Pair production from strong fields

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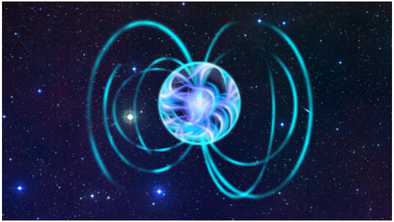
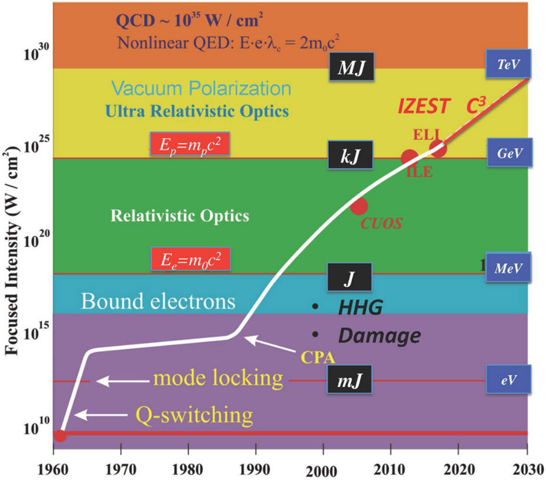
QED Motivation

- QED pair production from vacuum was predicted long ago, but was not yet observed $\rightarrow$ "Holy Grail of QED".
- QED pair production may take place in near miss heavy ion collisions.

QED pair production near massive astrophysical objects (black holes, magnetars, possible source of gamma ray bursts?)



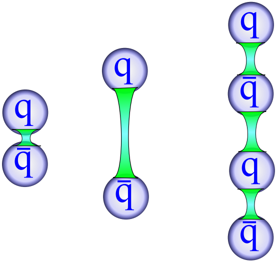
Laser Technology



For a list of present and upcoming facilities see: A. Di Piazza et. al, Rev. Mod. Phys. vol. 84, (2012) 1177

QCD Motivation

From QCD:
The success of color rope/string models in describing Heavy Ion collisions (e.g. at LHC):



Quark potential is linear with separation: if a $q - \bar{q}$ pair is separating, the interaction creates more and more quark pairs until energy is depleted.

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Theory

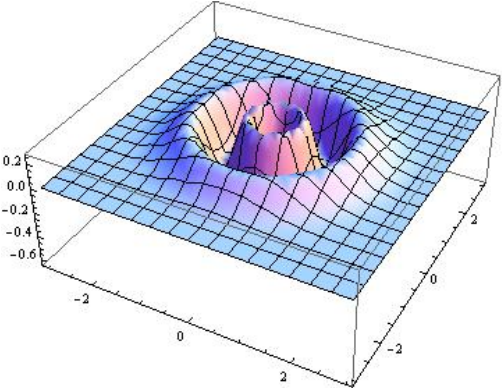
Relevant scales for QED:

- Field strength: $\mathcal{E}_c = \frac{m^2c^3}{e\hbar} \approx 1.3 \cdot 10^{18} \frac{\mathrm{V}}{\mathrm{m}}$
- Time: $t_c = \frac{\hbar}{mc^2} \approx 1 \cdot 10^{-21}\mathrm{s}$
- Frequency: $\omega_c = \frac{e\mathcal{E}}{mc} \approx 8 \cdot 10^{20}\mathrm{Hz}$ $(\mathcal{E} = \mathcal{E}_c)$
- Spatial gradient: $\partial_r = \frac{mc}{\hbar} \approx 6.6 \cdot 10^{10}\mathrm{m}^{-1}$

Wigner function

Tool of description: the Wigner function:

- Quantum analogue of the classical phase space distribution.



Wigner function of an n=3 Fock state.

Wigner function

How it is defined?

- Take the equal time density matrix in terms of 'center of mass' coordinates:

$$\hat{\rho}(\vec{x},\vec{s},t) = e^{-ig\int_{-1/2}^{1/2}\vec{\mathcal{A}}(\vec{x}+\lambda\vec{s},t)\vec{s}\mathrm{d}\lambda}\left[\Psi(\vec{x}+\frac{\vec{s}}{2},t),\bar{\Psi}(\vec{x}-\frac{\vec{s}}{2},t)\right] \quad (1)$$

- Take the expectation value.
- Fourier transform it w.r.t the coordinate difference:

$$W(\vec{x},\vec{p},t) = -\frac{1}{2}\int e^{-i\vec{p}\vec{s}}\langle\Omega|\hat{\rho}(\vec{x},\vec{s},t)|\Omega\rangle\mathrm{d}^3s \quad (2)$$

Wigner function

The evolution equation:

$$D_t W = -\frac{1}{2}\vec{D}_{\vec{x}}[\gamma^0\vec{\gamma}, W] - im[\gamma^0, W] - i\vec{P}\{\gamma^0\vec{\gamma}, W\} \tag{3}$$

The equation has the following non-local differential operators:

$$D_t = \partial_t + g\vec{\mathcal{E}}(\vec{x}, t)\vec{\nabla}_{\vec{p}} - \frac{g\hbar^2}{12}(\vec{\nabla}_{\vec{x}}\vec{\nabla}_{\vec{p}})^2\vec{\mathcal{E}}(\vec{x}, t)\vec{\nabla}_{\vec{p}} + ... \tag{4}$$

$$\vec{D}_{\vec{x}} = \vec{\nabla}_{\vec{x}} + g\vec{\mathcal{B}}(\vec{x}, t) \times \vec{\nabla}_{\vec{p}} - \frac{g\hbar^2}{12}(\vec{\nabla}_{\vec{x}}\vec{\nabla}_{\vec{p}})^2\vec{\mathcal{B}}(\vec{x}, t) \times \vec{\nabla}_{\vec{p}} + ... \tag{5}$$

$$\vec{P} = \vec{p} + \frac{g\hbar}{12}(\vec{\nabla}_{\vec{x}}\vec{\nabla}_{\vec{p}})\vec{\mathcal{B}}(\vec{x}, t) \times \vec{\nabla}_{\vec{p}} + ... \tag{6}$$

For spin-1/2, the 4x4 gamma matrix basis is used:

$$W(x, p, t) = \frac{1}{4}\left[\mathbb{1}\mathbb{s} + i\gamma_5\mathbb{p} + \gamma^\mu\mathbb{v}_\mu + \gamma^\mu\gamma_5\mathbb{a}_\mu + \sigma^{\mu\nu}\mathbb{t}_{\mu\nu}\right] \tag{7}$$

Equations of motion for the spin-1/2 Wigner function

We arrive at a system for 16 unknown real functions:

$$
\begin{aligned}
&D_t\mathbb{s} & & & &- \; 2\vec{P}\cdot\vec{\mathbb{t}}_1 &&= 0 && (8)\\
&D_t\mathbb{p} & & & &+ \; 2\vec{P}\cdot\vec{\mathbb{t}}_2 &&= 2m\mathbb{a}_0 && (9)\\
&D_t\mathbb{v}_0 &+& \vec{D}_{\vec{x}}\cdot\vec{\mathbb{v}} & & &&= 0 && (10)\\
&D_t\mathbb{a}_0 &+& \vec{D}_{\vec{x}}\cdot\vec{\mathbb{a}} & & &&= 2m\mathbb{p} && (11)\\
&D_t\vec{\mathbb{v}} &+& \vec{D}_{\vec{x}}\mathbb{v}_0 & &+ \; 2\vec{P}\times\vec{\mathbb{a}} &&= -2m\vec{\mathbb{t}}_1 && (12)\\
&D_t\vec{\mathbb{a}} &+& \vec{D}_{\vec{x}}\mathbb{a}_0 & &+ \; 2\vec{P}\times\vec{\mathbb{v}} &&= 0 && (13)\\
&D_t\vec{\mathbb{t}}_1 &+& \vec{D}_{\vec{x}}\times\vec{\mathbb{t}}_2 & &+ \; 2\vec{P}\mathbb{s} &&= 2m\vec{\mathbb{v}} && (14)\\
&D_t\vec{\mathbb{t}}_2 &-& \vec{D}_{\vec{x}}\times\vec{\mathbb{t}}_1 & &- \; 2\vec{P}\mathbb{p} &&= 0 && (15)
\end{aligned}
$$

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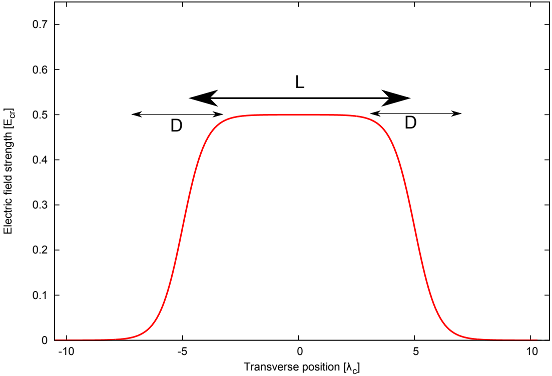
Inhomogeneous toy field

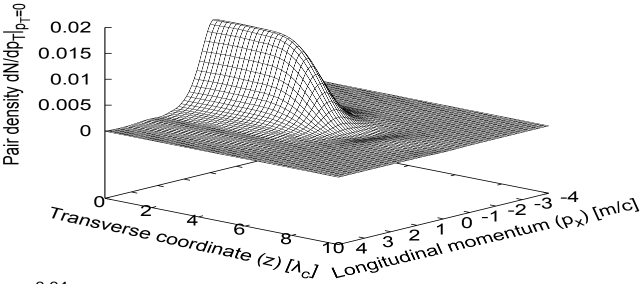
We would like to investigate the combined effect of space/time dependence.

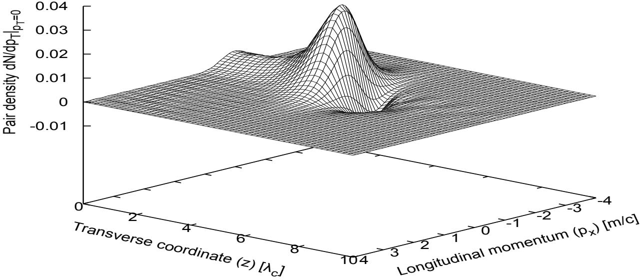
Consider an inhomogeneous plateu field in the transverse spatial direction and Sauter-like ($\cosh(t/\tau)^{-2}$) time dependence. ($E_0 = 0.5E_{cr}$).



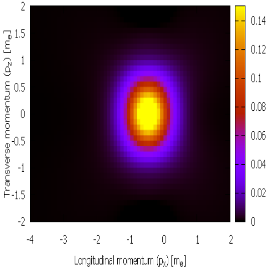
D. Berényi et al, Physics Letters B, Vol 749, pp. 210-214, DOI: 10.1016/j.physletb.2015.07.074.

Inhomogeneous toy field

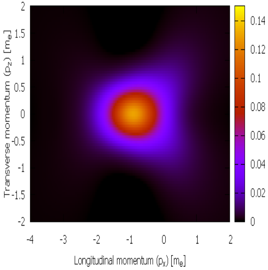




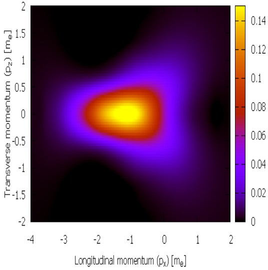
Inhomogeneous toy field



(a) $\tau = 1\frac{\lambda_c}{c}$



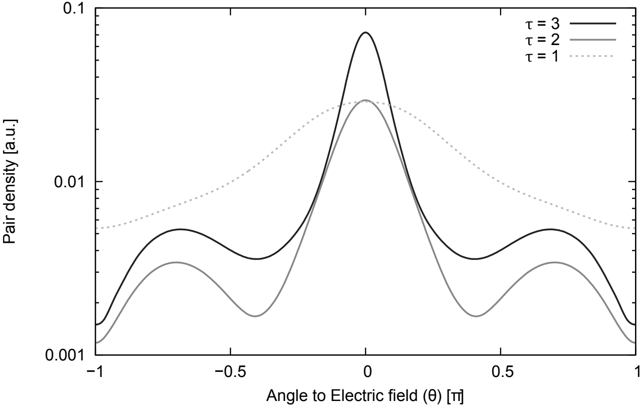
(b) $\tau = 2\frac{\lambda_c}{c}$



(c) $\tau = 3\frac{\lambda_c}{c}$

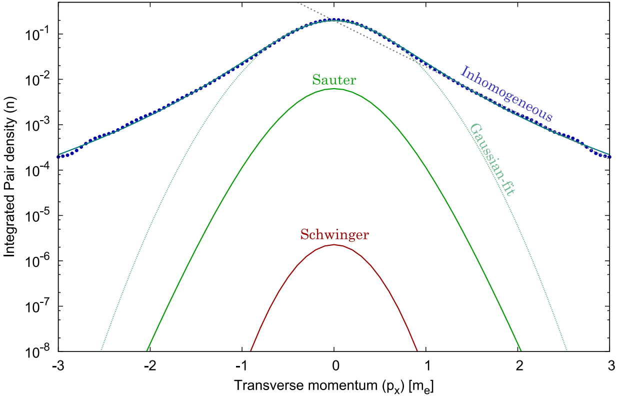
Inhomogeneous toy field

What would a calorimetric experiment see?
Longitudinal angle to the Electric field (θ, with $\theta = 0$ is longitudinal direction, $\theta = \pm\frac{\pi}{2}$ is the transverse direction):



Inhomogeneous toy field

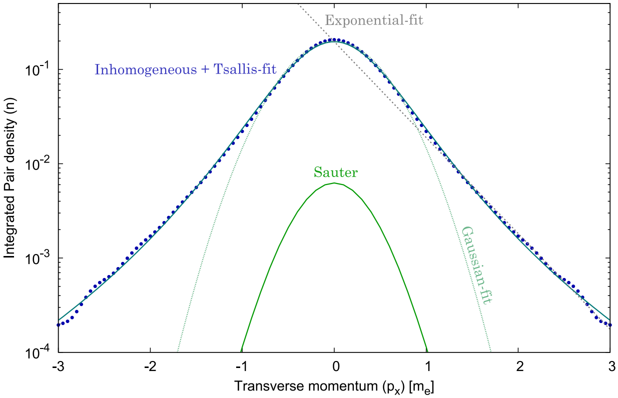
How does the transverse spectra compares to the homogeneous models?



Gaussianity: $Ae^{-\beta p_x^2}$, with $\beta_{Schwinger} = 6.6$, $\beta_{Sauter} = 4.3$, $\beta_{InHom} = 2.6$

Inhomogeneous toy field

How does the transverse spectra compares to the homogeneous models?



Tsallis: $A\left(1+(1-q)\frac{p_x}{p_{x0}}\right)^{\frac{1}{1-q}}$, with $q = 1.1, p_{x0} = 0.26$

Inhomogeneous toy field

Observations:

- The inhomogeneity increases the particle yield.
- The increase is further enhanced if the pulse lasts longer (as opposed to the homogeneous case).
- As a consequence, homogeneous models of string fragmentation may underestimate the particle production rates.
- The transverse spectra develops distinct higher-momenta "shoulders" at the boundaries.

Homogeneous E and B fields

Homogeneous E and B fields

When the E and B fields are homogeneous, the following equations are exact, that are the classical phase space evolution operators:

$$D_t = \partial_t + g\vec{\mathcal{E}}(\vec{x}, t)\vec{\nabla}_{\vec{p}} \tag{16}$$

$$\vec{D}_{\vec{x}} = g\vec{\mathcal{B}}(\vec{x}, t) \times \vec{\nabla}_{\vec{p}} \tag{17}$$

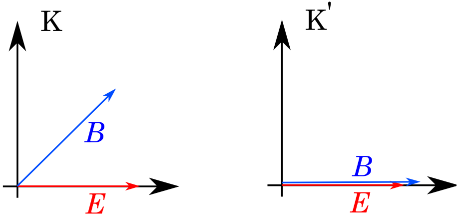
$$\vec{P} = \vec{p} \tag{18}$$

Homogeneous E and B fields

In the initial state of heavy ion collisions strong color E and B fields expected. Usually they are considered to be constant and homogeneous, but we can go further in the Wigner formalism. Potential applications relate to e.g. the Chiral Magnetic Effect c.f. arxiv:1002.2495

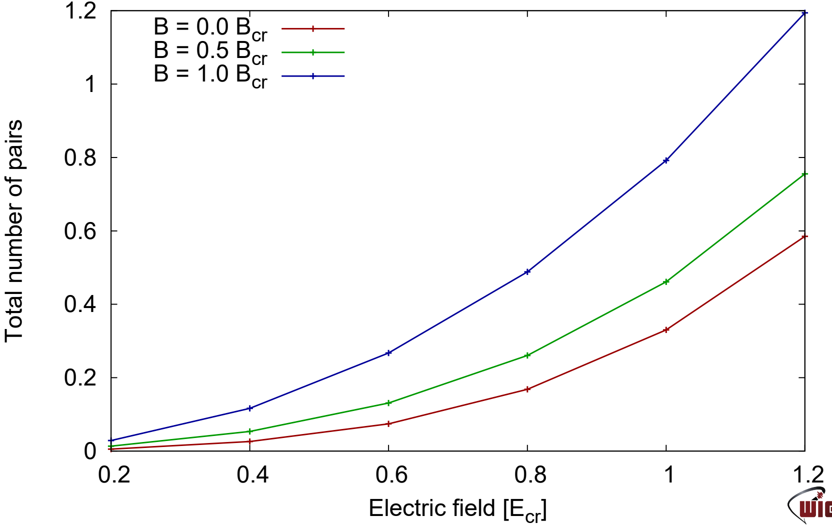
- Logic:

  SU(3) quark production can be decomposed into a QED calculation for each flavour, where $\vec{E} = (0, 0, E_z)$ and $\vec{B} = (E, B_y, B_z)$ but we can apply a Lorentz transformation and zero-out B_y also.



Preliminary result

$$E_z(t) = B_z(t) = [B, E] \times e^{-\left(\frac{t}{\tau}\right)^2}$$



Summary

- We have shown examples of pair production in high energy physics.
- The evolution of the Wigner function in multiple dimensions is possible including space and time dependent field configurations.
- We discussed a simple toy field that models the interplay of space and time localizations and showed characteristic differences from homogeneous results.
- We started to investigate strongly time dependent E and B fields.

  D. Berényi et al, Physics Letters B, Vol 749, pp. 210-214, DOI: 10.1016/j.physletb.2015.07.074.

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