The fluidity measure η/s from effective field theory

Miklós Horváth
Antal Jakovác

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A teaser

What does make matter more fluent?

A teaser

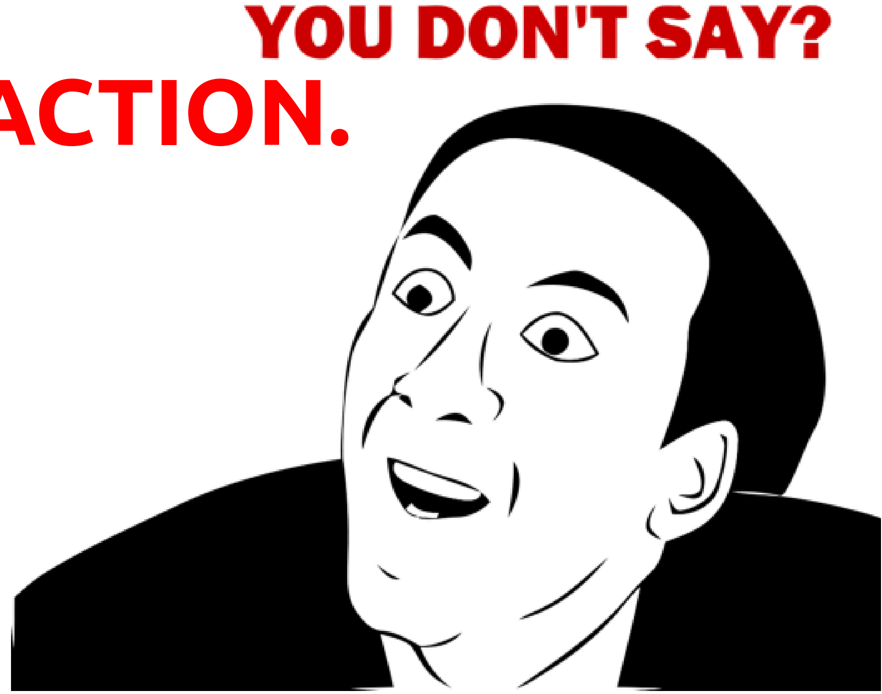
What does make matter more fluent?

INTERACTION.

A teaser

What does make matter more fluent?

INTERACTION.

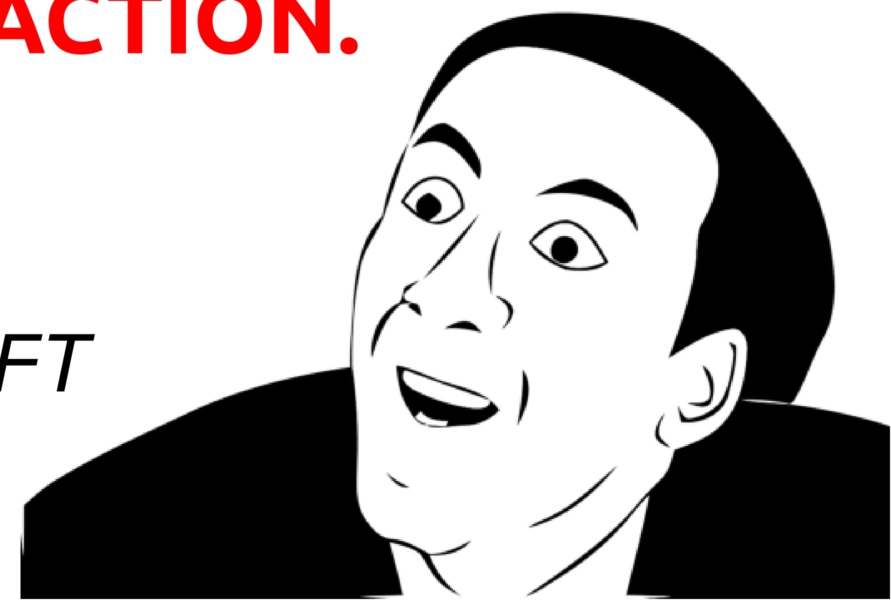


A teaser

What does make matter more fluent?

YOU DON'T SAY?

INTERACTION.



matter: *scalar channel of an integrable effective QFT*

fluidity: *η/s (shear viscosity to entropy density)*

The actual statement

micro

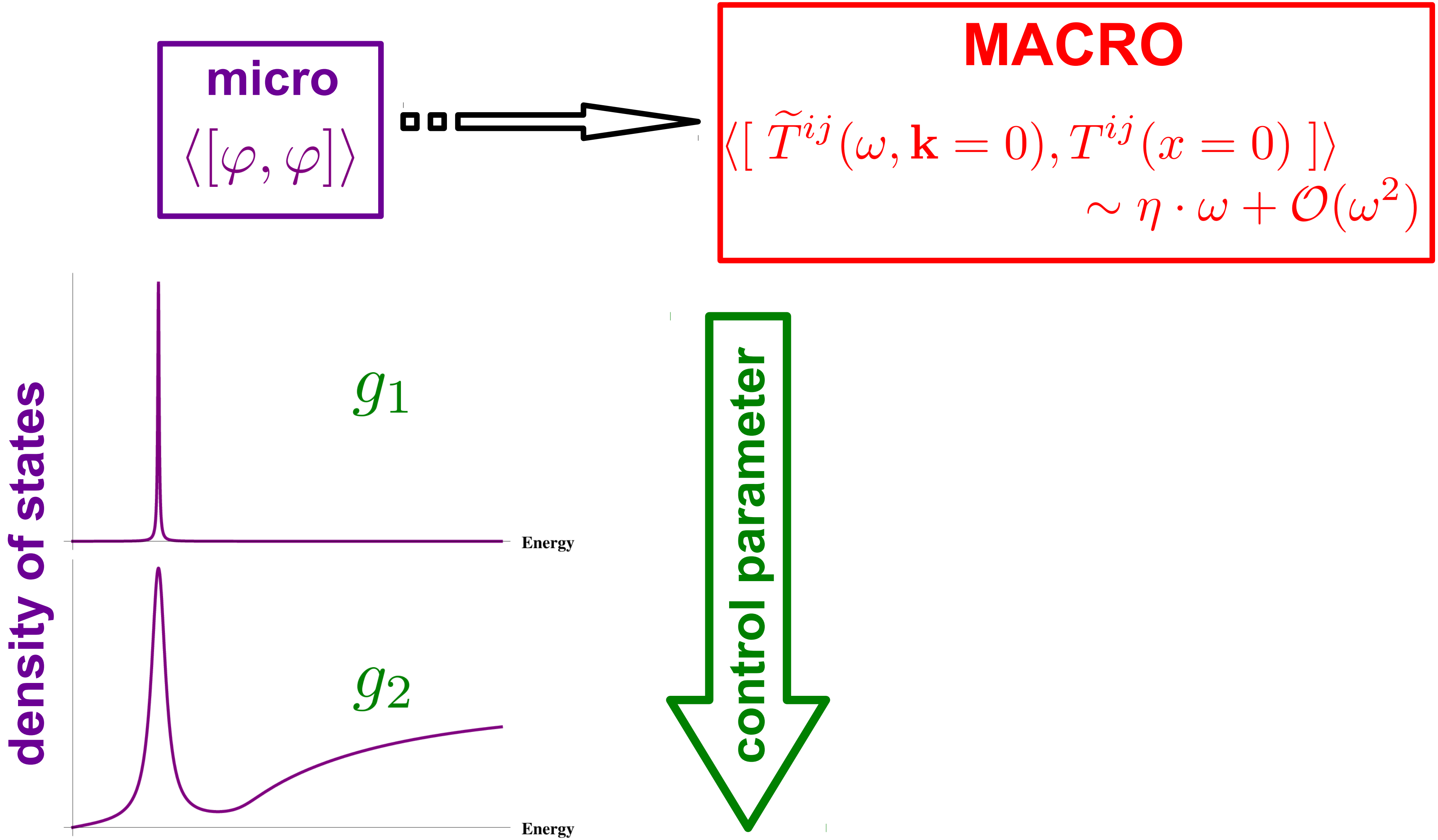
$$\langle [\varphi, \varphi] \rangle$$

$\Longrightarrow$

MACRO

$$\langle [\, \widetilde{T}^{ij}(\omega, \mathbf{k}=0), T^{ij}(x=0)\,] \rangle \sim \eta \cdot \omega + \mathcal{O}(\omega^2)$$

The actual statement



η/s from effective field theory

The actual statement

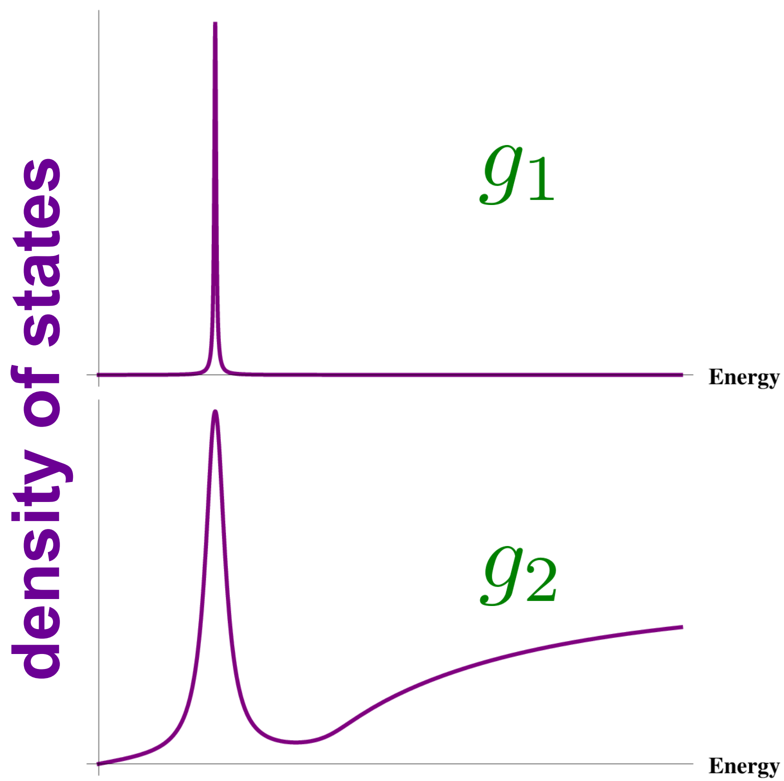
micro

$$\langle[\varphi,\varphi]\rangle$$

⟹

MACRO

$$\langle[\,\widetilde{T}^{ij}(\omega,\mathbf{k}=0),T^{ij}(x=0)\,]\rangle \sim \eta\cdot\omega+\mathcal{O}(\omega^2)$$



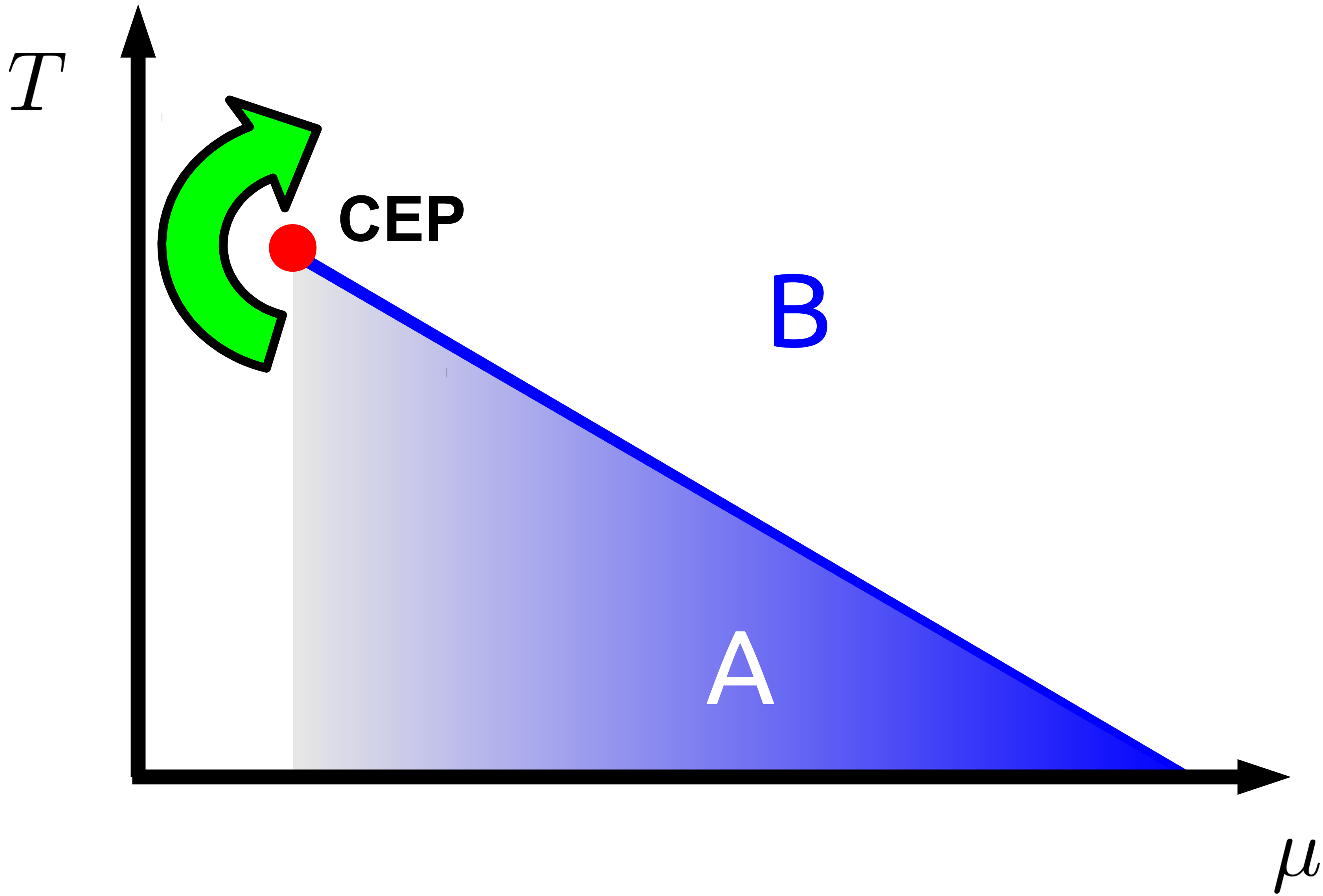
control parameter

$$\left.\frac{\eta}{s}\right|_{g_1} > \left.\frac{\eta}{s}\right|_{g_2}$$

measure of fluidity

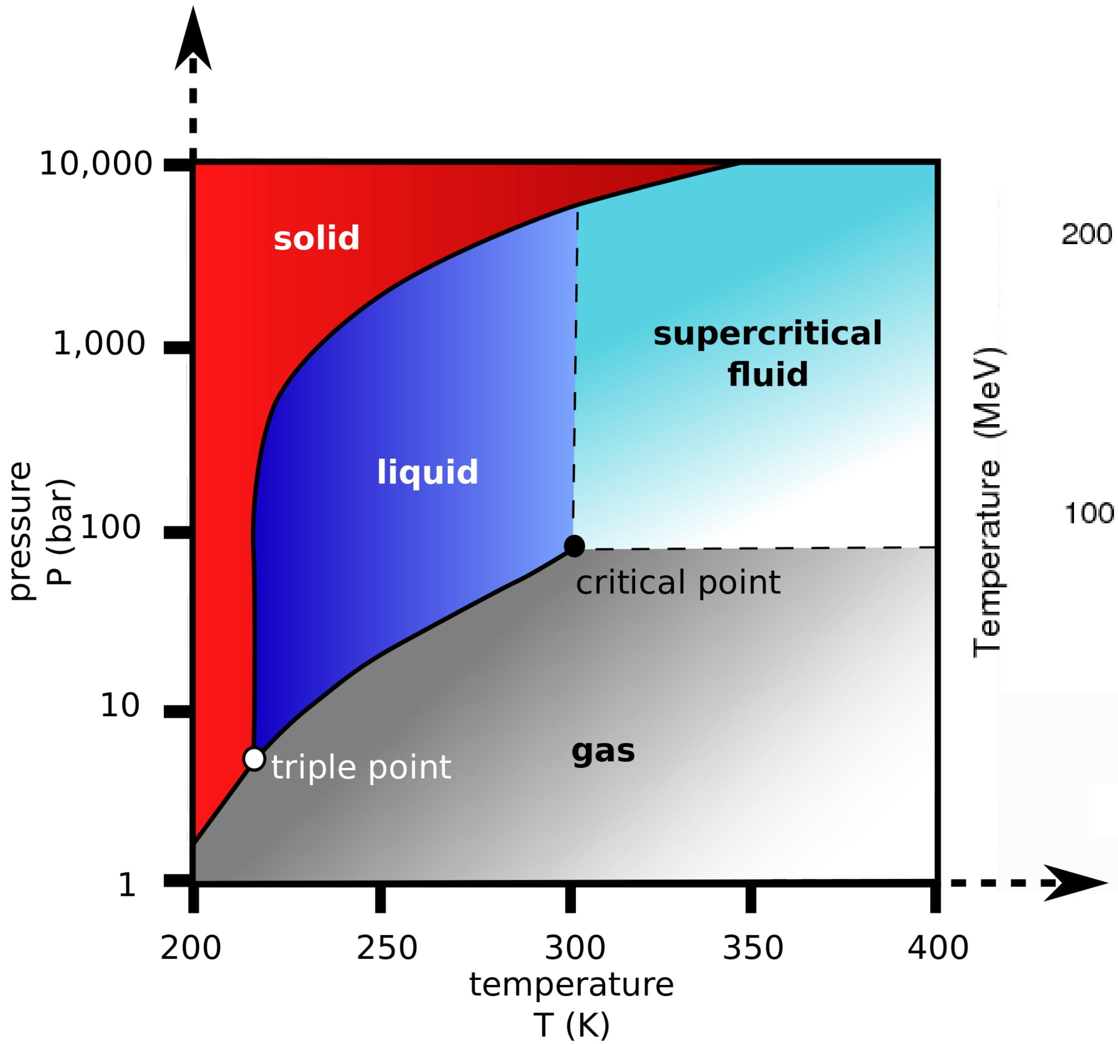
η/s from effective field theory

Near-critical behaviour

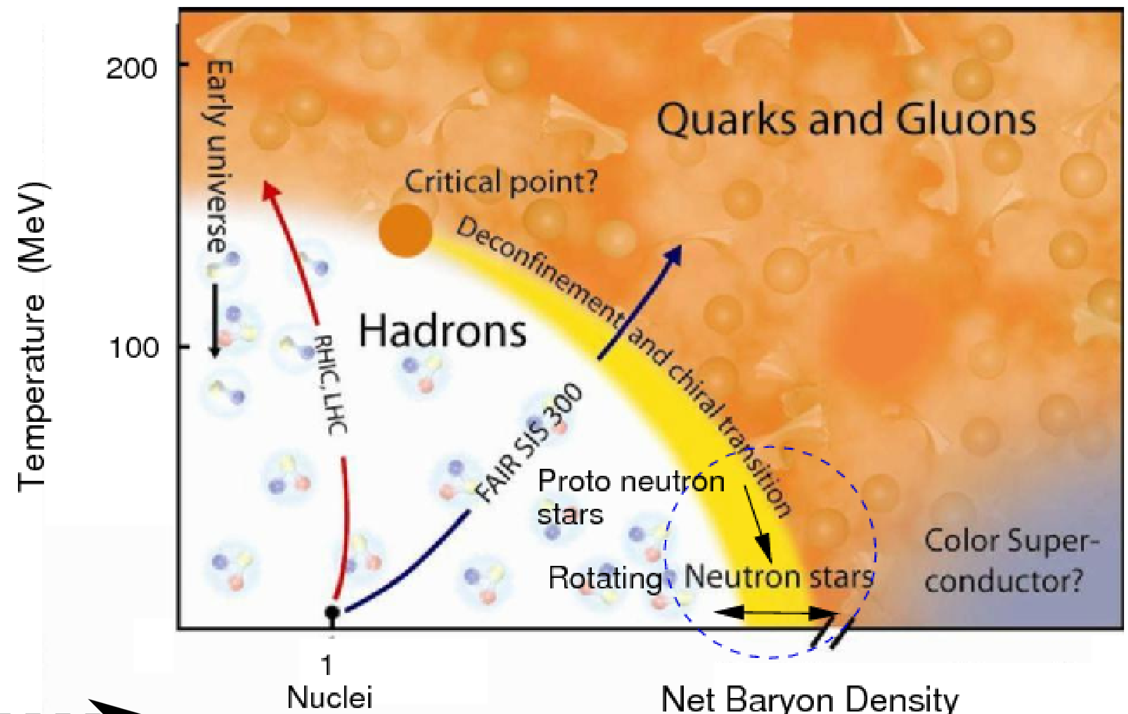


η/s from effective field theory

Near-critical behaviour



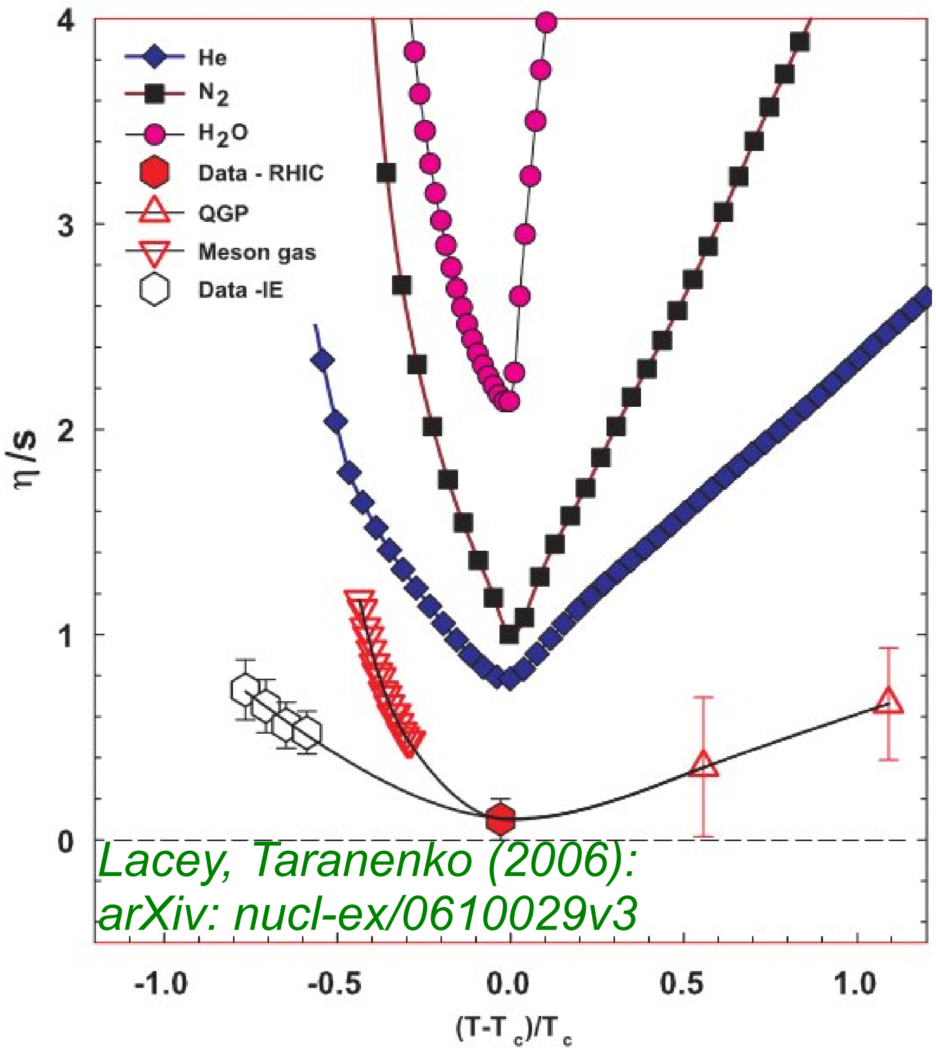
CO_2

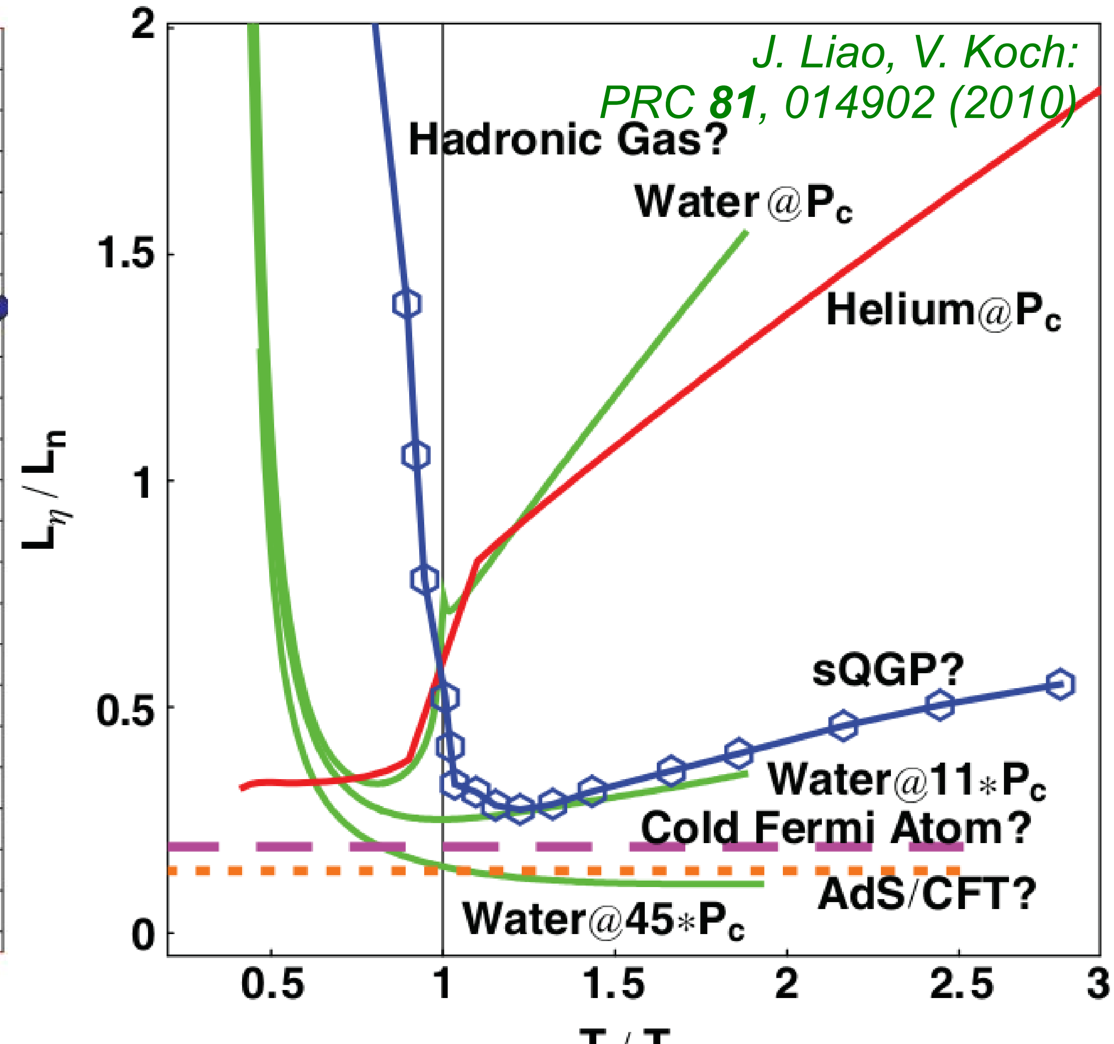


strongly interacting
matter (~QCD)

η/s from effective field theory

Measure of fluidity





$$\delta\pi_\perp^{\mu\nu}(k,t) = \exp\left(-\frac{\eta}{s}\frac{k^2 t}{T}\right)\cdot\delta\pi_\perp^{\mu\nu}(k,0)$$

$$\frac{L_\eta}{L_n} = \frac{n^{\frac{1}{3}}\eta}{c_s\rho} \sim \frac{\text{min. wavelength to propagate}}{\text{inner scale, prop. to } n^{-1/3}}$$

η/s from effective field theory

Measure of fluidity

gas of quasi-particles

(kinetic theory)

~momentum-diffusion coefficient

quasipart. approximation

$$\rho \frac{\delta v}{\tau} \sim \eta \frac{\delta v}{l^2} \quad \Rightarrow \quad \frac{\eta}{\rho} \sim \frac{\eta}{s} \sim \langle v \rangle \, l$$

hydrodynamics

damping of transverse hydrodynamic modes

$$\delta\pi_{\perp}^{\mu\nu}(k,t) = \exp\left(-\frac{\eta}{s}\frac{k^2 t}{T}\right) \cdot \delta\pi_{\perp}^{\mu\nu}(k,0)$$

cross section of scattering processes: $\sigma n l \sim 1$

$\sigma \to 0$

$l \to \infty$ **ideal gas**

weakly interacting, ***large*** η

$\sigma \to \infty$

$l \to 0$ **ideal liquid**

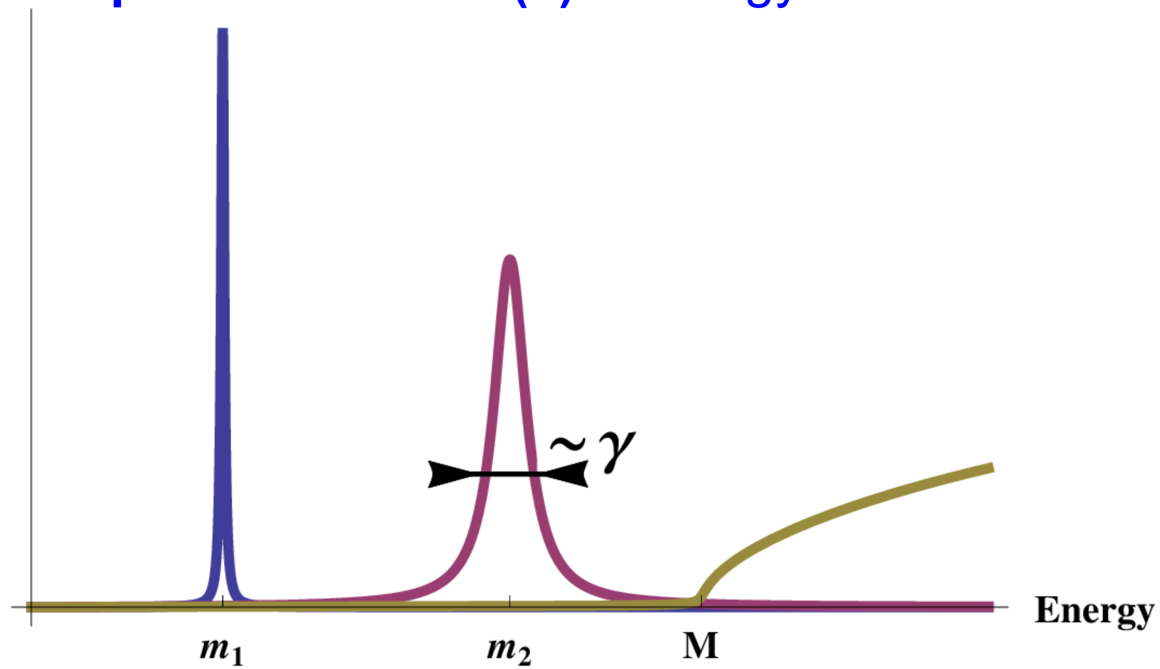
strongly interacting, ***small*** η

in linear response: $\eta = \left. \frac{\langle [T_{xy}(\omega, \mathbf{p}=0), T_{xy}(0)] \rangle}{\omega} \right|_{\omega \to 0}$

η/s from effective field theory

Extended quasi-particles

Parametrization: spectral function(s) = energy levels at fixed quantum numbers

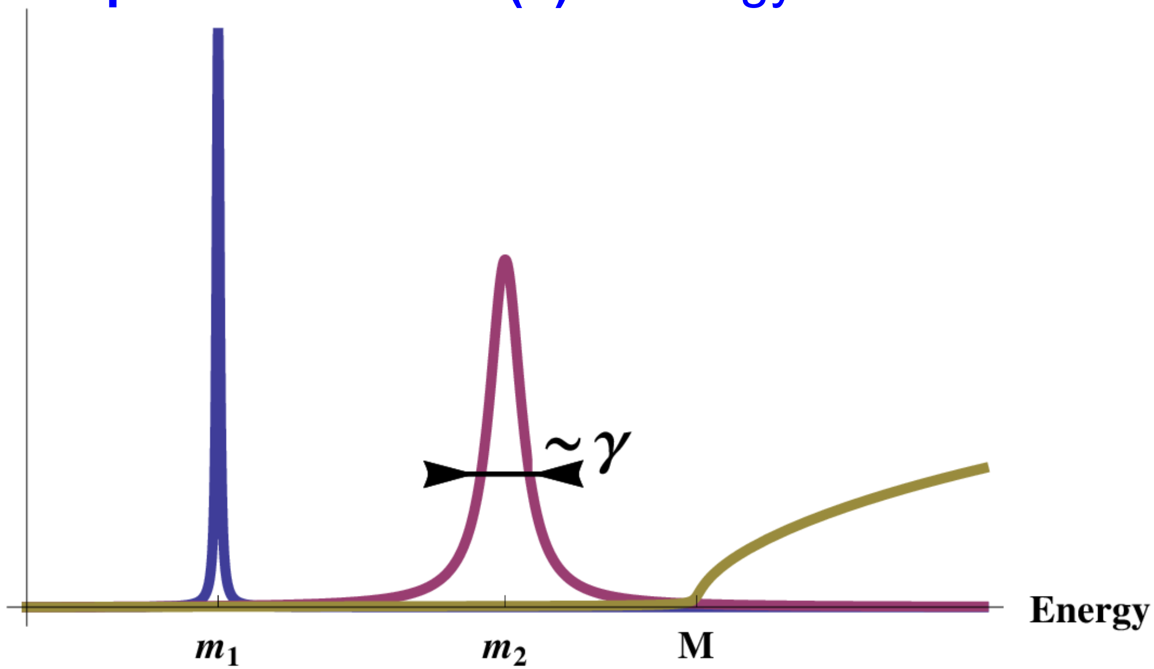


- *all dynamical informations at 2-point level*
- *response to local perturbations*

$$\rho(x) = \langle [\varphi_x , \varphi_{x=0}] \rangle$$

Extended quasi-particles

Parametrization: spectral function(s) = energy levels at fixed quantum numbers



$$S_{\text{eff}}[\varphi] = \int_x \int_y \varphi_x \mathcal{G}^{-1}_{x-y} \varphi_y + \int_x J_x \varphi_x$$

$$\rho(x) = \langle [\varphi_x, \varphi_{x=0}] \rangle$$

$$-\text{Im}\mathcal{G} = \rho$$

EQP – thermo. & transport

arXiv:1512.03001

$$\varepsilon = \int_0^\infty \mathrm{d}p \frac{\partial \mathrm{Re} G^{-1}}{\partial p} \rho(p) T^4 \chi_\varepsilon(p/T)$$

$$P = \int_0^\infty \mathrm{d}p \frac{\partial \mathrm{Re} G^{-1}}{\partial p} \rho(p) T^4 \chi_P(p/T)$$

$$\eta = \int_0^\infty \mathrm{d}p \left(\frac{\partial \mathrm{Re} G^{-1}}{\partial p} \rho(p) \right)^2 T^4 \lambda_\eta(p/T)$$

η/s from effective field theory

EQP – thermo. & transport

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$$P = \int_0^\infty \mathrm{d}p \frac{\partial \mathrm{Re} G^{-1}}{\partial p} \rho(p) T^4 \chi_P(p/T)$$

~mass distribution

$$\eta = \int_0^\infty \mathrm{d}p \left(\frac{\partial \mathrm{Re} G^{-1}}{\partial p} \rho(p) \right)^2 T^4 \lambda_\eta(p/T)$$

EQP – thermo. & transport

arXiv:1512.03001

$$\varepsilon = \int_0^\infty \mathrm{d}p \, \frac{\partial \mathrm{Re} G^{-1}}{\partial p} \rho(p) T^4 \chi_\varepsilon(p/T) + B(T)$$

$$P = \int_0^\infty \mathrm{d}p \, \frac{\partial \mathrm{Re} G^{-1}}{\partial p} \rho(p) T^4 \chi_P(p/T) - B(T)$$

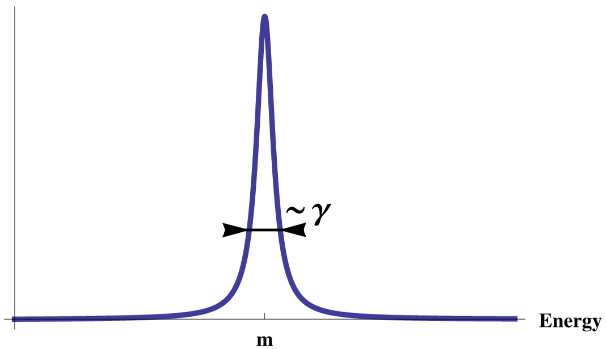
~mass distribution (marks $\frac{\partial \mathrm{Re} G^{-1}}{\partial p} \rho(p)$)

background (marks $\pm B(T)$)

$$\eta = \int_0^\infty \mathrm{d}p \left(\frac{\partial \mathrm{Re} G^{-1}}{\partial p} \rho(p) \right)^2 T^4 \lambda_\eta(p/T)$$

Broad QP-peak

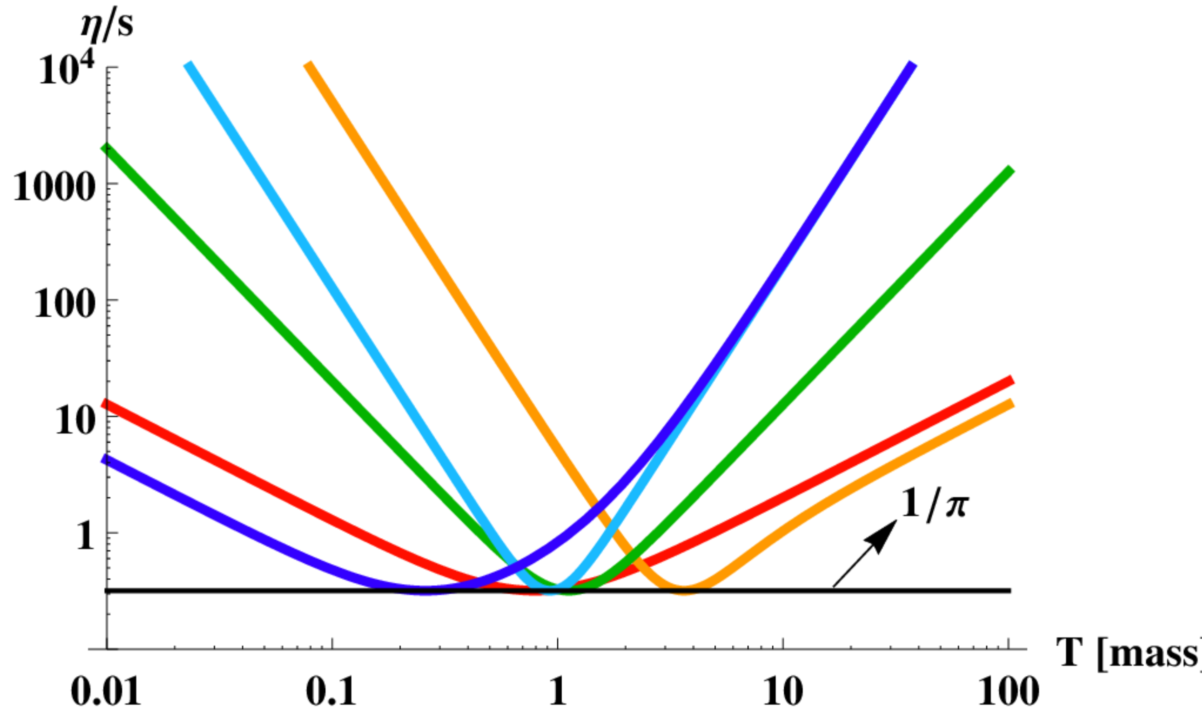
arXiv:1512.03001



$$\rho(\omega, \mathbf{p}) = \frac{4\gamma\omega}{(\omega^2 - \mathbf{p}^2 - \gamma^2)^2 + 4\gamma^2\omega^2}$$

$$s = \frac{2\pi^2}{45} T^3$$

$$\eta = \frac{\gamma T^2}{18} + \frac{2\pi^2}{225}\frac{T^4}{\gamma}$$



$$\frac{\eta}{s} = \frac{5}{4\pi^2}\frac{\gamma}{T} + \frac{1}{5}\frac{T}{\gamma}$$

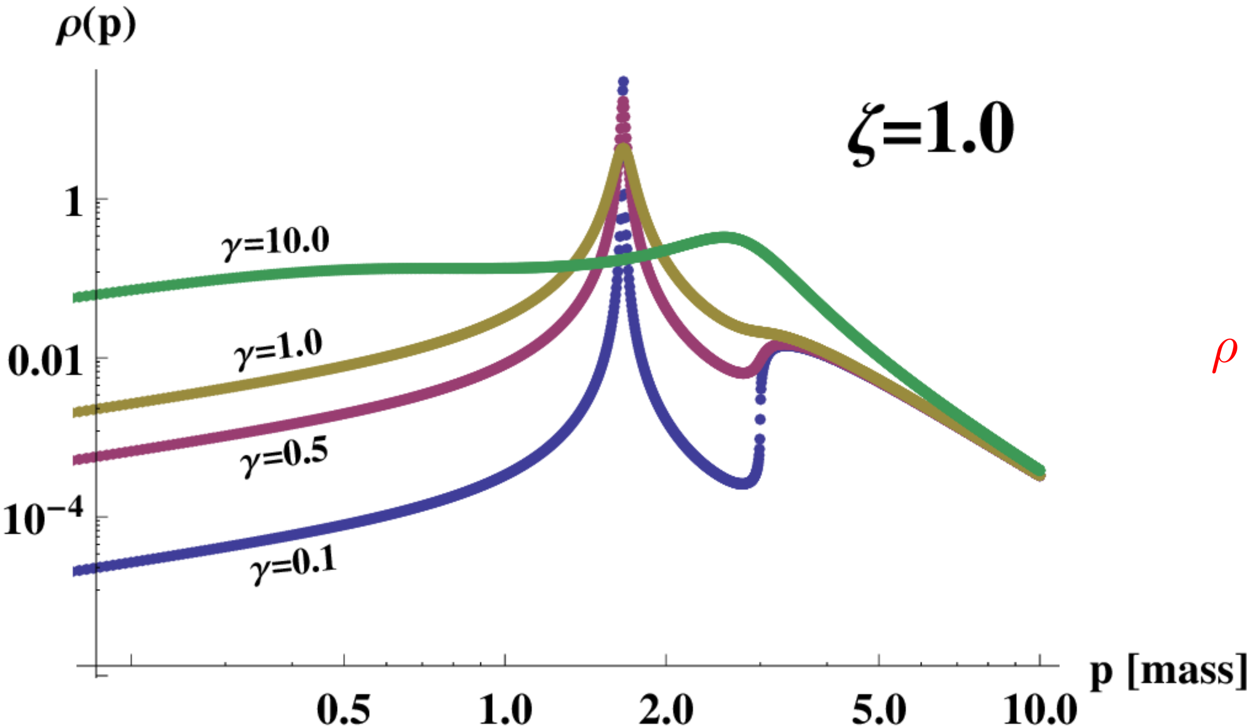
beyond the relaxation time approx.!

because of the term ~γ

η/s from effective field theory

Beyond the QP-peak

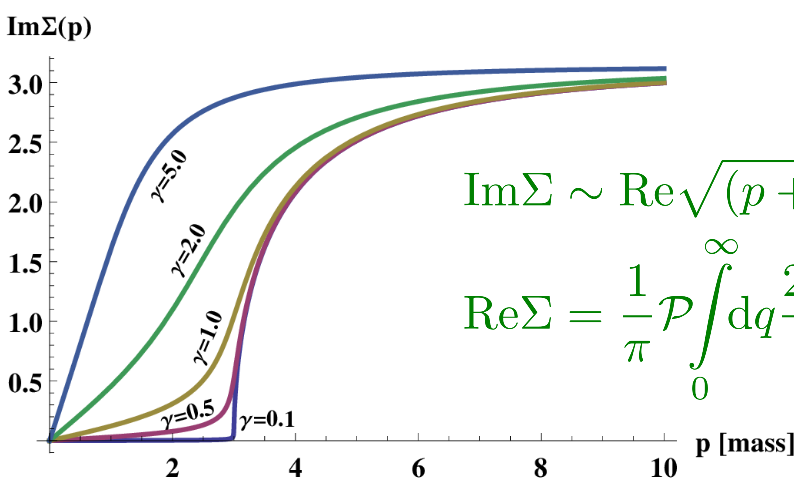
arXiv:1512.03001



$$G(p) = \frac{1}{p^2 - m^2 - \Sigma(p)}$$

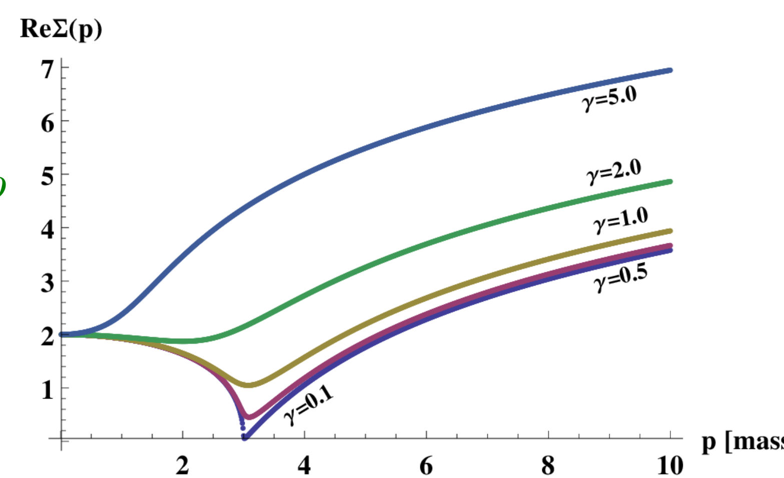
$$\rho = \frac{-\mathrm{Im}\Sigma}{(p^2 - m^2 - \mathrm{Re}\Sigma)^2 + (\mathrm{Im}\Sigma)^2}$$

$$\frac{\partial \mathrm{Re} G^{-1}}{\partial p} = 2p - \frac{\partial \mathrm{Re}\Sigma(p)}{\partial p}$$



$$\mathrm{Im}\Sigma \sim \mathrm{Re}\sqrt{(p+i\gamma)^2 - M^2}/p$$

$$\mathrm{Re}\Sigma = \frac{1}{\pi}\mathcal{P}\int_0^\infty \mathrm{d}q \frac{2q\,\mathrm{Im}\Sigma(q)}{p^2 - q^2}$$



η/s from effective field theory

Beyond the QP-peak

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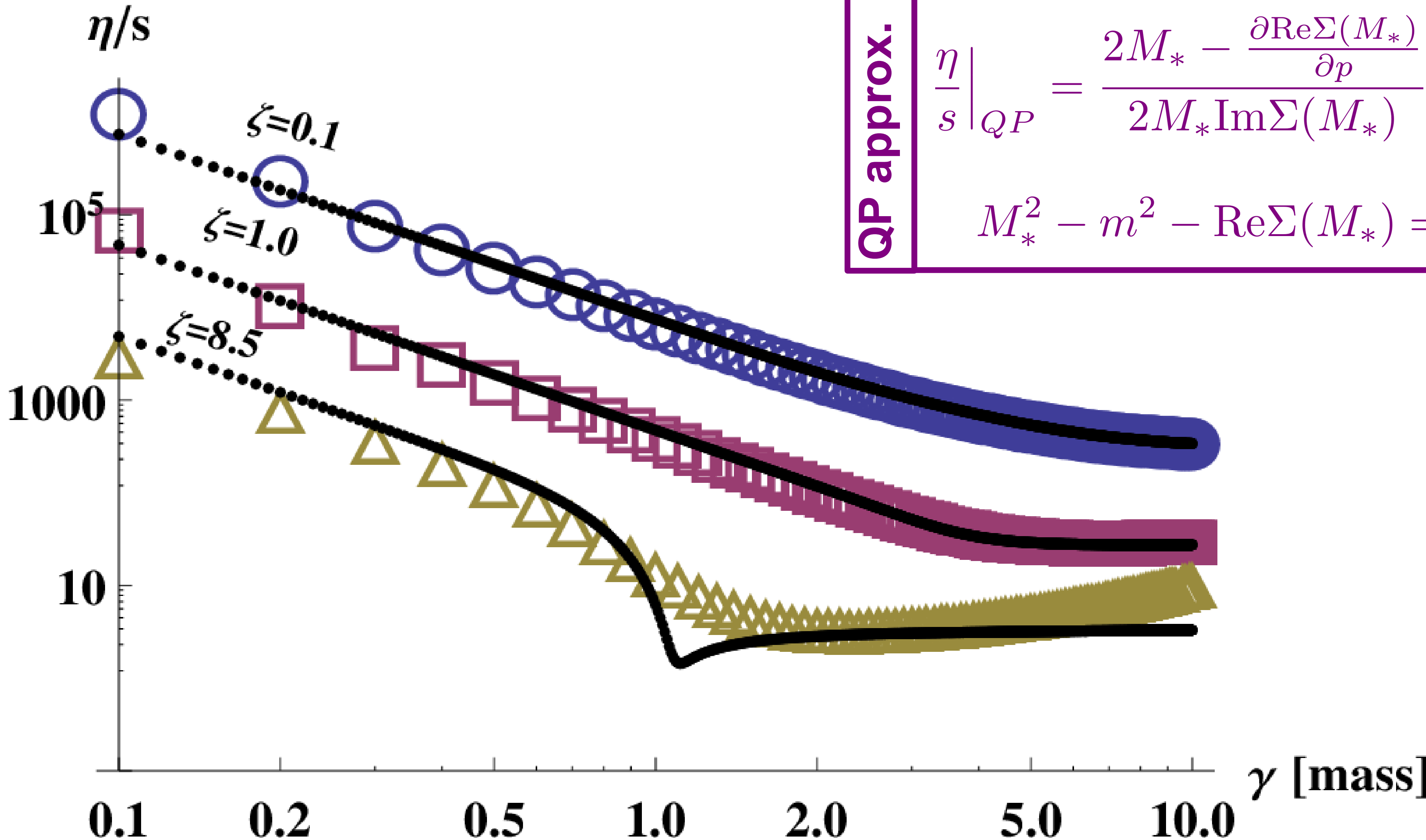

QP approx.

$$\frac{\eta}{s}\Big|_{QP} = \frac{2M_* - \frac{\partial \mathrm{Re}\Sigma(M_*)}{\partial p}}{2M_* \mathrm{Im}\Sigma(M_*)} T^2$$

$$M_*^2 - m^2 - \mathrm{Re}\Sigma(M_*) = 0$$

η/s from effective field theory

Non-universal lower bound to η/s

arXiv:1512.03001

$$s = \int_0^\infty \mathrm{d}p\, g(p) T^3 \chi_s(p/T)$$

$$g = \frac{\partial \mathrm{Re} G^{-1}}{\partial p} \rho$$

$$\eta = \int_0^\infty \mathrm{d}p\, g^2(p) T^4 \lambda_\eta(p/T)$$

Non-universal lower bound to η/s

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$$s = \int_0^\infty \mathrm{d}p g(p) T^3 \chi_s(p/T)$$

$$g = \frac{\partial \mathrm{Re} G^{-1}}{\partial p}\rho$$

$$\eta = \int_0^\infty \mathrm{d}p g^2(p) T^4 \lambda_\eta(p/T)$$

$$\frac{\delta}{\delta g}(\eta[g] - \alpha s[g]) \overset{!}{=} 0 \quad \Rightarrow \quad g^*(p) \sim p$$

$$\frac{\eta}{s} \geq \mathrm{const.} \cdot \frac{s}{T^3}$$

$$\rho^*(p) \sim e^{-\frac{\eta}{s}\frac{p^2}{T^2}}$$

no poles!

What have we learnt?

arXiv:1512.03001

- continuum of states besides the QP-peak:

  reduced η/s,

  the less QP-like the spectrum is,

  the more fluent the system it describes

$$\frac{\eta}{s} = A \cdot \frac{T}{\Gamma} + B \cdot \frac{\Gamma}{T} \geq 2\sqrt{AB}$$

- **non-universal lower bound on η/s**

  constrained by thermodynamics

$$\frac{\eta}{s} \geq C \cdot \frac{s}{T^3}$$

- possible estimation of transport properties from thermodynamic quantities taken from measurements

Thank you for the attention!

Questions? Comments?

arXiv: 1512.03001

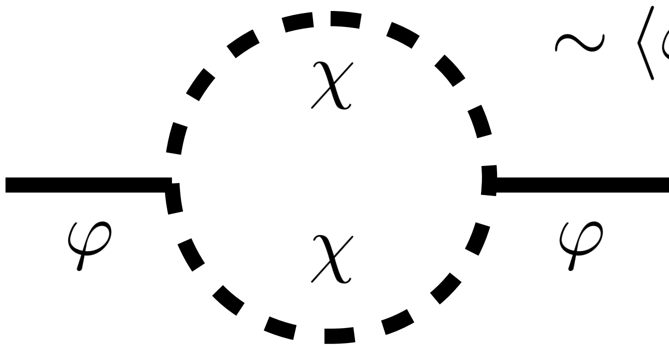
Back-up slides

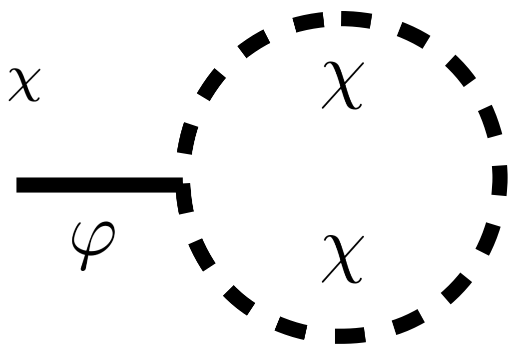
Extended quasi-particles

$$S_{\text{eff}}[\varphi] = \int_x \int_y \varphi_x \mathcal{K}_{x-y} \varphi_y + \int_x J_x \varphi_x$$

$\sim \langle \varphi\varphi \rangle_\chi$

$\sim \langle \varphi \rangle_\chi$





$$\mathcal{L}[\varphi, \chi] = \mathcal{L}[\chi] - \frac{1}{2}\varphi\Box\varphi + g\varphi\chi^2$$

"enviroment"

"test particle"

EQP – energy-momentum conservation

$$\int_x \frac{\delta S[e^{\alpha\partial}\varphi]}{\delta\alpha_x^\mu}\alpha_x\bigg|_{\alpha\equiv 0} = -\int_x \alpha_x(\partial_x\cdot T_x)^\mu$$

Noether's theorem

$$= \frac{\mathrm{d}}{\mathrm{d}\varepsilon}\left[\frac{1}{2}\int_x \varphi_{x^\mu+\varepsilon\alpha_x^\mu}\int_z \mathcal{K}_z e^{z\cdot\partial_x}\varphi_{x^\mu+\varepsilon\alpha_x^\mu}\right]_{\varepsilon=0}$$

The energy-momentum tensor in Fourier-space:

$$T_k^{\mu\nu} = \frac{1}{2}\int_p\int_q \varphi_{-p}\varphi_q\delta_{k+p-q}\frac{p^\mu(p+q)^\nu}{q^2-p^2}(K_q-K_p) \overset{k\to 0}{\to} \frac{1}{2}\int_p \varphi_{-p}\varphi_p\frac{p^\mu p^\nu}{|p|}\frac{\partial K_p}{\partial|p|}$$

EQP – energy-momentum conservation

$$\int_x \frac{\delta S[e^{\alpha\partial}\varphi]}{\delta\alpha_x^\mu}\alpha_x\Bigg|_{\alpha\equiv 0} = -\int_x \alpha_x(\partial_x \cdot T_x)^\mu$$

Noether's theorem

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The energy-momentum tensor in thermal equilibrium

$$\langle T^{\mu\nu}_{x=0}\rangle = \frac{1}{2}\int_p \frac{p^\mu p^\nu}{|p|}\frac{\partial K_p}{\partial|p|}\rho_p\left(n(p^0/T)+\frac{1}{2}\right)$$

EQP – linear response theory

perturbation in ***A***

$$\delta H = \int_y A_y h_y$$

change of avr. ***B*** to linear order in the strength of the perturbation

$$\delta \langle B_x \rangle = \int_y i\mathcal{G}^{ra}_{BA}(x-y) h_y$$

the linear response-function:

$$i\mathcal{G}^{ra}_{BA}(z) = \theta_{z^0} \langle [B_z, A_0] \rangle = \theta_{z^0} \rho_{BA}(z)$$

in case of the energy-momentum in EQP:

$$\rho_{T^{ij}T^{ij}}(k) = iG^{21}_{T^{ij}T^{ij}}(k) - iG^{12}_{T^{ij}T^{ij}}(k) =$$

$$= \frac{1}{4}\int_p \left((D^{ij}_{p,p+k})^2 + D^{ij}_{p,p+k} D^{ij}_{p+k,p} \right) \rho_p \rho_{p+k} (n_p - n_{p+k})$$

EQP – linear response theory

perturbation in ***A***

$$\delta H = \int_y A_y h_y$$

change of avr. ***B*** to linear order
in the strength of the perturbation

$$\delta \langle B_x \rangle = \int_y i\mathcal{G}^{ra}_{BA}(x-y) h_y$$

$$\eta = \lim_{\omega \to 0} \frac{\rho_{T^{ij}T^{ij}}(\omega, \mathbf{k}=0)}{\omega} = \frac{1}{2}\int_p \left(\frac{p^1 p^2}{p^0} \frac{\partial K_p}{\partial p^0} \rho_p \right)^2 \frac{-n'(p^0/T)}{T}$$

in case of the energy-momentum in EQP:

$$\rho_{T^{ij}T^{ij}}(k) = iG^{21}_{T^{ij}T^{ij}}(k) - iG^{12}_{T^{ij}T^{ij}}(k) =$$

$$= \frac{1}{4}\int_p \left((D^{ij}_{p,p+k})^2 + D^{ij}_{p,p+k} D^{ij}_{p+k,p} \right) \rho_p \rho_{p+k} (n_p - n_{p+k})$$