Transport Coefficients and Thermalisation in Classical Field Theories

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Homor, M. M. and Jakovac, A., *Shear viscosity of the Φ^4 theory from classical simulation*, PhysRevD.92.105011

Viscosity of hadronic matter

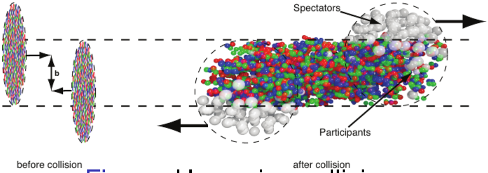


Figure: Heavy-ion collisions

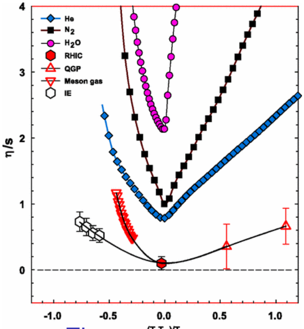


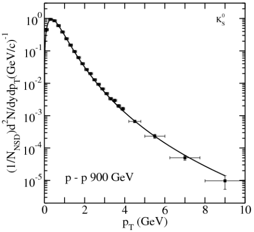
Figure: η/s [2]

- observation: nearly ideal liquid
- relevant quantity: η/s (damping of hydrodynamic waves)

Tsallis distribution in heavy-ion collisions and hadronization

Urmossy, K. and Barnaföldi, G.G. and Harangozó, Sz. and Biró, T.S. and Xu, Z., *A 'soft + hard' model for heavy-ion collisions*, arXiv:1501.02352 [hep-ph], 2015

Cleymans, J. and Worku, D., *The Tsallis Distribution in Proton-Proton Collisions at $\sqrt{s}$ = 0.9 TeV at the LHC*, J.Phys.G39:025006, 2012



Khachatryan, Vardan and others, *Transverse-momentum and pseudorapidity distributions of charged hadrons in* pp *collisions at* $\sqrt{s} = 7$ *TeV*, CMS Collaboration, Phys.Rev.Lett. 105.022002, 2010

Aamodt, K. and others, *Production of pions, kaons and protons in* pp *collisions at* $\sqrt{s} = 900$ *GeV with ALICE at the LHC*, Eur.Phys.J.C71:1655, 2011

Objectives

- thermalization properties
- viscosity (η) in classical field theory
- small transport coefficient, non-trivial distributions $\Leftrightarrow$ strongly interacting system $\rightarrow$ non-perturbative
- Boltzmann equation, MC (less sensitive for $\omega \rightarrow 0$)
- different approach:
- test: classical Φ^4 theory

$$\mathcal{H} = \frac{1}{2}\Pi^2 + \frac{1}{2}(\nabla\Phi)^2 + \frac{m^2}{2}\Phi^2 + \frac{\lambda}{24}\Phi^4 \qquad (1)$$

- "toy model" which may show interesting effects mentioned above

Simulation

- canonical equations, periodic boundary conditions, leap-frog algorithm
- initial conditions: $\left\{\Pi\left(t_0+\frac{\delta t}{2}\right), \Phi(t_0)\right\}$
- uniform and $f(\Pi) \sim \operatorname{sech}\left(\frac{\pi}{2}\Pi\right) \rightarrow$ hyperbolic secant distribution
- Canonical eq. of $\dot{\Phi}$ (1st part of time step): Initial condition $\rightarrow \Phi(t_0+\delta t)$
- Canonical eq. of $\dot{\Pi}$ (2nd part of time step): $\left\{\Phi(t_0+\delta t), \Pi\left(t_0+\frac{\delta t}{2}\right)\right\} \rightarrow \Pi\left(t_0+\frac{3\delta t}{2}\right)$
- input parameters: N^3 lattice size, $a=1$ (grid), λ (interaction), m^2 Lagrangian-mass
- test: total energy conserved
- temperature: $T=\frac{1}{2N^3}\langle|\Pi_k|^2\rangle$

Viscosity from Green-Kubo - formula

- Linear response theory (Kubo-formula for transport):

$$\eta = \lim_{\omega \to 0} \frac{\langle [T_{12}, T_{12}] \rangle (\omega, \mathbf{k} = 0)}{\omega}, \quad (2)$$

where $T_{12} = \partial_x \Phi \partial_y \Phi$.

- Correlation function for bosonic A and B operators: $S_{BA}(t) = \langle B(t) A(0) \rangle$.
- Fluctuation-dissipation theorem on finite temperature:

$$\langle [B, A] \rangle (\omega, \mathbf{k} = 0) = (1 - \mathrm{e}^{-\beta\omega}) S_{BA}(\omega, \mathbf{k} = 0). \quad (3)$$

- if $\beta\omega << 1$ ($\omega \to 0$):

$$\lim_{\omega \to 0} \frac{1}{\omega} \langle [B, A] \rangle (\omega, \mathbf{k} = 0) = \beta S_{BA}(\omega = 0, \mathbf{k} = 0). \quad (4)$$

- Green-Kubo - formula: $\eta = \beta \langle T_{12} T_{12} \rangle_{(k=0)}$

$\langle T_{12}T_{12}\rangle$

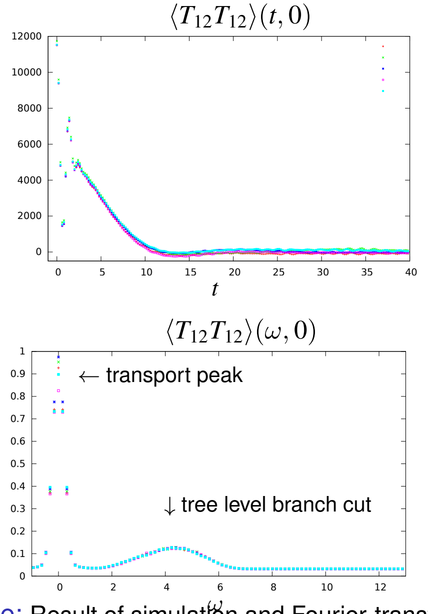


Figure: Result of simulation and Fourier-transform

Classical viscosity

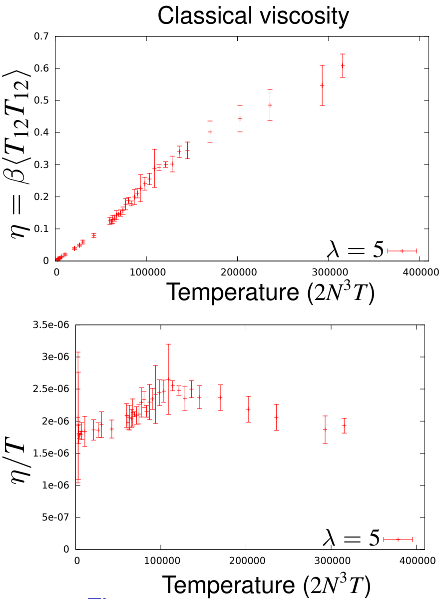


Figure: Classical viscosity

Expectation values of local quantities

- Local quantity: $A(\Phi, \Pi)$
- Real measurement is the time average:

$$\langle A(\Phi, \Pi)\rangle = \frac{1}{t}\int_{t_0}^{t_0+t} \mathrm{d}t' A(\Phi(t), \Pi(t)) \tag{5}$$

- Inserting δ integral $\langle A(\Phi, \Pi)\rangle =$

$$\int \mathcal{D}\bar{\Phi}\mathcal{D}\bar{\Pi} A(\bar{\Phi}, \bar{\Pi}) \frac{1}{t}\int_{t_0}^{t_0+t} \mathrm{d}t' \delta(\bar{\Phi} - \Phi(t))\delta(\bar{\Pi} - \Pi(t)) \tag{6}$$

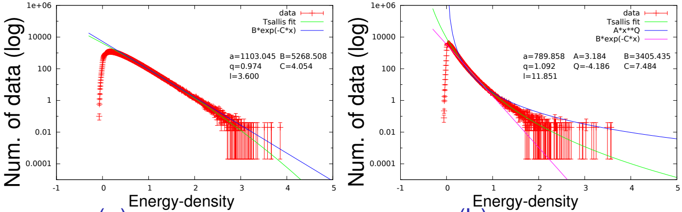
- where:
- $f(\bar{\Phi}, \bar{\Pi}) = \frac{1}{t}\int_{t_0}^{t_0+t} \mathrm{d}t' \delta(\bar{\Phi} - \Phi(t))\delta(\bar{\Pi} - \Pi(t))$
- Time-average $\rightarrow$ Ensemble average

$$\langle A(\Phi, \Pi)\rangle = \int \mathcal{D}\bar{\Phi}\mathcal{D}\bar{\Pi} A(\bar{\Phi}, \bar{\Pi}) f(\bar{\Phi}, \bar{\Pi}) \tag{7}$$

- e.g. canonical: $e^{-\beta\mathcal{H}}$

Energy histogram

Early time local energy-distribution function is not Boltzmannian



(a) uniform init. (b) sech init.

Figure: Different fits on logscale energy histogram

$$\mathcal{P}(\varepsilon) = \langle \delta \left(\varepsilon - \varepsilon_x \left[\bar{\Phi}, \bar{\Pi} \right] \right) \rangle \tag{8}$$

Tsallis distribution is an excellent fit!

$$\mathcal{P}(\varepsilon) = a \left[1 + (q-1)\beta\varepsilon \right]^{\frac{1}{1-q}} \tag{9}$$

$\rightarrow$ consider the time evolution of q

Not Tsallis?

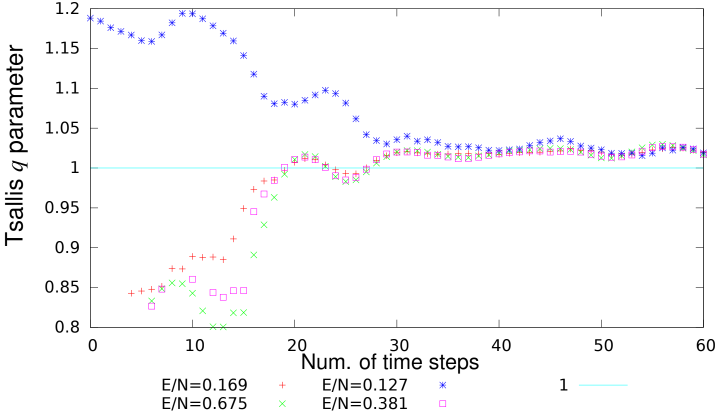


Figure: Time dependence of the Tsallis parameter

Tsallis?

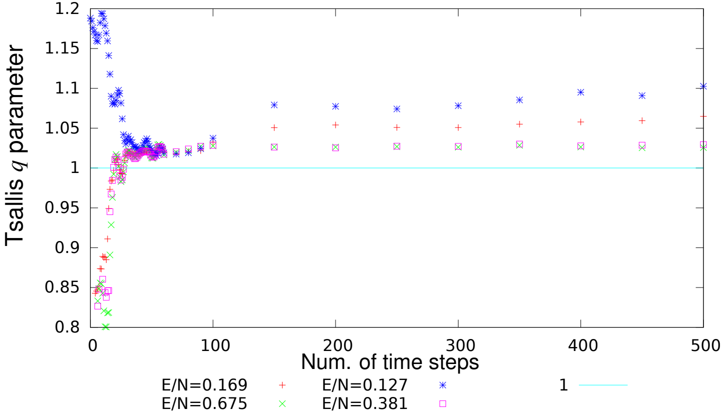


Figure: Time dependence of the Tsallis parameter

Just pre-thermal?

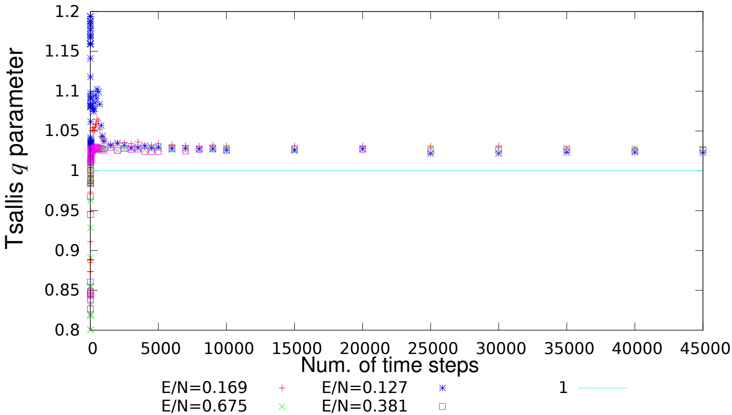


Figure: Time dependence of the Tsallis parameter

$$q \approx 1.026$$

$\Pi(x)$ histogram

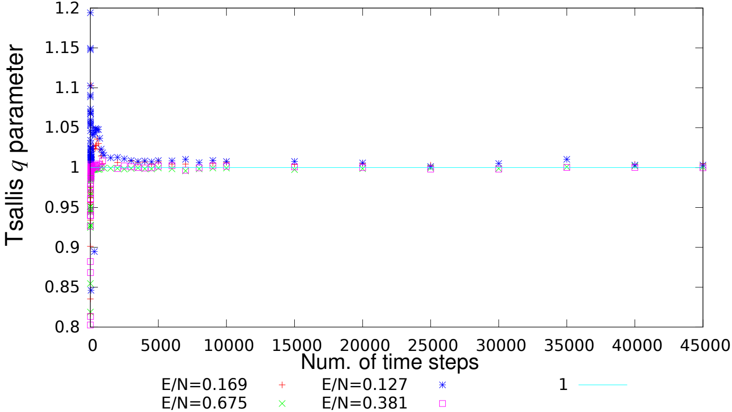
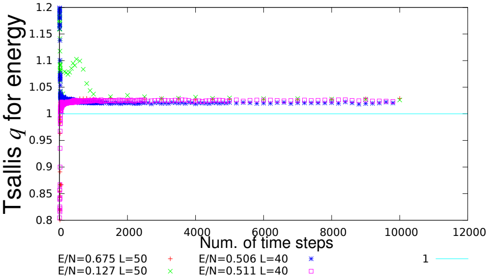
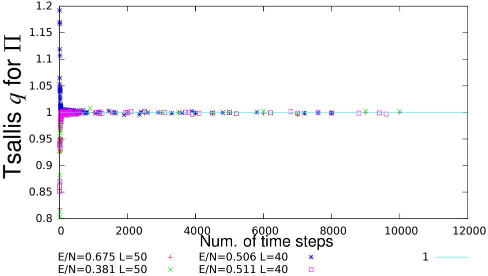


Figure: Time dependence of the Tsallis parameter for Π histogram

$$q \approx 0.999$$

Lattice size





Interpretation

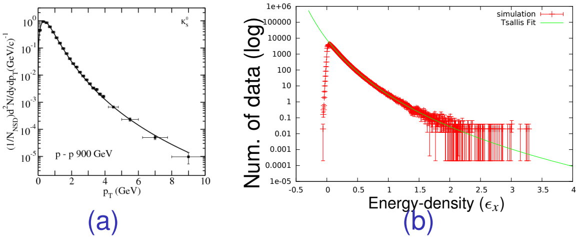


Figure: Experiment vs. Φ^4 simulation

- Π histogram $\Rightarrow$ canonical ens. $f(\bar{\Phi}, \bar{\Pi}) \sim e^{-\beta\mathcal{H}[\bar{\Phi},\bar{\Pi}]}$
- ε histogram $\Rightarrow$ local energy distr. is Tsallis
- ***if*** *hadron creation depends on local energy density* $\rightarrow$ hadron yields are Tsallis
- local energy distr. determined by computer simulations $\rightarrow$ tool for measuring hadron distr.
- QCD? Work in progress...

Total energy

$$E = \sum_{\mathbf{i}\in U} \frac{1}{2}\Pi_{\mathbf{i}}^2 + \frac{1}{2}(\nabla\Phi)_{\mathbf{i}}^2 + \frac{m^2}{2}\Phi_{\mathbf{i}}^2 + \frac{\lambda}{24}\Phi_{\mathbf{i}}^4, \qquad (10)$$

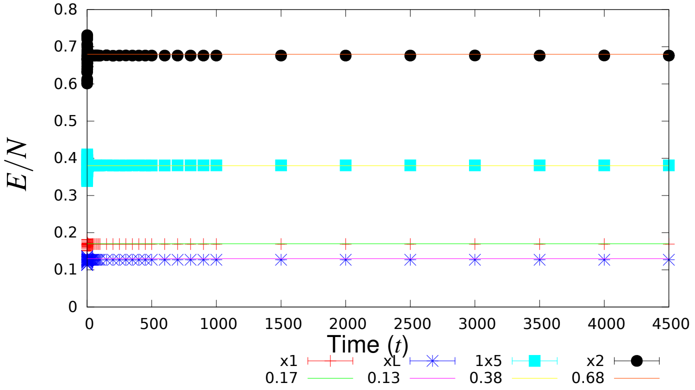


Figure: Time dependence of E/N where $N = 50^3$

Temperature $\langle|\Pi_k|^2\rangle = 2N^3T$

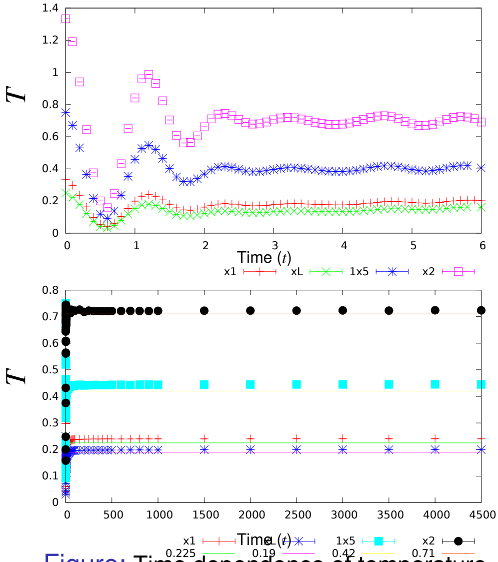
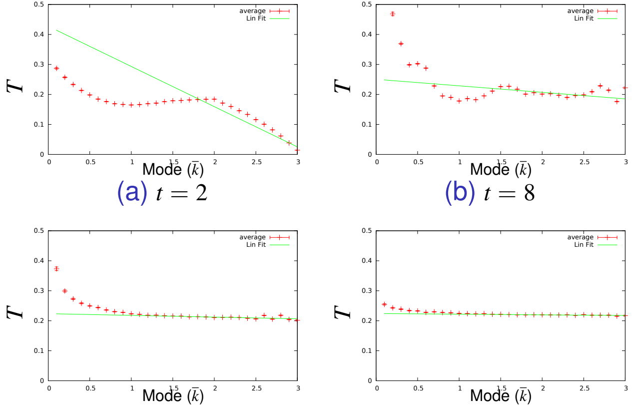


Figure: Time dependence of temperature

Equipartition

equilibrium $\rightarrow$ equipartition



(a) $t = 2$ (b) $t = 8$ (c) $t = 200.1$ (d) $t = 4500.1$

Figure: Equipartition during time evolution ($\lambda = 5$)

Bibliography

M. M. Homor and A. Jakovac. "Shear viscosity of the Φ^4 theory from classical simulation". In: *Phys. Rev. D* 92 (10 2015), p. 105011. DOI: 10.1103/PhysRevD.92.105011.

Roy A. Lacey et al. "Has the QCD Critical Point Been Signaled by Observations at the BNL Relativistic Heavy Ion Collider?" In: *Phys. Rev. Lett.* 98 (9 2007), p. 092301. DOI: 10.1103/PhysRevLett.98.092301.

Thank you for your attention!