Lattice QCD and hadron resonance gas equation of state

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Outline

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Introduction

The Monte Carlo results in lattice QCD for the pressure and energy density at small temperature $T < 155$ MeV and zero baryonic chemical potential are analyzed within the hadron resonance gas model. Two extensions of the ideal hadron resonance gas are considered: the excluded volume model which describes a repulsion of hadrons at short distances and Hagedorn model with the exponential mass spectrum. Considering both of these models *one by one* we do not find the conclusive evidences in favor of *any of them*. The controversial results appear because of rather different sensitivities of the pressure and energy density to both excluded volume and Hagedorn mass spectrum effects. On the other hand, we have found a clear evidence for a *simultaneous* presence of *both of them*. They lead to rather essential contributions: suppression effects for thermodynamical functions of the hadron resonance gas due to the excluded volume effects and enhancement due to the Hagedorn mass spectrum.

Introduction

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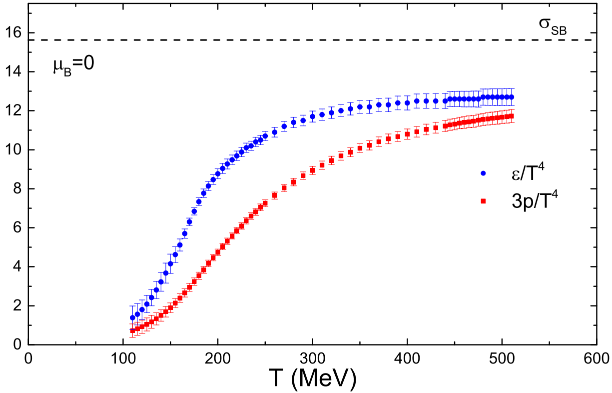
Introduction



The lattice results from [1] for $3p/T^4$ (squares) and ε/T^4 (circles) at zero baryonic chemical potential.

[1] S. Borsanyi, Z. Fodor, C. Hoelbling, S. D. Katz, S. Krieg, and K. K. Szabo, Phys. Lett. B **730**, 99 (2014).

Lattice data v.s. hadron EOS

From Fig. 1 we observe a steep increase of thermodynamical quantities near the crossover temperature T_c. This temperature is estimated in the range of 150-160 MeV. The values of $3p/T^4$ and ε/T^4 in the deconfined quark-gluon phase approach slowly from below the Stefan-Boltzmann limit

$$\frac{3p_{SB}}{T^4} = \frac{\varepsilon_{SB}}{T^4} = \sigma_{SB} ,$$

which equals to (in the 3-flavor QCD)

$$\sigma_{SB} = 19\pi^2/12 \cong 15.6$$

At $T < T_c$ the confined hadron phase emerges. We use the lattice data [1] to constrain EOS of the hadronic matter.

Ideal Hadron Resonance Gas

Ideal HRG pressure

$$p^{\rm id}(T,\mu) = \sum_i p_i^{\rm id}(T,\mu_i)$$

$$= \sum_i \frac{g_i}{3} \int dm\, f_i(m) \int \frac{d^3k}{(2\pi)^3} \frac{\boldsymbol{k}^2}{\sqrt{\boldsymbol{k}^2+m^2}} \left[\exp\left(\frac{\sqrt{\boldsymbol{k}^2+m^2}-\mu_i}{T}\right)+\eta_i\right]^{-1}$$

$\eta_i = -1$ for bosons, $\eta_i = 1$ for fermions, $\eta = 0$ for the Boltzmann statistics

$\mu_i = b_i\,\mu_B + s_i\,\mu_S + q_i\,\mu_Q$ with $b_i = 0, \pm 1$, $s_i = 0, \pm 1, \pm 2, \pm 3$, and $q_i = 0, \pm 1, \pm 2$

Ideal HRG energy density

$$\varepsilon^{\rm id}(T,\mu) = \sum_i \varepsilon_i^{\rm id}(T,\mu_i)$$

$$= \sum_i g_i \int dm\, f_i(m) \int \frac{d^3k}{(2\pi)^3} \sqrt{\boldsymbol{k}^2+m^2} \left[\exp\left(\frac{\sqrt{\boldsymbol{k}^2+m^2}-\mu_i}{T}\right)+\eta_i\right]^{-1}$$

This includes mesons up to $f_2(2340)$ and (anti-)baryons up to $N(2600)$

$f_i(m)$ = "the Breit-Wigner shape of resonance with finite width Γ_i around their average mass m_i"

$f_i(m) = \delta(m - m_i)$ for the stable hadrons

The motivations for HRG description

A description of hadron multiplicities in high-energy nucleus-nucleus collisions shows a surprisingly good agreement between the results of the HRG model and the experimental data, see [2]. In most statistical model formulations the ideal HRG is used. It is argued that a presence of all known resonance states in the thermal system takes into account attractive interactions between hadrons.

[2]
J. Cleymans and H. Satz, Z. Phys. C **57**, 135 (1993);
F. Becattini, J. Cleymans, A. Keranen, E. Suhonen and K. Redlich, Phys. Rev. C **64**, 024901 (2001);
P. Braun-Munzinger, D. Magestro, K. Redlich, and J. Stachel, Phys. Lett. B **518**, 41 (2001);
J. Rafelski and J. Letessier, Nucl. Phys. A **715**, 98c (2003);
A. Andronic, P. Braun-Munzinger, and J. Stachel, Nucl. Phys. A **772**, 167 (2006);
F. Becattini, J. Manninen and M. Gazdzicki, Phys. Rev. C **73**, 044905 (2006).

Extensions of the ideal HRG model

Two extensions of the ideal RG model have been widely discussed:

1) The first one is the excluded volume HRG model which introduces the effects of hadron repulsions at short distances [3].

2) The second extension of the HRG model is an inclusion of the exponentially increasing mass spectrum $\rho(m)$ proposed by Hagedorn about 50 years ago [4]. These excited colorless states (fireballs) are considered as a continuation of the resonance spectrum at masses m higher than 2 GeV.

[3]
M. I. Gorenstein, V. K. Petrov, and G. M. Zinovjev, Phys. Lett. B **106**, 327 (1981);
D. H. Rischke, M. I. Gorenstein, H. Stoecker, and W. Greiner, Z. Phys. C **51**, 485 (1991).

[4]
R. Hagedorn, Nuovo Cim. Suppl. **6**, 311 (1968);
R. Hagedorn and J. Ranft, Nuovo Cim. Suppl. **6**, 169 (1968).

Excluded volume procedure

The van der Waals excluded volume procedure can be regarded as a substitution of the total system volume V by the available volume $V_{\rm av}$

$$V \to V_{\rm av} = V - \sum_i v_i N_i \,, \qquad v_i = 4 \cdot (4\pi r_i^3/3)$$

v_i is the proper volume, r_i is the hard-core radius of a particle.

$$p^{\rm ev}(T,\mu) = \sum_i p_i^{\rm id}(T,\tilde{\mu}_i)\,, \qquad \tilde{\mu}_i = \mu_i - v_i p^{\rm ev}$$

$$\varepsilon^{\rm ev}(T,\mu) = \frac{\sum_i \varepsilon_i^{\rm id}(T,\tilde{\mu}_i)}{1 + \sum_j v_j n_j^{\rm id}(T,\tilde{\mu}_j)}$$

$$n_i^{\rm id}(T,\mu_i) = g_i \int dm\, f_i(m) \int \frac{d^3k}{(2\pi)^3} \left[\exp\left(\frac{\sqrt{\boldsymbol{k}^2 + m^2} - \mu_i}{T}\right) + \eta_i\right]^{-1}$$

Excluded volume procedure in the mean-field approach

These equations can be also obtained in the framework of thermodynamically self-consistent mean-field theory, which gives a sequential treating of the problem when one can examine various different mean fields that mimic the repulsive and attractive interactions (for details see [5])

Single-particle spectrum

$$\sqrt{m^2+\boldsymbol{k}^2} \quad \rightarrow \quad \sqrt{m^2+\boldsymbol{k}^2} + U(n,T)$$

$$p = Tn(T,\mu) + P^{\rm ex}(n,T)$$

$$n\frac{\partial U(n,T)}{\partial n} = \frac{\partial P^{\rm ex}(n,T)}{\partial n} \quad \Rightarrow \quad U(n,T) = \int_0^n \frac{dn'}{n'}\,\frac{\partial P^{\rm ex}(n',T)}{\partial n'}$$

The van der Waals mean field

$$U_{\rm VdW}(n,T) = T\frac{b\,n}{1-b\,n} - T\ln(1-b\,n) - 2a\,n$$

[5] D. Anchishkin, V. Vovchenko, J. Phys. G**42**, 105102 (2015); arXiv:1411.1444 [nucl-th].

V. Vovchenko, D.V. Anchishkin, M.I. Gorenstein, Phys. Rev. C**91**, 064314 (2015); arXiv:1504.01363 [nucl-th].

Excluded volume procedure in the mean-field approach: the Boltzmann statistics

For the Boltzmann statistics $\eta_i = 0$ all equations are simplified. The distribution function reads

$$f(T,\mu) = \exp\left[-\frac{\sqrt{\mathbf{k}^2+m^2}+U(n,T)-\mu}{T}\right]$$

with the van der Waals repulsive mean field $U_{\rm rep}$

$$U_{\rm rep}(n,T) \;=\; T\,\frac{b\,n}{1\,-\,b\,n} \;-\; T\,\ln(1-b\,n)$$

$$f(T,\mu) = (1-bn)\,\exp\left(-\frac{\sqrt{\mathbf{k}^2+m^2}+T\,\frac{bn}{1-bn}-\mu}{T}\right) \quad\rightarrow\quad n^{\rm id}(T,\mu^*) = \frac{n}{1-bn}$$

$$p(T,\mu) = \alpha\,p^{\rm id}(T,\mu) = \alpha\,T\,n^{\rm id}(T,\mu)\,, \qquad \mu^* = \mu - T\,\frac{bn}{1-bn}$$

$$\alpha \;\equiv\; \exp\left(-\frac{b\,p}{T}\right), \qquad n^{\rm id}(T,\mu) \;\equiv\; \sum_i n_i^{\rm id}(T,\mu_i)$$

$$\varepsilon(T,\mu) = \frac{\alpha\,\varepsilon^{\rm id}(T,\mu)}{1+b\,n^{\rm id}(T,\mu)}$$

Ideal HRG and excluded volume HRG versus the lattice data

$$\mu_B = \mu_S = \mu_Q = 0$$

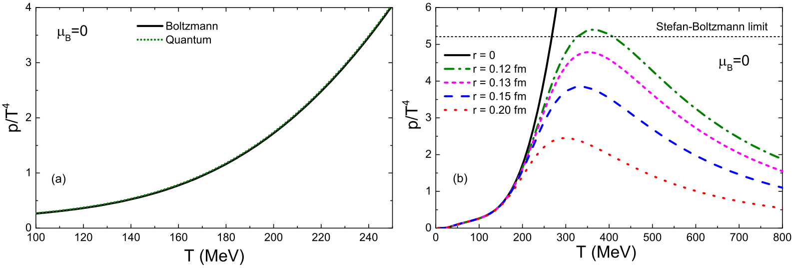


Figure: (a): The ideal HRG pressure, $p^{\rm id}(T)/T^4$, is shown as a function of temperature at $\mu = 0$ by the dotted line. The Boltzmann approximation $\eta_i = 0$ is shown by the solid line. (b): The ideal HRG pressure and excluded volume HRG pressure for several different values of hard-core radius r are presented. The Stefan-Boltzmann limit for the deconfined quark-gluon phase, $p_{SB}/T^4 = \sigma_{SB}/3 \cong 5.2$ is indicated by the horizontal dotted line.

Conclusion : $r \geq 0.13$

The excluded volume HRG versus the lattice data [1]

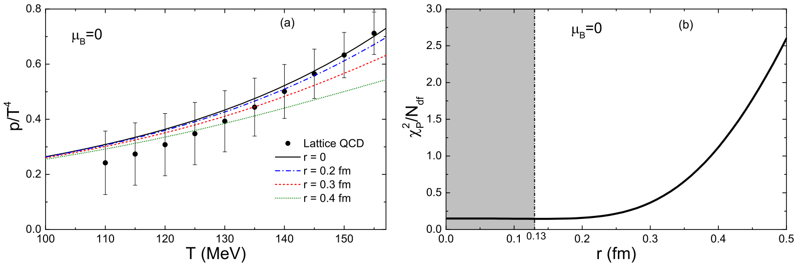


Figure: The results of excluded volume HRG model for different values of r are compared to the lattice data [1] for p/T^4 (a) and the values of $\chi^2_p/N_{\rm df}$ (b) are shown as functions of r. The shaded grey area corresponds to $r \leq 0.13$ fm.

The excluded volume HRG versus the lattice data [1]

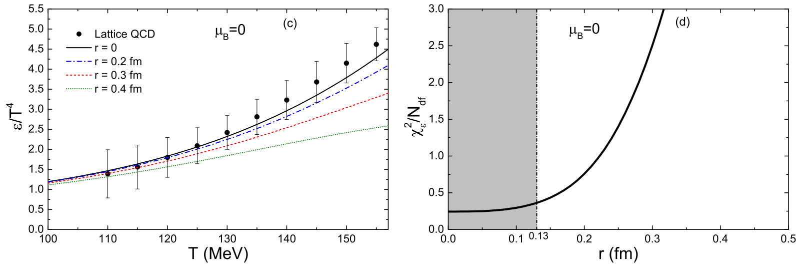


Figure: The results of excluded volume HRG model for different values of r are compared to the lattice data [1] for ε/T^4 (c). The values of $\chi^2_\varepsilon/N_{\mathrm{df}}$ are shown as functions of r (d). The shaded grey area corresponds to $r \leq 0.13$ fm.

The excluded Volume HRG with the Hagedorn mass spectrum

The above comparison with lattice data may indicate a presence of additional contributions to $p^{\rm ev}$ and $\varepsilon^{\rm ev}$ in the excluded volume HRG model. These contributions should be small enough for the pressure and much larger for the energy density. We argue that massive Hagedorn states are the ideal candidates for this role. Indeed, each heavy particle with $m \gg T$ gives its contribution, T, to the pressure, and much larger contribution, $m + 3T/2$, to the energy density.

the Hagedorn mass spectrum [4]

$$\rho(m) = C\,\frac{\theta(m - M_0)}{(m^2 + m_0^2)^a}\,\exp\left(\frac{m}{T_H}\right)$$

with $M_0 = 2$ GeV, $T_H = 160$ MeV, m_0=0.5 GeV, and $a = 5/4$

the Hagedorn pressure in the Boltzmann statistics

$$p^{\rm H} = \exp\left(-\frac{v p^{\rm H}}{T}\right) T \int dm \int \frac{d^3k}{(2\pi)^3} \exp\left(-\frac{\sqrt{m^2 + \boldsymbol{k}^2}}{T}\right) \left[\sum_i g_i f_i(m) + \rho(m)\right]$$

Selfconsistent equation with the Hagedorn resonances

$$p^{\mathrm{H}}(T) \;=\; \alpha^{\mathrm{H}}\, p_{\mathrm{H}}^{\mathrm{id}}(T) \;=\; \alpha^{\mathrm{H}}\, T\, n_{\mathrm{H}}^{\mathrm{id}}(T)\,, \qquad \alpha^{\mathrm{H}} \equiv \exp\left(-\frac{v\, p^{\mathrm{H}}}{T}\right)$$

$b = v$ is the proper volume for all particles.

$$\varepsilon^{\mathrm{H}}(T) \;=\; \frac{\alpha^{\mathrm{H}}\, \varepsilon_{\mathrm{id}}^{\mathrm{H}}}{1 + v\, \alpha^{\mathrm{H}}\, n_{\mathrm{id}}^{\mathrm{H}}}$$

$$\varepsilon_{\mathrm{id}}^{\mathrm{H}}(T) \;=\; \int dm \int \frac{d^3k}{(2\pi)^3}\, \sqrt{m^2 + \boldsymbol{k}^2}\, \exp\left(-\frac{\sqrt{m^2 + \boldsymbol{k}^2}}{T}\right) \left[\sum_i g_i f_i(m) + \rho(m)\right]$$

The excluded volume HRG Hagedorn model versus the lattice data

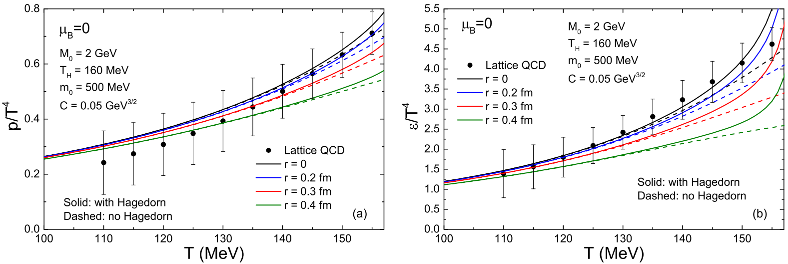


Figure: The results of excluded volume HRG Hagedorn model for different values of r are compared to the lattice data for p/T^4 and ε/T^4. The value of C is fixed as $C = 0.05$ GeV$^{3/2}$.

The excluded volume HRG Hagedorn model versus the lattice data

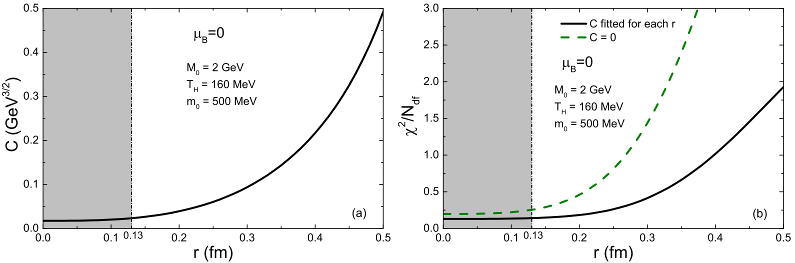


Figure: (a): Parameter C which minimizes $\chi^2/N_{\rm df}$ at each value of r is shown as a function of r. (b): The quantity $\chi^2/N_{\rm df}$ as a function of r. For each value of r parameter C is fitted in order to minimize $\chi^2/N_{\rm df}$. The shaded grey area corresponds to $r \leq 0.13$.

Conclusions

1. A condition that the pressure of the HRG should not exceed the Stefan-Boltzmann limit for quarks and gluons indicates that hadrons should have a non-zero hard-core radius of at least 0.13 fm.
2. A comparison of the excluded volume HRG model with the lattice data at $T < 155$ MeV yields no conclusive evidences in favor of a presence of the excluded volume effects. The fit of $p^{\rm lat}/T^4$ prefers values of $r \lesssim 0.4$ fm, while the best fit of $\varepsilon^{\rm lat}/T^4$ corresponds to $r \cong 0$.
3. Neither excluded volume HRG nor ideal HRG with additional Hagedorn states being considered separately demonstrates any advantages for fitting the lattice data in a comparison to the ideal HRG model.
4. There is a clear indication that both the hard core repulsion and the Hagedorn mass spectrum should be taken into account simultaneously in the framework of the hadron resonance gas model.

More details in:

V. Yu. Vovchenko, D.V. Anchishkin, M.I. Gorenstein, Phys. Rev. C**91**, 024905 (2015); arXiv:1412.5478 [nucl-th].

Thank you for attention!