Exact Solutions for Observables and Initial Conditions for a Rehadronizing, 3d Expanding Fireball with lattice QCD Equation of State

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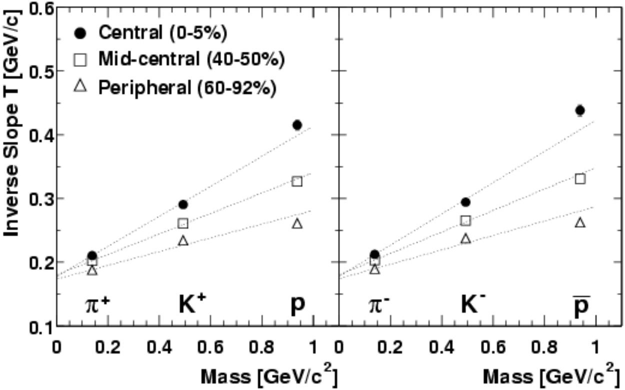
Motivation

- Deeper understanding of rehadronization
- More accurate description of the fireball evolution
- Previous analytic solutions describe a one component transition
- A multi-component scenario have to be more realistic
- First create a simplified, non-relativistic model
- Preparation of the relativistic calculation



$$T = T_f + m\langle u_t\rangle^2 \Longrightarrow T_i = T_f + m_i\langle u_t\rangle^2 \qquad (1)$$

[1] PHENIX Collaboration: arXiv:nucl-ex/0307022

Basic equations of non-relativistic hydrodynamics

$$\frac{\partial n}{\partial t} + \nabla (n\mathbf{v}) = 0, \tag{2}$$

$$\frac{\partial \varepsilon}{\partial t} + \nabla (\varepsilon \mathbf{v}) = -p\nabla \mathbf{v}, \tag{3}$$

$$mn \left(\frac{\partial}{\partial t} + \mathbf{v}\nabla \right) \mathbf{v} = -\nabla p. \tag{4}$$

Basic equations of relativistic hydrodynamics

$$\partial_\mu (nu^\mu) = 0, \tag{5}$$

$$\partial_\nu T^{\mu\nu} = 0. \tag{6}$$

Before the rehadronization

There's no particle conservation!

$$\varepsilon = T\sigma - p + \mu n \Longrightarrow \varepsilon = T\sigma - p + \sum_i \mu_i n_i, \qquad (7)$$

$$\mu_i = 0 \Longrightarrow \varepsilon + p = T\sigma \Longrightarrow d\varepsilon = Td\sigma. \qquad (8)$$

From the energy conservation:

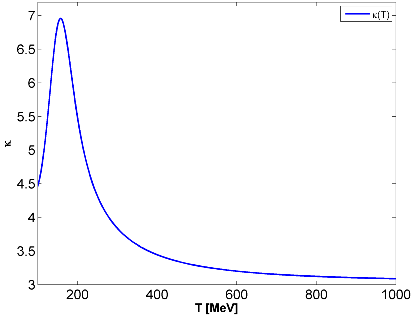
$$\frac{\partial\sigma}{\partial t} + \nabla\left(\mathbf{v}\sigma\right) = 0. \qquad (9)$$

Euler equation ($\mu = 0,\ n \to 0,\ v \ll c = 1$):

$$(\varepsilon + p)\left(\partial_t + \mathbf{v}\nabla\right)\mathbf{v} = T\sigma\left(\partial_t + \mathbf{v}\nabla\right)\mathbf{v} = -\nabla p. \qquad (10)$$

II. Basic equations

Equation of state (from lattice QCD [2]): $\varepsilon = \kappa_{QCD}(T)p$.



Energy conservation $\Longrightarrow$ diff. eq. of temperature [3]:

$$\frac{1+\kappa}{T}\left[\frac{d}{dT}\frac{\kappa T}{1+\kappa}\right]\left(\partial_t + \mathbf{v}\nabla\right)T + \nabla\mathbf{v} = 0. \qquad (11)$$

[2] Sz. Borsányi, G. Endrődi and others: arXiv:1007.2580

[3] T. Csörgő, M.I. Nagy: arXiv:1309.4390

After the rehadronization

It was known for one component ($T \ll m \Longrightarrow \mu \approx m$):

$$\varepsilon + p = \mu n + T\sigma \approx mn. \tag{12}$$

For the multi-component scenario:

$$\varepsilon \approx \sum_i m_i n_i \gg p = \sum_i p_i, \tag{13}$$

$$\sum_i m_i n_i \left(\partial_t + \mathbf{v}\nabla\right) \mathbf{v} = -\sum_i \nabla p_i. \tag{14}$$

Diff. equation of T [3]:

$$\left[\frac{d}{dT}\kappa T\right] \left(\partial_t + \mathbf{v}\nabla\right) T + T\nabla\mathbf{v} = 0, \tag{15}$$

$$\lim_{T\to 0} \kappa_{HRG}(T) = const. \tag{16}$$

III. Crossover

sQGP	Multi-component hadron gas
$\frac{\partial\sigma}{\partial t} + \nabla(\mathbf{v}\sigma) = 0$	$\frac{\partial n_i}{\partial t} + \nabla(\mathbf{v}n_i) = 0, \quad \forall i$
$T\sigma(\partial_t + \mathbf{v}\nabla)\mathbf{v} = -\nabla p$	$\sum_i m_i n_i (\partial_t + \mathbf{v}\nabla)\mathbf{v} = -\sum_i \nabla p_i$
$\frac{1+\kappa}{T}\left[\frac{d}{dT}\frac{\kappa T}{1+\kappa}\right](\partial_t + \mathbf{v}\nabla) T = -\nabla\mathbf{v}$	$\frac{1}{T}\left[\frac{d}{dT}\kappa T\right](\partial_t + \mathbf{v}\nabla) T = -\nabla\mathbf{v}$
$\kappa = \kappa_{QCD}(T)$	$\kappa = \kappa_{HRG}(T)$

Boundary conditions (B=before, A=after)

t_r: the estimated "moment" of the rehadronization

$$T_B(t_r, \not{r}) = T_A(t_r, \not{r}) \tag{17}$$

$$\mathbf{v}_B(t_r) = \mathbf{v}_A(t_r) \tag{18}$$

$$\kappa_{QCD}(T_B(t_r)) = \kappa_{HRG}(T_A(t_r)) \tag{19}$$

$$\{X_B(t_r), Y_B(t_r), Z_B(t_r)\} = \{X_A(t_r), Y_A(t_r), Z_A(t_r)\} \tag{20}$$

We are looking for a solution, which allows us to use the same scaling for each component of the hadron gas, therefore the gas expands collectively:

$$\{X_i, Y_i, Z_i\} = \{X, Y, Z\}, \quad \forall i. \tag{21}$$

Ideal gas approximation:

$$p = \sum_i p_i = T \sum_i n_i, \tag{22}$$

replace it to the Euler-equation:

$$\sum_i m_i n_i \left(\partial_t + \mathbf{v}\nabla\right) \mathbf{v} = -T \sum_i \nabla n_i. \tag{23}$$

Searching of the solution

The expression of entropy density [3]:

$$\sigma(\mathbf{r},t) = \sigma_r \frac{V_r}{V} e^{-s/2} = \frac{\sigma_r}{n_{i,r}} n_i(\mathbf{r},t) \tag{24}$$

$$s = \frac{r_x^2}{X^2} + \frac{r_y^2}{Y^2} + \frac{r_z^2}{Z^2} \tag{25}$$

Let's follow Landau's guess:

$$\frac{\sigma(\mathbf{r},t)}{\sigma_r} = \frac{n_i(\mathbf{r},t)}{n_{i,r}} \Longrightarrow \sigma \sim \sigma_r \tag{26}$$

$$n_i(\mathbf{r},t) = n_{i,r} \left(\frac{X_r Y_r Z_r}{XYZ} \right) e^{-\frac{r_x^2}{2X^2} - \frac{r_y^2}{2Y^2} - \frac{r_z^2}{2Z^2}} = n_{i,r} \frac{V_r}{V} \nu(s) \tag{27}$$

The velocity field (Hubble profile):

$$v_x = \frac{\dot{X}(t)}{X(t)} r_x, \quad v_y = \frac{\dot{Y}(t)}{Y(t)} r_y, \quad v_z = \frac{\dot{Z}(t)}{Z(t)} r_z. \qquad (28)$$

The temperature profile (at low T, $\kappa \to$ const.):

$$T(t) = T_r \left(\frac{X_r Y_r Z_r}{XYZ} \right)^{1/\kappa_0} . \qquad (29)$$

where

$$T_r = T_A(t_r) = T_B(t_r) \approx T_c \approx 155\ MeV.$$

is the temperature of the transition.

Calculate with the Euler equation

First replace $v_x(\mathbf{r}, t)$:

$$r_x \frac{\ddot{X}}{X} \sum_i m_i n_i = -T \sum_i \nabla_x n_i, \tag{30}$$

then replace $n_i(\mathbf{r}, t)$:

$$r_x \nu(s) \frac{V_r}{V} \frac{\ddot{X}}{X} \sum_i m_i n_{i,r} = r_x \nu(s) \frac{T}{X^2} \sum_i n_{i,r}. \tag{31}$$

Put through the eliminations:

$$\ddot{X} X \sum_i m_i n_{i,r} = T \sum_i n_{i,r}. \tag{32}$$

III. Crossover

The mean of the mass of the particles:

$$\langle m \rangle = \frac{\sum_i m_i n_{i,r}}{\sum_i n_{i,r}}. \tag{33}$$

The differential equation of the expanding fireball:

$$X\ddot{X} = Y\ddot{Y} = Z\ddot{Z} = \frac{T}{\langle m \rangle}. \tag{34}$$

In one component case:

$$X\ddot{X} = Y\ddot{Y} = Z\ddot{Z} = \frac{T}{m}. \tag{35}$$

Almost the same! One difference: $m \Longleftrightarrow \langle m \rangle$. As a conclusion we don't need to introduce several scales for each component of the medium!

IV. Observables

Number of particles:

$$N_i = \int_{-\infty}^{\infty} n_i(\mathbf{r}, t_r) d^3\mathbf{r} = n_{i,r}(2\pi)^{3/2} V_r \quad (36)$$

$$\sum_i N_i = (2\pi)^{3/2} V_r \sum_i n_{i,r} \quad (37)$$

V_r is the same for each particle.
The share of the jth component from the mix is:

$$\frac{n_{j,r}}{\sum_i n_{i,r}} = \frac{N_j}{\sum_i N_i}, \quad (38)$$

Thus $\langle m \rangle$ is the mass weighted by the number of particles:

$$\langle m \rangle = \frac{\sum_i m_i n_{i,r}}{\sum_i n_{i,r}} = \frac{\sum_i m_i N_i}{\sum_i N_i} \quad (39)$$

Momentum distribution

$$\frac{d^3 n}{dp_x dp_y dp_z} \propto \int d^3 r \, dt \, S(\mathbf{r}, \mathbf{p}, t) \delta(t - t_f), \tag{40}$$

where

$$S(\mathbf{r}, \mathbf{p}) \propto exp \left(- \sum_{j=x,y,z} \frac{\left(r_j - \frac{p_j}{m} t_f \right)^2}{2 R_T^2} - \frac{r_x^2}{2 X_f^2} - \frac{r_y^2}{2 Y_f^2} - \frac{r_z^2}{2 Z_f^2} \right), \tag{41}$$

Inverse slope

In one-component case, for a system with ellipsoidal symmetry [4]:

$$T_x = T_f + m \dot{X_f}^2, \tag{42}$$

$$T_y = T_f + m \dot{Y_f}^2, \tag{43}$$

$$T_z = T_f + m \dot{Z_f}^2. \tag{44}$$

[4] T. Csörgő, S.V. Akkelin and others: arXiv:hep-ph/0108067v4

In multi-component case:

$$m \to m_i, \quad \mathbf{p} \to \mathbf{p}_i, \quad S(\mathbf{r},\mathbf{p}) \to S_i(\mathbf{r},\mathbf{p}_i), \quad d^3n \to d^3n_i, \tag{45}$$

but the method of the calculation doesn't change. The momentum distribution is governed by

$$\frac{d^3 n_i}{dp_x dp_y dp_z} \propto exp\left(-\sum_{j=x,y,z} \frac{p_{j,i}^2}{2m_i T_{j,i}}\right), \tag{46}$$

where

$$T_{x,i} = T_f + m_i \dot{X_f}^2, \tag{47}$$

$$T_{y,i} = T_f + m_i \dot{Y_f}^2, \tag{48}$$

$$T_{z,i} = T_f + m_i \dot{Z_f}^2. \tag{49}$$

Two-particle correlations

$$C_2(\mathbf{q},\mathbf{p}) \simeq 1 \pm \frac{|\tilde{S}(\mathbf{q},\mathbf{p})|^2}{|\tilde{S}(0,\mathbf{p})|^2}. \tag{50}$$

HBT-radii

In one-component case, for a system with ellipsoidal symmetry:

$$C_2(\mathbf{q}) = 1 + e^{-q_x^2 R_x^2 - q_y^2 R_y^2 - q_z^2 R_z^2}, \tag{51}$$

where the HBT-radii are [4]

$$R_x^2 = \frac{X_f^2}{1 + \frac{m}{T_f}\dot{X}_f^2} = \frac{T_f}{T_x} X_f^2, \tag{52}$$

$$R_y^2 = \frac{Y_f^2}{1 + \frac{m}{T_f}\dot{Y}_f^2} = \frac{T_f}{T_y} Y_f^2, \tag{53}$$

$$R_z^2 = \frac{Z_f^2}{1 + \frac{m}{T_f}\dot{Z}_f^2} = \frac{T_f}{T_z} Z_f^2. \tag{54}$$

In multi-component case:

$$m \to m_i, \quad \mathbf{p} \to \mathbf{p}_i, \quad \tilde{S}(\mathbf{r},\mathbf{p}) \to \tilde{S}_i(\mathbf{r},\mathbf{p}_i), \quad C_2 \to C_{2,i}, \tag{55}$$

In this way the correlation function is

$$C_{2,i}(\mathbf{q}) = 1 + e^{-q_x^2 R_{x,i}^2 - q_y^2 R_{y,i}^2 - q_z^2 R_{z,i}^2}, \tag{56}$$

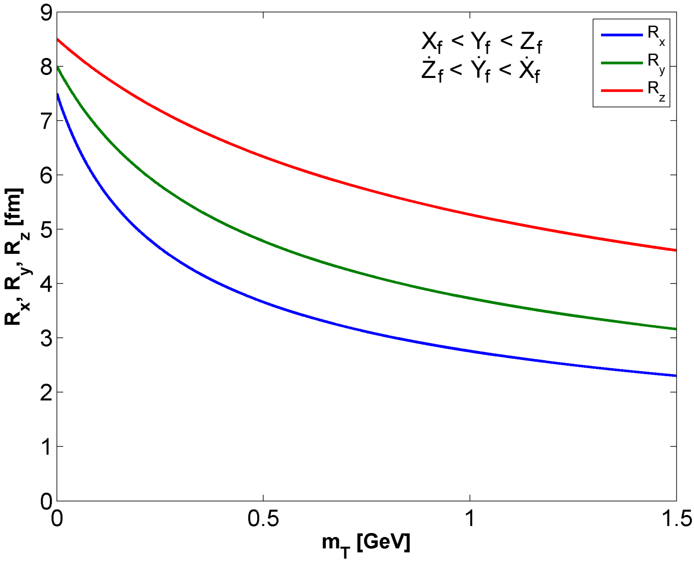
and consistently with the one-component case:

$$R_{x,i}^2 = \frac{X_f^2}{1 + \frac{m_i}{T_f}\dot{X}_f^2} = \frac{T_f}{T_{x,i}} X_f^2, \tag{57}$$

$$R_{y,i}^2 = \frac{Y_f^2}{1 + \frac{m_i}{T_f}\dot{Y}_f^2} = \frac{T_f}{T_{y,i}} Y_f^2, \tag{58}$$

$$R_{z,i}^2 = \frac{Z_f^2}{1 + \frac{m_i}{T_f}\dot{Z}_f^2} = \frac{T_f}{T_{z,i}} Z_f^2. \tag{59}$$

IV. Observables



Summary

- Rehadronization: crossover $\Longrightarrow$ simple boundary conditions
- Take into consideration the multi-component scenario
- Introduce scales independently from particle multiplicity
- Gain a similar dynamical equation to the one-component case
- The multi-component scenario does not complicate the description
- Difference: mean mass weighted by the number of particles
- Inverse slope parameters: $T \longrightarrow T_i$
- HBT-radii: $R \longrightarrow R_i$

The end

Thanks for your attention!