Statistical significance in CP violation

Mattias Blennow
emb@kth.se

KTH Theoretical Physics

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Parameter estimation sensitivity

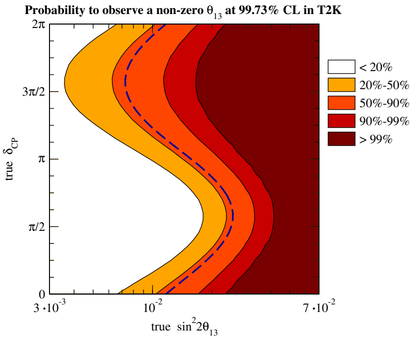
- Define the test statistic "$\Delta\chi^2$"

$$\Delta\chi^2(\theta) = -2\log\left[\frac{\mathcal{L}(\theta|d)}{\sup_{\theta'}\mathcal{L}(\theta'|d)}\right]$$

- Assume it is χ^2 distributed with n degrees of freedom
- Use the data set without statistical fluctuations (Asimov data)
- Quote result

The interpretation

- $\Delta\chi^2$ *is* asymptotically χ^2 (Wilks' theorem)
- The Asimov data (expected data without fluctuations) is representative
- Several requirements, not always fulfilled

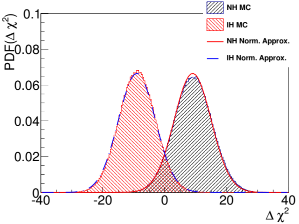


Schwetz, Phys.Lett. B648 (2007) 54

For the mass ordering

- Mass ordering is not nested
- Wilks' theorem not applicable
- Test statistic

$$T = \chi^2_{\mathrm{IO}} - \chi^2_{\mathrm{NO}}$$



Qian, *et al.*, Phys.Rev. D86 (2012) 113011

- T is approximately Gaussian for many situations

$$T \simeq \mathcal{N}(T_0, 2\sqrt{T_0})$$

T_0 = value for Asimov data

What is sensitivity?

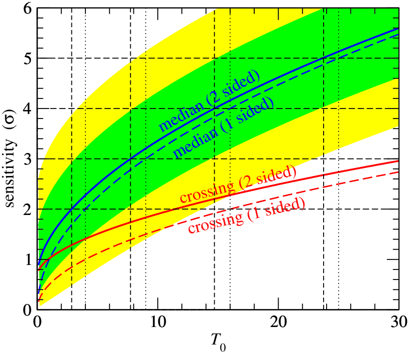
Sensitivity (median)

What is the expected rejection of a false ordering? (Given a parameter set)

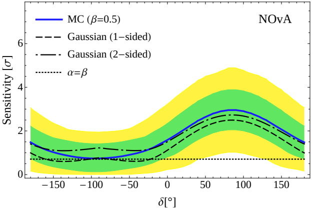
Interpretation:

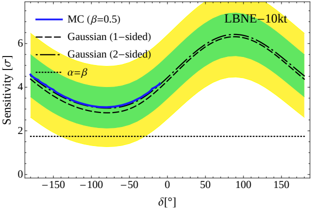
- It is *representative* for how well the experiment will do
- 50 % probability of not reaching it
- 50 % probability of *doing better*
- *Not* 50 % probability of "being wrong"
- Not the only relevant quantity, distribution matters (do Brazilian bands!)

Mass ordering results



MB, Coloma, Huber, Schwetz, JHEP 03(2014)028





Other measurements

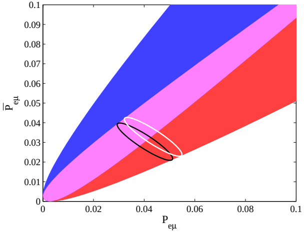
CP violation

- Nested hypothesis
- Does not mean that Wilk's theorem holds, cyclic parameters
- Rest of this talk

θ_{23} octant

- Degeneracies closer
- Wilk's theorem still violated
- A priori, a dedicated study is needed

Setup for CP violation



Blennow, Smirnov, Adv.High Energy Phys. 2013 (2013) 972485

295 km, 0.65 GeV

- Test statistic

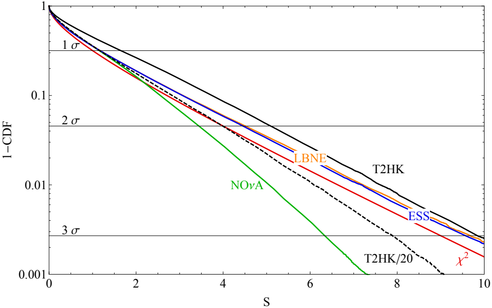
$$S = \min_{\delta=0,\pi} \chi^2 - \min_{\text{global}} \chi^2$$

- Why not necessarily gaussian?
  - Cyclic parameter
  - Several points in null hypothesis ($\delta = 0, \pi$)
  - Degeneracies
- Distribution should *always* be checked or argued for

Critical values

Expectation from null hypothesis

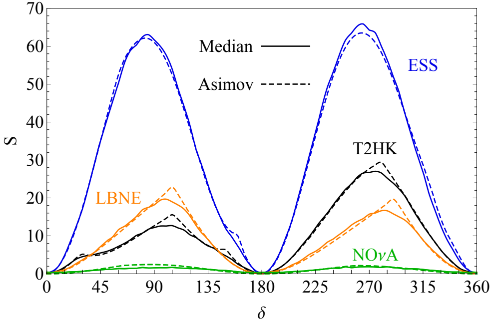
- Red line shows the χ^2 distribution
- NOvA to lower cutoff values
- More sensitive experiments to higher cutoff values



MB, Coloma, Fernandez-Martinez, JHEP 1503 (2015) 005

Median deviations

Need to consider the *expected* outcome

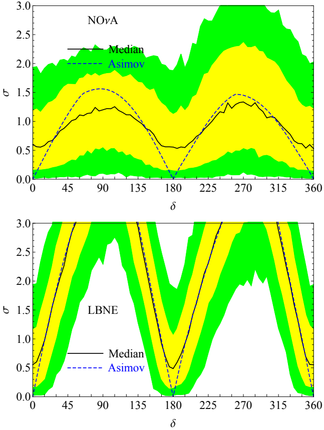


MB, Coloma, Fernandez-Martinez, JHEP 1503 (2015) 005

- Agrees quite well for most experiments
- Lower than Asimov data for NOvA
- Depending on δ for other experiments

Sensitivity results

Combining the distributions with the cutoffs



MB, Coloma, Fernandez-Martinez, JHEP 1503 (2015) 005

Handling of nuisance parameters

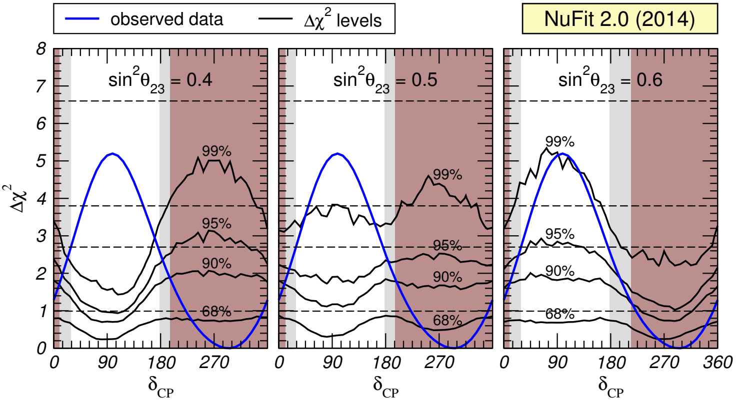
Systematics and previous measurements:

- Addition to the χ^2 function

$$\chi^2(\theta,\xi) = \chi_0^2(\theta,\xi) + \frac{(\xi-\bar{\xi})^2}{\sigma_\xi^2}$$

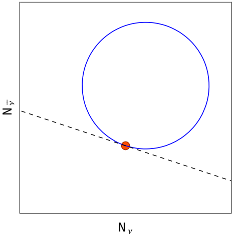
- ξ is the fit value of the nuisance parameter
- $\bar{\xi}$ is the experimental measurement or theoretical prediction
- A priori: Should calibrate χ^2 for all true values of $\xi_{\rm true}$
- In reality: Little dependence on the true value, calibrate for $\xi_{\rm true} = \bar{\xi}$ for existing experiments

Current hints



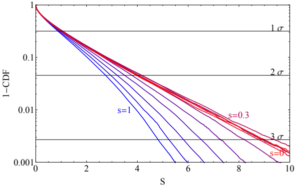
Gonzalez-Garcia, Maltoni, Schwetz, JHEP 1411 (2014) 052
www.nu-fit.org, 2014

Heuristic interpretation



- Parameter space is curved
- Do not expect χ^2
- Can we understand the deviations?

- Large errors on rates: Difference between best fit and null hypothesis small
- Medium errors: Curvature plays a role
- Small errors: Possible outcomes essentially linearly related to δ



Summary and conclusions

- Wilks' theorem is not a priori applicable to the neutrino CP violation, the test statistic is not χ^2 distributed
- More precise experiments $\to \chi^2$
- Critical values will depend on the experiments
- Generally: Lower critical values for low precision experiments
- Also expect lower χ^2 than Asimov for those
- The usual Asimov $+ \chi^2$ approximation is a relatively good estimator