An analytic hydrodynamical model of rotating 3D expansion

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Based on:
arXiv:1511.02593
arXiv:1512.00888

– Rotation in high-energy heavy-ion collision
– Details of exact hydrodynamical solutions with rotation
– Observables, consequences of rotating expansion

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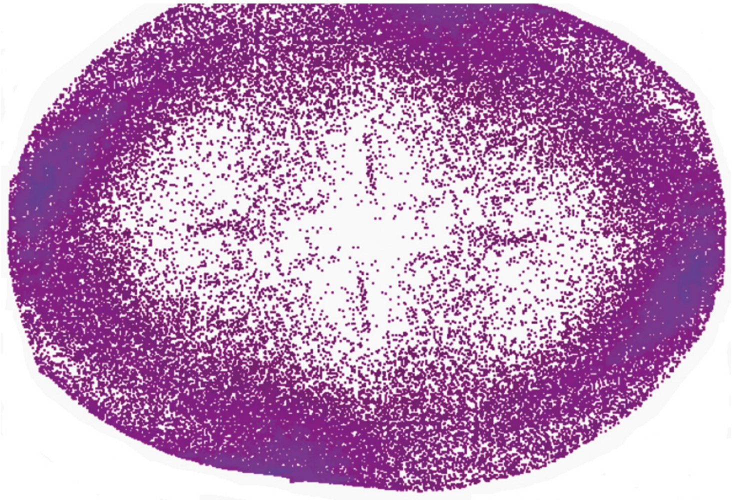
Introduction

Motivation:

Non-central collisions: angular momentum

Time evolution of rotation:

- final tilt of fireball observable (?)
- insight into expansion dynamics
- EoS stiffness observable?



Example from EPN 43/22 (2012) 91
(L. Cifarelli, L.P. Csernai, H. Stöcker)

Exact hydrodynamical solutions:

insight into the dynamics
using simple analytic formulas

This work follows the footsteps of:

Csizmadia, Csörgő, Lukács, PLB443 (1998) 21
Csörgő, Acta Phys. Polon. B37 (2006) 483 & references therein
Csörgő, Akkelin, Hama, Lukács, Sinyukov, PRC67 (2003) 034094
Csörgő, Nagy, PRC89 (2014) 044901

Recent work:

Csörgő, Nagy, Barna, arXiv:1511.02593
Nagy, Csörgő: arXiv:1512.00888

Equations of hydrodynamics

Basic equations (non-relativistic, perfect fluid)

$$\partial_t n + \mathbf{v}\nabla n = -n\nabla\mathbf{v}$$ Mass conservation

$$\partial_t \varepsilon + \mathbf{v}\nabla\varepsilon = -\left(\varepsilon + p\right)\nabla\mathbf{v}$$ energy conservation

$$\partial_t \mathbf{v} + \left(\mathbf{v}\nabla\right)\mathbf{v} = -\frac{1}{nm_0}\nabla p$$ Euler equation

Follow only from local thermal equilibrium & local conservation laws.

Equation of State (EoS):

$$p = nT \quad \varepsilon = \kappa\left(T\right)p$$

Rotating frame:

$$\mathbf{\Omega} = \begin{pmatrix} 0 \\ \dot{\vartheta}_{\mathbf{r}} \\ 0 \end{pmatrix} \qquad \begin{aligned} r'_x &= r_x \cos\vartheta_{\mathbf{r}} - r_z \sin\vartheta_{\mathbf{r}} \\ r'_y &= r_y \\ r'_z &= r_x \sin\vartheta_{\mathbf{r}} + r_z \cos\vartheta_{\mathbf{r}} \end{aligned}$$

in x-z plane

Equations in rotating frame:

$$\left(\partial'_t + \mathbf{v}'\nabla'\right)n + n\nabla'\mathbf{v}' = 0$$

$$\left(\frac{\kappa}{T} + \frac{d\kappa}{dT}\right)\left(\partial'_t + \mathbf{v}'\nabla'\right)T + \nabla'\mathbf{v}' = 0$$

$$\partial'_t \mathbf{v}' + \left(\mathbf{v}'\nabla'\right)\mathbf{v}' = -\frac{1}{nm_0}\nabla' p + \mathbf{f}'$$

$$\mathbf{f}' = 2\mathbf{v}' \times \mathbf{\Omega} + \mathbf{r}' \times \dot{\mathbf{\Omega}} + \mathbf{\Omega} \times \left(\mathbf{r}' \times \mathbf{\Omega}\right)$$

Hubble-like rotating solutions I. - velocity field

Hubble-type velocity + rotation: spheroidal solutions

$$\mathbf{v} = \begin{pmatrix} \frac{\dot{X}}{X} r_x \\ \frac{\dot{Y}}{Y} r_y \\ \frac{\dot{Z}}{Z} r_z \end{pmatrix} + \begin{pmatrix} \omega r_z \\ 0 \\ -\omega r_x \end{pmatrix}$$

Valid if: $X = Z \equiv R,$

$$\dot{X} = \dot{Z} \equiv \dot{R}$$

$$\omega(t) = \omega_0 \frac{R_0^2}{R^2(t)}$$

Csörgő, Nagy, PRC89 (2014) 044901

Similar to: Csernai, Wang, Csörgő, PRC 90 (2014) 024901

Hubble-type velocity + rotation *in a rotating K' frame:*

general rotating three-axis ellipsoidal solutions!

$$\mathbf{v}' = \begin{pmatrix} \frac{\dot{X}}{X} r'_x \\ \frac{\dot{Y}}{Y} r'_y \\ \frac{\dot{Z}}{Z} r'_x \end{pmatrix} + \begin{pmatrix} \frac{\omega}{2}\frac{X}{Z} r'_z \\ 0 \\ -\frac{\omega}{2}\frac{Z}{X} r'_x \end{pmatrix}$$

Valid if:

$$\omega(t) = \omega_0 \frac{R_0^2}{R^2(t)}$$

$$R \equiv \frac{X+Z}{2}$$

Nagy & Csörgő, arXiv:1512.00888

Hubble-like rotating solutions II. - time evolution

Temperature & number density:

$$n = n_0 \frac{V_0}{V} e^{-s/2} \quad s = \frac{{r'_x}^2}{X^2} + \frac{{r'_y}^2}{Y^2} + \frac{{r'_z}^2}{Z^2} \quad T = T_0 \left(\frac{V_0}{V}\right)^{1/\kappa}$$ (valid for constant κ)

Expanding rotating ellipsoids! (spheroids)

Eqn. of motion for axes: governed by a Hamiltonian:

Spheroidal case:

$$H = \frac{P_R^2 + 2P_Y^2}{4m_0} + \frac{m_0 \omega_0^2 R_0^4}{R^2} + T_0 \kappa \left(\frac{R_0^2 Y_0}{R^2 Y}\right)^{1/\kappa}$$

$$P_R = 2m\dot{R}, \; P_Y = m\dot{Y}$$

General case:

$$H = \frac{P_X^2 + P_Y^2 + P_Z^2}{2m_0} + \frac{m_0 \omega_0^2 R_0^4}{R^2} + T_0 \kappa \left(\frac{V_0}{V}\right)^{1/\kappa}$$

Generalizations to other EoS straightforward!

Hubble-like rotating solutions II. - time evolution

Conserved quantities:

Total particle number: $N_0 = (2\pi)^{3/2} n_0 V_0$

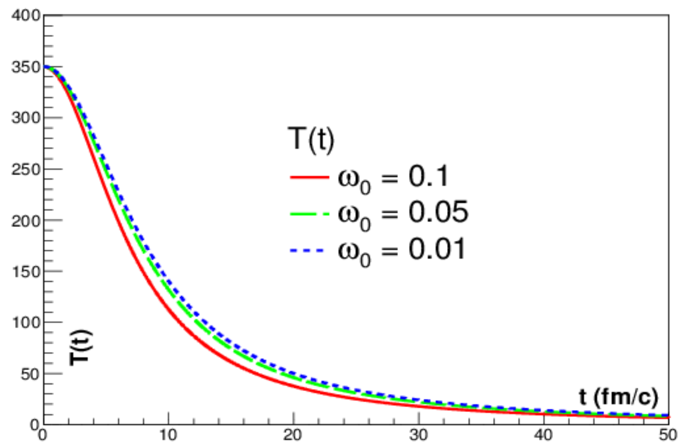
Total energy: $E = \frac{m_0}{2}\left(\dot{X}^2 + \dot{Y}^2 + \dot{Z}^2\right) + \frac{m_0 \omega_0^2 R_0^4}{R^2} + T_0 \kappa \left(\frac{V_0}{V}\right)^{1/\kappa}$

Total angular momentum: $M_y = \Theta_0 \omega_0 = \Theta(t)\,\omega(t)$

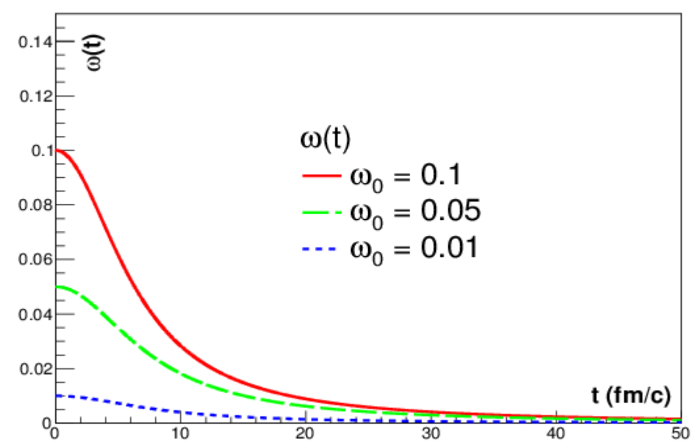
for Gaussian case: $\Theta(t) = 2 m_0 N_0 R^2(t)$

Time evolution:

Temperature:

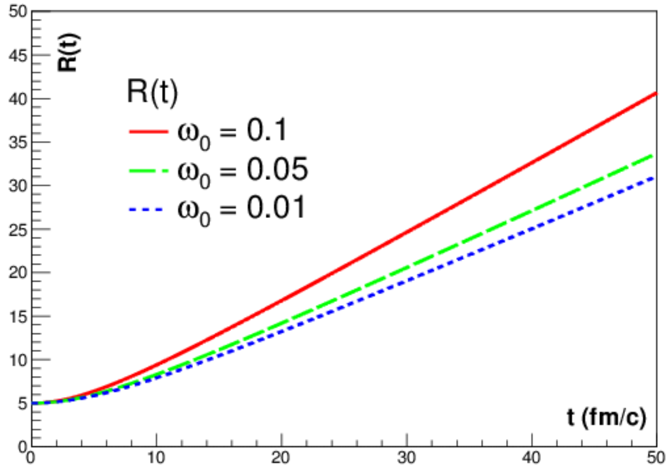


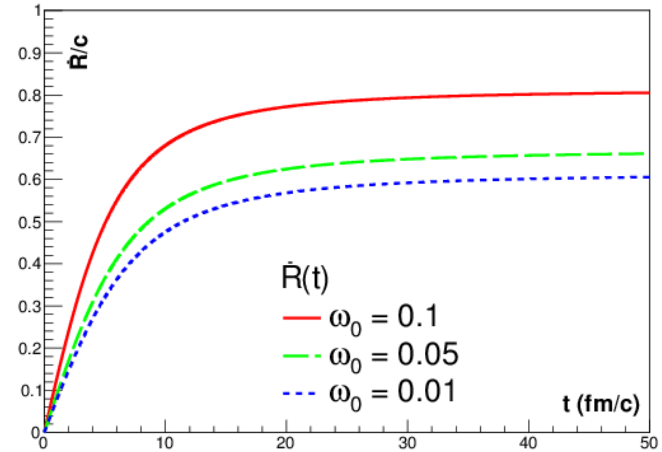
Angular velocity:

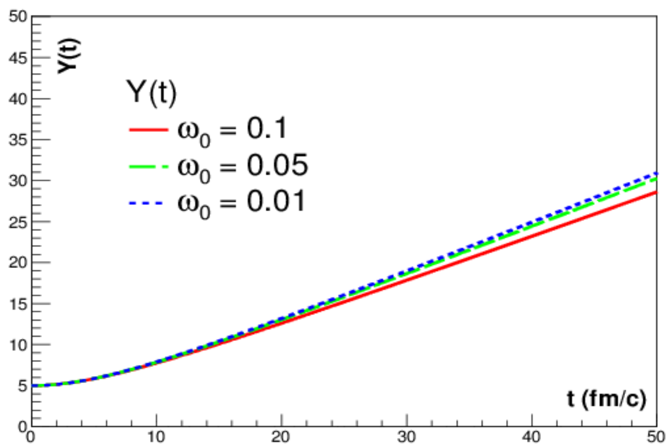


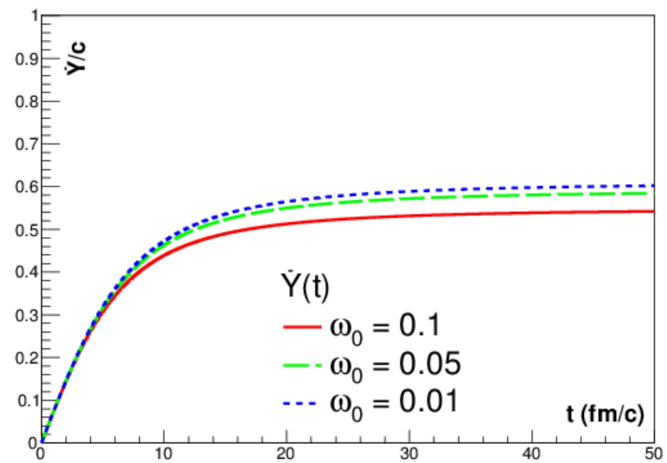
Hubble-like rotating solutions II. - time evolution

Principal axes (spheroidal case**):**









Observables

Hydro evolution until freeze-out:

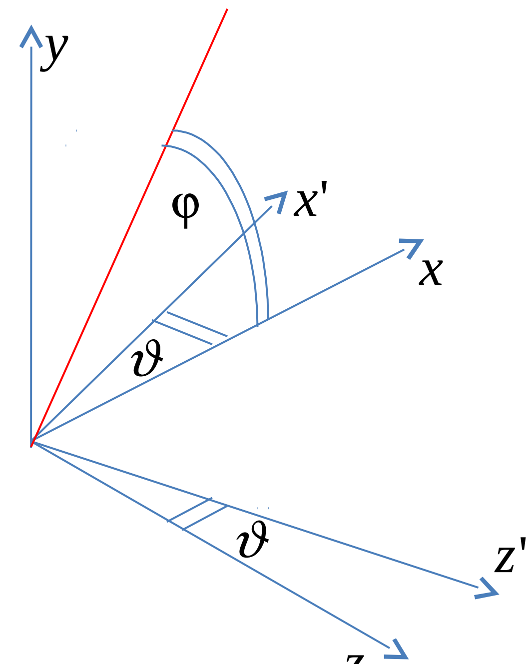
Dependent on EoS

Our freeze-out criterion:

Same freeze-out temperature

Same time everywhere

Analytic results!



Definitions & recipes:

Single-particle spectrum:

$$\frac{\mathrm{d}n}{\mathrm{d}^3\mathbf{p}} = \int \mathrm{d}^3\mathbf{r}\, S(\mathbf{r},\mathbf{p})$$

Bose-Einstein correlation:

$$C(\mathbf{K},\mathbf{q}) = 1 + \lambda \frac{\left|\tilde{S}(\mathbf{K},\mathbf{q})\right|^2}{\left|\tilde{S}(\mathbf{K},0)\right|^2}$$

Source function:

$$S(\mathbf{r},\mathbf{p}) = \frac{n(t_f,\mathbf{r})}{\left(2\pi m T_f\right)^{3/2}} \exp\left\{-\frac{(\mathbf{p}-m\mathbf{v}(t_f,\mathbf{r}))^2}{2mT_f}\right\}$$

Auxiliary quantity („speed of rotation"):

$$U = \frac{\omega_0}{2}\frac{(X_0+Z_0)^2}{X+Z}$$

Observables: single-particle spectrum

Definition:

$$\frac{dn}{d^3\mathbf{p}} = \int d^3\mathbf{r}\, S(\mathbf{r},\mathbf{p})$$

Result:

$$\frac{dn}{d^3\mathbf{p}} \propto \exp\left(-\frac{p_x^2}{2mT_x} - \frac{p_y^2}{2mT_y} - \frac{p_z^2}{2mT_z} - \frac{\beta_{xz}}{m} p_x p_z \right)$$

Auxiliary quantities:

$$T_y^* = T_f + m\dot{Y}^2$$

$$T_x^* = T_f + m\left(\dot{X}^2 + U^2\right), \quad T_z^* = T_f + m\left(\dot{Z}^2 + U^2\right), \quad \beta_* = mU(\dot{X} - \dot{Z})$$

Coefficients:

$$T'_x = T_x^* + \frac{\beta_*^2}{T_z^*}$$

$$T'_x = T_x^* + \frac{\beta_*^2}{T_z^*}$$

$$\beta'_{xz} = \frac{\beta_*}{T_x^* T_z^* - \beta_*^2}$$

In lab frame:

$$\frac{1}{T_x} = \frac{\cos^2\vartheta_f}{T'_x} + \frac{\sin^2\vartheta_f}{T'_z} + \beta'_{xz}\sin(2\vartheta_f)$$

$$\frac{1}{T_z} = \frac{\sin^2\vartheta_f}{T'_x} + \frac{\cos^2\vartheta_f}{T'_z} - \beta'_{xz}\sin(2\vartheta_f)$$

$$\beta_{xz} = \frac{\sin(2\vartheta_f)}{2}\left(\frac{1}{T'_z} - \frac{1}{T'_x}\right) + \beta'_{xz}\cos(2\vartheta_f)$$

Observables: average p_T-spectrum

Definition:

$$E\frac{\mathrm{d}n}{\mathrm{d}^3\mathbf{p}}=\frac{1}{2\pi p_T}\frac{\mathrm{d}n}{\mathrm{d}p_T\mathrm{d}y}\left\{1+2\sum_{n=1}^{\infty}v_n\cos\left[n\left(\varphi-\Psi_n\right)\right]\right\}$$

Auxiliary quantities:

$$w=\frac{p_T^2}{4m}\left(\frac{1}{T_y}-\frac{1}{T_x}\right)$$

$$T_{\mathrm{eff}}=\frac{1}{2}\left(\frac{1}{T_y}+\frac{1}{T_x}\right)$$

$$v=-\frac{\beta_{xz}}{m}p_z p_T$$

In this model:

All event planes coincide!

Result

(same as *Csörgő et al., PRC67 (2003) 034094*)

Azimuthal average, (up to 2nd order in v):

$$\frac{1}{2\pi p_T}\frac{\mathrm{d}n}{\mathrm{d}p_T\mathrm{d}y}\propto\exp\left(-\frac{p_z^2}{2mT_z}-\frac{p_T^2}{2mT_{\mathrm{eff}}}\right)\times$$

$$\times\left[I_0(w)+\frac{v^2}{4}\left(I_0(w)+I_1(w)\right)\right]$$

Observables: flow coefficients

Results: (same as in *Csörgő et al., PRC67 (2003) 034094*):

Flow coefficients (to 2nd order in ν):

$$v_1 = \frac{\nu}{2}\left[1+\frac{I_1(w)}{I_0(w)}\right]$$

$$v_2 = \frac{I_1(w)}{I_0(w)} + \frac{\nu^2}{8}\left[1+\frac{I_2(w)}{I_0(w)} - 2\frac{I_1^2(w)}{I_0^2(w)}\right]$$

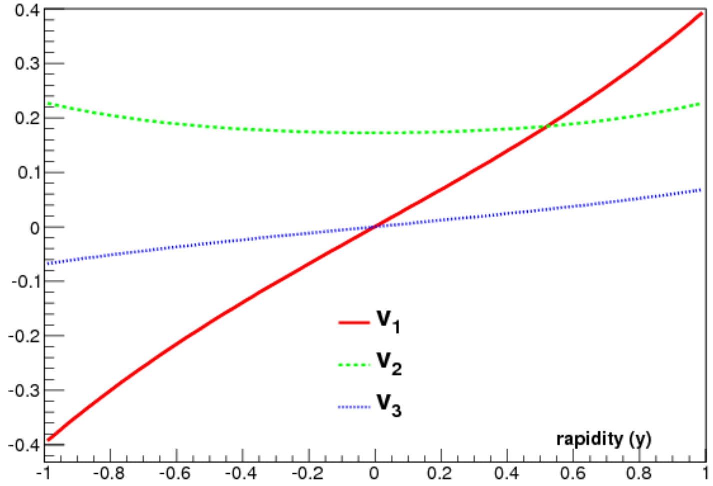
$$v_3 = \frac{\nu}{2}\left[\frac{I_1(w)}{I_0(w)} + \frac{I_2(w)}{I_0(w)}\right]$$

Modified Bessel function:

$$I_\nu(x) = \int_0^\pi \mathrm{d}\varphi \frac{\cos(\nu\varphi)}{\pi} e^{x\cos\varphi}$$

Universal scaling of v_2 is preserved
(*Csanád et al., Nucl.Phys. A742 (2004) 80*)

Directed flow: opposite in two directions!
Useful for telling the orientation of rotation



Observables: two-particle correlations

Bose-Einstein correlations (raw):

Definition:

$$C(\mathbf{K},\mathbf{q}) = 1 + \lambda \frac{\left|\tilde{S}(\mathbf{K},\mathbf{q})\right|^2}{\left|\tilde{S}(\mathbf{K},0)\right|^2}$$

$$\tilde{S}(\mathbf{K},\mathbf{q}) = \int \mathrm{d}^3\mathbf{r}\, e^{iq\mathbf{r}} S(\mathbf{r},\mathbf{K})$$

Result: Gaussian

$$C(\mathbf{K},\mathbf{q}) = 1 + \lambda \exp\left(-\sum_{ij} q_i q_j R_{ij}^2\right)$$

Radii in tilted frame:

$$R'^2_x = X^2 \frac{T_f}{T_x^*}$$

$$R'^2_y = Y^2 \frac{T_f}{T_y^*}$$

$$R'^2_z = Z^2 \frac{T_f}{T_z^*}$$

$$R'^2_{xz} = XZT_f \beta'_{xz}$$

In the lab frame:

$$R_x^2 = R'^2_x \cos^2\vartheta_f + R'^2_z \sin^2\vartheta_f + R'^2_{xz}\sin(2\vartheta_f)$$

$$R_y^2 = R'^2_y$$

$$R_x^2 = R'^2_x \sin^2\vartheta_f + R'^2_z \cos^2\vartheta_f - R'^2_{xz}\sin(2\vartheta_f)$$

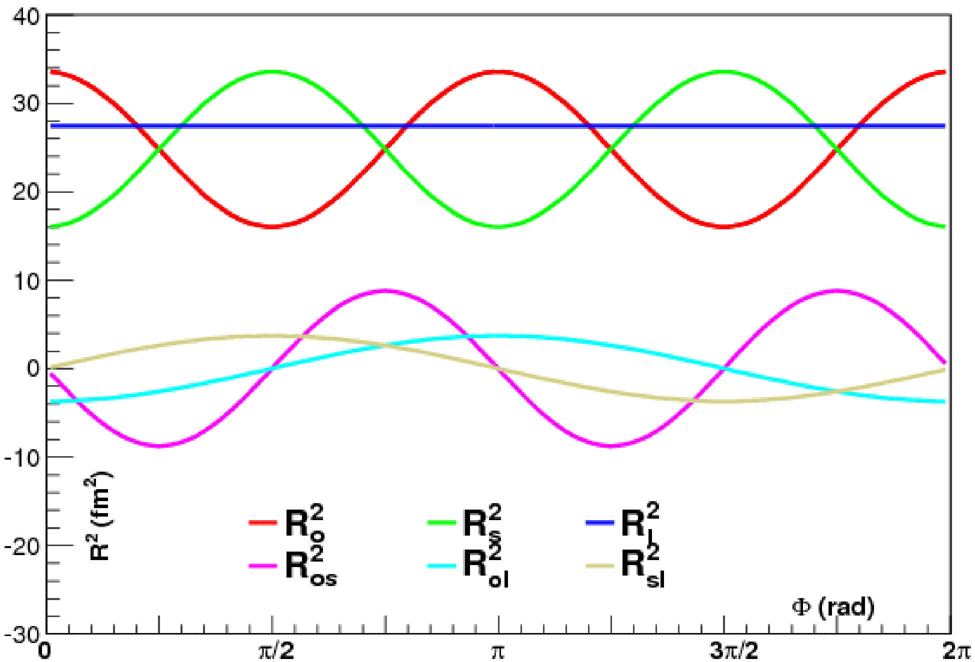
$$R_{xz}^2 = \frac{\sin(2\vartheta_f)}{2}\left(R'^2_x - R'^2_z\right) + \cos(2\vartheta_f) R'^2_{xz}$$

Coulomb correction treatment important!

Observables: two-particle correlations

Bertsch-Pratt parametrization:

$$C(\mathbf{K},\mathbf{q}) = 1 + \lambda \exp\left(- \sum_{i,j=o,s,l} q_i q_j R_{ij}^2 \right)$$



HBT Bertsch-Pratt radii:

$$R_o^2 = R_x^2 \cos^2\varphi + R_y^2 \sin^2\varphi$$

$$R_s^2 = R_x^2 \sin^2\varphi + R_y^2 \cos^2\varphi$$

$$R_l^2 = R_z^2$$

$$R_{os}^2 = \left(R_y^2 - R_x^2\right)\sin\varphi\cos\varphi$$

$$R_{ol}^2 = R_{xz}^2 \cos\varphi$$

$$R_{sl}^2 = -R_{xz}^2 \sin\varphi$$

Oscillations: s, o, os, ol, sl !

ol, sl values vanish for no tilt.

No 2nd order oscillations of ol, sl.

Signatures of rotation

Observables that appear because of tilt:

Flow coefficients vs. rapidity: v_3 and v_1 have characteristic y-dependence!

HBT-radii: azimuthal oscillation vs. reaction plane:

out-side: known, measured multiple times (present for ellipsoidal sources)

- eg. STAR beam energy scan HBT paper: (*arXiv:1403.4972*)

out-long, side-long: not measured yet above AGS energies (?)

Interesting ***ol, sl oscillations*** depend on $\cos\varphi$ and $\sin\varphi$:

1th order event plane needed! (fluctuations may interplay...)

Rotation detection with HBT measurements is possible!

Flow coefficients vs. rapidity: may help to establish 1th order event plane

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Effect of physical rotation (not jus final tilt):

Four differen ***eigen-frames:***

- of the rotated ***coordinate-space*** ellipsoid
- of the ***single particle spectrum***
- of the ***HBT correlation*** functions are ***three different*** frames!

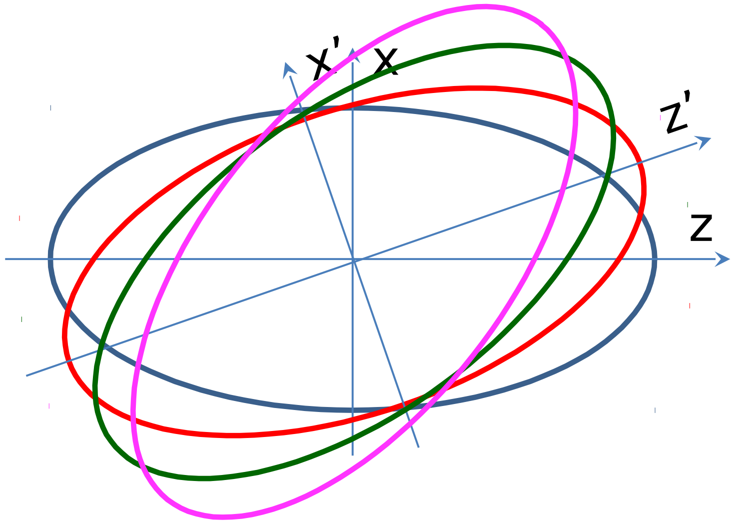
Signatures of rotation

Coordinate-ellipsoid at the start of time evolution

Final coordinate-space ellipsoid

„Momentum-space" ellipsoid (from single-particle spectrum)

„HBT-ellipsoid" (from HBT correlations):



$$\mathrm{tg}\left(2\vartheta_{\mathbf{p}}\right) = \omega_0 \frac{\left(X_0 + Z_0\right)^2}{X + Z} \frac{1}{\dot{X} + \dot{Z}}$$

$$\mathrm{tg}\left(2\vartheta_{HBT}\right) = \frac{m\omega_0 \frac{\left(X_0+Z_0\right)^2}{X+Z} X Z \left(\dot{Z} - \dot{X}\right)}{\left(T + \frac{m\omega_0^2}{2} \frac{\left(X_0+Z_0\right)^2}{X+Z}\right)\left(X^2 - Z^2\right) + m\left(X^2\dot{Z}^2 - Z^2\dot{X}^2\right)}$$

Summary and outlook

Hydrodynamical models:

Dynamically connect initial state with final state

Non-relativistic hydrodamical equations:

Extended the class of known parametric solutions

Found rotating, 3-axis ellipsoidal solutions, for arbitrary EoS

Simplified treatment

Relativistic generalization needed! (eg. HBT radii now do not depend on transverse momentum in NR setting)

Signals of rotation:

Directed & third flow vs. rapidity

1th order oscillations of out-long, side-long HBT radii parameters

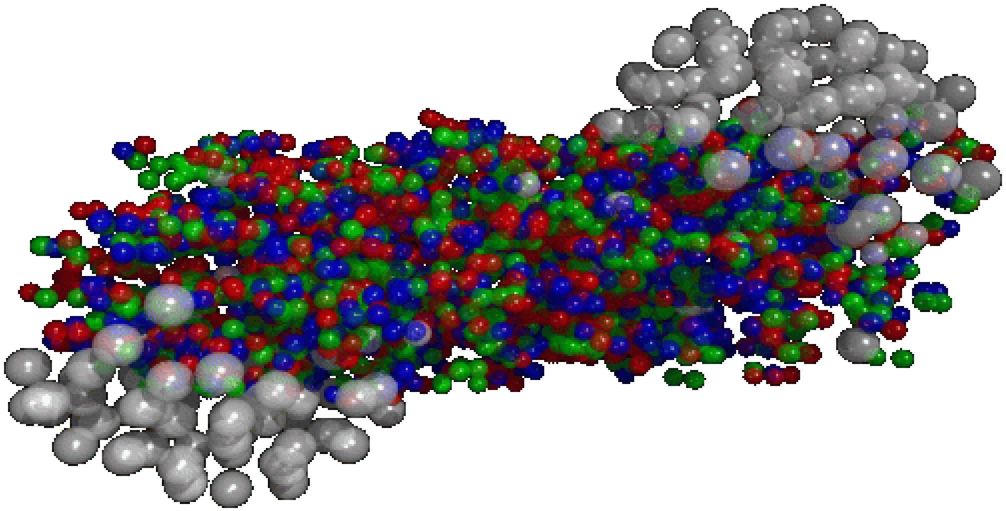
Measurement of tilt angle:

Combined knowledge of momentum space tilt angle, ,,HBT-space'' tilt angles is necessary! (coordinate-space tilt is not enough...)

A final state variable; initial angle known to be zero?

Possible to investigate EoS!

Thank you for your attention!



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