How to quantify the compatibility of DM direct detection experiments

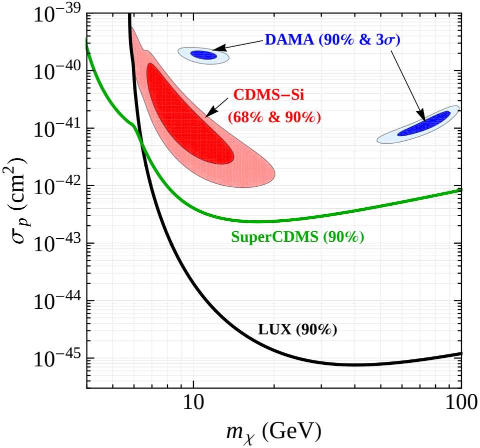
Nassim Bozorgnia

GRAPPA Institute
University of Amsterdam

Based on work done with Thomas Schwetz
[1410.6160]

Dark matter direct detection

- Strong tension between hints for a signal and exclusion limits:



- These kinds of plots assume the Standard Halo Model and a specific DM-nucleus interaction.

Dark matter direct detection

Very little is known about the details of the dark matter (DM) halo in the local neighborhood. $\Rightarrow$ *significant uncertainty when interpreting data from experiments.*

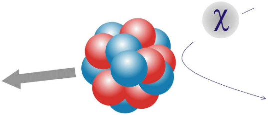
Dark matter direct detection

Very little is known about the details of the dark matter (DM) halo in the local neighborhood. $\Rightarrow$ *significant uncertainty when interpreting data from experiments.*

- **Astrophysics independent methods:** compare different experiments without making assumptions about the DM distribution. $\Rightarrow$ can say *qualitatively* if a $\oplus$ signal is in agreement with a $\ominus$ result.
- **Our aim:** present methods to *quantify* the compatibility of $\oplus$ and $\ominus$ results. $\Rightarrow$ calculate the probability for both experimental outcomes to happen simultaneously, assuming the DM hypothesis.

Dark matter direct detection

- Look for energy deposited in low-background detectors by the scattering of WIMPs in the dark halo of our galaxy.
- WIMP-nucleus collision:



- Minimum WIMP speed required to produce a recoil energy E_R:

$$v_m = \sqrt{\frac{m_A E_R}{2\mu_{\chi A}^2}}$$

The differential event rate

- The differential event rate (event/keV/kg/day):

$$R(E_R, t) = \frac{\rho_\chi}{m_\chi} \frac{1}{m_A} \int_{v > v_m} d^3v \, \frac{d\sigma_A}{dE_R} \; v \; f_{\rm det}(\mathbf{v}, t)$$

- For the standard spin-independent and spin-dependent scattering:

$$R(E_R, t) = \frac{\rho_\chi \sigma_0 \; F^2(E_R)}{2 m_\chi \mu_{\chi A}^2} \; \eta(v_m, t)$$

where

$$\boxed{\eta(v_m, t) \equiv \int_{v > v_m} d^3v \; \frac{f_{\rm det}(\mathbf{v}, t)}{v}}$$

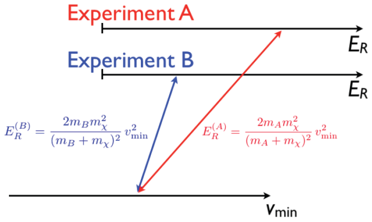
halo integral

Astrophysics independent method

Fox, Kribs, Tait, 1011.1910; Fox, Liu, Weiner, 1011.1915

$$\underbrace{\frac{2m_\chi \mu_{\chi A}^2}{\sigma_0 F^2(E_R)}}_{\text{particle physics}} R(E_R, t) = \underbrace{\rho_\chi \eta(v_m, t)}_{\text{astrophysics}}$$

- r.h.s. is independent of experiment. Compare experiments without specifying the r.h.s.



- Experimental $\oplus$ results $\Rightarrow$ measurement of the halo integral.
- Experimental $\ominus$ results $\Rightarrow$ upper bound on the halo integral.

Upper bound on η from $\ominus$ results

- The predicted number of events in a detected energy interval $[E_1, E_2]$:

$$N^{\text{pred}}_{[E_1,E2]} = MT\ A^2 \int_0^\infty dE_R F^2(E_R) G_{[E_1,E_2]}(E_R) \tilde{\eta}(v_m)$$

Upper bound on η from $\ominus$ results

- The predicted number of events in a detected energy interval $[E_1, E_2]$:

$$N^{\rm pred}_{[E_1,E2]} = MT\ A^2 \int_0^\infty dE_R F^2(E_R) G_{[E_1,E_2]}(E_R) \tilde{\eta}(v_m)$$

- $\tilde{\eta}(v_m)$ is a decreasing function; the minimum number of events is obtained for a step function.

$$N^{\rm pred}_{[E_1,E2]} > \mu(v_m) = MT\ A^2\ \tilde{\eta}(v_m) \int_0^{E(v_m)} dE_R F^2(E_R) G_{[E_1,E_2]}(E_R)$$

Upper bound on η from $\ominus$ results

- The predicted number of events in a detected energy interval $[E_1, E_2]$:

$$N^{\rm pred}_{[E_1,E2]} = MT\ A^2 \int_0^\infty dE_R F^2(E_R) G_{[E_1,E_2]}(E_R)\tilde{\eta}(v_m)$$

- $\tilde{\eta}(v_m)$ is a decreasing function; the minimum number of events is obtained for a step function.

$$N^{\rm pred}_{[E_1,E2]} > \mu(v_m) = MT\ A^2\ \tilde{\eta}(v_m) \int_0^{E(v_m)} dE_R F^2(E_R) G_{[E_1,E_2]}(E_R)$$

- From $N^{\rm obs}$ in a $\ominus$ result experiment, obtain an upper bound on $\tilde{\eta}$ at CL, by requiring:

$$e^{\mu} \sum_{n=0}^{N^{\rm obs}} \frac{\mu^n}{n!} = 1 - {\rm CL}$$

Compare the bound to $\oplus$ signals

- From N^{obs} and the expected background in a $\oplus$ result experiment, determine the halo integral in a given bin: $\langle \tilde{\eta}(v_m^i) \rangle$.

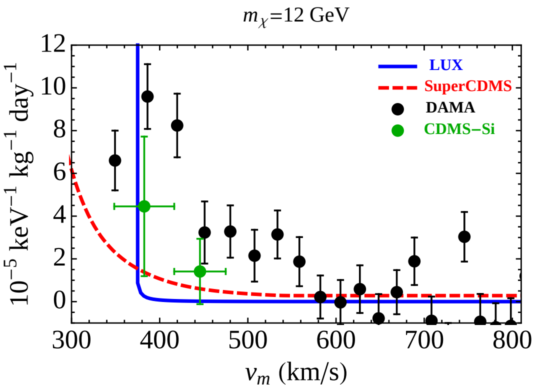
Compare the bound to $\oplus$ signals

- From $N^{\rm obs}$ and the expected background in a $\oplus$ result experiment, determine the halo integral in a given bin: $\langle \tilde{\eta}(v_m^i) \rangle$.

- If the DM interpretation is correct: $\boxed{\langle \tilde{\eta}(v_m) \rangle \leq \tilde{\eta}_{\rm bnd}(v_m)}$

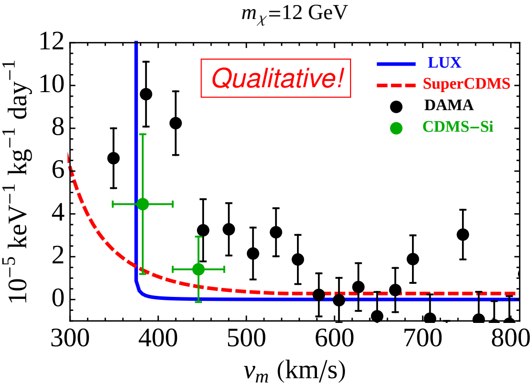
Compare the bound to ⊕ signals

- From $N^{\rm obs}$ and the expected background in a ⊕ result experiment, determine the halo integral in a given bin: $\langle \tilde{\eta}(v_m^i) \rangle$.
- If the DM interpretation is correct: $\langle \tilde{\eta}(v_m) \rangle \leq \tilde{\eta}_{\rm bnd}(v_m)$



Compare the bound to ⊕ signals

- From N^{obs} and the expected background in a ⊕ result experiment, determine the halo integral in a given bin: $\langle\tilde{\eta}(v_m^i)\rangle$.
- If the DM interpretation is correct: $\langle\tilde{\eta}(v_m)\rangle \leq \tilde{\eta}_{\text{bnd}}(v_m)$



Joint probability of $\oplus$ and $\ominus$ results

- *Need a quantitative way of reporting agreement or disagreement between the results of two experiments.*

Joint probability of $\oplus$ and $\ominus$ results

- *Need a quantitative way of reporting agreement or disagreement between the results of two experiments.*
- Consider two experiments:
  - Experiment **A**: excess of events
  - Experiment **B**: null-results
- Two methods to quantify the disagreement between **A** and **B**:
  - **Method 1:** using only total event numbers
  - **Method 2:** using in addition the energy information of the events

Joint probability of $\oplus$ and $\ominus$ results

- $p_{\mathbf{B}}$: prob. to obtain equal or less events than observed by exp. **B**.
- Derive an upper bound on $\tilde{\eta}$ from experiment **B** at $\mathrm{CL} = 1 - p_{\mathbf{B}}$. This gives an upper bound on the predicted number of events in experiment **A**,

$$N^{\mathrm{bnd},\mathbf{A}}_{[E_1,E2]} = MT\ A^2 \int_0^\infty dE_R F^2(E_R) G_{[E_1,E_2]}(E_R)\ \tilde{\eta}^{\mathbf{B}}_{\mathrm{bnd}}(v_m)$$

Joint probability of ⊕ and ⊖ results

- $p_{\mathbf{B}}$: prob. to obtain equal or less events than observed by exp. **B**.
- Derive an upper bound on $\tilde{\eta}$ from experiment **B** at $\mathrm{CL} = 1 - p_{\mathbf{B}}$. This gives an upper bound on the predicted number of events in experiment **A**,

$$N^{\mathrm{bnd},\mathbf{A}}_{[E_1,E2]} = MT\ A^2 \int_0^\infty dE_R F^2(E_R) G_{[E_1,E_2]}(E_R)\ \tilde{\eta}^{\mathbf{B}}_{\mathrm{bnd}}(v_m)$$

- Have to also include the expected number of background events:

$$\mu^{\mathbf{A}}_{\mathrm{bnd}} = N^{\mathrm{bnd},\mathbf{A}}{}_{[E_1,E_2]} + \beta^{\mathbf{A}}_{[E_1,E_2]}$$

$$N^{\mathrm{pred},\mathbf{A}}_{[E_1,E2]} \leq \mu^{\mathbf{A}}_{\mathrm{bnd}}$$

- Note: $N^{\mathrm{bnd},\mathbf{A}}$ depends on the CL that $\tilde{\eta}^{\mathbf{B}}_{\mathrm{bnd}}$ is obtained at, and thus it depends on $p_{\mathbf{B}}$.

Method 1 – total number of events

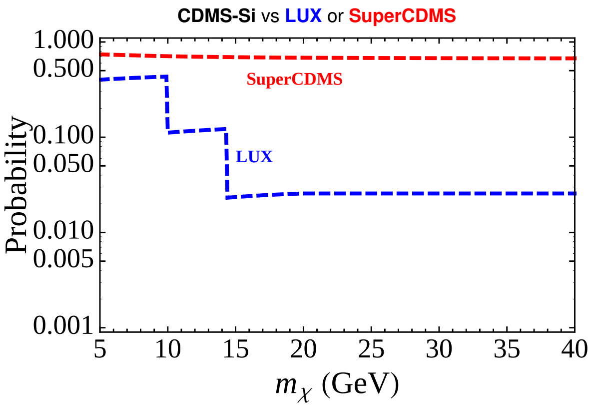
- Probability to obtain N^{obs} events or more by experiment **A**, given the bound:

$$p_{\mathbf{A}} = e^{\mu_{\mathrm{bnd}}^{\mathbf{A}}} \sum_{n=N^{\mathrm{obs},\mathbf{A}}}^{\infty} \frac{(\mu_{\mathrm{bnd}}^{\mathbf{A}})^n}{n!}$$

- Largest joint probability of obtaining the results of experiments **A** and **B**:

$$p_{\mathrm{joint}} = \max_{p_{\mathbf{B}}} \left[p_{\mathbf{A}}(p_{\mathbf{B}})\; p_{\mathbf{B}} \right]$$

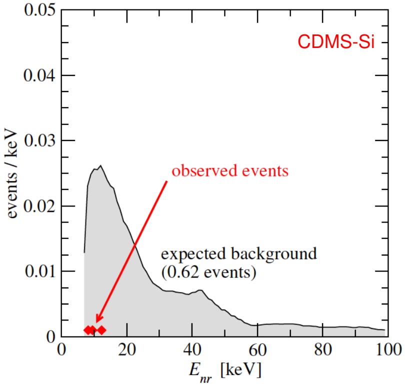
Method 1 – total number of events



- p_{joint} of **CDMS-Si** and **SuperCDMS** ~ 70 % $\Rightarrow$ compatible
- p_{joint} of **CDMS-Si** and **LUX** for $m_\chi \gtrsim 14$ GeV approaches probability for the background-only hypothesis which is 2.57%.

Method 2 – "signal length" method

- Take into account the energy information of the events in addition to the observed number of events.



- Design a method to discriminate a signal predicting clustered events from a more broadly distributed background.

Method 2 – "signal length" method

- Define **"signal length"** (SL) as:

  $\Delta \equiv$ expected # events in the energy interval between the two events with the *lowest* and *highest* energy

  $\mu =$ expected # events in the full energy interval

Method 2 – "signal length" method

- Define **"signal length"** (SL) as:

  $\Delta \equiv$ expected # events in the energy interval between the two events with the *lowest* and *highest* energy

  $\mu =$ expected # events in the full energy interval

- Joint probability of obtaining $N^{\rm obs}$ events or more, and a signal length of size Δ or smaller:

$$P_{\rm SL}(N^{\rm obs}, \Delta|\mu) = e^{-\mu} \sum_{n=N^{\rm obs}}^{\infty} \frac{1}{n!} \left[n\mu\Delta^{n-1} - (n-1)\Delta^{n} \right]$$

Method 2 – "signal length" method

- Define **"signal length"** (SL) as:

  $\Delta \equiv$ expected # events in the energy interval between the two events with the *lowest* and *highest* energy

  $\mu =$ expected # events in the full energy interval

- Joint probability of obtaining $N^{\rm obs}$ events or more, and a signal length of size Δ or smaller:

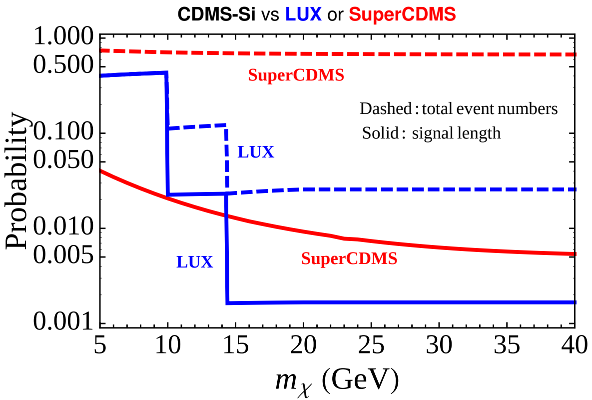
$$P_{\rm SL}(N^{\rm obs}, \Delta|\mu) = e^{-\mu} \sum_{n=N^{\rm obs}}^{\infty} \frac{1}{n!} \left[n\mu\Delta^{n-1} - (n-1)\Delta^{n} \right]$$

- Have upper bounds on μ and Δ from the null-result experiment.

- Combined probability of obtaining results of experiments **A** & **B**:

$$p_{\rm joint} = \max_{p_{\mathbf{B}}} \left[P_{\rm SL}^{\mathbf{A}}(p_{\mathbf{B}})\; p_{\mathbf{B}} \right]$$

Joint probability of ⊕ and ⊖ results



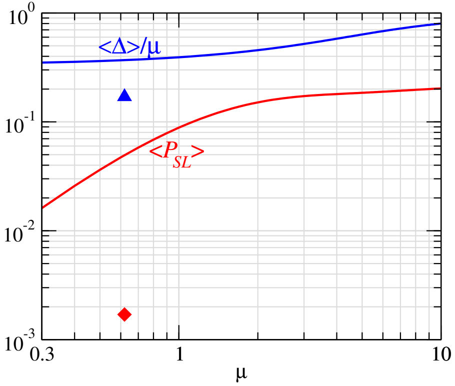
- Using energy information via SL method leads to much stronger tension.
- $p_{\rm joint}$ of **CDMS-Si** and **SuperCDMS** is 4% for 5 GeV, decreasing to 0.5% for 40 GeV.
- $p_{\rm joint}$ of **CDMS-Si** and **LUX** for $m_\chi \gtrsim$ 14 GeV is 0.17% (background-only hypothesis probability).

Summary

- Presented a method to evaluate the joint probability for the outcomes of two potentially conflicting experiments, under the assumption that the DM hypothesis is true, but ***completely independent of assumptions about the DM distribution***.
- For experiments observing an excess of events, the ***signal length method*** was developed to take into account energy information. Low joint probabilities of **CDMS-Si** with **SuperCDMS** and **LUX**.
- Our approach does not require Monte Carlo simulations, and is mostly based on Poisson statistics. The relevant probabilities can be analytically calculated and are relatively simple.

Additional slides

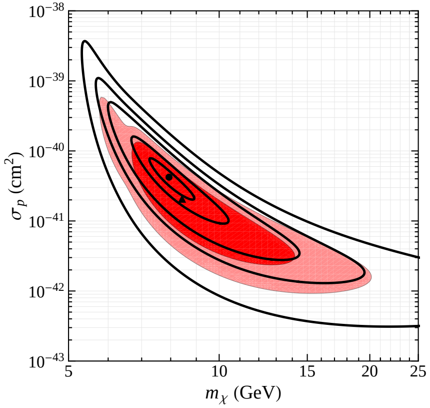
Expectation for $P_{\rm SL}$



- Expect that $P_{\rm SL}$ becomes small for $\mu \lesssim 2$, since it is unlikely to obtain at least 2 events.
- If for a given data the observed value of $P_{\rm SL}$ is much smaller than 0.2, the experimental outcome is considered to be unlikely.

Application to CDMS-Si data for the SHM

- Comparison of SL and Maximum Likelihood methods:

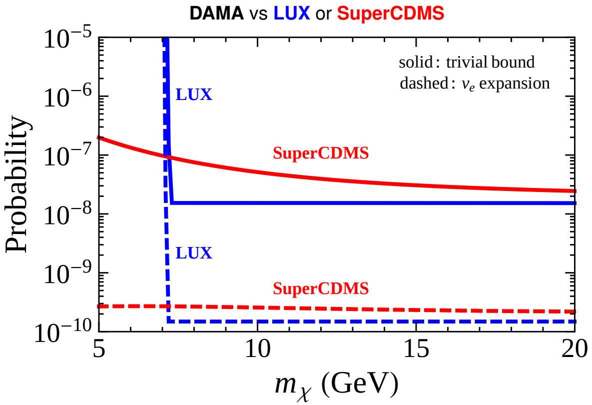


black curves:
$P_{\mathrm{SL}} = 0.01$, 0.05, 0.1, 0.2, 0.25

red regions:
68% and 90% CL regions

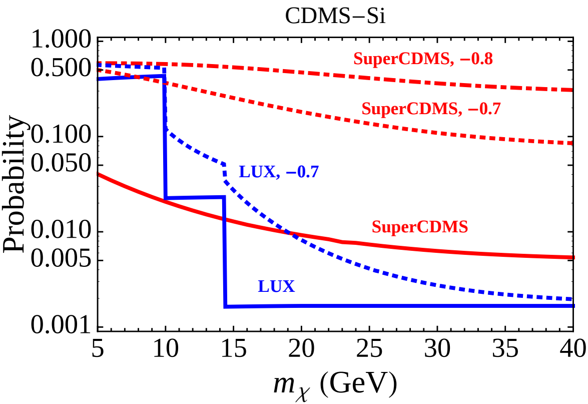
- SL method provides regions where the experimental outcome is likely, while the maximum likelihood method leads to confidence regions relative to the best fit point.

Joint probability of ⊕ and ⊖ results



Isospin violating interactions

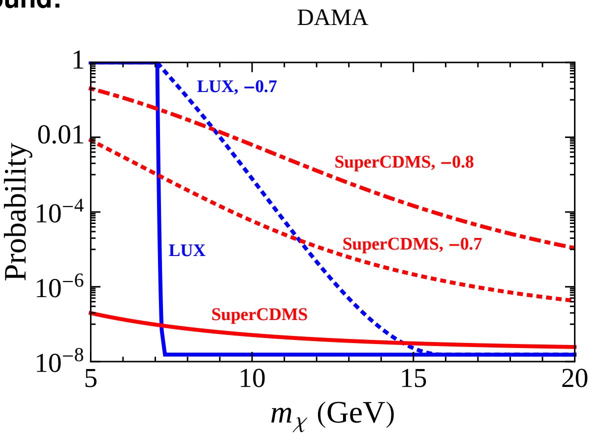
Signal length method:



- $f_n/f_p = -0.8$: **CDMS-Si** consistent with **SuperCDMS**; while **LUX** curve coincides with isospin conserving case.
- $f_n/f_p = -0.7$: P_{joint} of **CDMS-Si** and **SuperCDMS** decreases to 18% at 20 GeV; while P_{joint} with **LUX** remains below 1% for $m_\chi \geq 19$ GeV.

Isospin violating interactions

Trivial bound:



- Compatibility with **LUX** for $f_n/f_p = -0.7$ and with **SuperCDMS** for -0.8 increased by many orders of magnitude for $m_\chi \leq 10$ GeV.
- Compatibility cannot be improved considerably with both **LUX** and **SuperCDMS** for fixed f_n/f_p.