



EFT of Gravity and Dark Energy

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in collaboration with:

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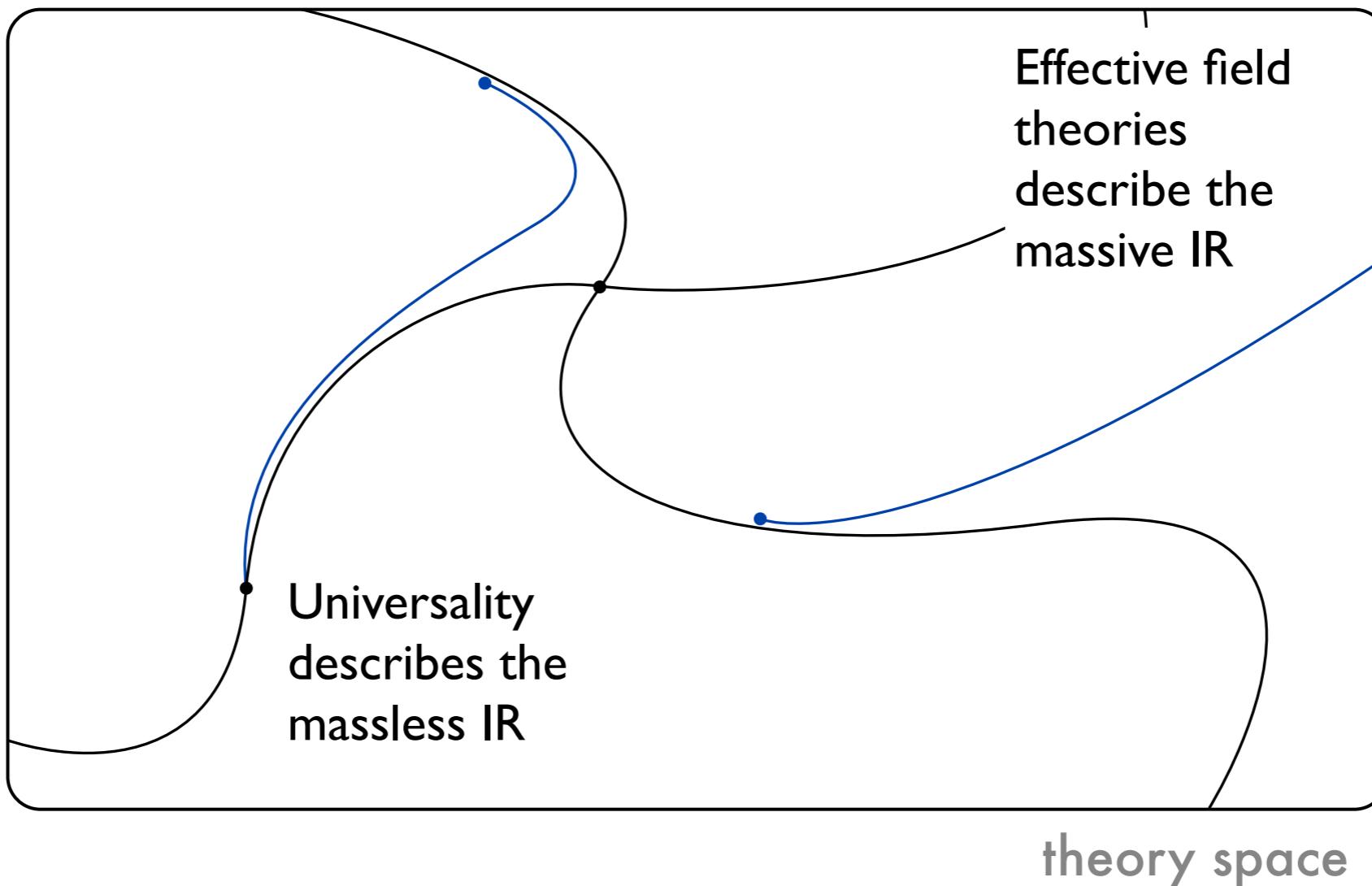
J Joergensen, F Sannino, O Svendsen

R Percacci, A Tonero, L Rachwal

Outline of the talk

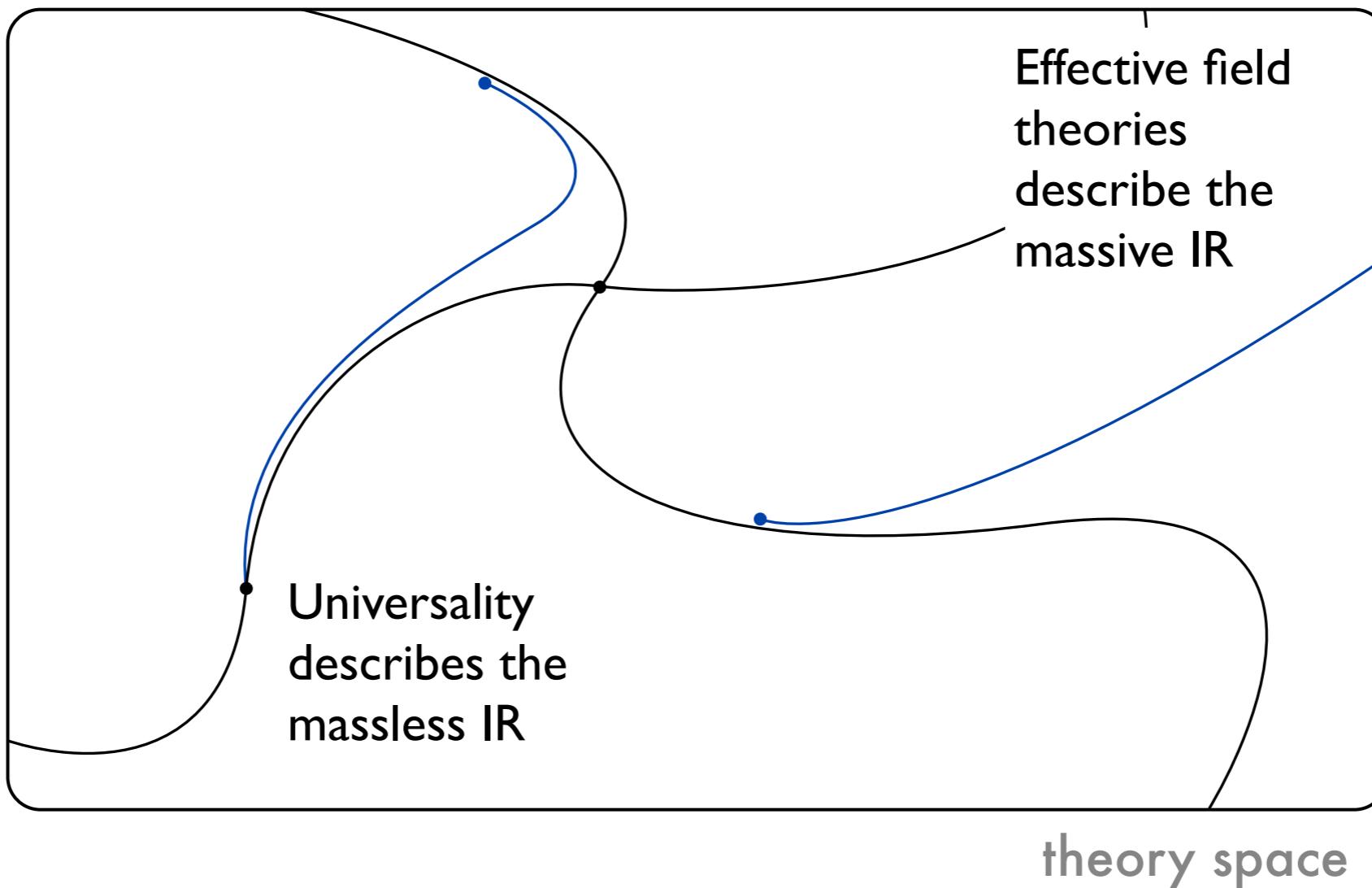
- Effectivity vs Universality
- Covariant EFT of gravity
- Phenomenological parameters
- Adding matter
- LO quantum corrections
- Effective Friedmann equations
- Dark energy
- Marginally deformed Starobinsky

Effectivity vs Universality



Two main reasons why mathematical modeling of nature actually works

Effectivity vs Universality



Massive IR lies in the broken phase (G to G/H)

Characteristic large scale M at which G is broken

EFT of Gravity

- The theory of small fluctuations of the metric

$$g_{\mu\nu} \rightarrow g_{\mu\nu} + \sqrt{16\pi G} h_{\mu\nu} = g_{\mu\nu} + \frac{1}{M} h_{\mu\nu}$$

- Planck's scale is the characteristic scale of gravity

$$M \equiv \frac{1}{\sqrt{16\pi G}} = \frac{M_{Planck}}{\sqrt{16\pi}}$$

$$M_{Planck} = \frac{1}{\sqrt{G}} = 1.2 \times 10^{19} \text{ GeV}$$

- Classical theory (CT) is successful over many orders of magnitude

EFT of Gravity

$$S_{eff}[g] = M^2 \left[I_1[g] + \frac{1}{M^2} I_2[g] + \frac{1}{M^4} I_3[g] + \dots \right]$$

$$I_1[g] = \int d^4x \sqrt{g} [M^2 c_0 - c_1 R]$$

$$I_2[g] = \int d^4x \sqrt{g} [c_{2,1} R^2 + c_{2,2} \text{Ric}^2 + c_{2,3} \text{Riem}^2]$$

$$I_3[g] = \int d^4x \sqrt{g} [c_{3,1} R \square R + c_{3,2} R_{\mu\nu} \square R^{\mu\nu} + c_{3,3} R^3 + \dots]$$

Derivative expansion of the bare (UV) action

Covariant EFT of Gravity

$$\begin{aligned} e^{-\Gamma[g]} &= \int_{1PI} \mathcal{D}h_{\mu\nu} e^{-S_{eff}[g + \frac{1}{M}h]} \\ &= \int_{1PI} \mathcal{D}h_{\mu\nu} e^{-M^2 \left\{ I_1[g + \frac{1}{M}h] + \frac{1}{M^2} I_2[g + \frac{1}{M}h] + \dots \right\}} \end{aligned}$$

EFT: saddle point expansion in $\frac{1}{M^2}$

Covariant EFT of Gravity

$$\Gamma[g] = I_1[g] \quad \text{CT}$$

$$+ \frac{1}{M^2} \left\{ I_2[g] + \frac{1}{2} \text{Tr} \log I_1^{(2)}[g] \right\} \quad \text{LO}$$

$$+ \frac{1}{M^4} \left\{ I_3[g] + \frac{1}{2} \text{Tr} \left[\left(I_1^{(2)}[g] \right)^{-1} I_2^{(2)}[g] \right] + \text{2-loops with } I_1[g] \right\} \quad \text{NLO}$$

$$+ \dots \quad \text{NNLO}$$

Covariant EFT of Gravity

$$\Gamma = \begin{array}{c} \text{CT} \\ \text{I}_1 \\ + \frac{1}{M^2} \left[\text{I}_2 + \frac{1}{2} \text{O} \right] \\ + \frac{1}{M^4} \left[\text{I}_3 + \frac{1}{2} \text{O} - \frac{1}{12} \text{S} + \frac{1}{8} \text{E} \right] \\ + \dots \end{array}$$

The equation shows the expansion of the Covariant EFT of Gravity. The first term is labeled CT (Counterterm) and consists of a single blue dot labeled I₁. The second term is labeled LO (Loop Order) and contains a blue dot labeled I₂ plus half of a blue circle labeled O. The third term is labeled NLO (Next-to-Loop Order) and contains a blue dot labeled I₃ plus half of a blue circle labeled O minus one-twelfth of a blue circle with a horizontal cross labeled S plus one-eighth of a blue circle labeled E. The fourth term is represented by three dots.

Covariant EFT of Gravity

The EFT recipe in three lines

$$\Gamma = \text{CT} + \frac{1}{M^2} \left[I_1 + \frac{1}{2} \text{LO} \right] + \frac{1}{M^4} \left[I_2 + \frac{1}{2} \text{NLO} \right] + \dots$$


 I_1

 I_2

 I_3

- I) the general lagrangian of order E^2 is to be used both at tree level and in loop diagrams
 - 2) the general lagrangian of order $E^{n \geq 4}$ is to be used at tree level and as an insertion in loop diagrams
 - 3) the renormalization program is carried out order by order

Covariant EFT of Gravity

What do we already know?

$$\Gamma = \text{CT} + \frac{1}{M^2} \left[\text{LO} + \frac{1}{2} \text{NLO} \right] + \frac{1}{M^4} \left[\text{NLO} - \frac{1}{12} \text{NNLO} + \frac{1}{8} \text{NNLO} \right] + \dots$$

The diagram shows the expansion of the effective action Γ . The first term is a blue dot. The second term is $\frac{1}{M^2}$ times the sum of a green box containing a purple dot and a blue circle divided by 2. The third term is $\frac{1}{M^4}$ times the sum of a pink dot, a blue circle divided by 2, a blue circle divided by 12, and a blue circle divided by 8. The ellipsis indicates higher-order terms.

UV divergencies and renormalization

G. 't Hooft and M. J. G. Veltman, Annales Poincare Phys. Theor. A 20 (1974) 69

G. W. Gibbons, S. W. Hawking and M. J. Perry, Nucl. Phys. B 138 (1978) 141

S. M. Christensen and M. J. Duff, Nucl. Phys. B 170 (1980)

Covariant EFT of Gravity

What do we already know?

$$\Gamma = \text{CT} + \frac{1}{M^2} \left[\text{LO} + \frac{1}{2} \text{NLO} \right] + \frac{1}{M^4} \left[\text{NLO} + \frac{1}{2} \text{NNLO} - \frac{1}{12} \text{UV} + \frac{1}{8} \text{UV} \right] + \dots$$

The diagram shows the expansion of the effective action Γ . The first term is a single blue dot labeled 'CT'. The second term is $\frac{1}{M^2}$ times a sum of two diagrams: a purple dot and a blue circle divided by a horizontal line. The third term is $\frac{1}{M^4}$ times a sum of three diagrams: a red box containing a pink dot, a blue circle with a purple dot inside, and a blue circle divided by a horizontal line. The fourth term is indicated by '...'.

Two loops UV divergencies

M.H. Goroff and A. Sagnotti, Nucl.Phys.B266, 709 (1986)
A. E. M. van de Ven, Nucl. Phys. B378, 309 (1992)

Covariant EFT of Gravity

What do we already know?

$$\Gamma = \text{CT} + \frac{1}{M^2} \left[\text{LO} + \frac{1}{2} \text{NLO} \right] + \frac{1}{M^4} \left[\text{NLO} + \frac{1}{2} \text{NNLO} \right] + \dots$$

The diagram shows the four-point vertex Γ as a sum of contributions. The first term is labeled "CT". The second term is labeled "LO" and contains a blue dot and a blue circle. The third term is labeled "NLO" and contains a purple dot, a pink circle, a blue circle, a blue circle with a horizontal line, and a blue circle with the number 8. The fourth term is labeled "NNLO" and contains three dots.

Finite LO terms

Leading logs

J.F. Donoghue, Phys. Rev. Lett. 72, 2996 (1994)

A. C., J. Joergensen, F. Sannino and O. Svendsen, JHEP 1502 (2015) 050

Conformal anomaly

S. Deser, M. J. Duff and C. J. Isham, Nucl. Phys. B 111, 45 (1976)

R.J. Riegert, Phys. Lett. B 134 (1984) 56

Four graviton vertex in Minkowski space

D. C. Dunbar and P. S. Norridge, Nucl. Phys. B 433, 181 (1995)

Curvature square terms

A. C. and R. K. Jain, in preparation

Covariant EFT of Gravity

LOQG: the only QG we will ever observe!

$$\Gamma = \text{CT} + \frac{1}{M^2} \left[\text{LO} + \frac{1}{2} \text{NLO} \right] + \frac{1}{M^4} \left[\text{NLO} - \frac{1}{12} \text{NNLO} \right] + \dots$$

The diagram shows the expansion of the effective coupling Γ . The first term is a single blue dot labeled "CT". The second term is enclosed in a blue box and consists of a purple dot plus half of a blue circle. The third term is a sum of three parts: a purple dot plus half of a blue circle, minus one-twelfth of a circle divided by two, plus one-eighth of a blue circle. The fourth term is followed by ellipses.

Even if we have a fundamental theory its is generally difficult to compute phenomenological parameters...

Phenomenological parameters

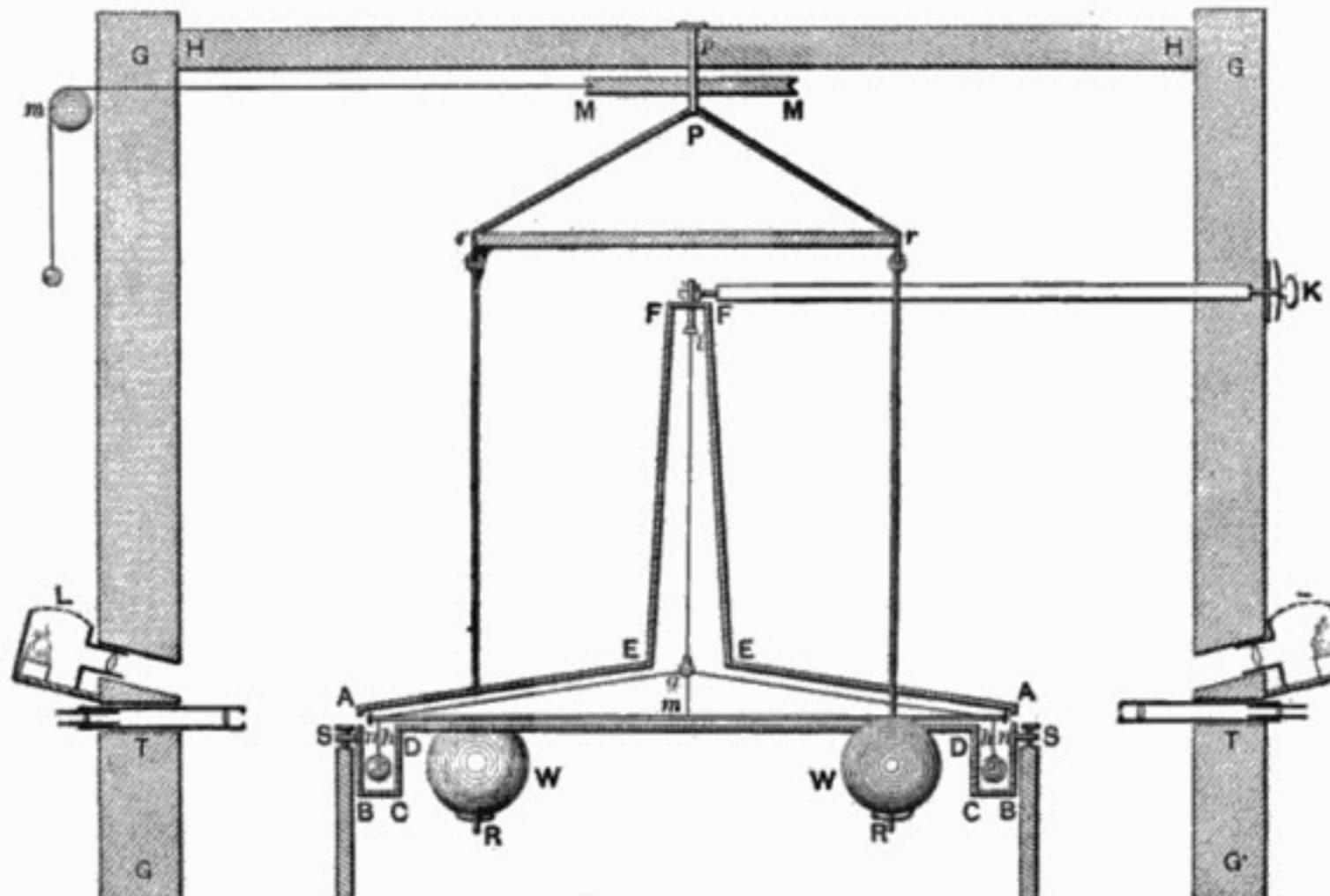
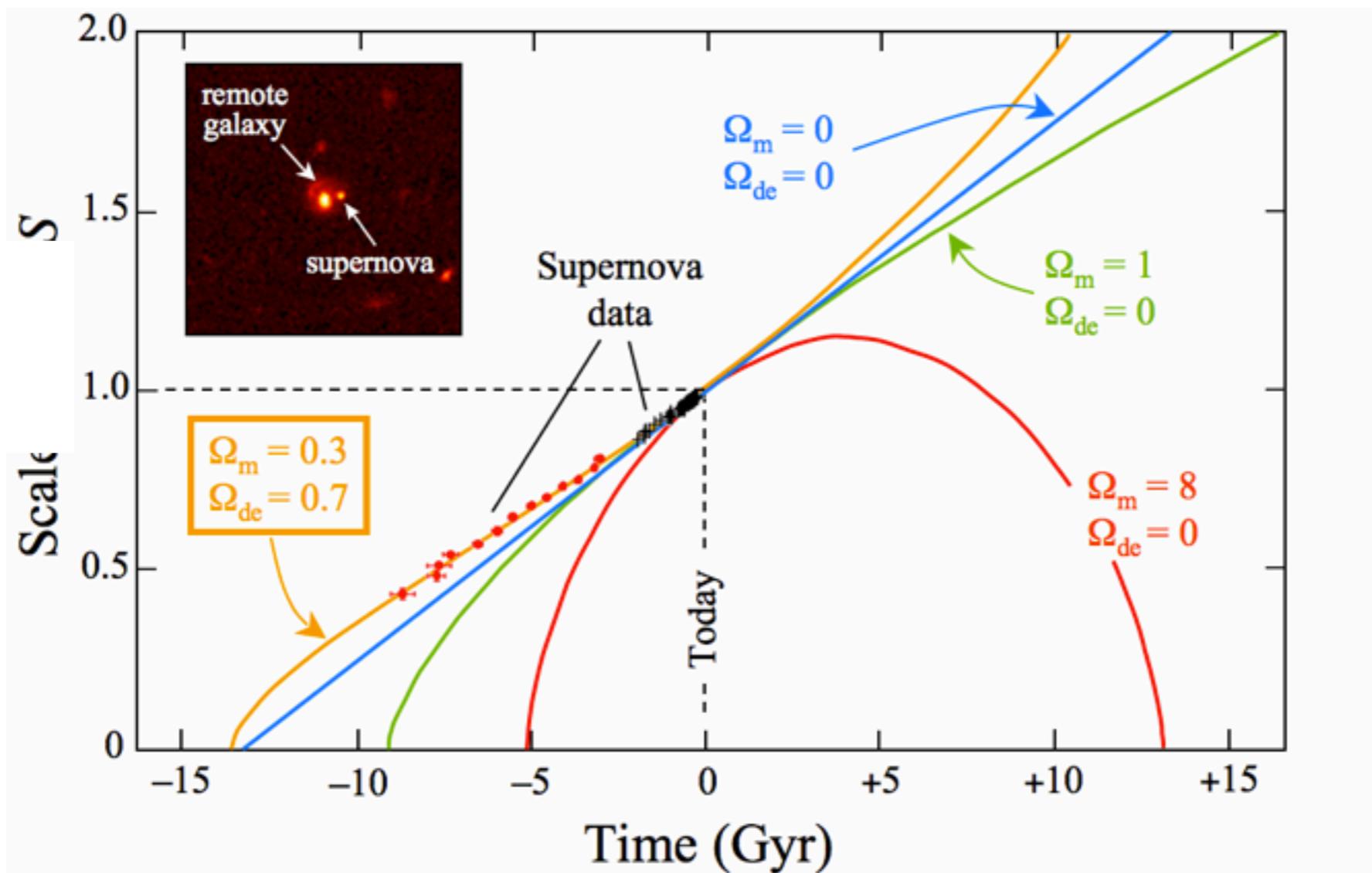


Fig. 1

$$G = 6.67428 \times 10^{-11} \text{ m}^3 \text{kg}^{-1} \text{s}^{-2}$$

Cavendish 1797 (1% off best value!)

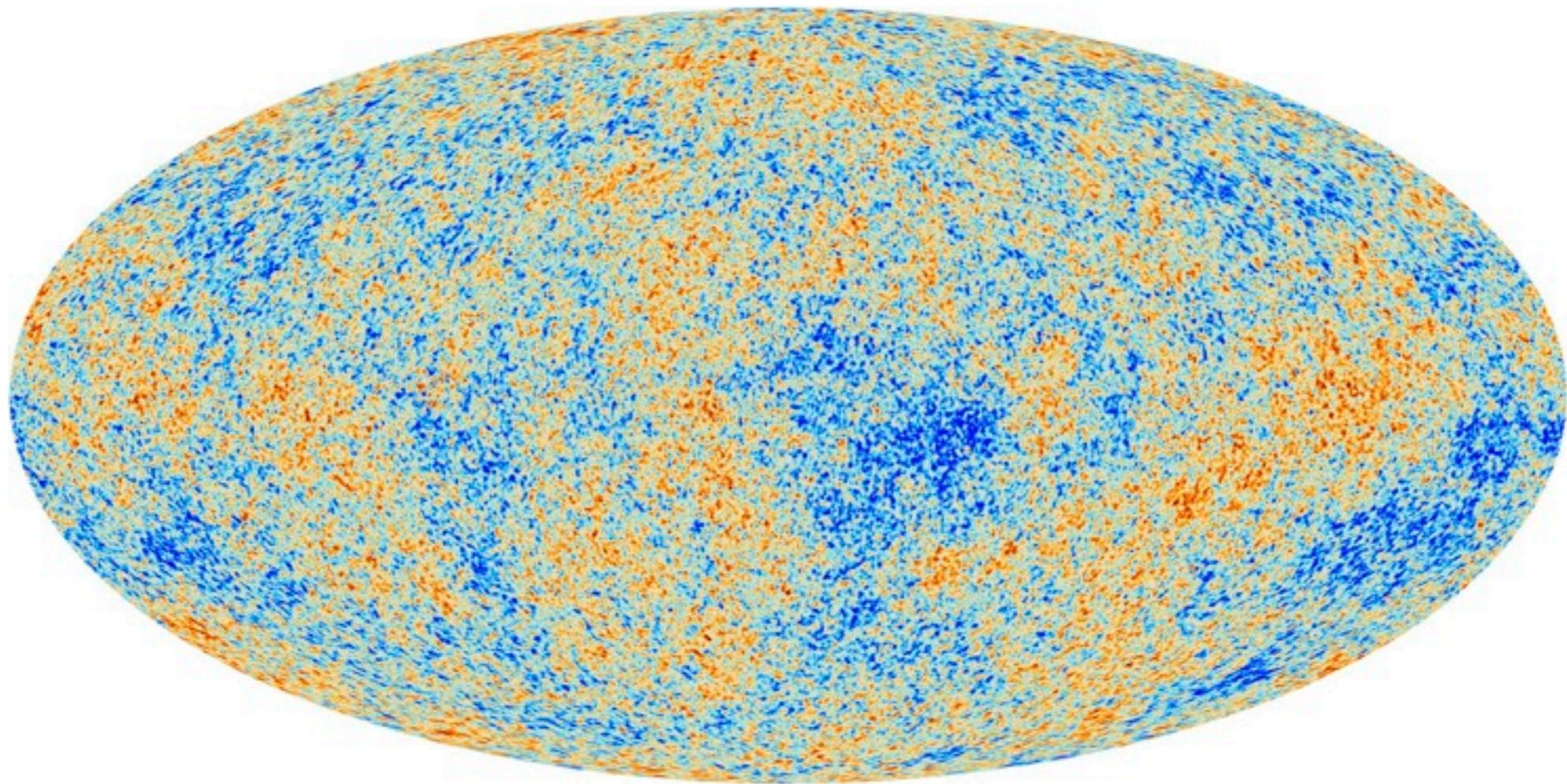
Phenomenological parameters



$$\Lambda = 10^{-47} \text{ GeV}^4$$

Supernova Cosmology Project

Phenomenological parameters



$$\xi \equiv c_{R^2}$$

$$\xi(k_*) \sim 10^9$$

$$k_* \equiv 0.05 \text{ Mpc}^{-1} \sim 10^{-40} \text{ GeV}$$

Planck mission

Adding matter

$$\Gamma = \text{CT} \\ + \frac{1}{M^2} \left[\textcolor{purple}{\bullet} + \textcolor{black}{\bullet} + \frac{1}{2} \textcolor{black}{\bigcirc} + \frac{1}{2} \textcolor{cyan}{\bigcirc} - \textcolor{black}{\bigcirc} \right] \text{LO} \\ + \dots \text{NLO}$$

Adding matter

$$\Gamma = \text{CT} \\ + \frac{1}{M^2} \left[\text{LO} \right] + \frac{1}{2} \dots \text{NLO}$$

The equation shows the definition of the effective action Γ . It consists of three parts: a counterterm (CT) represented by a single blue dot, a loop diagram labeled LO enclosed in a green box, and a series of terms starting with $\frac{1}{M^2}$ followed by a plus sign, a green box containing two vertices (one purple dot and one black dot), another plus sign, a black circle, another plus sign, a blue circle, a minus sign, and a black circle.

**UV divergencies and renormalization with matter
Scalars**

A. O. Barvinsky, A. Y. Kamenshchik and I. P. Karmazin, Phys. Rev. D 48 (1993) 3677

...

Gauge

S. P. Robinson and F. Wilczek, Phys. Rev. Lett. 96, 231601 (2006)

...

Yukawa

A. Rodigast and T. Schuster, Phys. Rev. Lett. 104, 081301 (2010)

...

Adding matter

Finite LO terms with matter

Flat space corrections to Newton's potential

J.F. Donoghue, Phys. Rev. Lett. 72, 2996 (1994)

N.E.J. Bjerrum-Bohr, J.F. Donoghue and B.R. Holstein (2003b) Phys. Rev. D 67

I.B. Khriplovich, G.G. Kirilin (2004) J. Exp. Theor. Phys. 98, 1063-1072

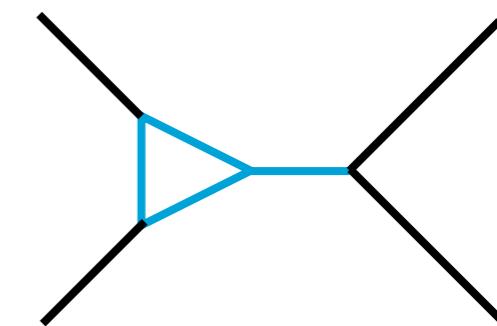
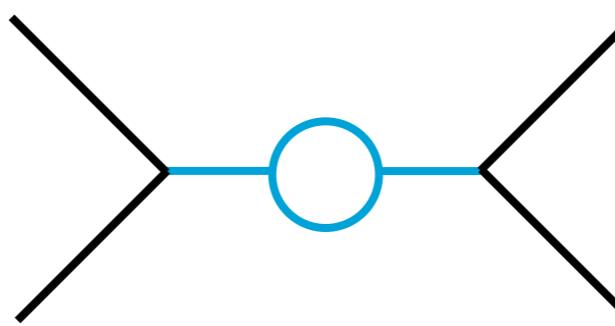
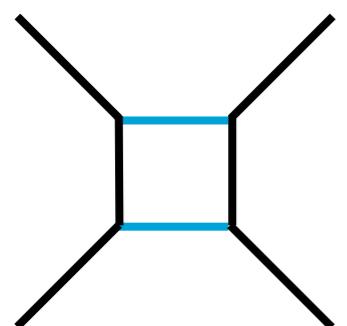
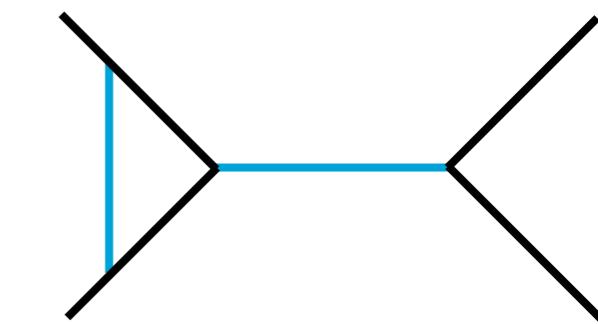
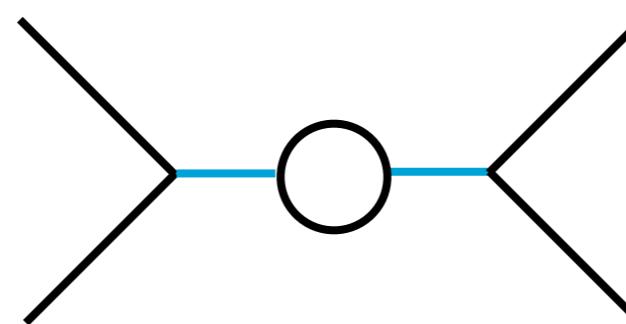
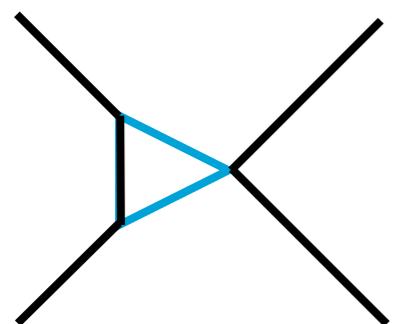
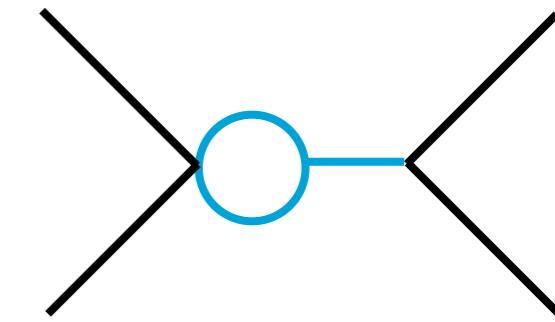
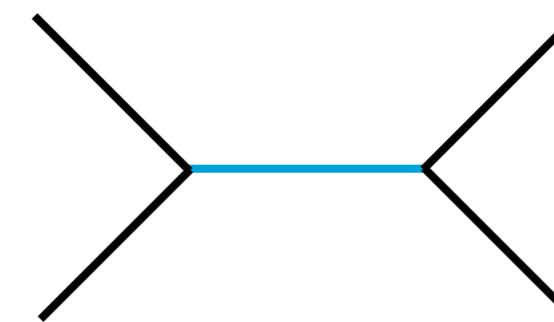
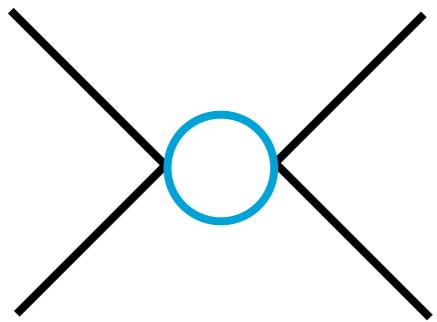
Covariant leading logs

A C, R Percacci, L Rachwal and A Tonero in preparation

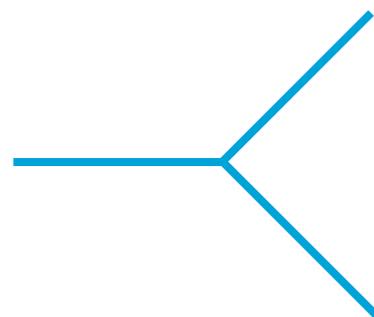
Adding matter

Matter induced effective action

Corrections to Newton's interaction



Corrections to Newton's interaction

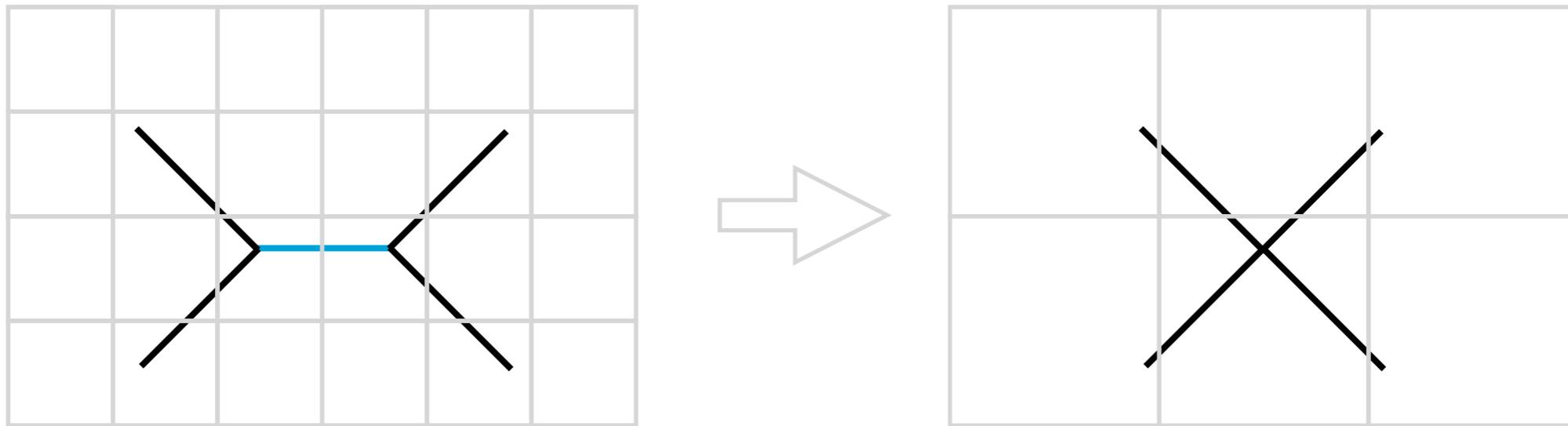


=

$$\begin{aligned}
 & \frac{i\kappa}{2} \left(P_{\alpha\beta,\gamma\delta} \left[k^\mu k^\nu + (k-q)^\mu (k-q)^\nu + q^\mu q^\nu - \frac{3}{2} \eta^{\mu\nu} q^2 \right] \right. \\
 & + 2q_\lambda q_\sigma \left[I^{\lambda\sigma,}_{\alpha\beta} I^{\mu\nu,}_{\gamma\delta} + I^{\lambda\sigma,}_{\gamma\delta} I^{\mu\nu,}_{\alpha\beta} - I^{\lambda\mu,}_{\alpha\beta} I^{\sigma\nu,}_{\gamma\delta} - I^{\sigma\nu,}_{\alpha\beta} I^{\lambda\mu,}_{\gamma\delta} \right] \\
 & + \left[q_\lambda q^\mu \left(\eta_{\alpha\beta} I^{\lambda\nu,}_{\gamma\delta} + \eta_{\gamma\delta} I^{\lambda\nu,}_{\alpha\beta} \right) + q_\lambda q^\nu \left(\eta_{\alpha\beta} I^{\lambda\mu,}_{\gamma\delta} + \eta_{\gamma\delta} I^{\lambda\mu,}_{\alpha\beta} \right) \right. \\
 & - q^2 \left(\eta_{\alpha\beta} I^{\mu\nu,}_{\gamma\delta} + \eta_{\gamma\delta} I^{\mu\nu,}_{\alpha\beta} \right) - \eta^{\mu\nu} q^\lambda q^\sigma (\eta_{\alpha\beta} I_{\gamma\delta,\lambda\sigma} + \eta_{\gamma\delta} I_{\alpha\beta,\lambda\sigma}) \Big] \\
 & + \left[2q^\lambda \left(I^{\sigma\nu,}_{\alpha\beta} I_{\gamma\delta,\lambda\sigma} (k-q)^\mu + I^{\sigma\mu,}_{\alpha\beta} I_{\gamma\delta,\lambda\sigma} (k-q)^\nu \right. \right. \\
 & \quad \left. \left. - I^{\sigma\nu,}_{\gamma\delta} I_{\alpha\beta,\lambda\sigma} k^\mu - I^{\sigma\mu,}_{\gamma\delta} I_{\alpha\beta,\lambda\sigma} k^\nu \right) \right. \\
 & + q^2 \left(I^{\sigma\mu,}_{\alpha\beta} I_{\gamma\delta,\sigma}{}^\nu + I_{\alpha\beta,\sigma}{}^\nu I^{\sigma\mu,}_{\alpha\delta} \right) + \eta^{\mu\nu} q^\lambda q_\sigma \left(I_{\alpha\beta,\lambda\rho} I^{\rho\sigma,}_{\gamma\delta} + I_{\gamma\delta,\lambda\rho} I^{\rho\sigma,}_{\alpha\beta} \right) \Big] \\
 & + \left\{ (k^2 + (k-q)^2) \left(I^{\sigma\mu,}_{\alpha\beta} I_{\gamma\delta,\sigma}{}^\nu + I^{\sigma\nu,}_{\alpha\beta} I_{\gamma\delta,\sigma}{}^\mu - \frac{1}{2} \eta^{\mu\nu} P_{\alpha\beta,\gamma\delta} \right) \right. \\
 & \quad \left. - \left(k^2 \eta_{\gamma\delta} I^{\mu\nu,}_{\alpha\beta} + (k-q)^2 \eta_{\alpha\beta} I^{\mu\nu,}_{\gamma\delta} \right) \right\}
 \end{aligned}$$

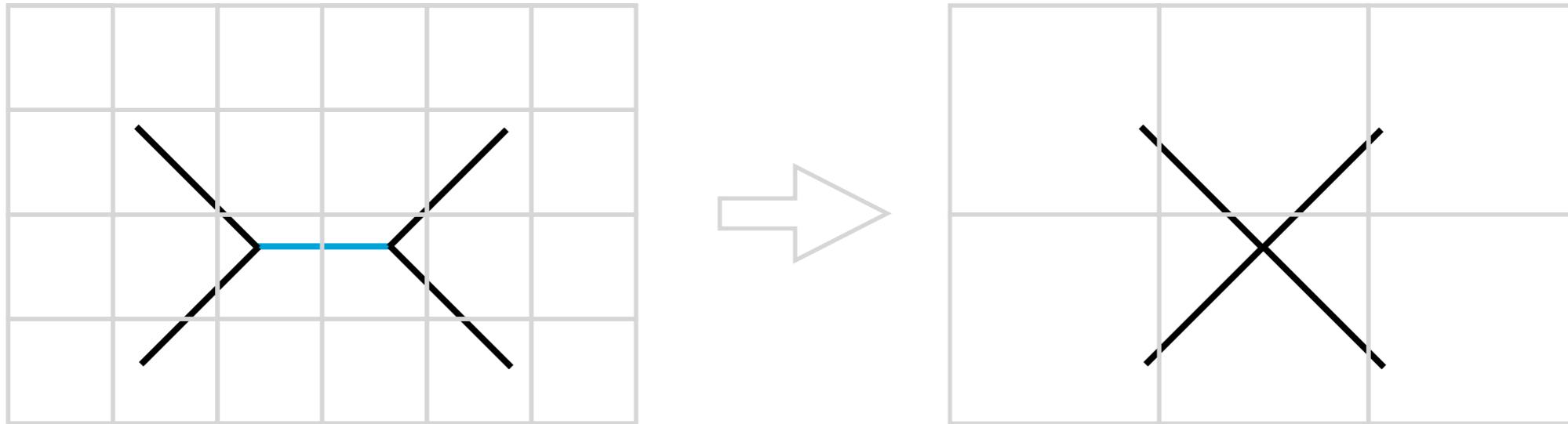
the truth behind Feynman diagrams...

Corrections to Newton's interaction



$$V = -\frac{GMm}{r} \left[1 + a \frac{G(M+m)}{c^2 r} + b \frac{G\hbar}{c^3 r^2} + \dots \right]$$

Corrections to Newton's interaction



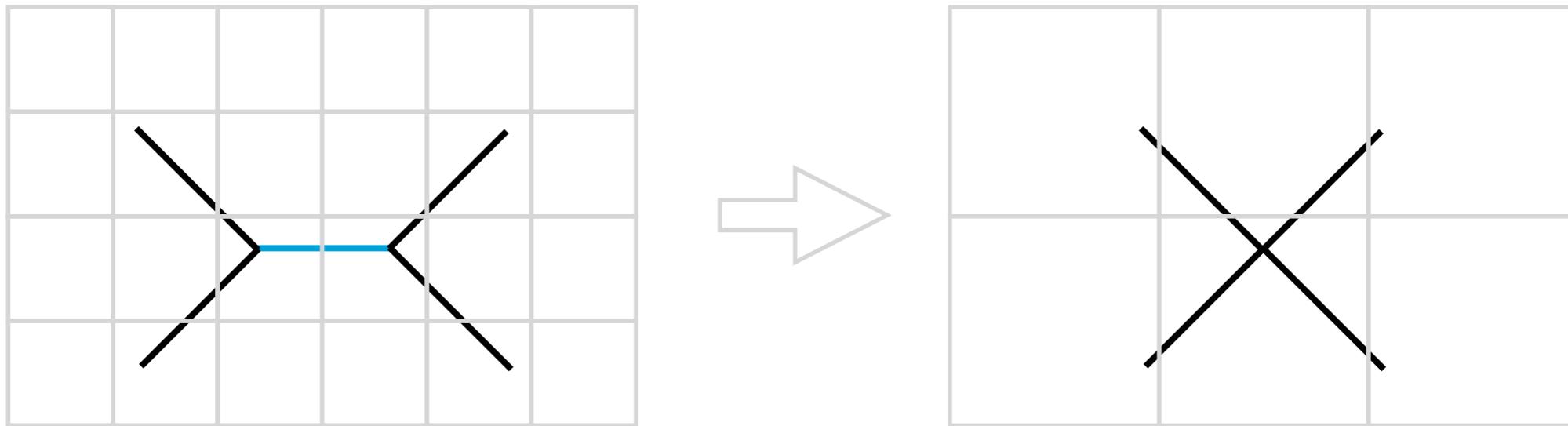
$$V = -\frac{GMm}{r} \left[1 + a \frac{G(M+m)}{c^2 r} + b \frac{G\hbar}{c^3 r^2} + \dots \right]$$

$$[G] = \frac{\text{m}^3}{\text{Kg s}^2}$$

$$[\hbar] = \frac{\text{m}^2 \text{ Kg}}{\text{s}}$$

$$[c] = \frac{\text{m}}{\text{s}}$$

Corrections to Newton's interaction

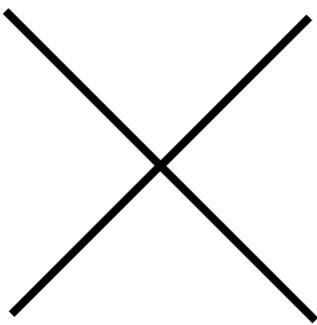


$$V = -\frac{GMm}{r} \left[1 + 3\frac{G(M+m)}{c^2 r} + \frac{41}{10\pi} \frac{G\hbar}{c^3 r^2} + \dots \right]$$

Leading quantum corrections to Newton's potential

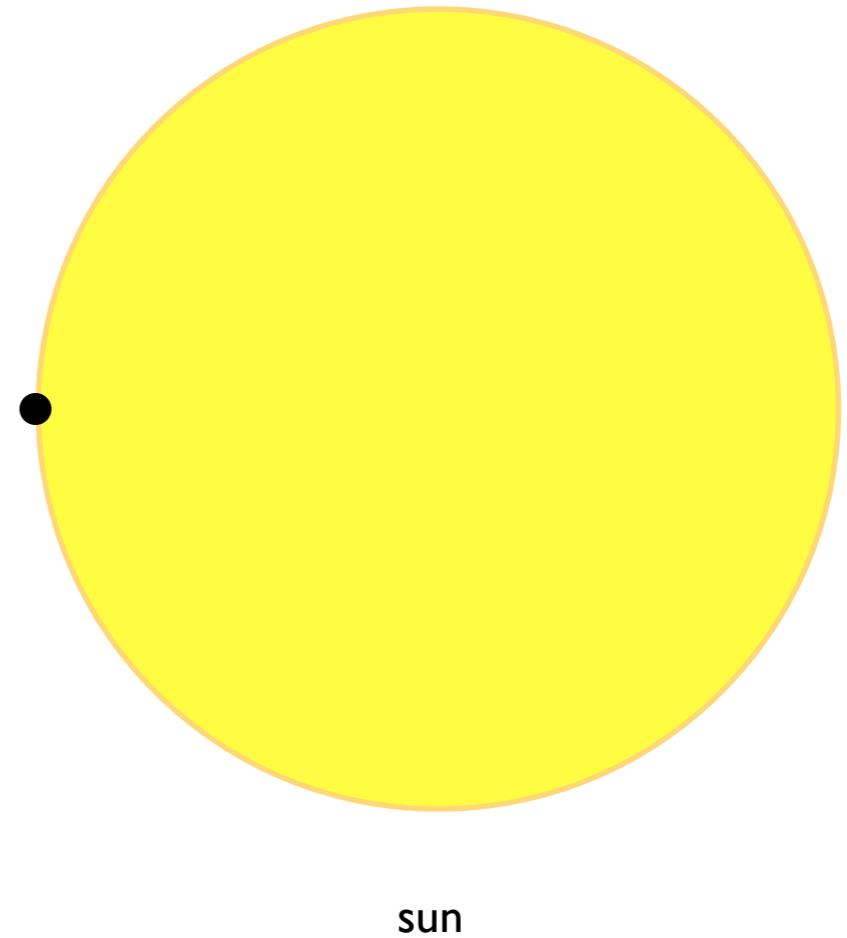
J.F. Donoghue, Phys. Rev. Lett. 72, 2996 (1994)

Corrections to Newton's interaction



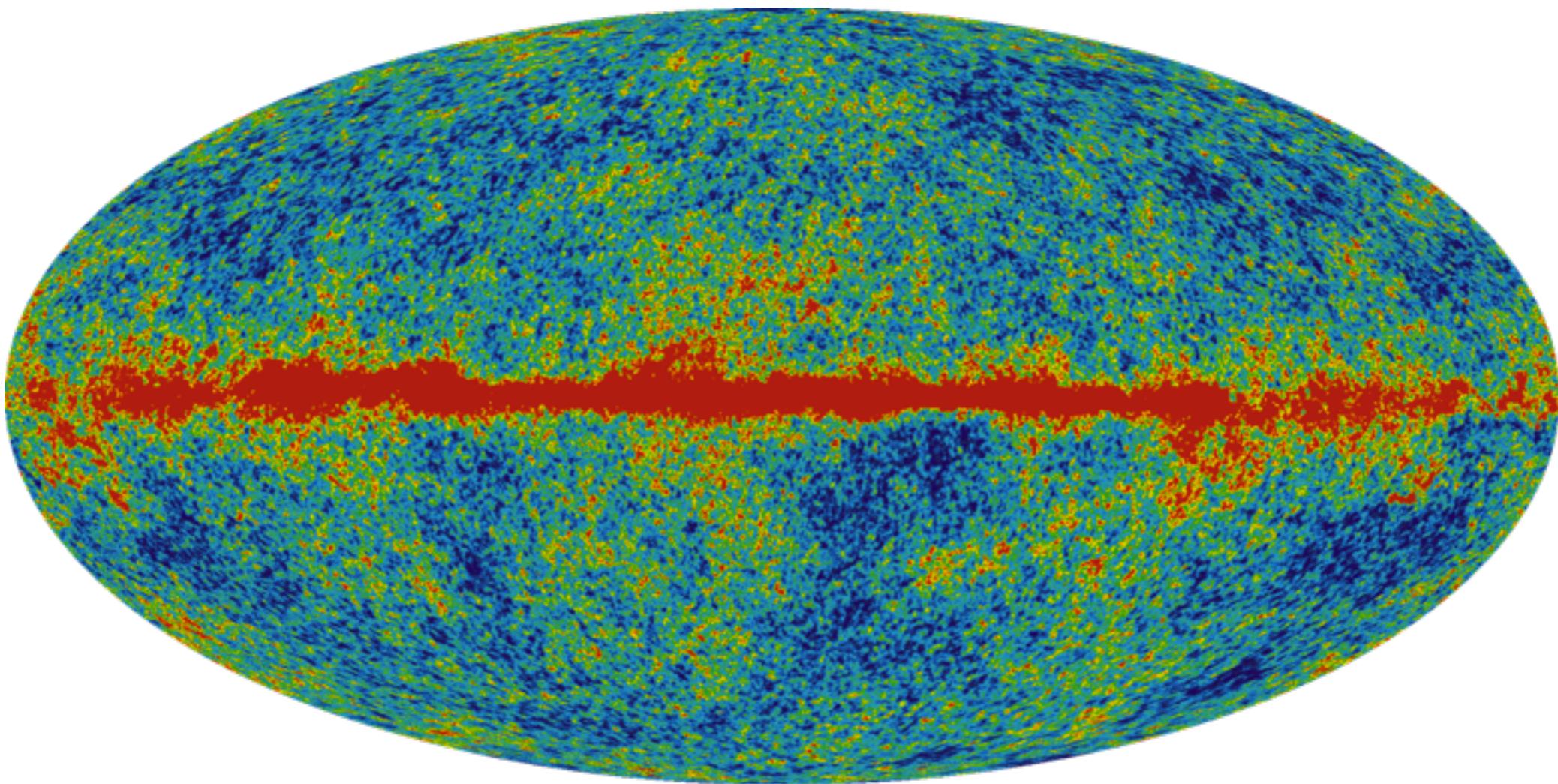
$$\frac{GM_{\odot}}{c^2 r_{\odot}} \sim 10^{-6}$$

$$\frac{G\hbar}{c^3 r_{\odot}^2} \sim 10^{-88}$$



Leading quantum corrections to Newton's law are incredibly small!

Can we ever observe quantum gravity effects?



Look for physical situations where
LO corrections are enhanced

Curvature expansion

- $+ \frac{1}{2} \circ = -\frac{1}{2(4\pi)^{d/2}} \int d^d x \sqrt{g} \operatorname{tr} \mathcal{R} \gamma_i \left(\frac{-\square}{m^2} \right) \mathcal{R} + \dots$

The finite physical part of the effective action is covariantly encoded in the structure functions which can be computed using the non-local heat kernel expansion

$$\gamma_i \left(\frac{X}{m^2} \right) \equiv \lim_{\Lambda_{UV} \rightarrow \infty} \int_{1/\Lambda_{UV}^2}^{\infty} \frac{ds}{s} s^{-d/2+2} [f_i(sX) - f_i(0)] e^{-sm^2}$$



Non-local heat kernel

A. O. Barvinsky and G. A. Vilkovisky, Nucl. Phys. B 282 (1987) 163

I. G. Avramidi, Lect. Notes Phys. M 64 (2000) 1

A. Codello and O. Zanusso, J. Math. Phys. 54 (2013) 013513

Non-local heat kernel structure functions

Curvature expansion

- $+ \frac{1}{2} \circlearrowleft = -\frac{1}{2(4\pi)^2} \int d^4x \sqrt{g} \operatorname{tr} \left[\mathbf{1} R_{\mu\nu} \gamma_{Ric} \left(\frac{-\square}{m^2} \right) R^{\mu\nu} + \frac{1}{120} R \gamma_R \left(\frac{-\square}{m^2} \right) R \right.$
 $\left. - \frac{1}{6} R \gamma_{RU} \left(\frac{-\square}{m^2} \right) \mathbf{U} + \frac{1}{2} \mathbf{U} \gamma_U \left(\frac{-\square}{m^2} \right) \mathbf{U} + \frac{1}{12} \boldsymbol{\Omega}_{\mu\nu} \gamma_\Omega \left(\frac{-\square}{m^2} \right) \boldsymbol{\Omega}^{\mu\nu} \right]$

Explicit form for the structure functions

$$\gamma_{Ric}(u) = \frac{1}{40} + \frac{1}{12u} - \frac{1}{2} \int_0^1 d\xi \left[\frac{1}{u} + \xi(1-\xi) \right]^2 \log [1 + u \xi(1-\xi)]$$

$$\begin{aligned} \gamma_R(u) = & -\frac{23}{960} - \frac{1}{96u} + \frac{1}{32} \int_0^1 d\xi \left\{ \frac{2}{u^2} + \frac{4}{u} [1 + \xi(1-\xi)] \right. \\ & \left. - 1 + 2\xi(2-\xi)(1-\xi^2) \right\} \log [1 + u \xi(1-\xi)] \end{aligned}$$

$$\gamma_{RU}(u) = \frac{1}{12} - \frac{1}{2} \int_0^1 d\xi \left[\frac{1}{u} - \frac{1}{2} + \xi(1-\xi) \right] \log [1 + u \xi(1-\xi)]$$

$$\gamma_U(u) = -\frac{1}{2} \int_0^1 d\xi \log [1 + u \xi(1-\xi)]$$

$$\gamma_\Omega(u) = \frac{1}{12} - \frac{1}{2} \int_0^1 d\xi \left[\frac{1}{u} + \xi(1-\xi) \right] \log [1 + u \xi(1-\xi)]$$

$$u \equiv \frac{-\square}{m^2}$$

Curvature expansion

- $+ \frac{1}{2} \circlearrowleft = -\frac{1}{2(4\pi)^2} \int d^4x \sqrt{g} \operatorname{tr} \left[\mathbf{1} R_{\mu\nu} \gamma_{Ric} \left(\frac{-\square}{m^2} \right) R^{\mu\nu} + \frac{1}{120} R \gamma_R \left(\frac{-\square}{m^2} \right) R \right.$
 $\left. - \frac{1}{6} R \gamma_{RU} \left(\frac{-\square}{m^2} \right) \mathbf{U} + \frac{1}{2} \mathbf{U} \gamma_U \left(\frac{-\square}{m^2} \right) \mathbf{U} + \frac{1}{12} \boldsymbol{\Omega}_{\mu\nu} \gamma_\Omega \left(\frac{-\square}{m^2} \right) \boldsymbol{\Omega}^{\mu\nu} \right]$

Large energy expansion $u \gg 1$

$$\gamma_{Ric}(u) = -\frac{u}{840} + \frac{u^2}{15120} - \frac{u^3}{166320} + O(u^4)$$

$$\gamma_R(u) = -\frac{u}{336} + \frac{11u^2}{30240} - \frac{19u^3}{332640} + O(u^4)$$

$$\gamma_{RU}(u) = \frac{u}{30} - \frac{u^2}{280} + \frac{u^3}{1890} + O(u^4)$$

$$\gamma_U(u) = -\frac{u}{12} + \frac{u^2}{120} - \frac{u^3}{840} + O(u^4)$$

$$\gamma_\Omega(u) = -\frac{u}{120} + \frac{u^2}{1680} - \frac{u^3}{15120} + O(u^4)$$

$$u \equiv \frac{-\square}{m^2}$$

Curvature expansion

- $+ \frac{1}{2} \circ = -\frac{1}{2(4\pi)^2} \int d^4x \sqrt{g} \operatorname{tr} \left[\mathbf{1} R_{\mu\nu} \gamma_{Ric} \left(\frac{-\square}{m^2} \right) R^{\mu\nu} + \frac{1}{120} R \gamma_R \left(\frac{-\square}{m^2} \right) R \right.$
 $\left. - \frac{1}{6} R \gamma_{RU} \left(\frac{-\square}{m^2} \right) \mathbf{U} + \frac{1}{2} \mathbf{U} \gamma_U \left(\frac{-\square}{m^2} \right) \mathbf{U} + \frac{1}{12} \boldsymbol{\Omega}_{\mu\nu} \gamma_\Omega \left(\frac{-\square}{m^2} \right) \boldsymbol{\Omega}^{\mu\nu} \right]$

Low energy expansion $u \ll 1$

$$\gamma_{Ric}(u) = \frac{23}{450} - \frac{1}{60} \log u + \frac{5}{18u} - \frac{\log u}{6u} + \frac{1}{4u^2} - \frac{\log u}{2u^2} + O\left(\frac{1}{u^3}\right)$$

$$\gamma_R(u) = \frac{1}{1800} - \frac{1}{120} \log u - \frac{2}{9u} + \frac{\log u}{12u} + \frac{1}{8u^2} + \frac{\log u}{4u^2} + O\left(\frac{1}{u^3}\right)$$

$$\gamma_{RU}(u) = -\frac{5}{18} + \frac{1}{6} \log u + \frac{1}{u} - \frac{1}{2u^2} - \frac{\log u}{u^2} + O\left(\frac{1}{u^3}\right)$$

$$\gamma_U(u) = 1 - \frac{1}{2} \log u - \frac{1}{u} - \frac{\log u}{u} - \frac{1}{2u^2} + \frac{\log u}{u^2} + O\left(\frac{1}{u^3}\right)$$

$$\gamma_\Omega(u) = \frac{2}{9} - \frac{1}{12} \log u + \frac{1}{2u} - \frac{\log u}{2u} - \frac{3}{4u^2} - \frac{\log u}{2u^2} + O\left(\frac{1}{u^3}\right)$$

$$u \equiv \frac{-\square}{m^2}$$

$$C_{\alpha \beta \gamma \delta}=0$$

Cosmological effective action

$$\Gamma[g] = \frac{1}{16\pi G} \int d^4x \sqrt{-g} (R - 2\Lambda) + \xi \int d^4x \sqrt{-g} R^2 + \int d^4x \sqrt{-g} RF(\square)R$$

$$C_{\alpha\beta\gamma\delta} = 0$$

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$$F(\square) = \alpha \log \frac{-\square}{m^2}$$

$$+ \beta \frac{m^2}{-\square}$$

$\alpha, \beta, \gamma, \delta$

are calculable
constants
depending on
effective gravitons
and matter content

$$+ \gamma \frac{m^2}{-\square} \log \frac{-\square}{m^2}$$

$$+ \delta \frac{m^4}{(-\square)^2}$$

+ ...

$$C_{\alpha\beta\gamma\delta} = 0$$

Cosmological effective action

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Leading logs

J. F. Donoghue and B. K. El-Menoufi, Phys. Rev. D 89, 104062 (2014)

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Non-local cosmology

S. Deser and R. P. Woodard, Phys. Rev. Lett. 99, 111301 (2007)

Non-local gravity and dark energy

M. Maggiore and M. Mancarella, Phys. Rev. D 90, 023005 (2014).

Effective non-local cosmology

A. C. and K. J. Jain in preparation

Effective Friedmann equations

$$G_{\mu\nu} + \Delta G_{\mu\nu} = 8\pi G T_{\mu\nu}$$

Effective Friedmann equations

$$G_{\mu\nu} + \Delta G_{\mu\nu} = 8\pi G T_{\mu\nu}$$



$$F(\square) = \beta \frac{m^2}{-\square} \quad \text{and} \quad ds^2 = -dt^2 + a^2(t)d\mathbf{x}^2$$

$$H^2 - 16\pi G \beta m^2 \left(\frac{1}{6} \dot{U}^2 - 2H\dot{U} - 2H^2 U \right) = \frac{8\pi G}{3} \rho$$

$$\ddot{U} + 3H\dot{U} = 6 \left(2H^2 + \dot{H} \right)$$

Effective Friedmann equations

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$$a(t) = (t/t_0)^{2/3} \quad \text{or} \quad a(t) = a_0 e^{\sqrt{\frac{\Lambda}{3}} t}$$

$$H^2 + \frac{128\pi G}{9} \beta m^2 \Lambda \left(1 + \sqrt{3\Lambda} (t - t_0) - e^{-2\sqrt{3\Lambda}(t-t_0)} \right) = \frac{\Lambda}{3}$$

$$H^2 + \frac{128\pi G}{27} \frac{\beta m^2}{t^2} \left(1 - \frac{t_0^2}{t^2} - 4 \log \frac{t_0}{t} \right) = \frac{8\pi G}{3} \rho_m(t_0) \left(\frac{t_0}{t} \right)^2$$

Dark Energy

$$\log \frac{-\square}{m^2} \quad \rho_{DE}^{\alpha,m}(t) = -\frac{32\alpha}{t^4} \left[\log mt + \log \left(\frac{t}{t_0} - 1 \right) + \frac{2}{3} \left(\frac{t}{t_0} - 1 \right) \right]$$

$$\frac{m^2}{-\square} \quad \rho_{DE}^{\beta,\Lambda}(t) = -\frac{16}{3}\beta m^2 \Lambda \left(1 + \sqrt{3\Lambda} (t - t_0) - e^{-2\sqrt{3\Lambda}(t-t_0)} \right)$$

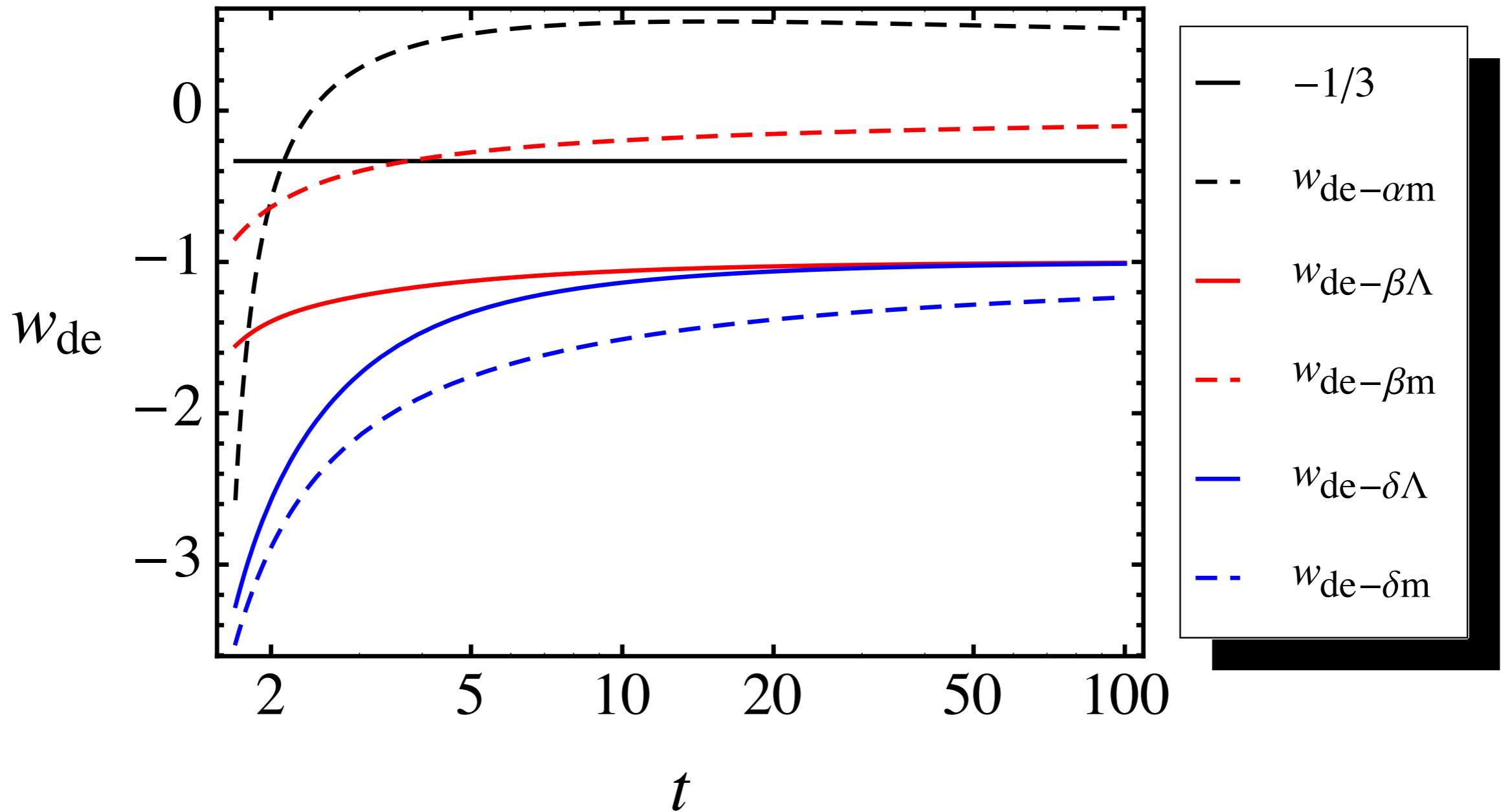
$$\rho_{DE}^{\beta,m}(t) = -\frac{16}{9}\beta m^2 \frac{1}{t^2} \left(1 - \frac{t_0^2}{t^2} - 4 \log \frac{t_0}{t} \right)$$

$$H^2 = \frac{8\pi G}{3} (\rho + \rho_{DE})$$

$$\frac{m^4}{\square^2} \quad \rho_{DE}^{\delta,\Lambda}(t) = \frac{16}{9}\delta m^4 \left\{ 4 - 3\sqrt{3\Lambda} (t - t_0) + 3\Lambda(t - t_0)^2 - \left[8 - \sqrt{3\Lambda} (t - t_0) \right] e^{-\sqrt{3\Lambda}(t-t_0)} + \left[4 + 2\sqrt{3\Lambda} (t - t_0) \right] e^{-2\sqrt{3\Lambda}(t-t_0)} \right\}$$

$$\rho_{DE}^{\delta,m}(t) = 12\delta m^4 \left\{ -\frac{119}{243} - \frac{26}{81} \log \frac{t_0}{t} + \frac{t_0}{t} \left(\frac{130}{243} - \frac{8}{81} \log \frac{t_0}{t} \right) - \frac{1}{27} \left(\frac{t_0}{t} \right)^2 + \frac{2}{243} \left(\frac{t_0}{t} \right)^3 - \frac{4}{243} \left(\frac{t_0}{t} \right)^4 \right\}$$

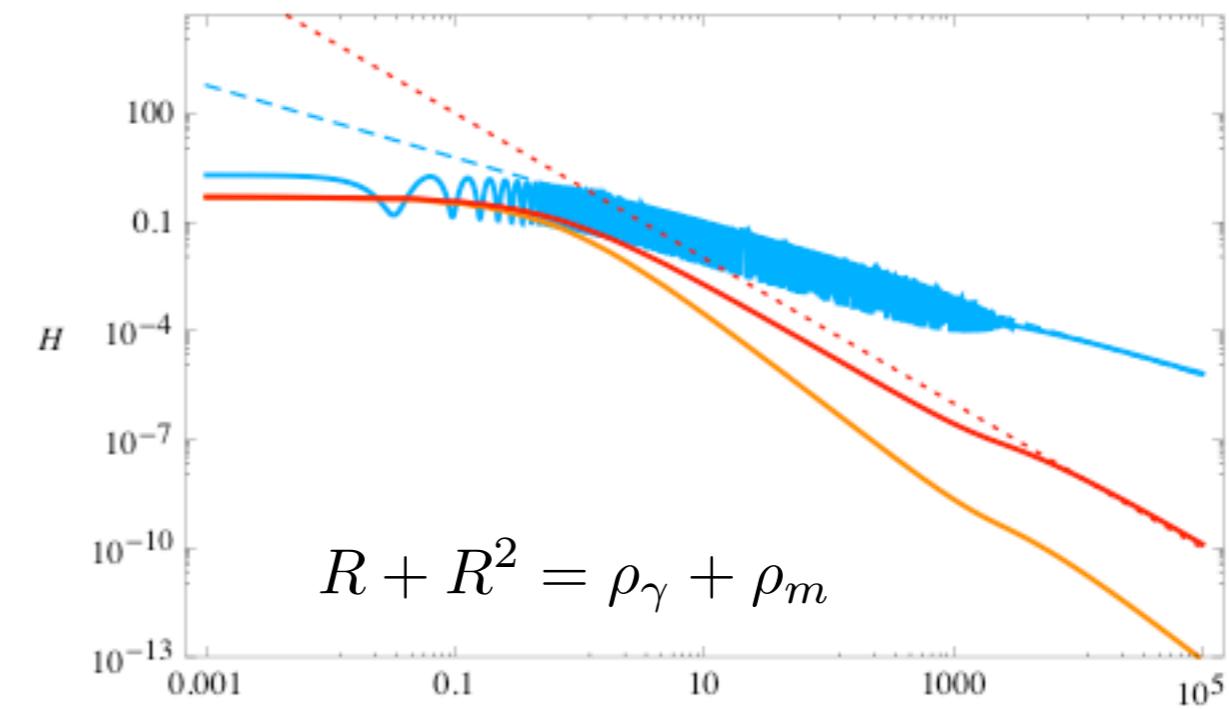
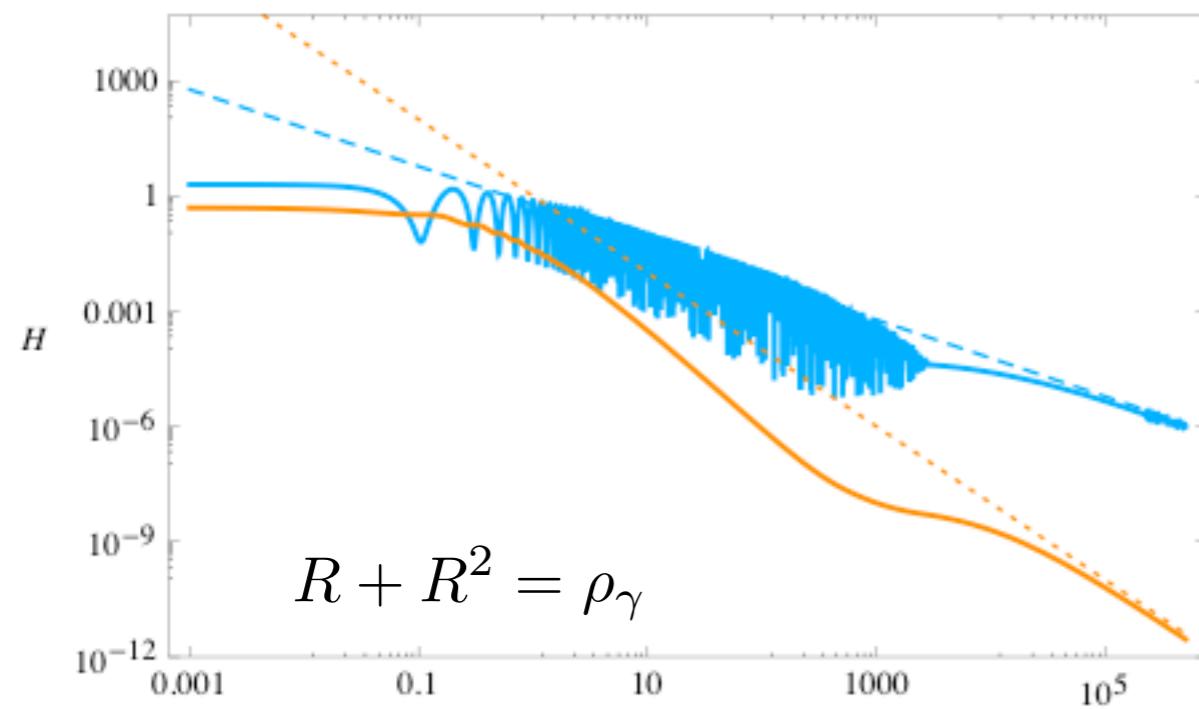
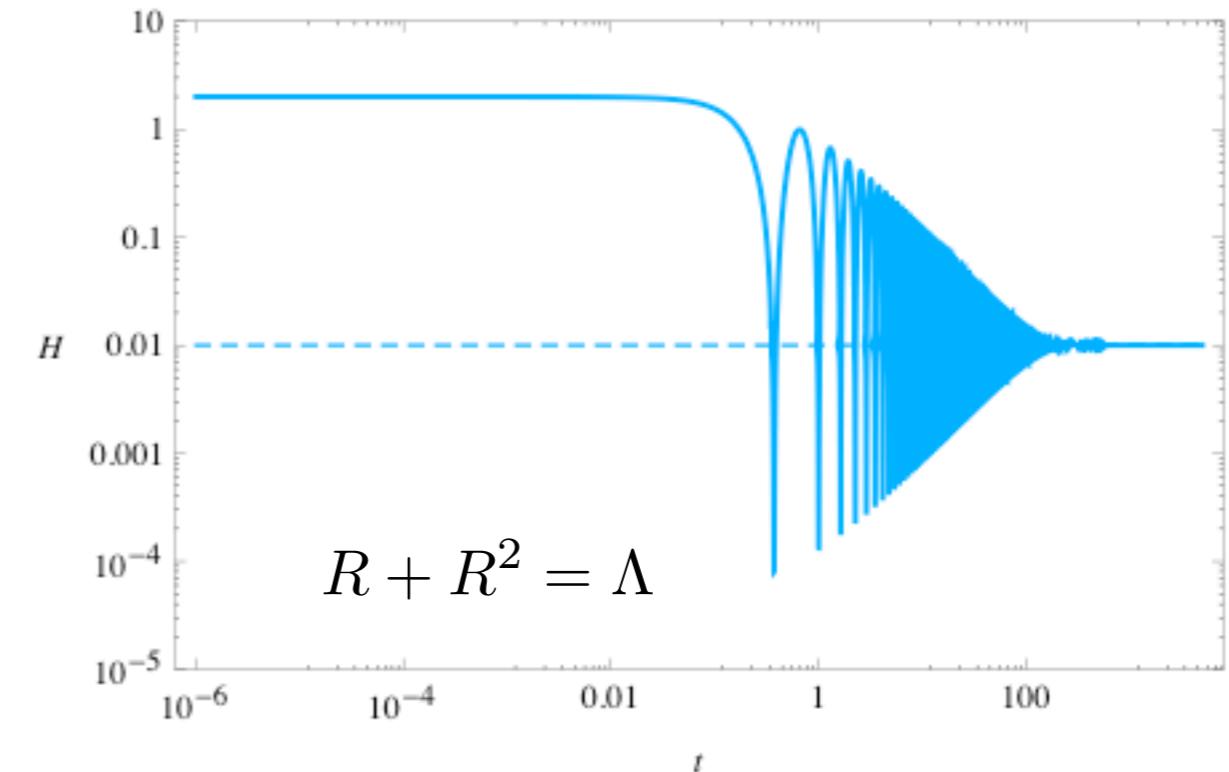
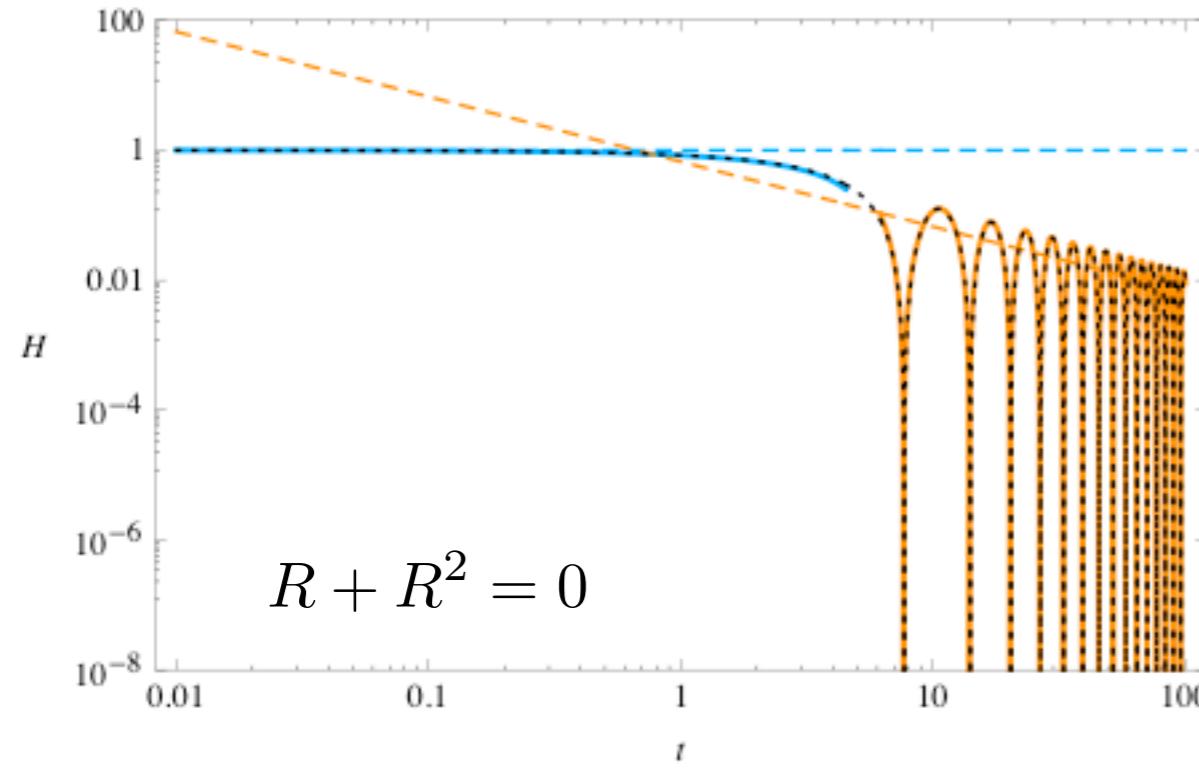
Dark Energy



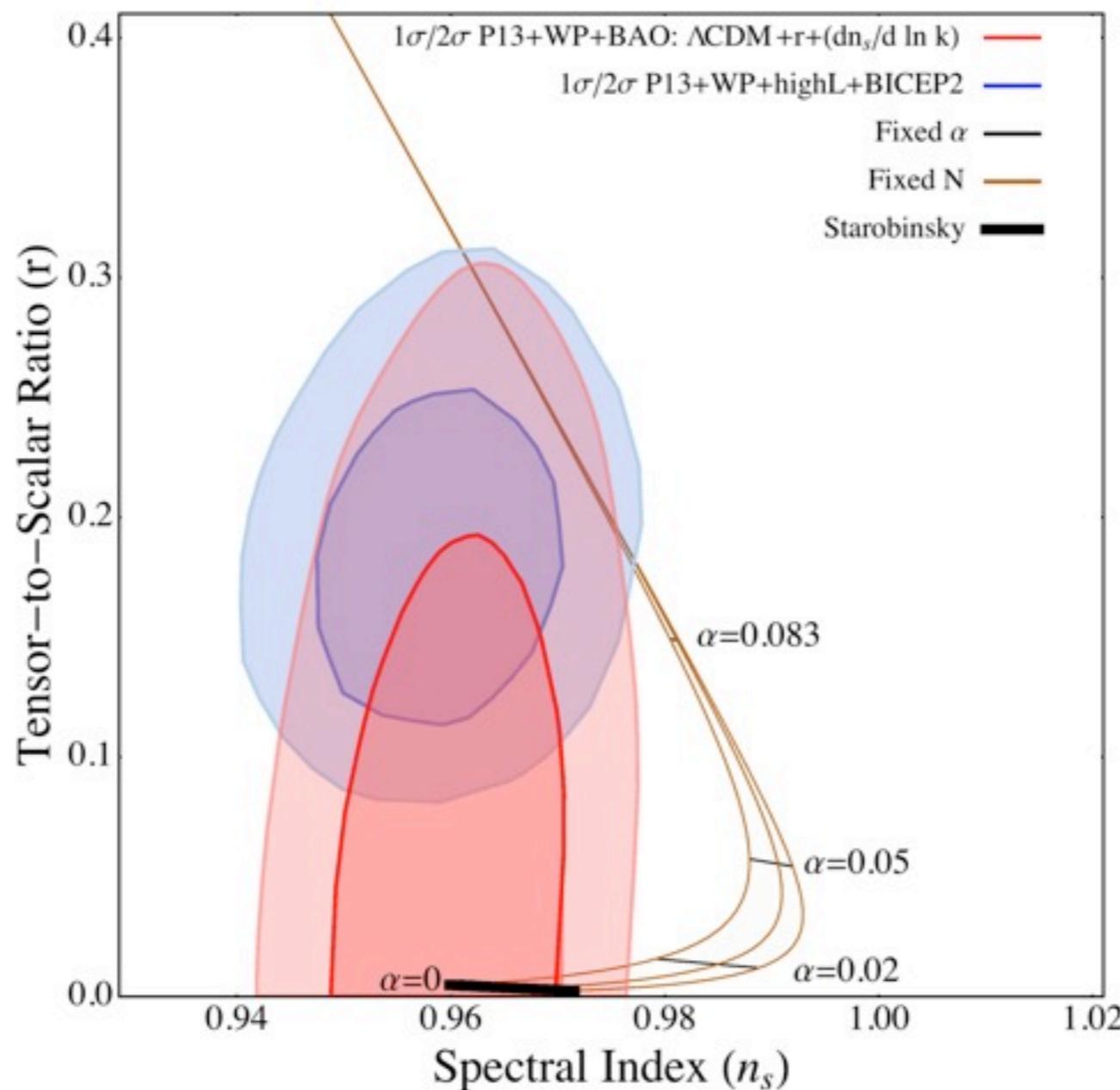
$$\dot{\rho}_{DE} + 3H(1 + w_{DE})\rho_{DE} = 0$$

$$w_{DE} = -1 - \frac{1}{3H} \left(\frac{\dot{\rho}_{DE}}{\rho_{DE}} \right)$$

Effective Friedmann equations



Marginally deformed Starobinsky



$$F(R) = \log \frac{R}{m^2}$$

Leading quantum corrections to tensor-to-scalar ratio

A. C. J. Joergensen, F. Sannino and O. Svendsen, JHEP 1502, 050 (2015)

Conclusions and Outlook

- Compute all LO terms
- Renormalization of NLO
- Conformal anomaly contribution
- Apply to cosmology
- Apply to stars/black holes
- Add the SM and constrain BSM
- Connection with high energy quantum gravity
- Falsify!

Thank you