The Strong CP Problem and Axions

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The $U(1)_A$ Problem of QCD

- In the 1970's the strong interactions had a puzzling problem, which became particularly clear with the development of QCD.
- The QCD Lagrangian for N flavors

$$L_{QCD} = -1/4 F_a^{\mu\nu} F_{a\mu\nu} - \Sigma_f \, \bar{q}_f (-i\gamma^\mu D_\mu + m_f) \, q_f$$

in the limit $m_f \rightarrow 0$ has a large global symmetry: $U(N)_V \times U(N)_A$

$$q_f \rightarrow [e^{i\alpha_a T_a/2}]_{ff'} \, q_{f'} \;\; ; \;\; q_f \rightarrow [e^{i\alpha_a T_a \gamma_5/2}]_{ff'} \, q_{f'}$$

Vector Axial

- Since $m_u, m_d << \Lambda_{QCD}$, for these quarks the $m_f \rightarrow 0$ limit is sensible. Thus one expect strong interactions to be approximately $U(2)_V \times U(2)_A$ invariant.

- Indeed, one knows experimentally that

$$U(2)_V = SU(2)_V \times U(1)_V \equiv \text{Isospin x Baryon \#}$$

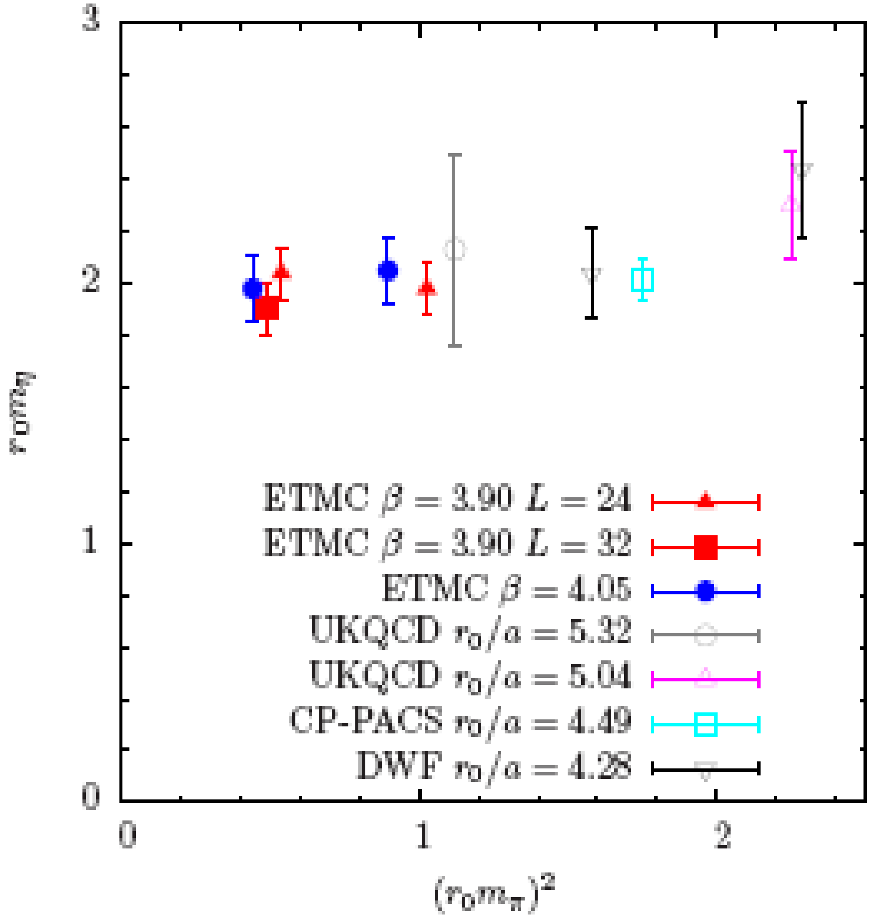
is a good approximate symmetry of nature

$\Rightarrow$ (p, n) and ($\pi^{\pm}$, π°) multiplets in spectrum

- For axial symmetries, however, things are different. Dynamically, quark condensates $<\bar{u}u> = <\bar{d}d> \neq 0$ form and break $U(2)_A$ down spontaneously and, as a result, there are no mixed parity multiplets

- However, because $U(2)_A$ is a spontaneously broken symmetry, one expects now the appearance in the spectrum of approximate Nambu-Goldstone bosons, with $m \approx 0$ [$m \to 0$ as $m_u, m_d \to 0$]
- For $U(2)_A$ one expects 4 such bosons (π, η). Although pions are light, $m_\pi \approx 0$, there is no sign of another light state in the hadronic spectrum, since $m^2_\eta >> m^2_\pi$.
- Weinberg dubbed this the $U(1)_A$ problem and suggested that, somehow, there was no $U(1)_A$ symmetry in the strong interactions

That there is no $U(1)_A$ symmetry emerges explicitly in lattice QCD calculations, which show that indeed, as $m_\pi \to 0$, $m_\eta \to$ constant



(b) The η_2 mass (the analogue of the η' mass for two flavours of quarks) as a function of the pseudo scalar mass. The flatness in the mass dependence allows an estimate at the physical point of $\eta_2 \approx 865$MeV.

- It is useful to describe the $U(1)_A$ problem in the language of Chiral Perturbation Theory, which describes the QCD dynamics for the (π, η)- sector
- The effective Chiral Lagrangian needs to be augmented by an additional term which breaks explicitly $U(1)_A$, beyond the breaking term induced by the quark mass terms.
- Defining $\Sigma = \exp i/F_\pi [\tau_a \pi_a + \eta]$ and including a symmetry breaking pion mass $m^2_\pi \sim (m_u + m_d)$ one has:

$$L_{eff} = \tfrac{1}{4}F^2_\pi \, Tr\, \partial_\mu \Sigma\, \partial^\mu \Sigma^\dagger + \tfrac{1}{4}F^2_\pi \, m^2_\pi \, Tr\, (\Sigma + \Sigma^\dagger) - \tfrac{1}{2} M^2_o \, \eta^2$$

- Provided $M^2_o >> m^2_\pi$ this allows $m^2_\eta >> m^2_\pi$, but what is the origin of this last term?

The 't Hooft Solution

- The resolution of the $U(1)_A$ problem is due to 't Hooft who realized the crucial dynamical role played by the gluon pseudoscalar density

$$Q = \frac{g^2}{32\pi^2} F_a^{\mu\nu} \tilde{F}_{a\mu\nu}$$

- Even though in the massless quark limit $U(1)_A$ is an apparent symmetry of the QCD Lagrangian, the current J^μ_5 associated with the $U(1)_A$ symmetry is anomalous [Adler Bell Jackiw]

$$\partial_\mu J_5^\mu = \frac{g^2 N}{32\pi^2} F_a^{\mu\nu} \tilde{F}_{a\mu\nu} = N\, Q$$

where N is the number of massless quarks

- Since Q enters in the anomaly equation, if it is dynamically important the $U(1)_A$ problem should be resolved, because then there is really no conserved $U(1)_A$ current
- This can be checked by explicitly including Q in the Chiral Lagrangian describing the low energy behavior of QCD [Di Vecchia Veneziano]
- Taking into account the anomaly in the $U_A(1)$ current and keeping terms up to $O(Q^2)$ one has:

$$L_{eff} = \tfrac{1}{4}F^2_\pi \,\mathrm{Tr}\, \partial_\mu\Sigma\, \partial^\mu\Sigma^\dagger + \tfrac{1}{4}F^2_\pi\, m^2_\pi\, \mathrm{Tr}\,(\Sigma + \Sigma^\dagger)$$
$$+ \tfrac{1}{2}\, i\, Q\, \mathrm{Tr}\,[\ln \Sigma - \ln \Sigma^\dagger] + [1/ F^2_\pi M^2_o]\, Q^2 + \ldots$$

- In this Lagrangian, Q is essentially a background field and can be eliminated through its equation of motion:

$$Q = -i/4\, [F^2_\pi M^2_o]\, \mathrm{Tr}\,[\ln \Sigma - \ln \Sigma^\dagger] = \tfrac{1}{2}\, [F_\pi M^2_o]\, \eta + \ldots$$

- Using this result for Q, the last two terms in L_{eff} reduce to:

$$\tfrac{1}{2}\, i\, Q\, \mathrm{Tr}\,[\ln \Sigma - \ln \Sigma^{\dagger}] + [1/ F^2_{\pi} M^2_{o}]\, Q^2 \rightarrow -\tfrac{1}{2} M^2_{o}\, \eta^2$$

providing an effective gluonic mass term for the η meson, and thus resolving the $U_A(1)$ problem

- Although one can see directly from the Chiral Lagrangian how the dynamical role of Q removes an apparent Nambu Goldstone boson (the η) from the spectrum, this follows also directly from QCD
- It can be traced to the non trivial properties of the QCD vacuum which involve a new dimensionless parameter – the vacuum angle θ [‘t Hooft]
- I’ll sketch below the principal points

- Because the integral of Q is a topological invariant:

$$\nu = \int d^4x\, Q = \int \frac{g^2}{32\pi^2} F_a^{\mu\nu} \tilde{F}_{a\mu\nu} \text{ with } \nu = 0, \pm 1, \pm 2, \ldots,$$

all transition amplitudes in QCD contain sums over distinct sectors characterized by the winding number ν.

- The contributions of the $\nu \neq 0$ sectors break the $U(1)_A$ symmetry
- Furthermore, one can show that gauge invariance introduces a parameter θ associated with the sum over the distinct ν sectors in the QCD transition amplitudes [$e^{i\nu\theta}$ is Bloch phase]

$$A = \Sigma_\nu\, e^{i\nu\theta} A_\nu$$

- The parameter θ can be connected with the structure of the QCD vacuum and its presence gives an additional contribution to the QCD Lagrangian

- This can be seen as follows. In the case of QCD, by having to sum over the distinct ν sectors, the usual path-integral representation for the vacuum-vacuum transition amplitude is modified to read:

$$_{+}\langle vac|vac\rangle_{-} = \Sigma_{\nu}\, e^{i\nu\theta} \int \delta A\, e^{iS[A]}\, \delta\left(\nu - \int \frac{g^2}{32\pi^2} F_a^{\mu\nu}\tilde{F}_{a\mu\nu}\right)$$

- Denoting the QCD vacuum as | θ >, one can re-interpret the θ term in the sum over ν as an addition to the usual QCD action
- That is:

$$_{+}<\theta|\theta>_{-} = \sum_{\nu}\int \delta A e^{iS_{eff}[A]}\delta(\nu - \frac{g^2}{32\pi^2}\int d^4x F_a^{\mu\nu}\tilde{F}_{a\mu\nu})$$

where

$$S_{eff} = S_{QCD} + \theta\frac{g^2}{32\pi^2}\int d^4x F_a^{\mu\nu}\tilde{F}_{a\mu\nu}$$

The Strong CP Problem and its Resolution

- The resolution of the $U(1)_A$ problem, however, engenders another problem: the strong CP problem
- As we have seen, effectively the QCD vacuum structure adds and extra term to L_{QCD}

$$L_\theta = \theta \frac{g^2}{32\pi^2} F_a^{\mu\nu} \tilde{F}_{a\mu\nu}$$

This term conserves C but violates P and T, and thus it also violates CP

- This is problematic, as there is no evidence of CP violation in the strong interactions!

- In fact, the θ term produces an electric dipole moment for the neutron of order:

$$d_n \approx e\, m_q/M_n^2\, \theta \approx 10^{-16}\, \theta \text{ ecm}$$

- The strong experimental bound $d_n < 2.9 \times 10^{-26}$ ecm requires the angle θ to be very small $\theta < 10^{-9} - 10^{-10}$ [Baluni; Crewther Di Vecchia Veneziano Witten].
- Why θ should be this small is the strong CP problem
- Problem is actually worse if one considers the effect of chiral transformations on the θ-vacuum
- Because of the chiral anomaly, these transformations change the θ-vacuum [Jackiw Rebbi]:

$$e^{i\alpha Q_5} \,|\, \theta > = |\, \theta + \alpha >$$

- If besides QCD one includes the weak interactions, in general the quark mass matrix is non-diagonal and complex

$$L_{Mass} = - \bar{q}_{iR} M_{ij} q_{jL} + h.c.$$

- To diagonalize M one must, among other things, perform a chiral transformation by an angle of Arg det M which, because of the Jackiw Rebbi result, changes θ into

$$\theta_{total} = \theta + \text{Arg det } M$$

- Thus, in full generality, the strong CP problem can be stated as follows: why is the angle θ_{total} , coming from the strong and weak interactions, so small?

- There are only three known classes of solutions to the strong CP problem:

i. Anthropically θ_{total} is small

ii. CP is broken spontaneously and the induced θ_{total} is small

iii. A chiral symmetry drives $\theta_{total} \to 0$

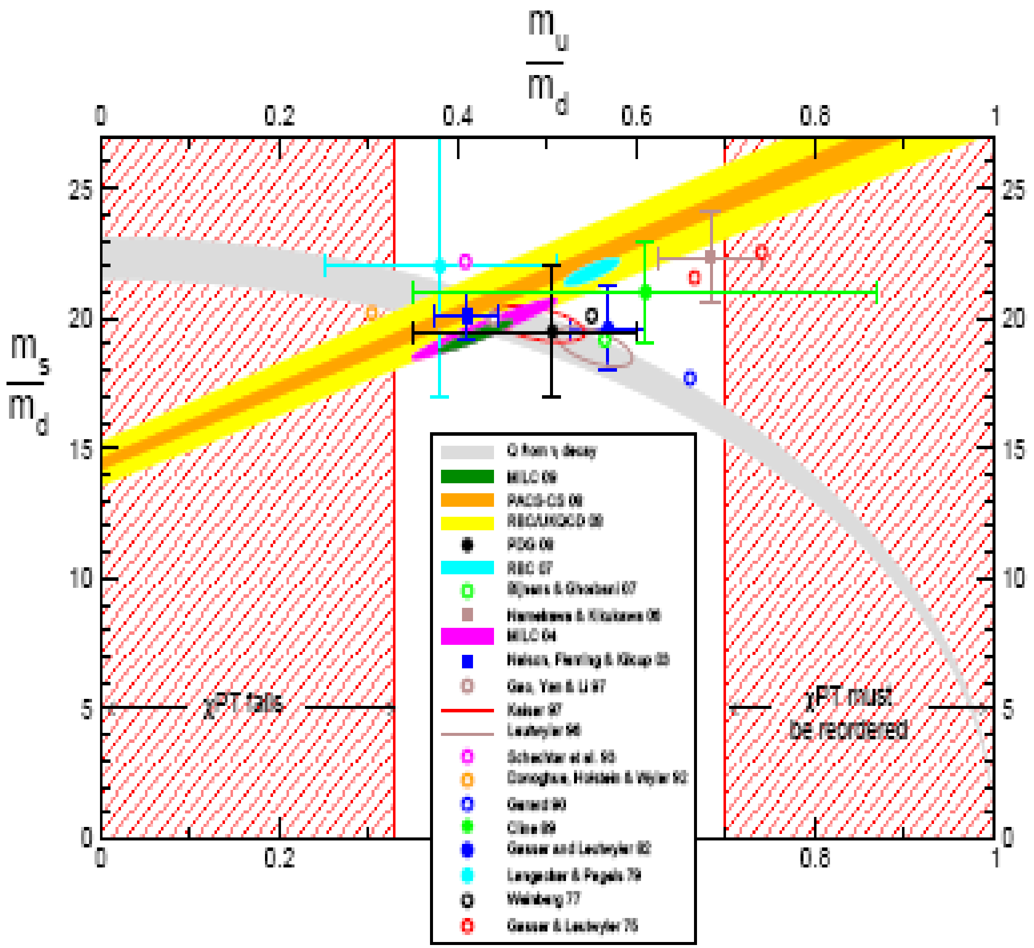
- I will make no comments on i
- Although ii. is interesting, the models which lead to $\theta_{total} \approx 10^{-10}$ are rather complex and often are at odds with the CKM paradigm and/or cosmology
- In my opinion, only iii. is a viable solution, although it necessitates introducing a new global, spontaneously broken, chiral symmetry

- Helen Quinn and I proposed the first prototype chiral solution [38 years ago!] suggesting that the SM had an additional U(1) chiral symmetry (now called $U(1)_{PQ}$) which drives $\theta_{total} \rightarrow 0$
- Very recently 4 other variant chiral solutions have been proposed:
  - Hook and independently Fukuda Harigaya Ibe Yanagida use a Z_2 symmetry which takes SM <-> SM' and an anomalous U(1) symmetry to drive $\theta_{total} \rightarrow 0$
  - Ahn uses a flavored version of $U(1)_{PQ}$ [A_4 x $U(1)_{PQ}$] to accomplish the same
  - Kawasaki Yamada Yanagida use instead [SU(3)x $U(1)_{PQ}$] as a flavor group
- These are all very natural solutions to the strong CP problem, since chirality effectively rotates the θ -vacua away:

$$e^{-i\,\theta Q_5}\,|\,\theta\,\rangle = |\,0\,\rangle$$

- Of course, in principle, this additional chiral symmetry could be intrinsic to QCD, if the u-quark had no mass, $m_u = 0$ [Kaplan Manohar]. However, calculations on the lattice exclude the $m_u = 0$ solution

Leutwyler



MILC Collaboration

rules out m_u=0

at 10σ

Figure 3: Ratios of the light quark masses

Axions and their Role in Cosmology

- Introducing a global $U(1)_{PQ}$ symmetry, which is necessarily spontaneously broken, replaces:

θ $\Rightarrow$ $a(x) / f_a$

Static CP Viol. Angle Dynamical CP conserving Axion field

and, effectively, eliminates CP violation in the strong sector

$$L_\theta = \theta \frac{g^2}{32\pi^2} F_a^{\mu\nu} \widetilde{F}_{a\mu\nu} \Rightarrow L_a = \frac{a}{f_a} \frac{g^2}{32\pi^2} F_a^{\mu\nu} \widetilde{F}_{a\mu\nu}$$

- Here f_a is the scale of the breaking of the $U(1)_{PQ}$ symmetry , while $a(x)$ is the Nambu Goldstone axion field associated with the broken symmetry [Weinberg Wilczek]

- The property and interactions of axions depend on f_a the scale of the breaking of the $U(1)_{PQ}$ symmetry
- In particular, the axion mass, its coupling to two photons and its isoscalar and isovector couplings are inversely proportional to f_a

$$m_a = \lambda_m \, m_a^{st} \, [v_F / f_a] \; ; \quad L_{a\gamma\gamma} = \frac{\alpha}{4\pi} K_{a\gamma\gamma} \frac{a_{\text{phys}}}{f_a} F^{\mu\nu} \widetilde{F}_{\mu\nu}$$

$$\xi_{a\eta} = \lambda_0 \, [f_\pi / f_a] \quad ; \quad \xi_{a\pi} = \lambda_3 \, [f_\pi / f_a]$$

where $m_a^{st} \approx 25$ KeV; $f_\pi = 92$ MeV ; and λ_m ; $K_{a\gamma\gamma}$; λ_0 ; λ_3 are of O(1)

- Initially, one assumed that $f_a = v_F = (\sqrt{2}\, G_F)^{-1/2} \approx 250$ GeV but weak scale axions are ruled out by experiment. For example, one predicts

$$BR(K^+ \to \pi^+ + a) > 1.2 \times 10^{-4} \text{ [Bardeen Peccei Yanagida]}$$

well above the bound obtained at KEK

$$BR(K^+ \to \pi^+ + \text{nothing}) < 3.8 \times 10^{-8}$$

- The choice $f_a = v_F$ is not necessary to solve the strong CP problem
- If $f_a >> v_F$ then the axion is very light, very weakly coupled and very long lived and such invisible axion models remain viable
- These models introduce fields which carry PQ charge but are SU(2)XU(1) singlets . Two different generic models exist:

i) DFSZ Models [Dine Fischler Srednicki; Zhitnisky]

These models add to the PQ model a scalar field σ which carries PQ charge and $f_a = \langle\sigma\rangle >> v_F$

ii) KSVZ Models [Kim; Shifman Vainshtein Zakharov]

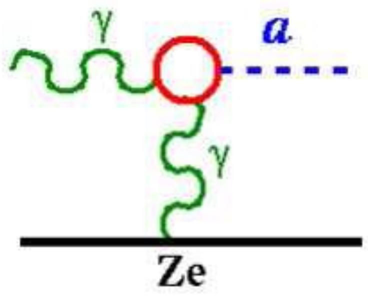
Only a superheavy quark Q and a scalar field σ carry PQ charge.

The dynamics is such that $f_a = \langle\sigma\rangle >> v_F$ and $M_Q \approx f_a$

- For both the KSVZ and the DFSZ models $\lambda_m=1$, hence:

$$m_a = m_a^{st} [v_F / f_a] \approx [10^6 \text{ GeV} / f_a] \; 6.3 \text{ eV}$$

- The KSVZ and DFSZ axions are very light, very weakly coupled and very long-lived, but are not totally invisible
- Upper bounds on m_a can be inferred from astrophysics since axion emission, through Primakoff and other processes, causes energy loss ~ $1/f_a$ affecting stellar evolution.
- Typically these astrophysical bounds, which I will not discuss in detail here, allow axions lighter than



$$m_a \leq 1\text{-}10^{-3} \text{ eV}$$

- Rather remarkably, cosmology gives a lower bound for the axion mass (upper bound on f_a) [Preskill Wise Wilczek; Abbott Sikivie; Dine Fischler] and axions can have a significant cosmological role
- Physics is simple to understand. When Universe goes through the $U(1)_{PQ}$ phase transition at $T \sim f_a >> \Lambda_{QCD}$ the QCD anomaly is ineffective and θ is arbitrary. Eventually, when Universe cools to $T \sim \Lambda_{QCD}$ the axion gets a mass and $\theta \to 0$.
- The coherent $\mathbf{p}_a=0$ axion oscillations towards this minimum contribute to the Universe's energy density and act as cold dark matter
- The detailed results depend on whether the PQ phase transition occurs before or after inflation and I'll sketch the main issues in both cases

Case i: the PQ phase transition happens before (or during) inflation

During inflation the axion field is homogenized over enormous distances. Thus only the evolution of the $p_a=0$ mode is relevant

- A typical calculation of the axion contribution to Universe's energy density [Hannestad et al] then gives

$$\Omega_a h^2 = 0.195\,[f_a/10^{12}\text{ GeV}]^{1.184}\,[\theta_i^2]$$

where θ_i is the initial misalignment angle

- This quantity is bounded by the density of Cold Dark Matter in the Universe:

$$\Omega_{CDM} h^2 = 0.120 \pm 0.003 \qquad \text{WMAP Planck}$$

- If one assumes that axions are the dark matter in the Universe, this then gives a relation between θ_i and f_a :

$$\theta_i = 0.748\,[10^{12}\text{ GeV}/ f_a]^{0.592}$$

- The table below gives some typical values for θ_i and f_a

f_a (GeV)	10^{12}	10^{15}	10^{18}
θ_i	0.75	1.3×10^{-2}	2.1×10^{-4}
$f_a \theta_i$ (GeV)	7.5×10^{11}	1.3×10^{13}	2.1×10^{14}

- One often assumes that θ_i is an average angle $\theta_i^2 = \langle\theta^2\rangle = \pi^2/3$
  Then assuming that axions are the dark matter in the Universe

$$\Omega_a h^2 = \Omega_{CDM} h^2 = 0.120 \pm 0.003$$

gives the following value for the PQ scale and the axion mass:

$$f_a = 0.24 \times 10^{12} \text{ GeV} \text{ and } m_a = 26 \times 10^{-6} \text{ eV}$$

- These results for $f_a\, \theta_i$ give an interesting bound, suggested long ago by Lyth, which originates because inflation induces measurable quantum fluctuations in the axion field
- These isocurvature axion perturbations correspond to fluctuations in the initial misalignment angle θ_i and have a power spectrum given by:

$$\Delta^2_a(k) = [2\,|\delta\theta_i|\,/\,\theta_i\,]^2 = [H_I\,/\pi\,\theta_i\,f_a\,]^2$$

where H_I is the expansion rate during inflation

- Both WMAP and Planck have put bounds on the ratio:

$$\beta_{iso} = \Delta^2_a(k)\,/(\Delta^2_R(k)\; + \Delta^2_a(k)\,)$$

where $\Delta^2_R(k)$ measures the curvature perturbation spectrum.

- At $k = 0.002\ \mathrm{Mpc}^{-1}$ these collaborations find:

$\beta_{iso} < 0.036$ (95% CL) Planck $\beta_{iso} < 0.047$ (95% CL) WMAP

- Using the best fit result of Planck for $\Delta^2_R(k)$:

$$\Delta^2_R(k) = 2.2 \times 10^{-9} \, (k/\, 0.05 \text{ Mpc}^{-1})^{-0.04}$$

the bound on β_{iso} implies a bound on the isocurvature axion perturbations at $k = 0.002 \text{ Mpc}^{-1}$:

$$\Delta^2_a(k) < 9.25 \times 10^{-11}$$

- Hence the fluctuation in the initial misalignment angle is very small:

$$|\delta\theta_i| / \theta_i < 4.8 \times 10^{-6}$$

and there is a strong bound on the expansion rate during inflation:

$$H_I < 3 \times 10^{-5} \, \theta_i \, f_a$$

- For a sensible range of PQ scales [10^{12} GeV $< f_a < 10^{18}$ GeV] this Lyth bound on H_I ranges from 2.25×10^{7} GeV to 6.3×10^{9} GeV .

- The Lyth bound makes only low energy scale inflation models tenable, predicting a very small contribution of the tensor perturbation spectrum $\Delta^2_h(k)$ to the CMB anisotropy.
- This tensor spectrum is given by the ratio of H_I to the Planck mass

$$\Delta^2_h(k) = 2\,(H_I / \pi M_P)^2$$

and, for example, for $f_a = 10^{18}$ GeV, $\Delta^2_h(k) < 5.4 \times 10^{-18}$

- This implies a negligibly small tensor to scalar ratio:

$$r(k) = \Delta^2_h(k) / \Delta^2_R(k) < 2.5 \times 10^{-9}$$

orders of magnitude below the recent joint bound of Planck and BICEP 2 [$r < 0.11$ at $k = 0.002\ \mathrm{Mpc}^{-1}$]

- The Lyth bound can be avoided, if f_a is not fixed during inflation [Linde]
- Imagine that $f_a = f_a(t)$, starting out very large before inflation $f_a > M_P$ and very slowly rolling down to its present value. In this case, the isocurvature perturbations have a power spectrum given by:

$$\Delta^2_a(k) = [H_I/\pi\, \theta_i\, f_a(t)\,]^2 \approx [H_I / \pi\, \theta_i\, M_P]^2$$

and the Lyth bound on H_I is much weakened:

$$H_I < 10^{-4}\, \theta_i\, M_P$$

- This bound is weakest when θ_i is large. For example, if $\theta_i = \langle\theta\rangle = \pi/\sqrt{3}$, then

$$H_I < 2 \times 10^{15} \text{ GeV } [\theta_i /(\pi/\sqrt{3})]$$

similar to the bound from the tensor to scalar ratio r(k) :

$$H_I < 0.4 \times 10^{15} \text{ GeV } [r(k)/0.11]^{1/2}$$

- However, the dynamics after inflation, may render irrelevant the solution proposed by Linde to allow for high scale inflation.
- In fact, if in the “preheating” stage after inflation large fluctuations in the axion field occur, they can lead to a non-thermal restoration of $U(1)_{PQ}$ [Kofman Linde Starobinsky Tkachev]
- To avoid this problem one needs to have, in effect, that:

$$f_a > \delta a$$

- This problem was studied by Kawasaki Yanagida Yoshino who found that, in general inflation models, $U(1)_{PQ}$ restoration could be avoided if:

$$f_a = (10^{12} – 10^{16})\ \mathrm{GeV}$$

Case ii: the PQ phase transition happens after inflation

- Because the PQ phase transition occurred after inflation, no isocurvature fluctuations ensue in this case
- However, as emphasized originally by Sikivie, in this case other dynamical issues arise due to the formation of axionic strings and domain walls, which are not erased by inflation
- At $T \approx f_a$ $U(1)_{PQ}$ gets spontaneously broken, and one-dimensional defects: axionic strings, around which $\theta = a/f_a$ winds by 2π, are formed
- These axionic strings have an energy per unit length $\mu \approx f_a^2 \ln L f_a$, where L is the inter-string separation. These strings decay very efficiently into axions up to temperatures $T \approx \Lambda_{QCD}$

- When $T \approx \Lambda_{QCD}$ $U(1)_{PQ}$ is explicitly broken by the gluon anomaly. However, since under a PQ transformation $\theta \rightarrow \theta + 2\alpha N_{fl}$ (where N_{fl} is the number of quarks carrying $U(1)_{PQ}$) a $Z(N_{fl})$ discrete symmetry is preserved
- Because of this $Z(N_{fl})$ symmetry there are N_{fl} degenerate vacuum states for the axion field. As a result, neighboring regions in the Universe which are in different axion vacua are separated by domain walls.
- At $T \approx \Lambda_{QCD}$, since θ winds by 2π as one goes around an axionic string, the axion field passes through each minimum. As a result each axionic string becomes the edge to N_{fl} domain walls, and the process of axion radiation stops

- If $N_{fl.} > 1$ (like in the DFSZ model) this string-wall network is stable and has a sizable surface energy density

$$\sigma \approx m_a f_a^{\ 2} \approx 6.3 \times 10^9 \text{ GeV}^3 [f_a / 10^{12} \text{ GeV}]$$

- This is a real problem, since the energy density in these walls dissipates slowly as the Universe expands [Zeldovich Kobzarev Okun]

$$\rho_{wall} = \sigma T$$

and ρ_{wall} now would vastly exceed the closure density of the Universe

- This disaster is avoided if $N_{fl} = 1$. Even though there is a unique vacuum, domain walls still form and attach to an axionic string –one wall per string.
- However, as Everett and Vilenkin showed, these walls very rapidly get chopped up into pieces, each enclosed by strings. These structures are unstable and disappear by radiating axions

- There has been an ongoing controversy on how much the axions radiated by these $N_{fl}=1$ walls, as well as by axionic strings, contribute to the Universe's energy density
- A recent compilation of Gondolo and Visinelli gives the following range for the ratio α of the contributions to the energy density of string/wall decay compared to that for $p_a=0$ oscillations :

$$0.16 \text{ [Sikivie et al]} < \alpha = \Omega_{str/wall} / \Omega_{p=0} < 186 \text{ [Battye Shellard]}$$

- In what follows, I will use the recent estimate of Hiramatsu et al:

$$\alpha = 19 \pm 10$$

and note that it alters the prediction for the axion mass (if $\theta_i^2=\pi^2/3$) for CDM axions to:

$$m_a = 26 \times 10^{-6}\, \text{eV}\, (1 + \alpha)^{.884} = 26 \times 10^{-6}\, \text{eV}\, (20 \pm 10)^{.884}$$

pushing it quite close to the astrophysical upper bound

- So there may be trouble also in the $N_{fl}=1$ case!

Concluding Remarks

- Making θ a dynamical parameter solves the strong CP problem naturally, but requires the existence of axions
- In turn, if axions exist, they have interesting cosmological properties that can be probed experimentally
- Expectations vary depending on whether $U(1)_{PQ}$ breaking occurs before or after inflation and on various dynamical assumptions

Before	After
Axion DM: $\theta_i = 0.748\,[10^{12}\mathrm{GeV}/f_a]^{0.592}$	$N_{fl.} = 1$; $\theta_i^2 = \langle\theta^2\rangle$:
$\theta_i^2 = \langle\theta^2\rangle$: $f_a = 2.4 \times 10^{11}$ GeV	$f_a = (1.71\,[1 \pm .5]^{-.884})$
No Lyth Bound: $f_a = (10^{12} - 10^{16})$ GeV	$\times\, 10^{10}$ GeV

Broad search needed!