ENHANCING THE EFFICIENCY OF PLASMA WAKEFIELD ACCELERATION

Mihály András Pocsai

Wigner Research Centre for Physics of the HAS
University of Pécs, Faculty of Sciences, Departement of Physics

7$^{\text{th}}$–11$^{\text{th}}$ of December
15$^{\text{th}}$ Zimányi School, 2015
Wigner RCP, Budapest

Outline

Outline

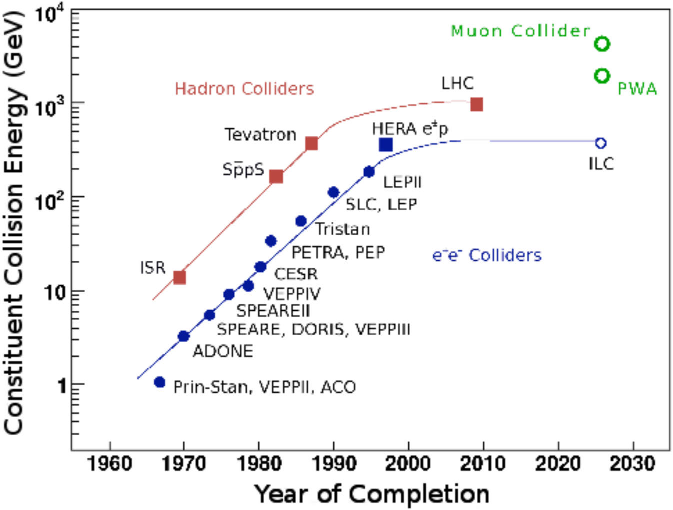
1. Introduction

2. Theoretical Basics
   - Gaussian Pulses
   - Generalization for Bichromatic Fields
   - Equations of Motion
   - The Presence of an Underdense Plasma

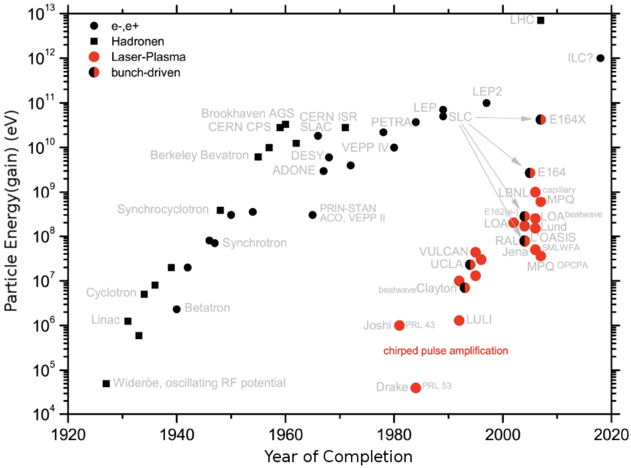
3. Results
   - General Remarks
   - Monochromatic Fields
   - Comparison with Experimental Data
   - Bichromatic Fields

4. Summary

- State-of-the-Art technology: circular accelerator, 8.3 T, 14 TeV
- Limit: accelerating field $< 50\,\mathrm{MV/m}$



How further → VLHC? Extremely Expensive!
(Future Circular Collider Kick Off Meeting, Feb. 2014, Geneva)
New, cheaper technologies are needed → Plasma based acceleration!



New technology: particle acceleration by **plasma waves.** The possible methods are (E. Esarey *et. al:* Rev. Mod. Phys. **81** (2009), 1229):

- **PWFA:** electron/proton bunch drives the wakes.
- **LWFA:** Short ($\approx 1\,\mathrm{ps}$), ultra intense $I \geq 10^{18}\mathrm{W}\cdot\mathrm{cm}^{-2}$ pulse. $L = c\tau_p \approx \lambda_p = 2\pi c/\omega_p$, $n = 10^{15}\,\mathrm{cm}^{-3}$.
- **PBWA:** Two laser pulses, $\omega_1 - \omega_2 \sim \omega_p$, $n = 10^{16} - 10^{17}\,\mathrm{cm}^{-3}$. An alternative for LWFA.
- **SMLWFA:** LWFA on higher plasma densities. $n = 10^{19}\,\mathrm{cm}^{-3}$, $I \approx 10^{19}\,\mathrm{W}\cdot\mathrm{cm}^{-2}$, $L > \lambda_p$. The plasma "chops up" the long laser pulse. The length of the equidistently spaced train of smaller pulses mathces the plasma wavelength. This train of pulses resonantly excites the plasma.
- **Multiple bunches or pulses:** larger amplitude plasma waves.

Short term solution: **PWFA** (CERN AWAKE Experiment)
Long term solution: **LWFA**

Outline

1. Introduction
2. Theoretical Basics
   - Gaussian Pulses
   - Generalization for Bichromatic Fields
   - Equations of Motion
   - The Presence of an Underdense Plasma
3. Results
   - General Remarks
   - Monochromatic Fields
   - Comparison with Experimental Data
   - Bichromatic Fields
4. Summary

The following theoretical basics are going to be over-viewed:

1. Mathematical form of a monochromatic Gaussian Pulse
2. Mathematical form of a bichromatic Gaussian Pulse
3. EOM in electromagnetic fields
4. Dealing with the presence of an Underdense Plasma

Gaussian beams can be derived from the paraxial approximation. For a Gaussian pulse, the electric field has the following form:

$$E_x = E_0 \frac{W_0}{W(z)} \exp\left[-\frac{r^2}{W^2(z)}\right] \exp\left(-\frac{\Theta^2}{T^2}\right) \times \cos\left[\frac{kr^2}{2R(z)} - \Phi(z) + \omega\Theta + \sigma\Theta^2 + \varphi\right] \tag{1a}$$

$$E_y = 0 \tag{1b}$$

$$E_z = -\frac{x}{R(z)} E_x + E_0 \frac{2x}{kW^2(z)} \cdot \frac{W_0}{W(z)} \exp\left[-\frac{r^2}{W^2(z)}\right] \times \exp\left[-\frac{\Theta^2}{T^2}\right] \sin\left[\frac{kr^2}{2R(z)} - \Phi(z) + \omega\Theta + \sigma\Theta^2 + \varphi\right] \tag{1c}$$

For details, see L.W. Davis: Phys. Rev. A **19** (1979), 1177

Gaussian beams can be derived from the paraxial approximation. For a Gaussian pulse, the magnetic field has the following form:

$$B_x = 0 \tag{2a}$$

$$B_y = \frac{E_x}{c} \tag{2b}$$

$$B_z = \frac{y}{cR(z)} E_x + \frac{1}{c} E_0 \frac{2y}{kW^2(z)} \cdot \frac{W_0}{W(z)} \exp\left[-\frac{r^2}{W^2(z)}\right] \times \exp\left[-\frac{\Theta^2}{T^2}\right] \sin\left[\frac{kr^2}{2R(z)} - \Phi(z) + \omega\Theta + \sigma\Theta^2 + \varphi\right] \tag{2c}$$

For details, see L.W. Davis: Phys. Rev. A **19** (1979), 1177

A Gaussian pulse given with eqs. (1) and (2) is an approximate solution of Maxwell's equations.

The parameters of the Gaussian pulse are the following:

$$W(z) = W_0 \left[1 + \left(\frac{z}{z_R}\right)^2\right]^{1/2} \quad \text{the spot size,} \tag{3a}$$

$$R(z) = z \left[1 + \left(\frac{z_R}{z}\right)^2\right] \quad \text{the radius of curvature,} \tag{3b}$$

$$\Phi(z) = \tan^{-1} \frac{z}{z_R} \quad \text{the Gouy phase, and} \tag{3c}$$

$$W_0 = \left(\frac{\lambda z_R}{\pi}\right)^{1/2} \quad \text{the beam waist.} \tag{3d}$$

and z_R being the Rayleigh-length. The wavenumber has the form of

$$k = \frac{\omega_0}{c} (1 + \sigma\Theta) \tag{4}$$

with ω_0 being the initial frequency.

At the Rayleigh-length, the area of the beam spot is twice as the minimal size:

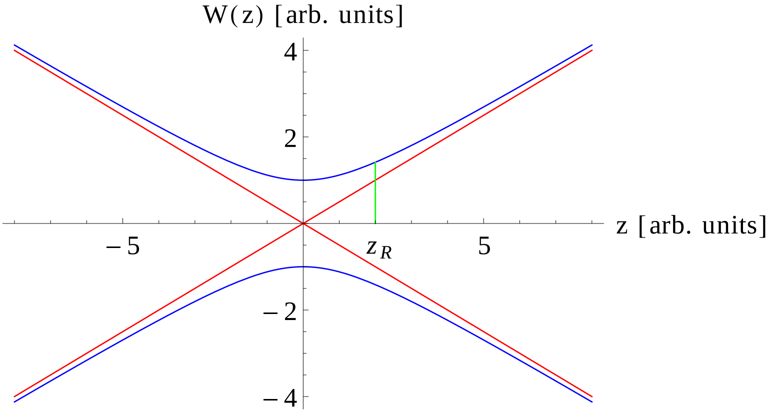
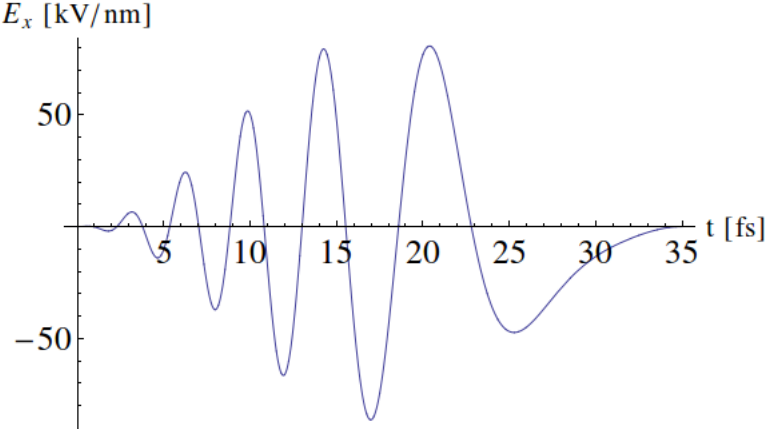


FIGURE : The width of a Gaussian beam as a function of distance along the direction of propagation.

A chirped laser pulse looks like:



- Gaussian beams may have higher order TEM modes! See: H. Kogelik and T. Li: Appl. Opt. **5** (1966), 1550.
- For a system with cylindrical symmetry, it is useful to express the beam in cylindrical coordinates. Cylindrical waves also may have higher order TEM modes.
- Is it possible, based on L.W. Davis: Phys. Rev. A **19** (1979), 1177 to obtain an exact solution of Maxwell's equations?
- If so, does it worth the work?
- Exact beam solutions of the Maxwell's equations can be produced using the Hertz-vector! See: P. Varga and P. and Török, Opt. Commun. **152** (1998), 108–118.

The q^{th} harmonic of a Gaussian pulse has the following form:

$$E_{q,x} = E_0 \frac{W_0}{W_q(z)} \exp\left[-\frac{r^2}{W_q^2(z)}\right] \exp\left[-\frac{\Theta^2}{T^2}\right] \times \cos\left[\frac{k_q r^2}{2R_q(z)} - \Phi_q(z) + q\Theta + q^2\sigma\Theta^2 + \varphi_q\right], \tag{5a}$$

$$E_{q,y} = 0, \tag{5b}$$

$$E_{q,z} = -\frac{x}{R_q(z)} E_{q,x} + \frac{2x}{k_q W_q^2(z)} \cdot \frac{W_0}{W_q(z)} \exp\left[-\frac{r^2}{W_q(z)}\right] \exp\left[-\frac{\Theta^2}{(\omega_0 T)^2}\right] \times \sin\left[\frac{k_q r^2}{2R_q(z)} - \Phi_q(z) + q\Theta + q^2\sigma\Theta_q^2 + \varphi_q\right] \tag{5c}$$

The q^{th} harmonic of a Gaussian pulse has the following form:

$$B_{q,x} = 0, \tag{6a}$$

$$B_{q,y} = \frac{E_{q,x}}{c}, \tag{6b}$$

$$B_{q,z} = \frac{y}{cR_q(z)} E_{q,x} + \frac{1}{c} E_0 \frac{2y}{k_q W_q^2(z)} \cdot \frac{W_0}{W_q(z)} \exp\left[-\frac{r^2}{W_q^2(z)}\right] \exp\left[-\frac{\Theta^2}{T^2}\right] \times \sin\left[\frac{k_q r^2}{2R_q(z)} - \Phi_q(z) + q\Theta + q^2\sigma\Theta^2 + \phi_q\right] \tag{6c}$$

The parameters of the higher harmonic are:

$$W_q(z) = W_{q,0}\left[1+\left(\frac{z}{z_{q,R}}\right)^2\right]^{1/2} \quad \text{the spot size,} \tag{7a}$$

$$R_q(z) = z\left[1+\left(\frac{z_{q,R}}{z}\right)^2\right] \quad \text{the radius of curvature,} \tag{7b}$$

$$\Phi_q(z) = \tan^{-1}\frac{z}{z_{q,R}} \quad \text{the Gouy phase, and} \tag{7c}$$

$$W_{q,0} = \left(\frac{\lambda z_{q,R}}{\pi}\right)^{1/2} \quad \text{the beam waist.} \tag{7d}$$

and $z_{q,R}$ being the Rayleigh-length.The wavenumber has the form of

$$k = q\frac{\omega_0}{c}\left(1+q\sigma_q\Theta\right) \tag{8}$$

with $q\omega_0$ being the initial frequency of the q^{th} harmonic.

Finally, the general form of a bichromatic field:

$$\mathbf{E} = \mathbf{E}_1 + \frac{A}{q}\mathbf{E}_q, \tag{9a}$$

$$\mathbf{B} = \mathbf{B}_1 + \frac{A}{q}\mathbf{B}_q \tag{9b}$$

with $\mathbf{E}_1$ and $\mathbf{B}_1$ being the electric and magnetic fields of the main harmonic and $0 \leq A \leq 1$ the relative amplitude of the harmonics.

A bichromatic pulse looks like:

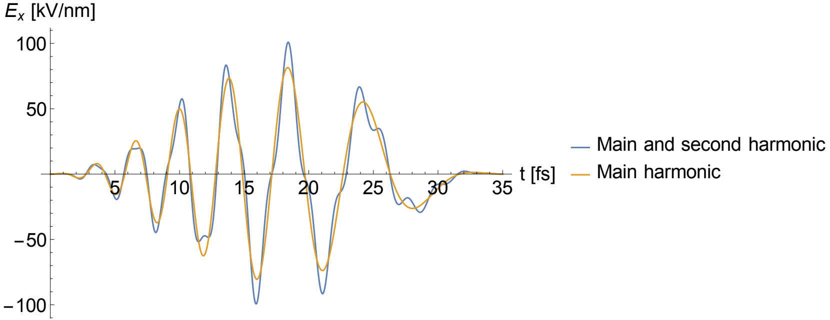


FIGURE : A bichromatic (main and second harmonic) pulse, compared with the corresponding monochromatic (main harmonic) component.

The Lorentz-Force acting on the electron:

$$\mathbf{F} = e\left(\mathbf{E} + \mathbf{v} \times \mathbf{B}\right) \tag{10}$$

Equations of Motion for a relativistic electron:

$$\frac{\mathrm{d}\gamma}{\mathrm{d}t} = \frac{1}{m_e c^2}\mathbf{F} \cdot \mathbf{v} \tag{11a}$$

$$\frac{\mathrm{d}\mathbf{p}}{\mathrm{d}t} = e\left(\mathbf{E} + \frac{\mathbf{p}}{m_e\gamma} \times \mathbf{B}\right) \tag{11b}$$

$\mathbf{E}(t, \mathbf{r}) = \mathbf{E}(\Theta(t, \mathbf{r}))$ and $\mathbf{B}(t, \mathbf{r}) = \mathbf{B}(\Theta(t, \mathbf{r}))$, respectively, with

$$\Theta(t, \mathbf{r}) := t - \mathbf{n} \cdot \frac{\mathbf{r}}{c}. \tag{12}$$

being the retarded time

The presence of an Underdense Plasma can be taken into account via it's n_m index of refraction!

$$n_m = \sqrt{1 - \frac{\omega_p^2}{\omega_L^2}} \quad \text{and} \quad \omega_p^2 = \frac{n_e e^2}{\varepsilon_0 m_e} \tag{13}$$

The retarded time, including the index of refraction:

$$\Theta(t, \mathbf{r}, n_m) := t - n_m \mathbf{n} \cdot \frac{\mathbf{r}}{c}. \tag{14}$$

Outline

The relevant plasma densities are far below the critical density. At $\lambda = 800\,\mathrm{nm}$, $n_c = 1.74196 \cdot 10^{21}\,\mathrm{cm}^{-3}$.

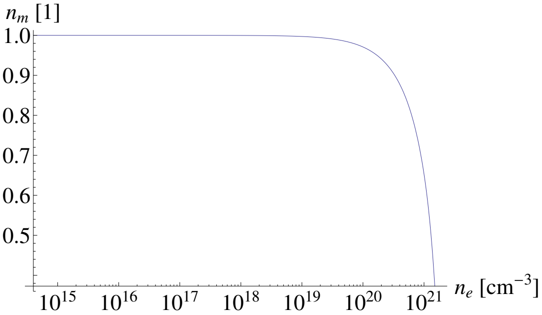


FIGURE : The index of refraction as a function of plasma electron density.

$n_m(10^{15}\,\mathrm{cm}^3) \approx n_m(0) \Rightarrow \Theta(t, \mathbf{r}, n_m) \approx \Theta(t, \mathbf{r})$

ACCELERATION IN UNDERDENSE PLASMAS CAN BE WELL APPROXIMATED BY ACCELERATION IN VACUUM!

For Gaussian laser pulses, the following results can be obtained (see: M.A. Pocsai, S. Varró, I.F. Barna: Laser and Particle Beams **33** (2015), 307–313):

- Only chirped pulses provide non-negligible acceleration.
- The electron has to be on-axis and propagate parallel with the pulse.
- Larger beam waists provide more energy gain.
- Shorter pulses provide more energy gain.
- With a Gaussian pulse of $\lambda = 800\,\mathrm{nm}$ wavelength, $T = 30\,\mathrm{fs}$ pulse duration, $I = 10^{21}\,\mathrm{W} \cdot \mathrm{cm}^{-2}$ intensity and $W_0 = 100\lambda$ beam waist, an energy gain of $270\,\mathrm{MeV}$ pro pulse can be achieved.

	Our Results	Kneip *et. al* Phys. Rev. Lett. **103** (2009), 035002
Wavelength	800 nm	800 nm
Pulse Duration	30 fs	55 fs
Intensity	$10^{21}\,\mathrm{W}\cdot\mathrm{cm}^{-2}$	$10^{19}\,\mathrm{W}\cdot\mathrm{cm}^{-2}$
Beam Waist	100λ	10 mm
Total Pulse Energy	9.6 J	10 J
Average Power	320 TW	180 TW
Energy gain	275 MeV (on 5 mm)	420 MeV (on 5 mm) 800 MeV (on 10 mm)
Accelerating Gradient	$58\,\mathrm{GVm}^{-1}$	$80\,\mathrm{GVm}^{-1}$

OUR RESULTS AGREE WITHIN A FACTOR OF TWO WITH THE EXPERIMENTAL DATA!

Summarizing the results (see: doi: 10.1016/j.nimb.2015.10.013; (M.A. Pocsai, S. Varró, I.F. Barna, article in press))

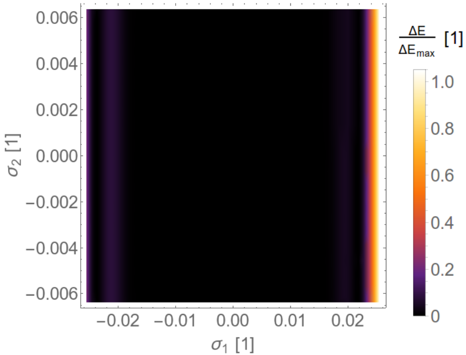


FIGURE : The relative energy gain as a function of the chirp parameters. ΔE depends very weakly on σ_2. $\lambda = 800\,\mathrm{nm}$, $T = 5\,\mathrm{fs}$, $I = 8.544 \cdot 10^{18}\,\mathrm{W} \cdot \mathrm{cm}^2$, $W_0 = 0.8\,\mu\mathrm{m}$, $A = 0.1$, $\varphi_1 = 2.058$.

Summarizing the results (see: doi: 10.1016/j.nimb.2015.10.013; (M.A. Pocsai, S. Varró, I.F. Barna, article in press))

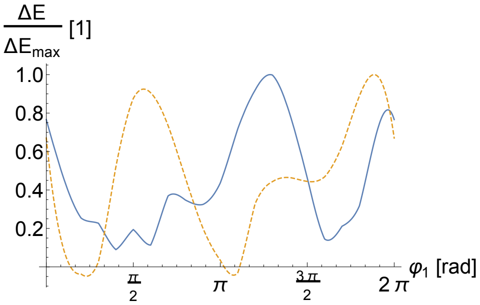


FIGURE : The relative energy gain as a function of the carrier–envelope phase. $\lambda = 800\,\mathrm{nm}$, $T = 5\,\mathrm{fs}$, $\sigma_1 = 0.0471\mathrm{fs}^{-2}$, $\sigma_2 = 0$, $A = 0.24$. $I = 10^{20}\,\mathrm{W}\cdot\mathrm{cm}^{-2}$, $W_0 = 8.976\,\mu\mathrm{m}$ (solid line), $a_0 = 3\cdot 10^{20}\,\mathrm{W}\cdot\mathrm{cm}^{-2}$, $W_0 = 8\,\mu\mathrm{m}$ (dashed line).

Summarizing the results (see: doi: 10.1016/j.nimb.2015.10.013; (M.A. Pocsai, S. Varró, I.F. Barna, article in press))

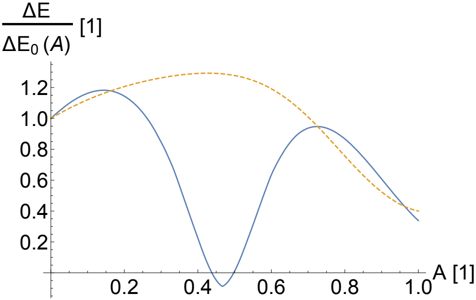


FIGURE : The relative energy gain as a function of the relative amplitude of the harmonics. The second harmonic enhanced the energy gain by about $20 - 30\,\%$, compared to the monochromatic case. $\lambda = 800\,\mathrm{nm}$, $T = 5\,\mathrm{fs}$, $\sigma_1 = 0.0471\,\mathrm{fs}^{-2}$, $\sigma_2 = 0$. $I = 10^{20}\,\mathrm{W} \cdot \mathrm{cm}^{-2}$, $W_0 = 8\,\mu\mathrm{m}$, $\varphi_1 = 0$ (solid line), $a_0 = 3 \cdot 10^{20}\,\mathrm{W} \cdot \mathrm{cm}^{-2}$, $W_0 = 8.976\,\mu\mathrm{m}$, $\varphi_1 = 0$ (dashed line).

Outline

- A simple but (computationally) efficient model has been presented.
- Chirped Gaussian pulses can transfer up to 270 MeV energy to a single electron.
- The addition of the second harmonic boosts the energy transfer to the electron by even 30 %—it is tempting to use a bichromatic driver pulse for electron acceleration.
- The results obtained with our simple model agree quite well with the experimental data.

THANK YOU FOR YOUR ATTENTION!