \geq 3 \text{ and } G^\nu = Z_2 \times Z_2$$

  (only $n = 3$ turns out to be viable)
- U^e is completely determined
- Every $G_i \in Z_2 \times Z_2$ fixes one column of the mixing matrix:

$$G_i U_i = +U_i$$

- $\Rightarrow$ The mixing matrix is completely fixed by symmetry

$\Delta(6n^2)$ groups[1]

- The groups $\Delta(6n^2)$ are non-abelian discrete subgroups of $U(3)$ of order $6n^2$ and are isomorphic to a semidirect product: $\Delta(6n^2) \cong (Z_n \times Z_n) \rtimes S_3$
- The left-handed leptons transform (without loss of generality) under a 3-dimensional representation with the generators a, b, c, d (where $\eta = e^{2\pi i/n}$).
- $U(3) \supset \Delta(6n^2) \supset \Delta(3n^2)$
- $\Delta(6 \times 1^2) = S_3$, $\Delta(6 \times 2^2) = S_4$, $\Delta(6 \times 3^2) = \Delta(54)$, $\Delta(6 \times 4^2) = \Delta(96)$

$$a = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix},$$

$$b = -\begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix},$$

$$c = \begin{pmatrix} \eta & 0 & 0 \\ 0 & \eta^{-1} & 0 \\ 0 & 0 & 1 \end{pmatrix},$$

$$d = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \eta & 0 \\ 0 & 0 & \eta^{-1} \end{pmatrix}$$

[1] Luhn, Escobar

(Generalised) CP[2]

- "Canonical" CP:

$$\nu_i(x) \mapsto e^{i\varphi_i}\nu_i^*(x^P)$$

  Basis-dependent?

- Generalised CP:

$$\nu_i(x) \mapsto X_{ij}\nu_j^*(x^P)$$

  Not basis-dependent: Canonical CP can be violated while physical CP is conserved

- For every unbroken gCP transformation: Additional constraint on mass matrix

$$X^T M X = M^*$$

  $\rightarrow$ Majorana phases from a symmetry principle

[2]Holthausen, Linder, Schmidt, Feruglio, Grimus, Rebelo, Chen, Trautner, etc etc

CP in presence of another symmetry

- Consistency condition:

$$X^\dagger G_i^* X = G_j$$

- G_F broken $\rightarrow$ CP violated through separate consistency for ν and e
- Some cases: $\{X\} \simeq G_F \rightarrow \Delta(6n^2)$ (Also e.g. continous groups: Only $SO(8)$ non-trivial)
- Current Work:
  - When is a lagrangian CP-conserving/violating?
  - Can CP be violated spontaneously?

Mixing results in direct models from $\Delta(6n^2)$ and generalised CP[3]

$$U_{\text{PMNS}} = \begin{pmatrix} \sqrt{\frac{2}{3}}\cos\left(\frac{\pi\gamma}{n}\right) & \frac{1}{\sqrt{3}} & \sqrt{\frac{2}{3}}\sin\left(\frac{\pi\gamma}{n}\right) \\ -\sqrt{\frac{2}{3}}\sin\left(\pi\left(\frac{\gamma}{n}+\frac{1}{6}\right)\right) & \frac{1}{\sqrt{3}} & \sqrt{\frac{2}{3}}\cos\left(\pi\left(\frac{\gamma}{n}+\frac{1}{6}\right)\right) \\ \sqrt{\frac{2}{3}}\sin\left(\pi\left(\frac{1}{6}-\frac{\gamma}{n}\right)\right) & -\frac{1}{\sqrt{3}} & \sqrt{\frac{2}{3}}\cos\left(\pi\left(\frac{1}{6}-\frac{\gamma}{n}\right)\right) \end{pmatrix} \times \begin{pmatrix} 1 & 0 & 0 \\ 0 & [i] i e^{-i6\pi(\gamma+x)/n} & 0 \\ 0 & 0 & [i] i \end{pmatrix}$$

In direct models, ALL lepton mixing matrices have the above form.

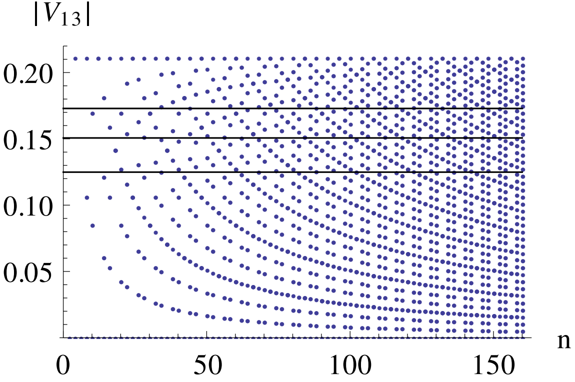
[3] King, TN, Stuart '13,; King, TN '14

Remarks about the mixing matrix

- The mixing matrix can only predicted up to permutations of rows and columns
- Fix ordering partially by demanding that the smallest entry becomes V_{13}
- Trimaximal with θ_{13} fixed up to a discrete choice, CP phase 0 or π,
- 2nd/3rd row ordering not determined experimentally $\rightarrow$ sum rule: $\theta_{23} = 45 \pm \theta_{13}/\sqrt{2}$
- (Barring loop-corrections:) Not only testing a single model, but the paradigm of Direct Flavour Models

Mixing results (contd.)

One can plot the possible values of $|V_{13}|$, the lines denote the present approximate 3σ range of $|V_{13}|$ (from Fogli et al,1205.5254):



Ex.:$|V_{13}| = 0.211, 0.170, 0.160, 0.154$ for $n = 4, 10, 16, 22$

Semidirect models (1)

- When in one sector only a single Z_2 is unbroken, only one column of that sector's U is fixed
- Can write the mixing matrix as $U_{Z_2} = U_{Z_2 \times Z_2} U_{2\times 2}$ with $U_{2\times 2} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos(\theta) & \sin(\theta)e^{i\phi} \\ 0 & -\sin(\theta)e^{-i\phi} & \cos(\theta) \end{pmatrix}$ and $U_{Z_2\times Z_2}$ from the full $Z_2 \times Z_2$
- We considered new breaking patterns in $\Delta(6n^2)$[4]:
  - $G^e = Z_2 \times Z_2, Z_p$ and $G^\nu = Z_2$: "Neutrino-semidirect"[5]
  - $G^e = Z_2$ and $G^\nu = Z_2 \times Z_2$: "Charged-lepton-semidirect"
- $\Rightarrow$ Only one column of the physical mixing matrix is determined from symmetry ($\rightarrow$ next slide)

[4] Ding, King, TN '14

[5] Hagedorn, Meroni, Molinari '14

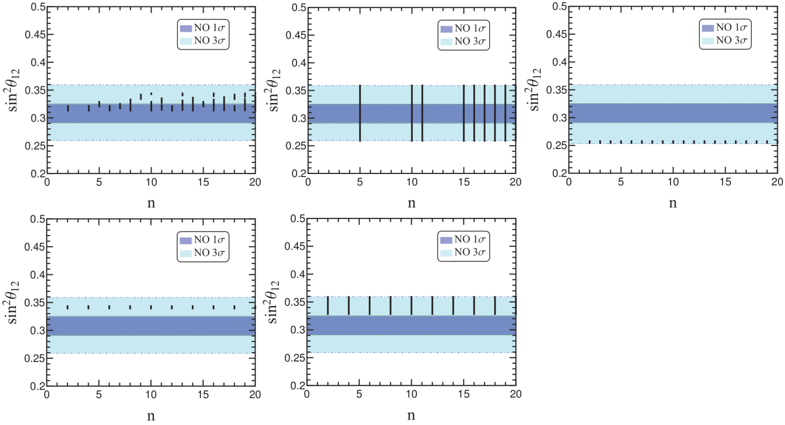
Semidirect models (2): $\Delta(6n^2)$ neutrino-semidirect mixing

	$G_\nu = Z_2^{bc^x d^x}$	$G_\nu = Z_2^{c^{n/2}}$
$G_l = \langle c^s d^t \rangle$	$\frac{1}{\sqrt{2}}\begin{pmatrix}0\\-1\\1\end{pmatrix}$ ✗	$\begin{pmatrix}0\\0\\1\end{pmatrix}$ ✗
$G_l = \langle bc^s d^t \rangle$	$\begin{pmatrix}0\\\cos\left(\frac{s+t-2x}{2n}\pi\right)\\\sin\left(\frac{s+t-2x}{2n}\pi\right)\end{pmatrix}$ ✗	$\frac{1}{\sqrt{2}}\begin{pmatrix}0\\-1\\1\end{pmatrix}$ ✗
$G_l = \langle ac^s d^t \rangle$	$\sqrt{\frac{2}{3}}\begin{pmatrix}\sin\left(\frac{s-x}{n}\pi\right)\\\cos\left(\frac{\pi}{6}-\frac{s-x}{n}\pi\right)\\\cos\left(\frac{\pi}{6}+\frac{s-x}{n}\pi\right)\end{pmatrix}$ ✓	$\frac{1}{\sqrt{3}}\begin{pmatrix}1\\1\\1\end{pmatrix}$ ✓
$G_l = \langle abc^s d^t \rangle$	$\frac{1}{2}\begin{pmatrix}1\\1\\-\sqrt{2}\end{pmatrix}$ ✓	$\begin{pmatrix}0\\0\\1\end{pmatrix}$ ✗
$G_l = \langle a^2bc^s d^t \rangle$	$\frac{1}{2}\begin{pmatrix}1\\1\\-\sqrt{2}\end{pmatrix}$ ✓	$\frac{1}{\sqrt{2}}\begin{pmatrix}0\\-1\\1\end{pmatrix}$ ✗

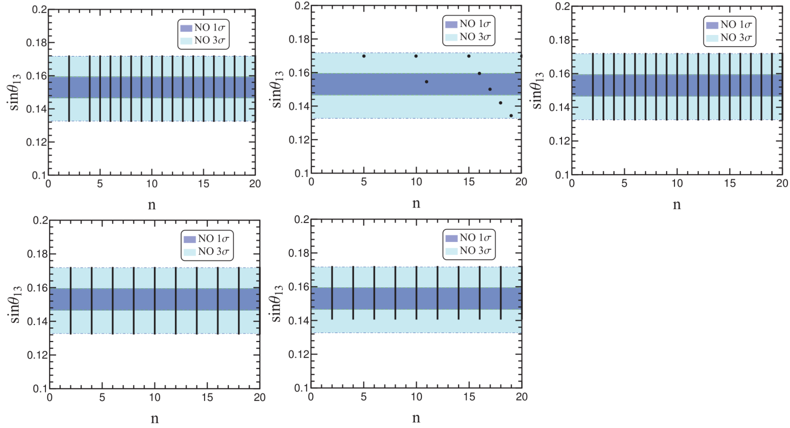
Semidirect models (3): $\Delta(6n^2)$ charged-lepton-semidirect mixing

	$G_l = Z_2^{bc^{x'}d^{x'}}$	$G_l = Z_2^{c^{n/2}}$
$G_\nu = K_4^{(c^{n/2},d^{n/2})}$	$\frac{1}{\sqrt{2}}\begin{pmatrix}1\\-1\\0\end{pmatrix}^T$ ✗	$\begin{pmatrix}1\\0\\0\end{pmatrix}^T$ ✗
$G_\nu = K_4^{(c^{n/2},abc^y)}$	$\frac{1}{2}\begin{pmatrix}1\\1\\-\sqrt{2}\end{pmatrix}^T$ ✓	$\begin{pmatrix}1\\0\\0\end{pmatrix}^T$ ✗
$G_\nu = K_4^{(d^{n/2},a^2bd^z)}$	$\frac{1}{2}\begin{pmatrix}1\\1\\-\sqrt{2}\end{pmatrix}^T$ ✓	$\frac{1}{\sqrt{2}}\begin{pmatrix}1\\-1\\0\end{pmatrix}^T$ ✗
$G_\nu = K_4^{(c^{n/2}d^{n/2},bc^xd^x)}$	$\begin{pmatrix}\cos\left(\frac{x-x'}{n}\pi\right)\\-i\sin\left(\frac{x-x'}{n}\pi\right)\\0\end{pmatrix}^T$ ✗	$\frac{1}{\sqrt{2}}\begin{pmatrix}1\\-1\\0\end{pmatrix}^T$ ✗

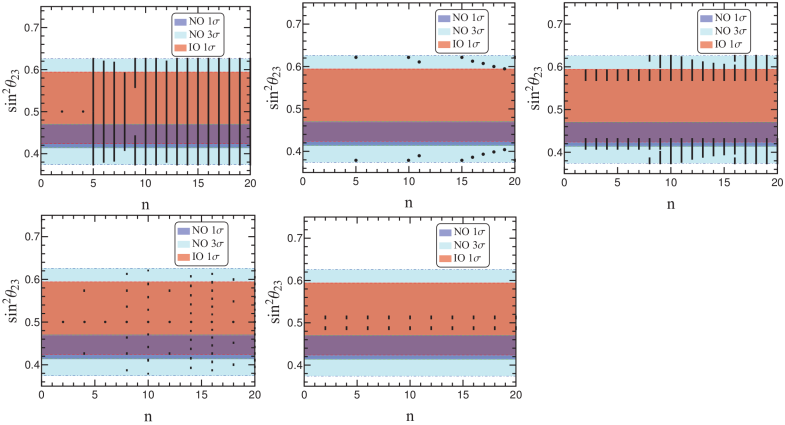
$\Delta(6n^2)$ semidirect mixing: Predictions for θ_{12}



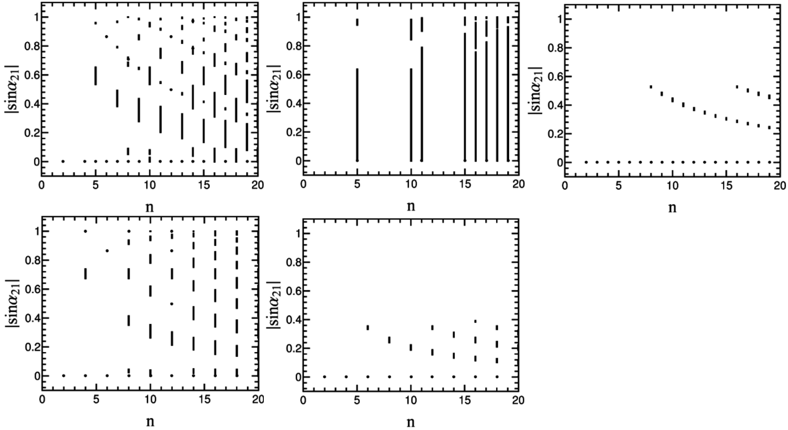
$\Delta(6n^2)$ semidirect mixing: Predictions for θ_{13}



$\Delta(6n^2)$ semidirect mixing: Predictions for θ_{23}



$\Delta(6n^2)$ semidirect mixing: Predictions for α_{21}



$\Delta(6n^2)$ semidirect mixing: Predictions for α_{31}