Interacting Fermion Stars in Kaluza–Klein World

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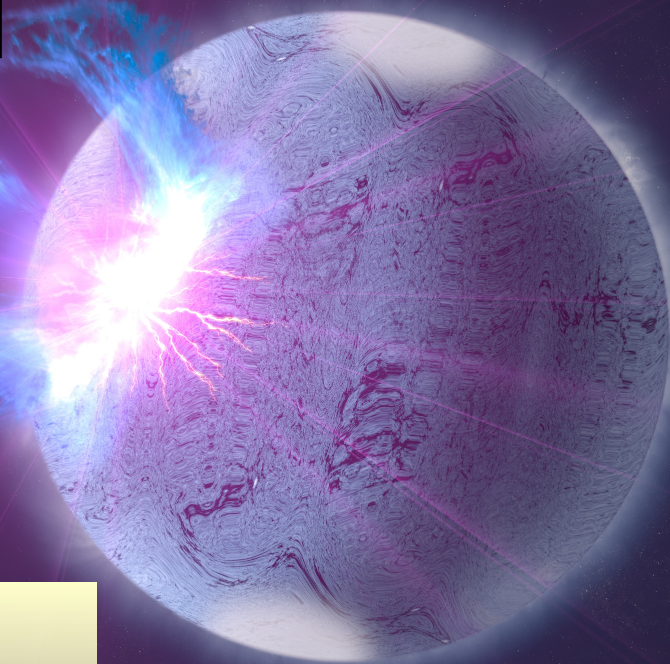


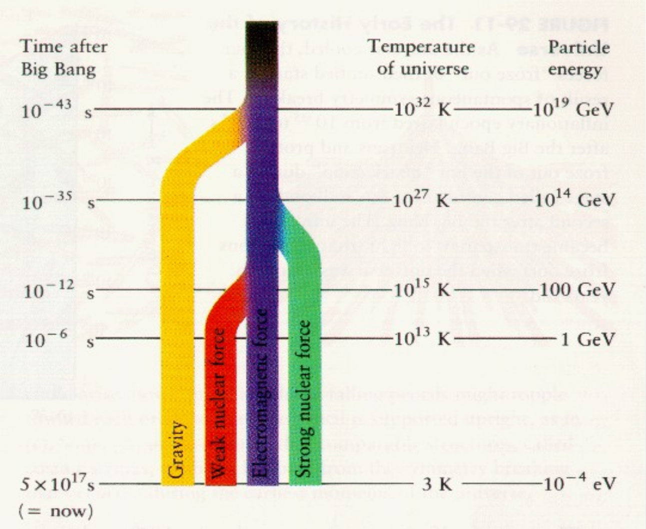
- Motivation for K–K theory & Stars
- Special solution in 1+4D spacetime
- **Interacting Fermion Star in 1+4D**
- **M-R relation via varying coupling constant**

Zimányi Winter School 2015 Budapest

Wigner RCP of the HAS
Eötvös Loránd University Budapest

1. Motivation for introducing extra dimensions

- Standard matter by Standard Model
  - Electromagnetic;
  - Weak and
  - Strong interactions
- Grand Unified Theory...
  - Gravity and QFT are not fitting into the same picture
  - GR locally valid; curved space-time
  - QFT globally valid; Minkowski



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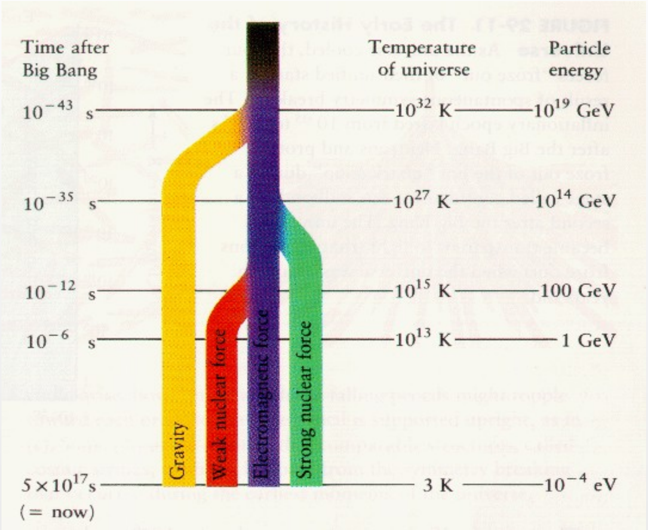
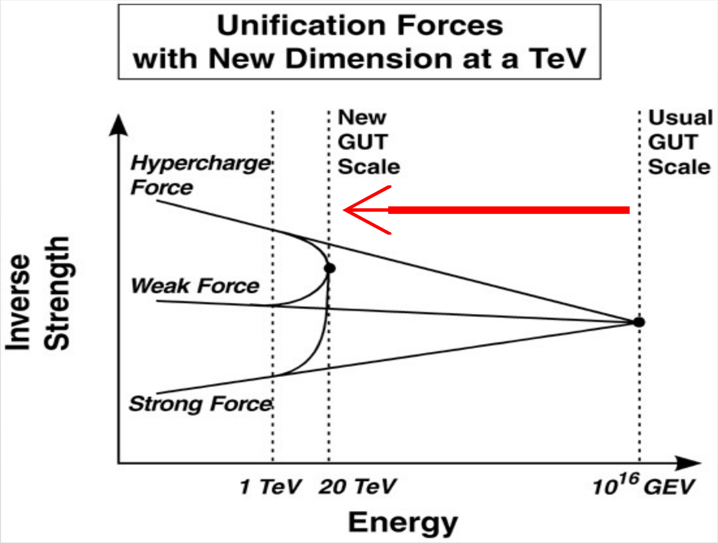




- **Possible way:**
  - Geometrization of elementary forces
  - Introducing new dimensions
  - Let's see the simplest case: $d_c=1$!

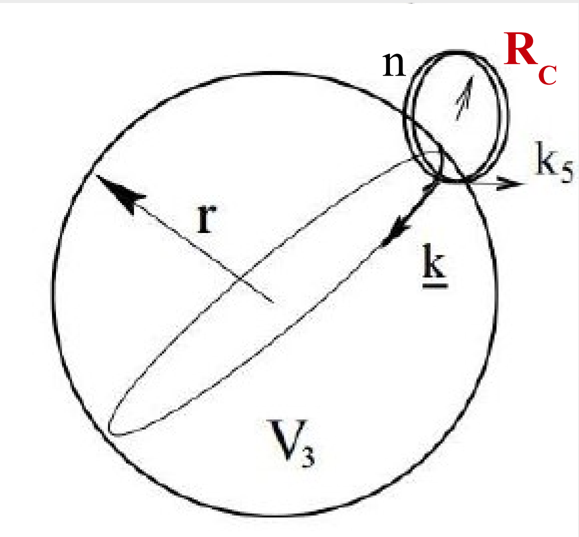
2. Extra space-dimension as a new degree of freedom

- Possible existence of extra dimensions at microscopical scales at extreme energies
- Particles with enough energy are able to move in the extra direction
- Motion in the extra direction generates an extra mass term (excited mass)

- **Strangeness as a new degree of freedom:**
  Connection between $\bar{m} = m_S$ and R_c radius:

$$E_5 = \sqrt{\underline{k}^2 + \left(\frac{n}{R_C}\right)^2 + m^2} = \sqrt{\underline{k}^2 + \bar{m}^2}$$

$$\bar{m}^2 = \left(\frac{n}{R_C}\right)^2 + m^2$$



m: light (u, d) quark mass
n: excitation number, n=1
$\bar{m}$: e.g. **heavy (s) quark mass**

2. Extra space-dimension as a new degree of freedom

- **Extra 5thD is compactified** in an **S^1 circle** with radius ***R$_c$***
  → periodical boundary condition → quantization condition

$$\psi(x_5) \approx e^{ik_5 \cdot x_5} \quad \text{and} \quad \psi(x_5 + 2\pi \boldsymbol{R_C}) \sim \psi(x_5) \quad \rightarrow \quad k_5 = \frac{n}{\boldsymbol{R_C}}$$

k_5: momentum in the 5th direction; $\mathbf{x}_5$: coordinate in the 5th direction; $\boldsymbol{n} \in \mathbb{Z}^+$

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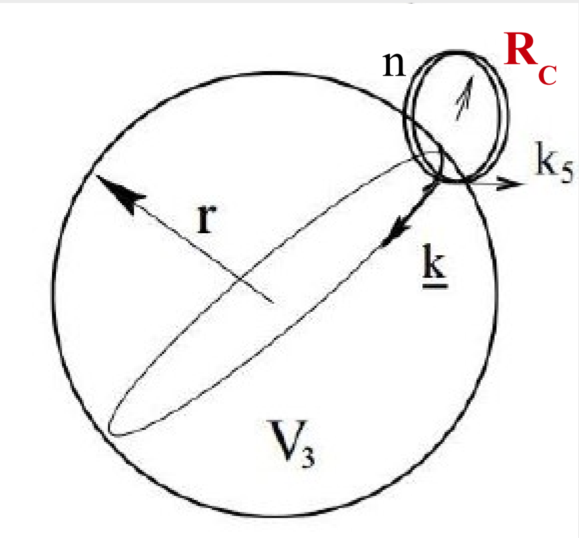

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3. Thermodynamics: potential for a 1+4D Fermion gas

- **1+3D two component** Fermi-gas → **1+4D one component** Fermi-gas
- (n^0,~~Λ~~) → n^0 and its extra dimensional excitation
- Excited state: new degree of freedom appearing in the momentum space of the particle on a given energy
- In equilibrium: $\mu = \mu_n + \mu_{n\_exc}$

$$\Omega_5 = -2\frac{V_4}{\beta}\int \frac{\mathrm{d}^4 k}{(4\pi)^4}\left[\ln\left(1+e^{-\beta\left(\sqrt{\underline{k}^2+\bar{\boldsymbol{m}}^2}-\mu\right)}\right)\left(+\mu \leftarrow\rightarrow -\mu\right)\right]$$

$$\bar{\boldsymbol{m}}^2 = (n/\boldsymbol{R_C})^2 + m^2 \qquad \textbf{excited mass}$$

$$\int_0^\infty \mathrm{d}^4\boldsymbol{k} = \int_0^\infty \mathrm{d}^3\boldsymbol{k}\,\mathrm{d}k_5 \rightarrow \frac{1}{\boldsymbol{R_C}}\sum_{i=min(n)}^{max(n)}\int_0^\infty \mathrm{d}^3\boldsymbol{k} \qquad \text{discretization}$$

$$V_5 = 2\pi\boldsymbol{R_C} V_4 \qquad \text{1+4D volume}$$

$$\Omega_5 = \sum_n \Omega_4\left(m^2 + \frac{n^2}{\boldsymbol{R_C^2}}\right) = \Omega_4(\bar{\boldsymbol{m}})$$

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For 1+4D EOS

$$\widetilde{\epsilon} = \frac{\widetilde{g}}{(2\pi)^4} \int_0^{\widetilde{k_F}} \widetilde{\varepsilon}\, \mathrm{d}^4\widetilde{\mathrm{k}} \Big|_{T=0} =$$

$$= \frac{\widetilde{g}}{16\pi^3 R_C} \sum_{\kappa=0}^{1} \left[\widetilde{\mu}\sqrt{\widetilde{\mu}^2 - \widetilde{m}^2} \left(\widetilde{\mu}^2 - \frac{1}{2}\widetilde{m}^2 \right) + \frac{\widetilde{m}^4}{2} \ln \left| \frac{\widetilde{m}}{\widetilde{\mu} + \sqrt{\widetilde{\mu}^2 - \widetilde{m}^2}} \right| \right],$$

$$\widetilde{p} = -\frac{1}{2\pi R_C} \frac{\partial \widetilde{\Omega}}{\partial V} \Big|_{T=0} =$$

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4. GR: Special solution in 1+4D spacetime

Assumptions for generalization:

i. **1+(3+d_c) dimensional space time:** dimensions are space-like, except first one: time-like
ii. **GR is the same** as in 1+3 D 'Equivalence Principle' is unchanged
iii. **All causality postulates are the same** as in 1+3 D
iv. **Extra space-like** dimensions are microscopical
v. **Complete Killing-symmetry** in the extra microscopical subspace

- **Static, spherically symmetric compact object**
- **Ideal relativistic fluid with isotropy**

$$g_{\mu\nu} = \begin{bmatrix} g_{00} & g_{01} & 0 & 0 & g_{05} \\ g_{01} & g_{11} & 0 & 0 & g_{15} \\ 0 & 0 & g_{22} & 0 & 0 \\ 0 & 0 & 0 & g_{22}\sin^2\vartheta & 0 \\ g_{05} & g_{15} & 0 & 0 & g_{55} \end{bmatrix}$$

[G. G. Barnaföldi, P. Lévai, B. Lukács et al. Astron. Nachr. **328**, 809 (2007)]

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- **Spherical symmetry** → O(3) symmetry
- **Static picture** → $g_{\mu 0}=0$ and $g_{\mu\nu,0}=0$
- **4D** $g^{\mu\nu}$ is x^5 independent
- **Killing transformations** → $g_{01}=0$ and $g_{51}=0$

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<u>Coordinates:</u>

$t = x^0$; $r = x^1$; $\vartheta = x^2$; $\varphi = x^3$; $\boldsymbol{\chi = x^5}$

$$g_{\mu\nu} = \mathrm{diag}\left(e^{2\nu}, -e^{2\lambda}, -r^{2,} -r^2 \sin^2\vartheta, \mathbf{e^{2\Phi}}\right)$$

<u>Radial functions for the metric components:</u>

$\nu(r)$, $\lambda(r)$, $\boldsymbol{\Phi(r)}$

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5. GR: TOV equation in 1+4D spacetime

- **'dΦ/dr = 0' special case:**

$$\frac{\mathrm{d}\,\boldsymbol{p}(r)}{\mathrm{d}\,r}=-\frac{[\boldsymbol{p}(r)+\boldsymbol{\epsilon}(r)][\boldsymbol{M}(r)+4\pi r^3\boldsymbol{p}(r)]}{r[r-2\boldsymbol{M}(r)]}$$

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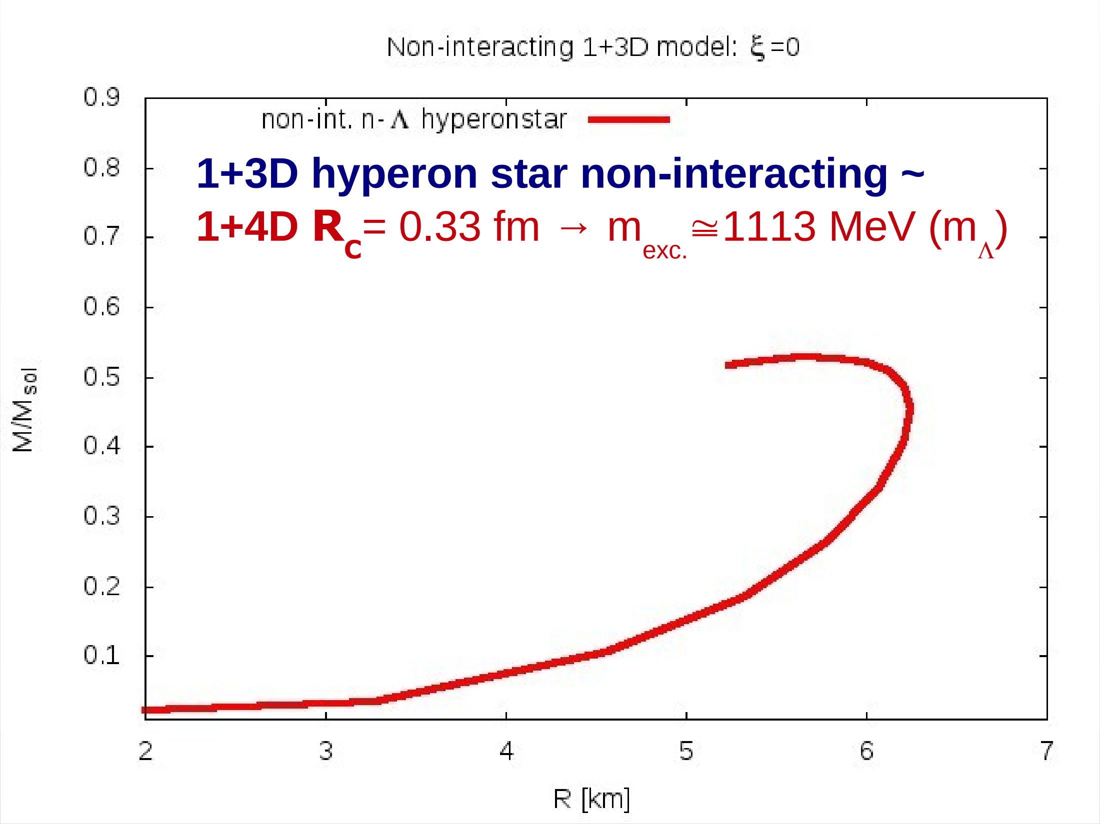
1+3D hyperon star:
n (u,d,d), Λ (u,d,s)
(non-interacting) vs.
1+4D fermion star: with

$$k_F > \hbar / R_C$$

1_C extra dimension
(non-interacting)

constraint: **n=1**



6. M – R relations with different R_C

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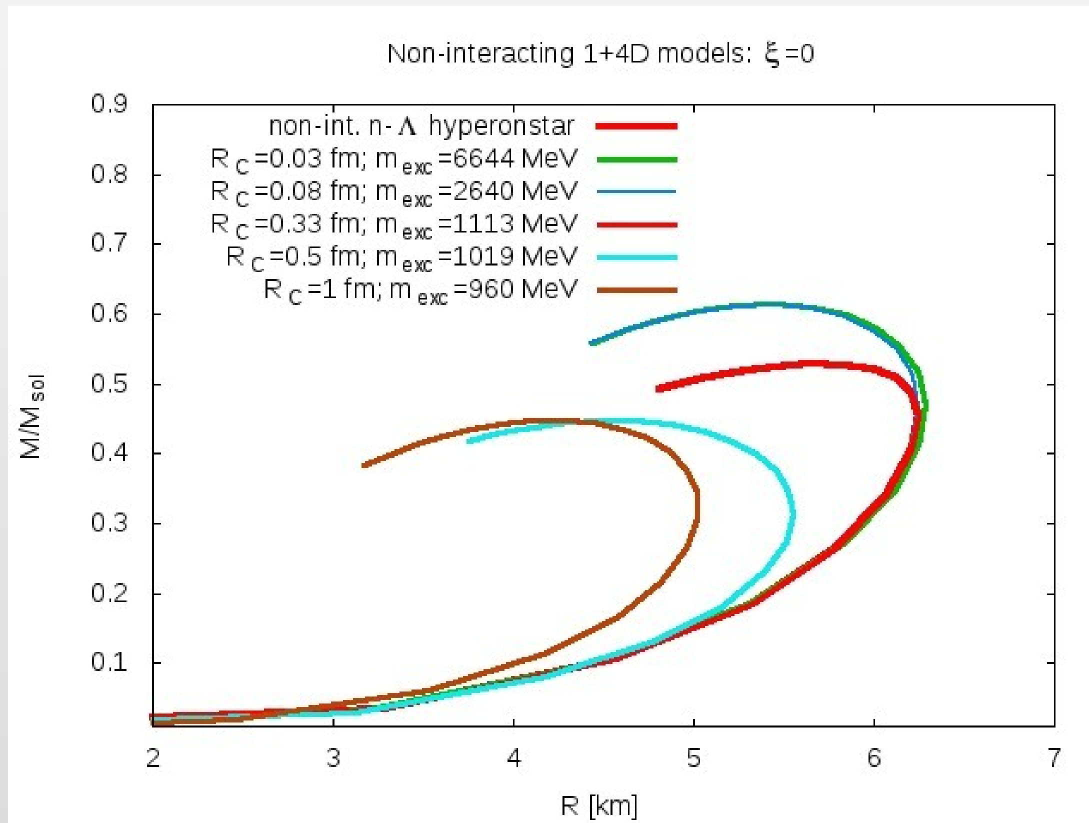
- larger the
  $\mathbf{R_C} \rightarrow \Delta m_n \sim \frac{n}{R_C}$
  smaller the excitations
  to m_n
- similar solutions



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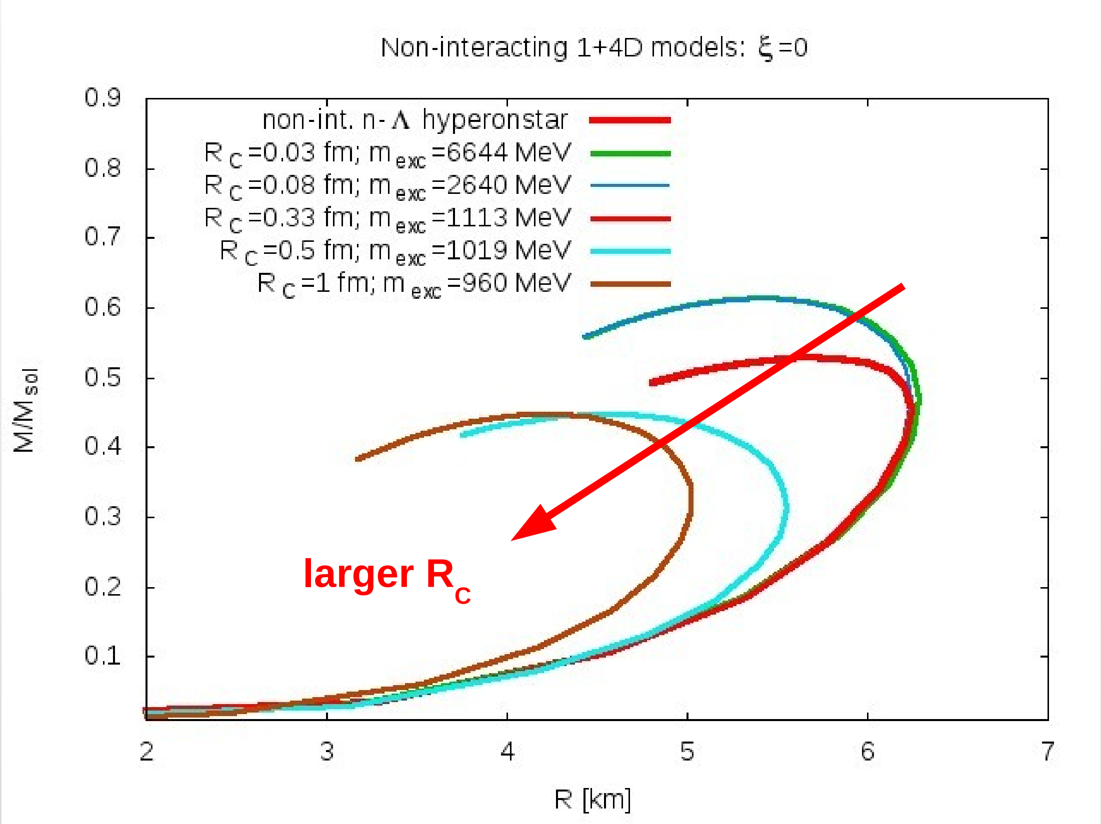

6. M – R relations with different R_C

- **But without interaction M_{max} too small!**
- On a larger scale compared to a **realistic 1+3D interacting hyperon eos** (n,Λ,Σ,Ξ) (Petrik et al. 2012)

1+3D hyperon star:
n (u,d,d), Λ (u,d,s)
(non-interacting) vs.
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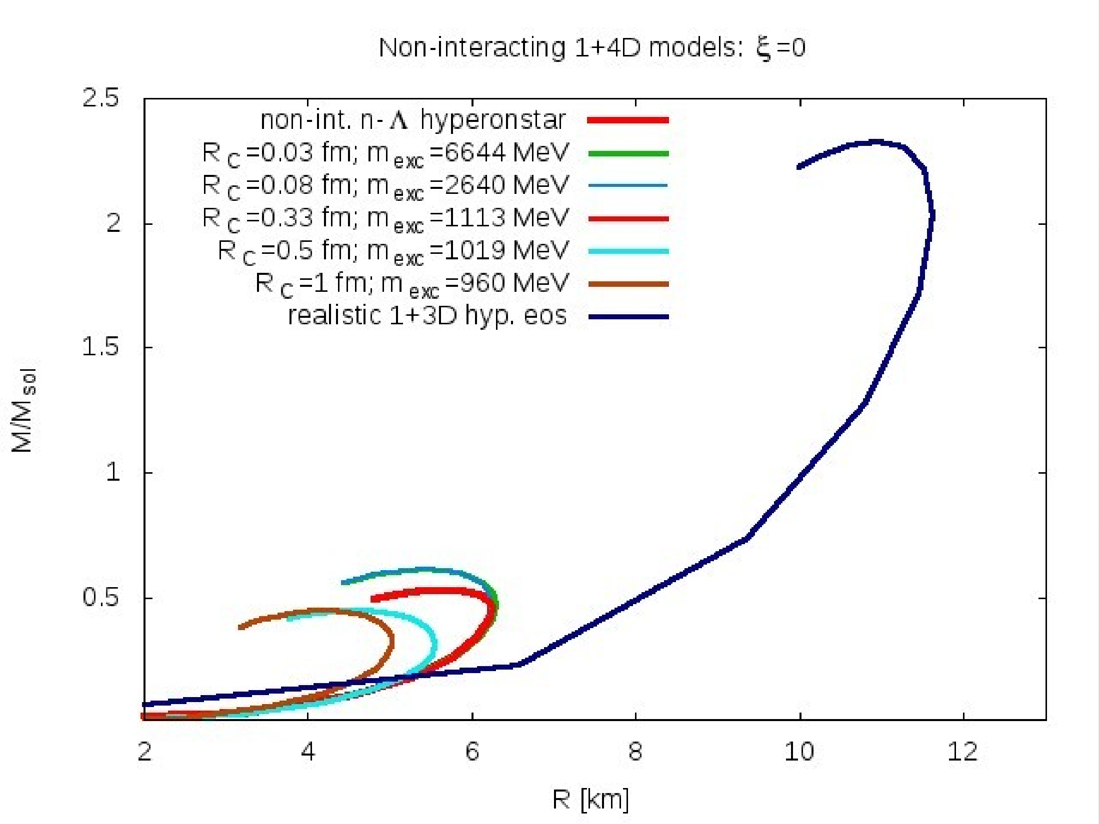

7. Introducing a repulsive interaction for 1+3D

- Introducing *u(n)* *repulsive potential* depending on density: *u(n)~n*
- In a linear approximation: $u(n)=\xi n$ → results a contribution of the coupling constant in **p, ε**

J. Zimanyi, B. Lukacs, P. Levai, J.P. Bondorf: „An Interpretable Family of Equation of State for Dense Hadronic Matter", Nucl.Phys. A484 (1988) 647

$$\varepsilon(\mu)=\varepsilon_0[\mu-u(n)]+\boldsymbol{\varepsilon_{\text{int}}}$$

$$p(\mu)=p_0[\mu-u(n)]+\boldsymbol{p_{\text{int}}}$$

$$n(\mu)=u_0[\mu-u(n)]=u_0[\mu_0]$$

$$\boldsymbol{\varepsilon_{\text{int}}}=\int_0^n u(n)\,\mathrm{d}n=\int_0^n \xi n\,\mathrm{d}n=\frac{1}{2}\xi n^2$$

$$\boldsymbol{p_{\text{int}}}=nu(n)-\int_0^n u(n)\,\mathrm{d}n=n\xi n\int_0^n \xi n\,\mathrm{d}n=\xi n^2-\frac{1}{2}\xi n^2=\frac{1}{2}\xi n^2$$

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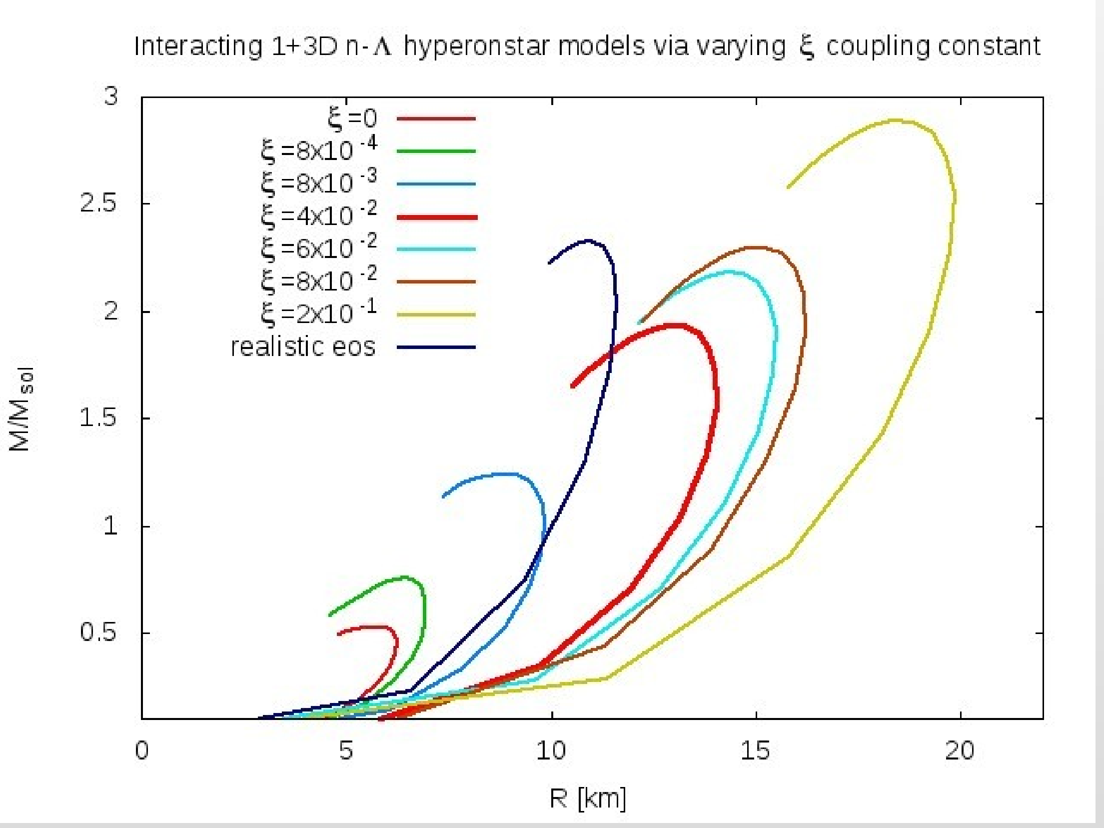
- for the **reference 1+3 D (n^0, Λ^0) eos** ξ with smoothly varying values from $10^{-4}<\xi<10^{-1}$
- increasing the magnitude of ξ → results increasing M_{max}, R_{max}



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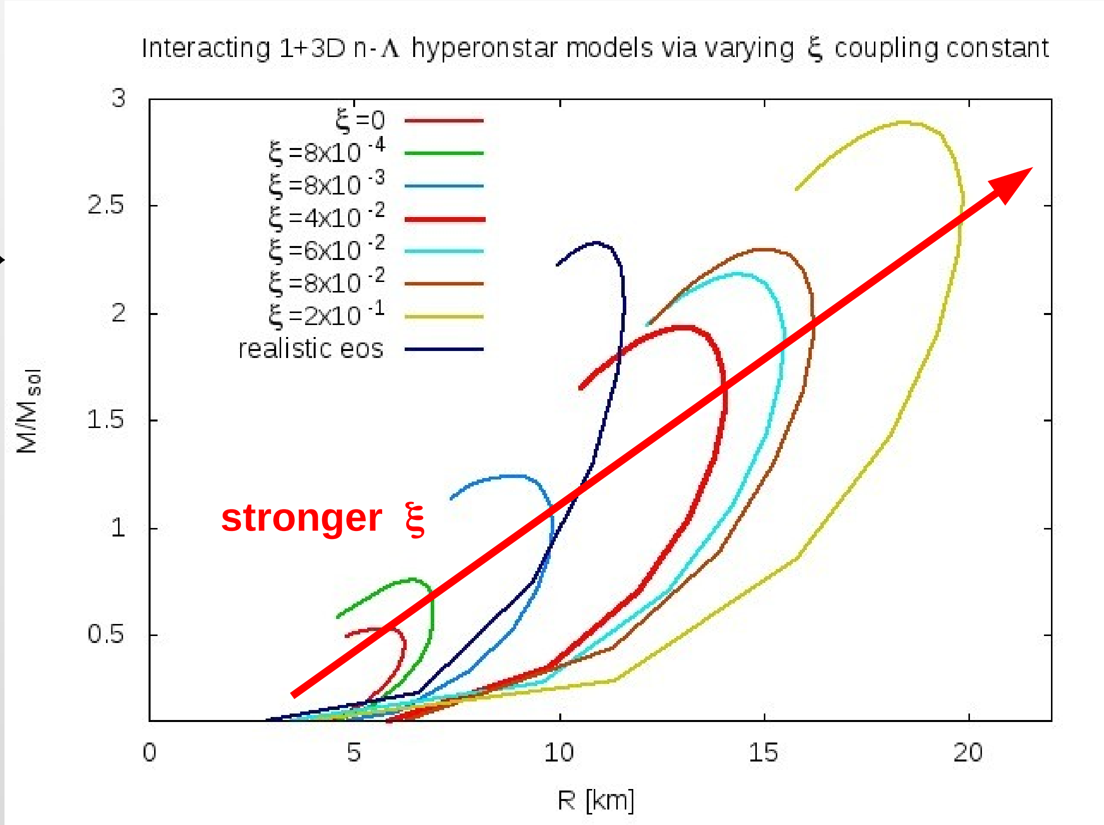

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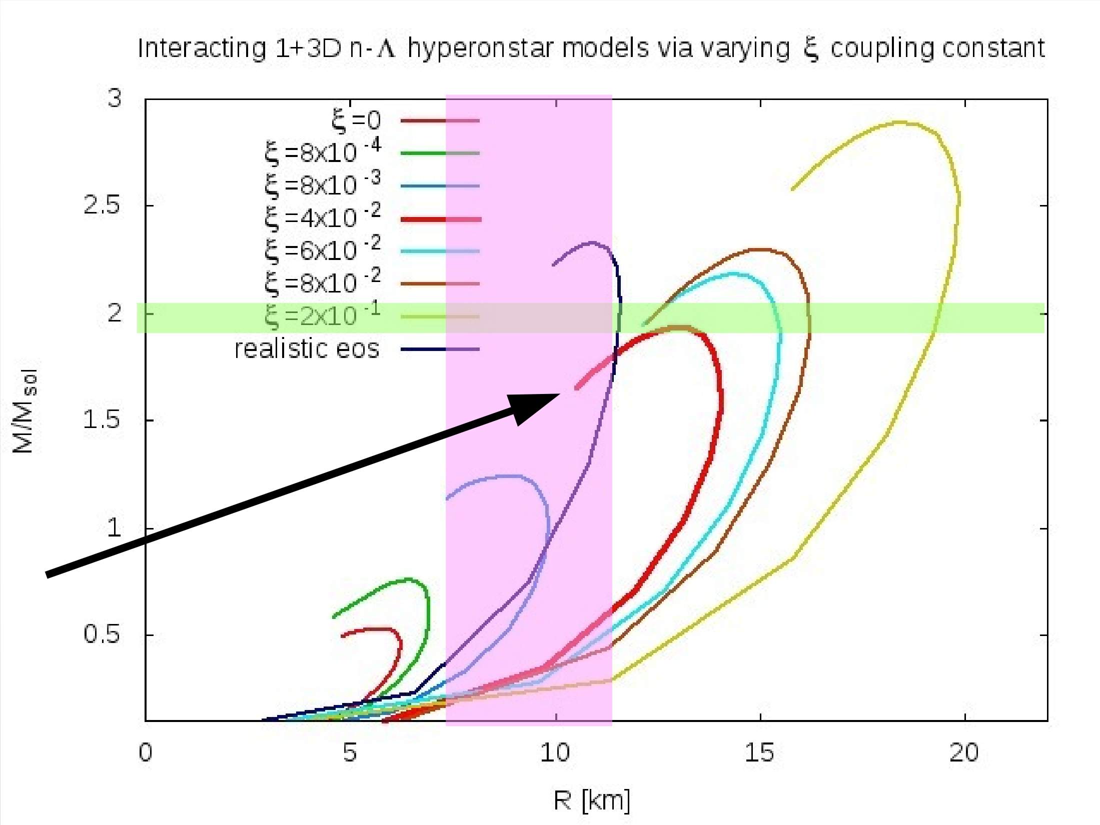
- increasing the magnitude of ξ → results increasing M_{max}, R_{max}
- Based on meauserements select the most reasonable one which nearly approaches
  - **2 M_{sol}** (Demorest et al. 2010)
  - **7-11 km** (Guillot et al. 2013)

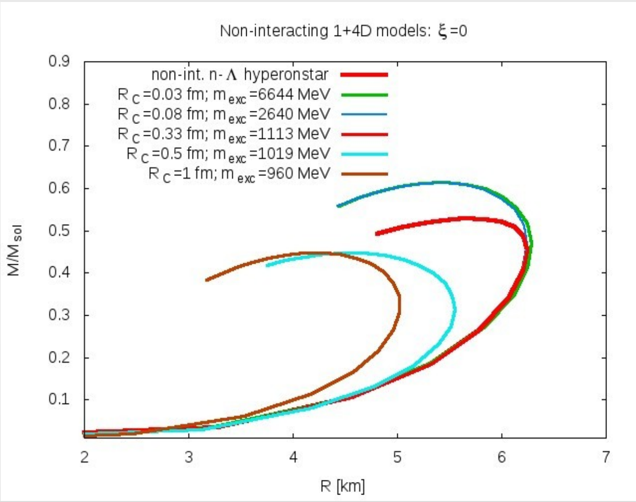
  → **ξ=0.04!**

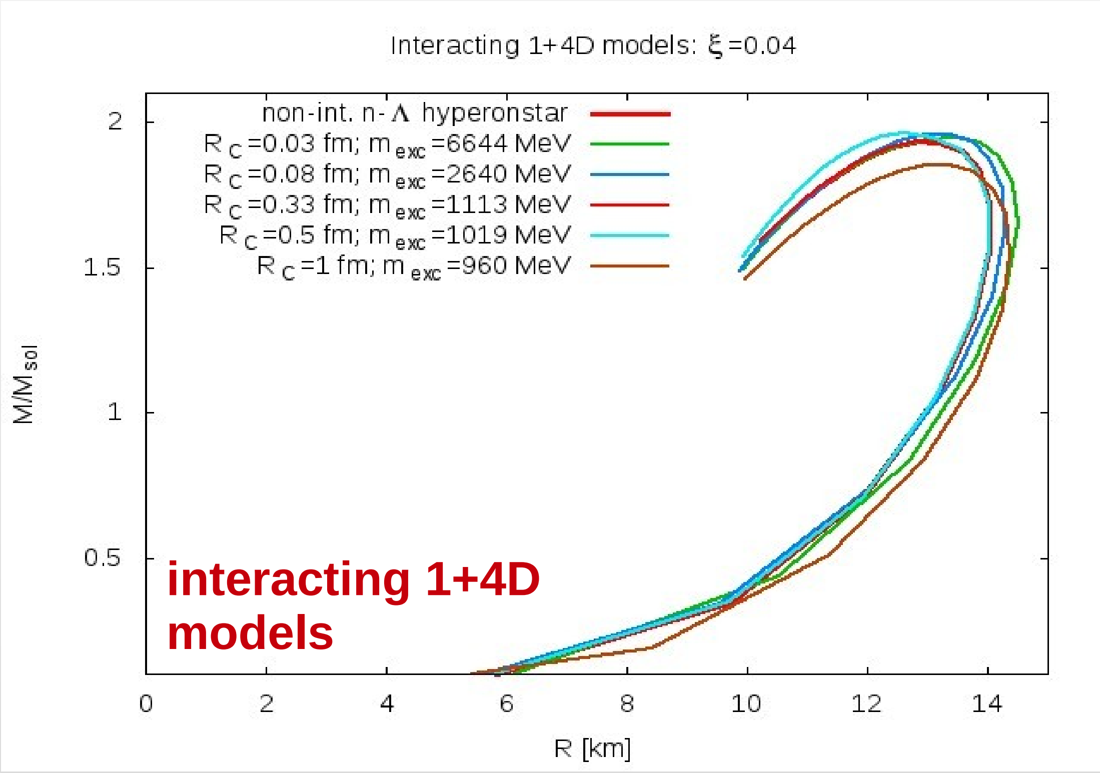


9. Selecting ξ =0.04 eos → applying for 1+4D

- **1+4 D fermion star** models: trying different **R_C** (from 0.03 fm to 1 fm) for **ξ=0.04 case**
- compared to the non-interacting case → when *interaction switched in*: varying the size of the **microsopical extra dimension (R_C)** *has less effect on M_{max}*

non-interacting 1+4D models





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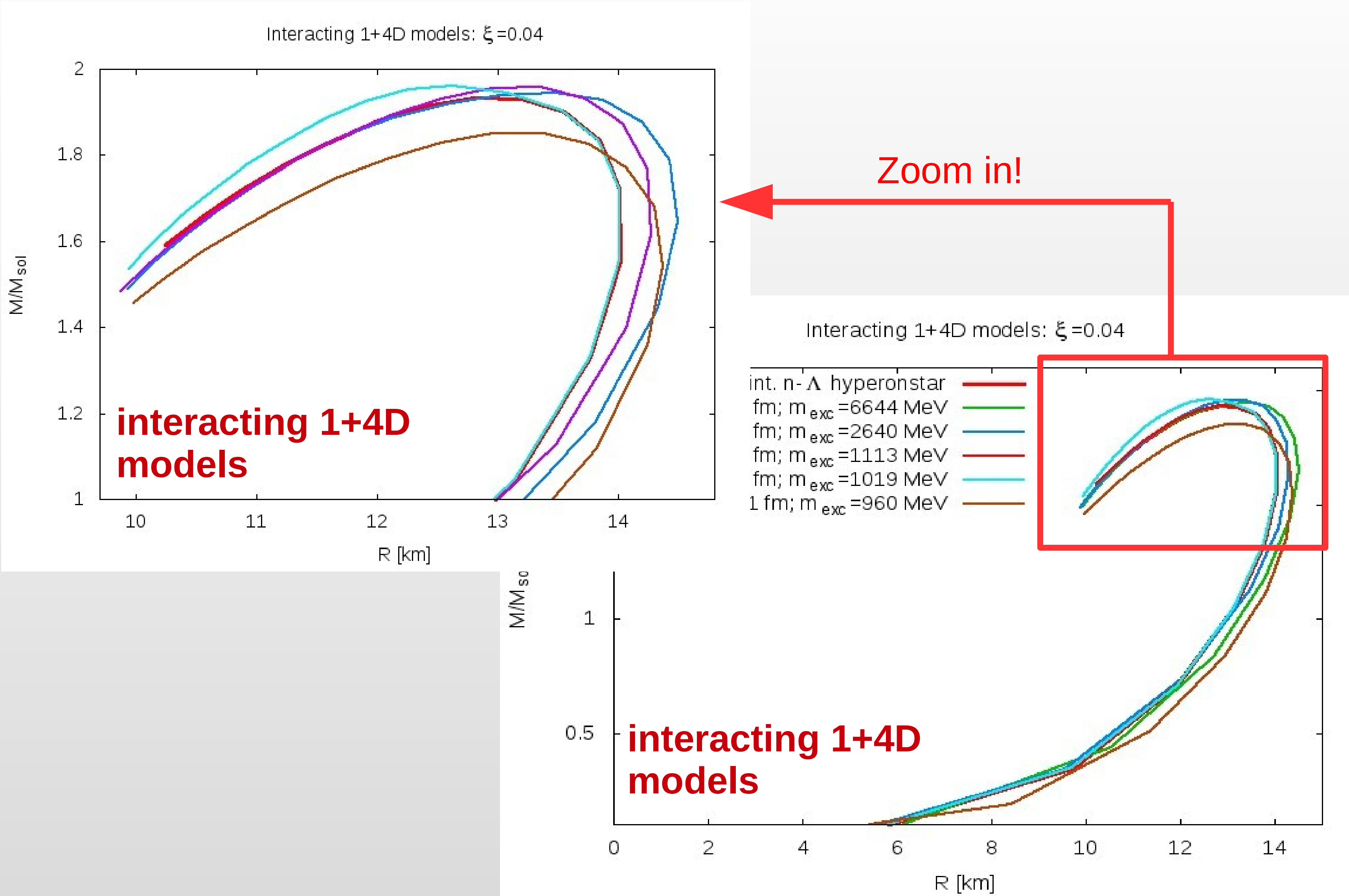


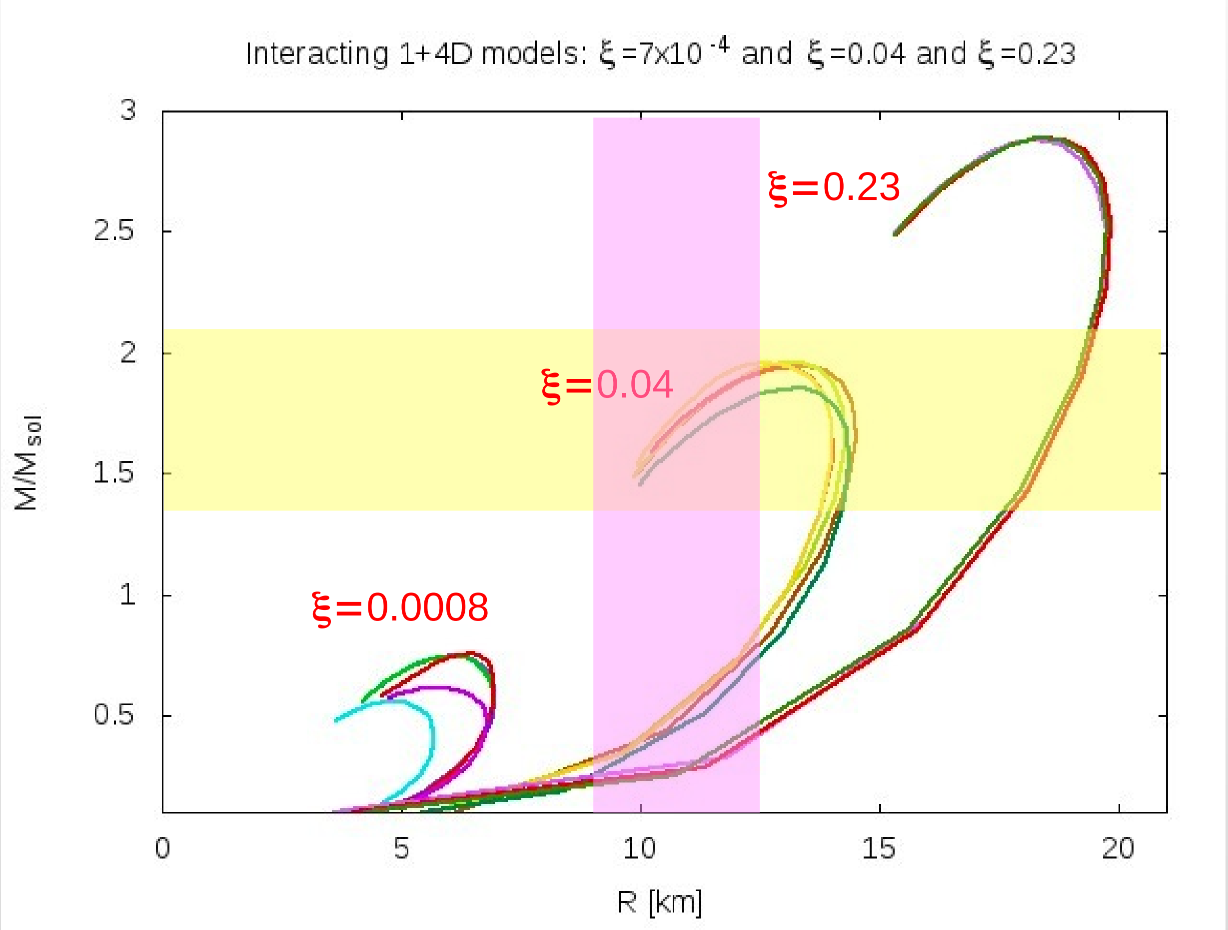
10. Probing different magnitudes of ξ

- For the same set of $\mathbf{R_c}$ values

R_C=0.08 fm → $m_{exc.}$=6644 MeV
R_C=0.05 fm → $m_{exc.}$=2640 MeV
R_C=0.33 fm → $m_{exc.}$=1113 MeV
R_C=0.5 fm → $m_{exc.}$=1019 MeV
R_C=1 fm → $m_{exc.}$=960 MeV

- greater the ξ ~ stronger the interaction
- varying the size of the microscopical extra dimension ($\mathbf{R_c}$) has less effect on M_{max}

- Appropriate setting of parameters can result realistic models between measured bounds of neutron star masses (1.4-2.1 M_{sol}) and radii (7-11km)



Summary

- **Fermion stars in 1+4D** were analyzed:
- Assumption:
  - Static, spherical Schwarzschild-like space-time; ideal fluid
  - TOV-like eqs. with specific, but exact (stable) solution

- Solution of **non-interacting**:
  - overlaps with (n^0, Λ^0) 1+3D strange star models if $\mathbf{R_c}$ is set to m_s
  - larger $\mathbf{R_c}$ results → decrease in star's mass
  - smaller $\mathbf{R_c}$ generates a saturation in M_{max}: mass limit
  - without interaction M_{max} is too small (under 1 M_{sol})

- Introducing a **repulsive interaction** for **1+3D** and **1+4D eos**:
  - repulsive potential is proportional with the density, $u(n)= \xi n$
  - models can reach the realistic mass range (1.4 – 2 M_{sol}) by setting ξ coupling constant in an appropriate range
  - stronger the interaction (increasing ξ) → larger the star, M_{max}
  - strong interaction supresses the effect of size variation of the extraD

Thank You for Your attention!

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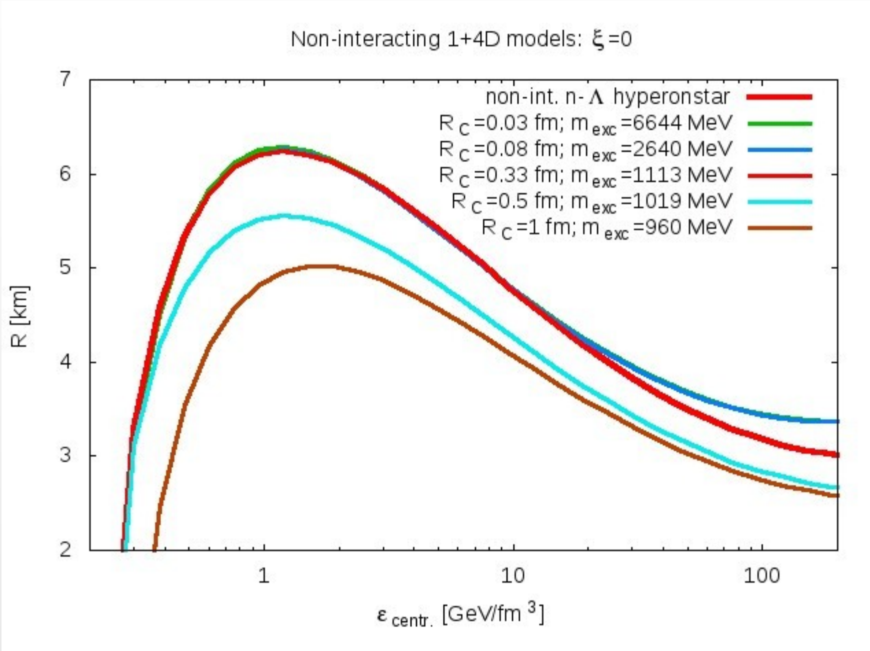
Backup slides

ε_{centr} – R relations with different R_C



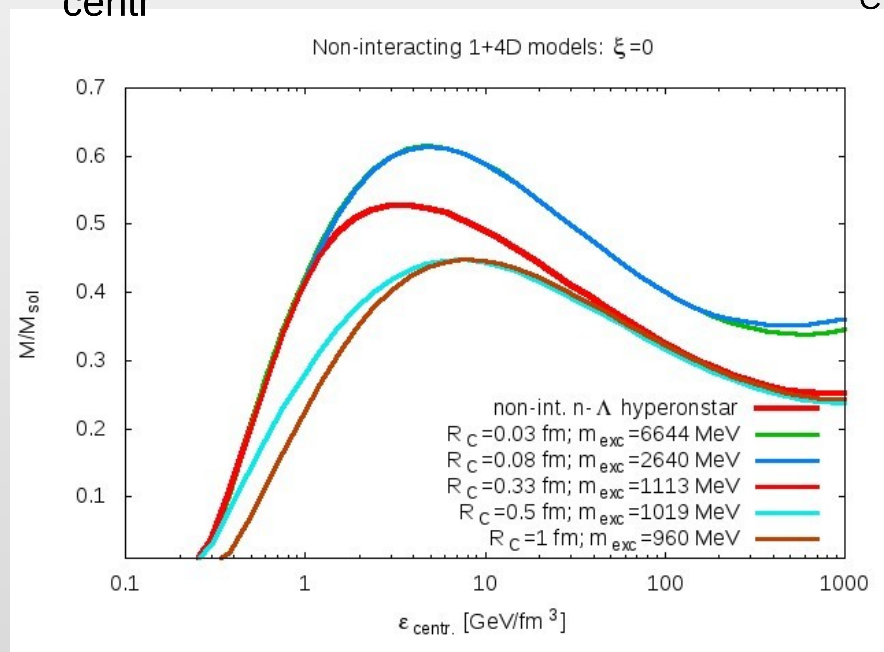
1+3D hyperon star: n^0, Λ^0 (non-interacting) vs.
1+4D fermion star: with

$k_F > \hbar / R_C$

1_C extra dimension (non-interacting)

constraint: **n=1**

ε_{centr} – M relations with different R_C



- excitation with:
- larger the **R_c** $\rightarrow \Delta m_n \sim \frac{n}{R_C}$
- smaller the excitations to m_n
- similar solutions

Thermodynamics for 1+4D

(a) **Thermodynamical potential for 1+4D Fermion gas**

$$\Omega_5 = -2\frac{V_4}{\beta}\int\frac{\mathrm{d}^4 k}{(4\pi)^4}\left[\ln\left(1+e^{-\beta\left(\sqrt{\underline{k}^2+\bar{m}^2}-\mu\right)}\right)(+\mu \leftarrow\rightarrow -\mu)\right]$$

$\bar{m}^2 = (n/R_C)^2 + m^2$ **excited mass**

$\int \mathrm{d}k_5 \rightarrow \frac{1}{R_C}\Sigma_n$ discretization

$V_5 = 2\pi R_C V_4$ volume

(b) Thermodynamical potential and its quantities

$$\Omega_5 = \sum_n \Omega_4\left(m^2 + \frac{n^2}{R_C}\right) = \Omega_4(\bar{m})$$

$$p = -\frac{1}{2\pi R_C}\frac{\partial\Omega_5}{\partial V} \qquad p_5 = -\frac{1}{2\pi V}\frac{\Omega_5}{\partial R_C} \qquad \epsilon = \frac{U}{V_4}$$

5. GR: Energy-momentum tensor for 1+4D matter

(a) **Energy-momentum tensor** for anisotrop liquid:

$$T_{\mu\nu} := \epsilon u_\mu u_\nu - p\left(g_{\mu\nu} - u_\mu u_\nu + v_\mu v_\nu\right) - p_5 v_\mu v_\nu$$

Liquid is isotrop for 3 dimension: $T^1_1 = T^2_2 = T^3_3 = p$

BUT **pressure** $\boldsymbol{T^5_5 = p_5}$ in the 5th direction is anisotrop,
with energy-density:

$$T_{\mu\nu} = \mathrm{diag}\left(\epsilon\,\mathrm{e}^{2\nu}, p\,\mathrm{e}^{2\lambda}, p\,r^2, pr^2\sin^2\vartheta, p_5\,\mathrm{e}^{2\Phi}\right)$$

(b) Let's construct the $\boldsymbol{R_{\mu\nu}}$ **Ricci tensor** and $\boldsymbol{R}$ **Ricci scalar**:

$$R := R^i_i = R^1_1 + R^2_2 + R^3_3 + R^4_4 + R^5_5$$

[B. Lukács, T. Pacher: KFKI-1985-74, Budapest, Hungary]

6. GR: Einstein equation in 1+4D spacetime

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = -8\pi G T_{\mu\nu}$$

- according to new dimensions 10 → 15 equations BUT because of symmetries: 5 components of the **Einstein equation**:

$$-8\pi G \epsilon = e^{-2\lambda}\left[\Phi'' + \Phi'^2 - \lambda'\Phi' + \frac{2\Phi'}{r} - \frac{2\lambda'}{r} + \frac{1}{r^2}\right] - \frac{1}{r^2}$$

$$-8\pi G p = e^{-2\lambda}\left[\nu'\Phi' - \frac{2\Phi'}{r} - \frac{2\nu'}{r} - \frac{1}{r^2}\right] + \frac{1}{r^2}$$

$$-8\pi G p = e^{-2\lambda}\left[-\nu'' - \nu'^2 - +\nu'\lambda' + \Phi'' - \Phi'^2 - \nu'\Phi' + \lambda'\Phi' - \frac{\nu'}{r} + \frac{\lambda'}{r} - \frac{\Phi'}{r}\right]$$

$$-8\pi G p_5 = e^{-2\lambda}\left[-\nu'' - \nu'^2 + \nu\lambda - \frac{2\nu'}{r} + \frac{2\lambda'}{r} - \frac{1}{r^2}\right] + \frac{1}{r^2}$$

- **Extra variables: p_5, $\Phi(r)$**

$$p_5 = 1 - \frac{2M(r)}{r} + \frac{1}{r}\ln\left[1 - \frac{2M(r)}{r}\right] - 2p$$