Anomaly–safe discrete groups

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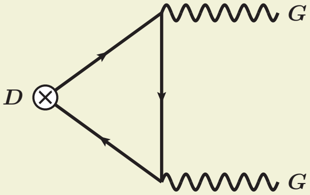
Anomalies of global symmetries

symmetry is anomalous ⟺ it is violated by quantum effects.

- Have in mind:



$D :=$ (discrete) global symmetry, $G :=$ gauge symmetry

- Anomaly is non–perturbative effect $\curvearrowright$ derive directly from path integral (PI) measure. [Fujikawa '79,'80][Araki et al. '07,'08]

$\Rightarrow$ for a global symmetry only DGG anomalies matter.

(**no** cubic anomaly DDD, as long as we do not want to gauge)

Transformation of the path integral (PI)

Anomaly is given by the transformation of the PI measure.

Symmetry(?) trafo: $\Psi_{\mathrm{L}} \longrightarrow U\,\Psi_{\mathrm{L}}$

$$\Longrightarrow \qquad \mathcal{D}\Psi\,\mathcal{D}\overline{\Psi} \longrightarrow J_{\Psi}^{-2}\,\mathcal{D}\Psi\,\mathcal{D}\overline{\Psi}\;,$$

Anomaly $\Leftrightarrow$ non–trivial Jacobian:

$$J_{\Psi}^{-2} \;=\; \det\left(U\right)^{\text{Integer}(\boldsymbol{r}_G^{(\Psi)},\,F_{\mu\nu})}$$

Note: $\det(U) \equiv$ 1D rep & product of 1D reps $\equiv$ 1D rep.

Insight: PI measure transforms as a one–dimensional (1D) representation of the global symmetry $\boldsymbol{D}$!

$\Rightarrow$ If $\boldsymbol{D}$ has none but the trivial 1D representation, it cannot exhibit anomalous behavior !

Consequences

Message 1: **Groups without non–trivial 1D representations are free from (DGG) anomalies.**

This includes:

- all semi–simple Lie groups. [Georgi, Glashow '72]
  (well–known equvialent statement: traces of all generators vanish
  $$\det(U) = \det(\mathrm{e}^{\lambda}) = \mathrm{e}^{\mathrm{tr}\,\lambda} = \mathrm{e}^{0} = 1\,,\text{ e.g. SU}(N) \quad)$$
- all "perfect" discrete groups (perfect $\Leftrightarrow$ only trivial 1D rep).

However, also the opposite conclusion works:

Message 2: **Groups which have non–trivial 1D representations generically can suffer from anomalies.**

- Whether settings based on non–perfect groups are anomalous depends on the field content, as usual.

Summary: Anomaly–(un–)safe discrete groups

Anomaly–safe	**not safe**
All perfect groups, including	All non–perfect groups, including
– all non–Abelian finite simple groups,	– all Abelian groups,
– all sporadic groups,	– all non–Abelian groups with non–trivial 1D reps.
– $\mathrm{PSL}(n>1,\|k\|>3)$,	(incl. A_4, T', T_7, S_n, D_n, ...)
– $\mathrm{SL}(n>1,\|k\|>3)$,	
– A_n for $n\geq 5$.	

Statements based on anomalous "symmetries" have to be taken with care.

Thank You!

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Backup slides

Transformation of the PI measure

$$\Psi_\mathrm{L} \;\longrightarrow\; U\,\Psi_\mathrm{L} \;=\; \mathrm{e}^{\lambda}\,\Psi_\mathrm{L}$$

$$\mathcal{D}\Psi\,\mathcal{D}\overline{\Psi} \;\longrightarrow\; J_\Psi^{-2}\,\mathcal{D}\Psi\,\mathcal{D}\overline{\Psi}$$

non–trivial Jacobian: e.g. [Araki '08]

$$\begin{aligned} J_\Psi^{-2} &= \exp\left\{\mathrm{tr}[\lambda]\cdot\ell(\boldsymbol{R})\cdot\int\mathrm{d}^4x\,\frac{1}{16\pi^2}\,F^{a,\mu\nu}\,\widetilde{F}^a_{\mu\nu}\right\} \\ &= \mathrm{e}^{\mathrm{tr}[\lambda]\cdot\ell(\boldsymbol{R})\cdot 2\cdot p} \\ &= \det\left(U\right)^{\overbrace{}}\underbrace{{}^{2\,\ell(\boldsymbol{R})\cdot p}}_{\in\mathbb{Z}} \end{aligned}$$

$\ell(\boldsymbol{R})$: Dynkin index of $\boldsymbol{R}$ of $\boldsymbol{G}$

$\mathbb{Z}\ni p := \int\mathrm{d}^4x\,\frac{1}{32\pi^2}\,F^{a,\mu\nu}\,\widetilde{F}^a_{\mu\nu}$: winding # of gauge configuration [Belavin '85][Bernard '87]

G	$\mathrm{SU}(N)$	$\mathrm{Sp}(N)$	$\mathrm{SO}(N)$	G_2	F_4	E_6	E_7	E_8
$\ell(\boldsymbol{F})$	$1/2$	$1/2$	1	1	3	3	6	30