15. Zimányi WINTER SCHOOL ON HEAVY ION PHYSICS

Dec. 7. - Dec. 11., Budapest, Hungary

Quark-anti-quark potential in the N=4 super gauge theory

Z. Bajnok

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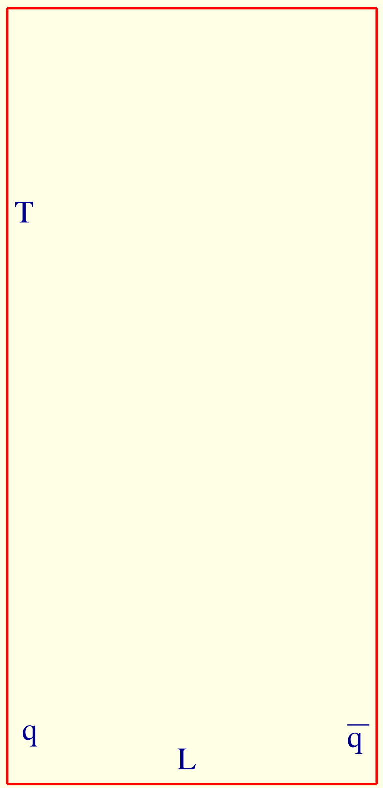
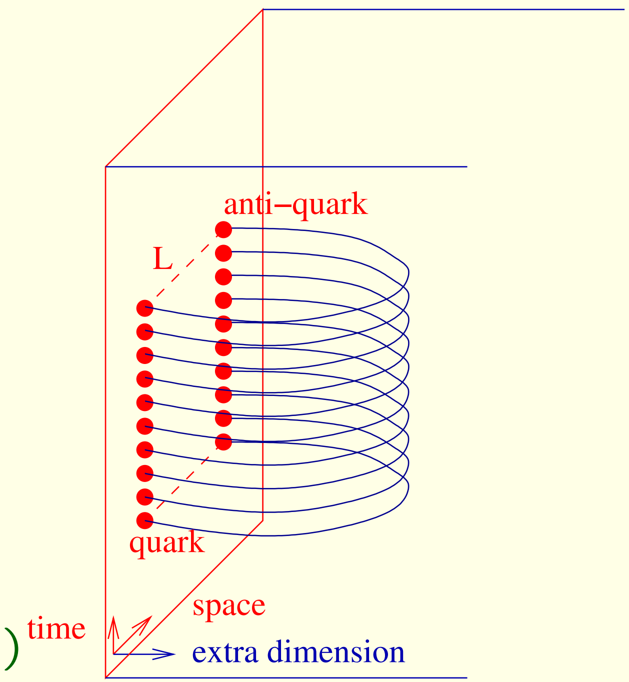
$$\langle W\rangle = e^{-TV_{q\bar{q}}(L,\lambda)}$$



$$= e^{-TE_0(L,\lambda)}$$



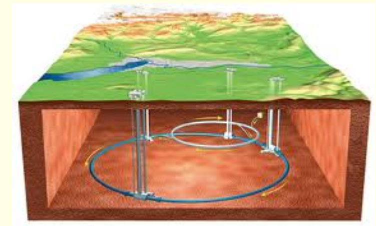
Elementary constituents of matter

Elementary constituents of matter

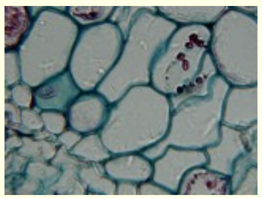
microscope

electron microscope

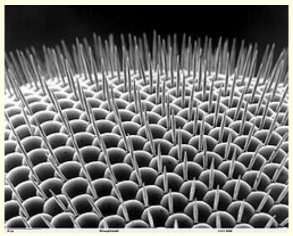
synchrotron



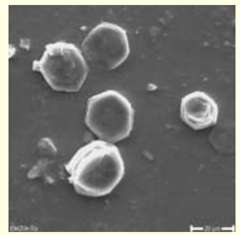
LHC



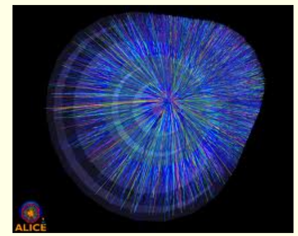
micrometer



nanometer



atomic scale



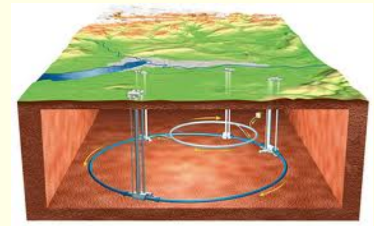
quarks, leptons

Elementary constituents of matter

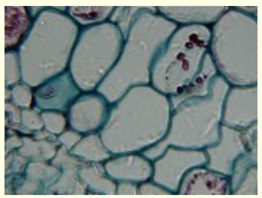
microscope

electron microscope

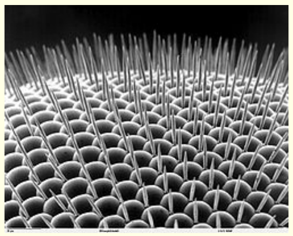
synchrotron



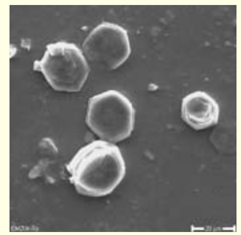
LHC



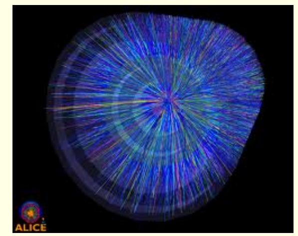
micrometer



nanometer

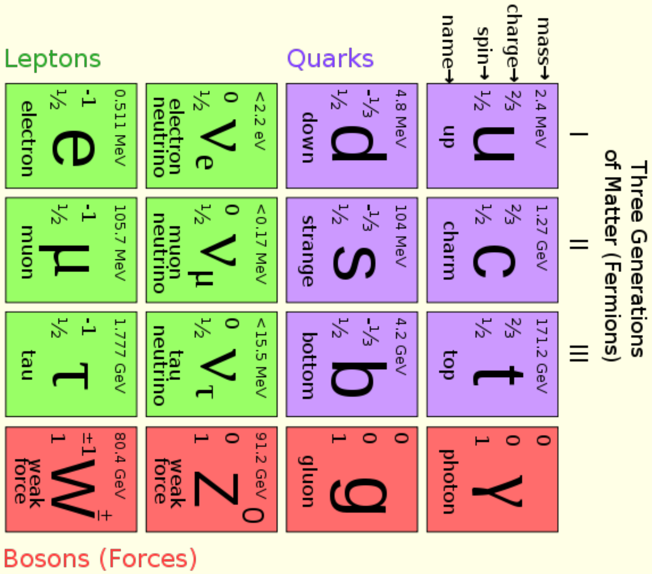


atomic scale

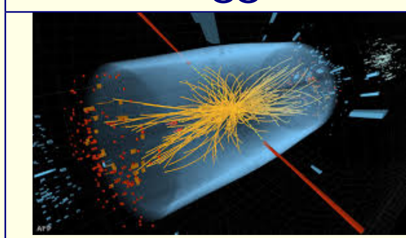


quarks, leptons

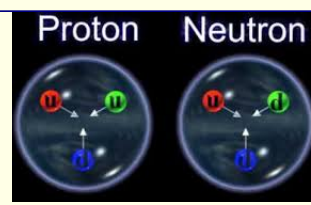
periodic table of particles



Higgs

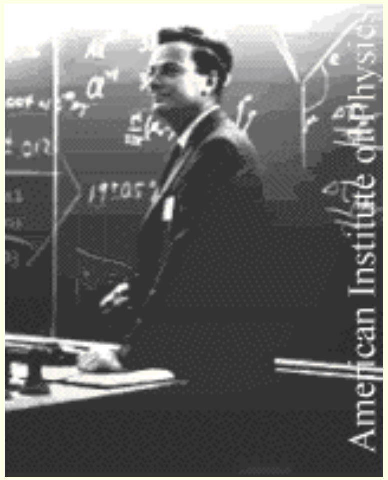


matter



	Interaction
γ	electromagnetic
$W^{\pm}, Z$	weak
g	strong
gr	gravitational

Quantum electrodynamics



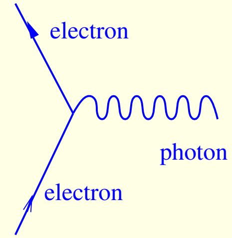
Feynman: *If you want to learn about nature, to appreciate nature, it is necessary to understand the language that she speaks in.*

Quantum

gauge theory

Electromagnetic interaction: $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$

$U(1)$ gauge theory: $A_\mu \to A_\mu + \partial_\mu \Lambda$



$$\mathcal{L} = -\tfrac{1}{4}F^2 + \bar{\Psi}(i\partial\!\!\!/ - m)\Psi - e\bar{\Psi}A\!\!\!/\Psi$$

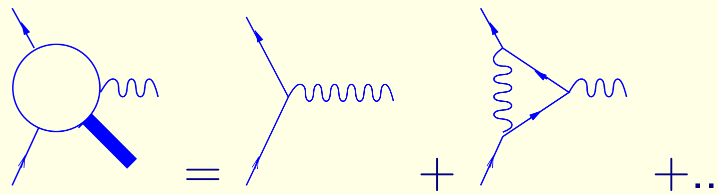
experiment: $\underline{\mu} = g\frac{e\hbar}{2mc}\underline{s}$ where $g = 2(1+a)$

[Gabrielse 2006]: $a = 1159652180.85(.76) \times 10^{-12}$

perturbation theory:

Feynman graphs

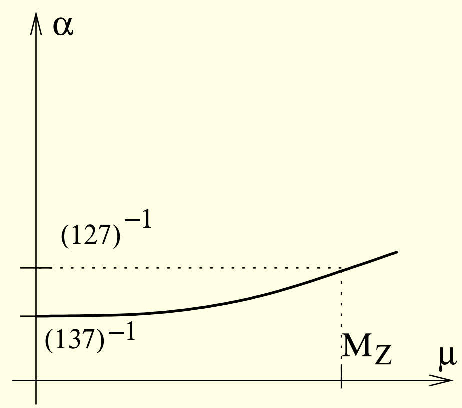
$\frac{\alpha}{2\pi} = \frac{e^2}{2\pi\hbar c} = 0.001161$



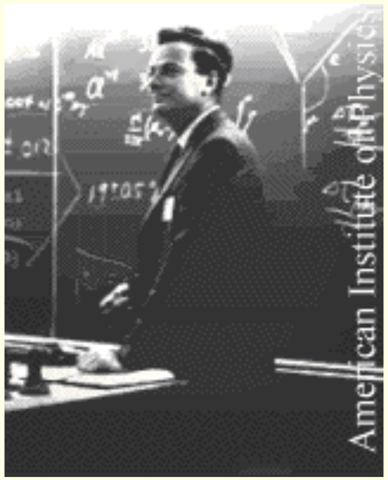
$\frac{g}{2} = 1 - 1.3140\frac{\alpha}{2\pi}$

running coupling:

$\beta(\alpha) = \mu\frac{\partial\alpha}{\partial\mu} > 0$

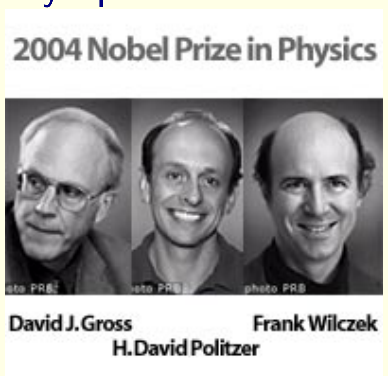


Quantum chromodynamics



Quantum gauge theory

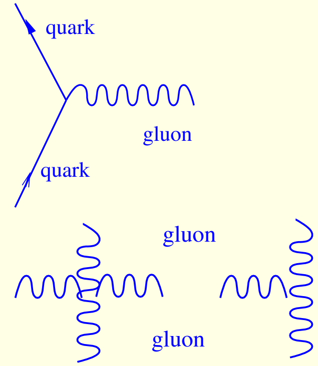
asymptotic freedom



photon $A_\mu \leftrightarrow G_\mu^{1..8}$ gluon $\rightarrow F_{\mu\nu}^{1..8}$

electron $\Psi_e \leftrightarrow \Psi_{quark}$ quark

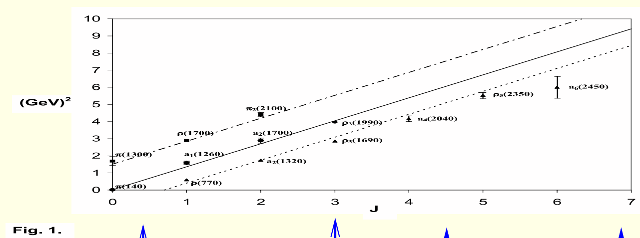
$SU(3)$ gauge theory: $G_\mu \rightarrow g^{-1} G_\mu g + g^{-1} \partial_\mu g$



$$\mathcal{L} = -\frac{1}{4} F^2 + \bar{\Psi}(i\not{\partial} - m)\Psi - g\bar{\Psi}\not{G}\Psi$$

experiments:

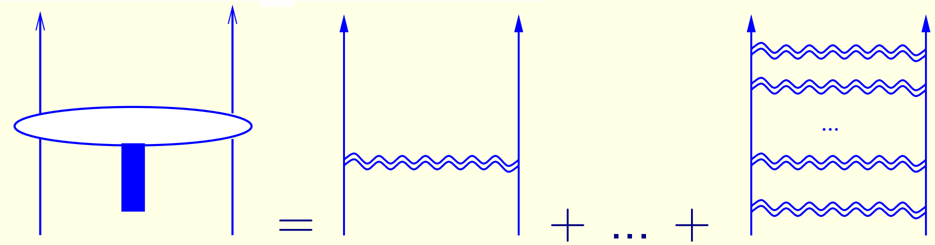
hadron spectrum



perturbation theory:

Feynman graphs

$0.001 = \frac{\alpha}{2\pi} \leftrightarrow \frac{\alpha_s}{4\pi} = O(1)$

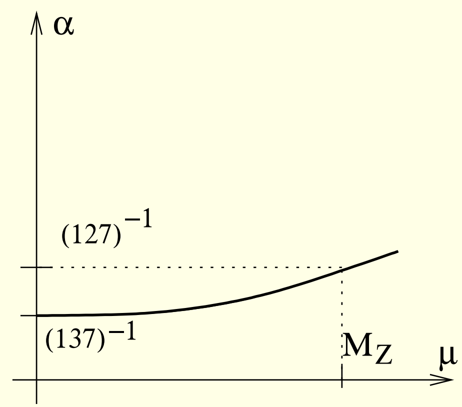


running coupling:

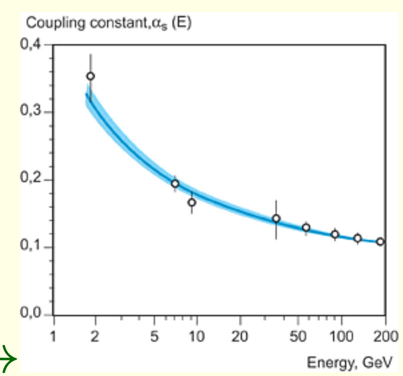
$\beta(\alpha_s) = \mu \frac{\partial \alpha_s}{\partial \mu} < 0$

asymptotic freedom

confinement



$\leftrightarrow$



Strong interaction=QCD

SU(3) gaugetheory

$$\mathcal{L} = -\frac{1}{4}F^2 + \bar{\Psi}(i\partial\!\!\!/ - m)\Psi - g\bar{\Psi}G\!\!\!/\Psi$$

Wigner Jenő:

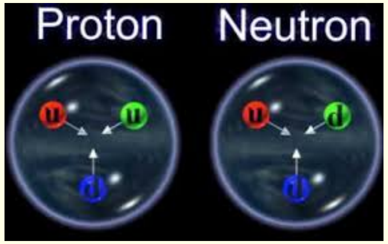
The simplicities of natural laws arise through the complexities of the language we use for their expression.

low energies: strongly coupled gauge theory

confinement

hadron spectrum,

heavy ion collision



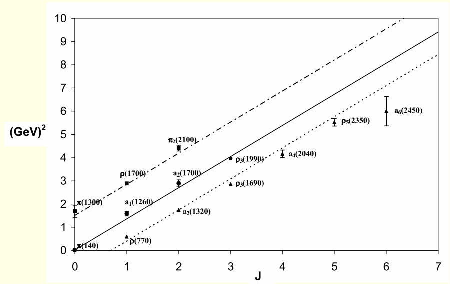
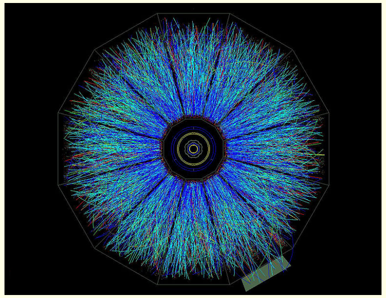
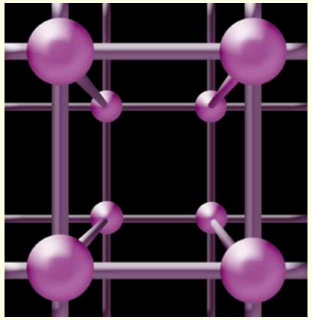


Fig. 1.



Lattice gauge theory: world 32^4

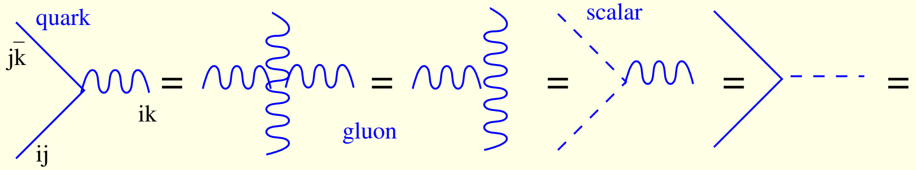


protonmass $\surd$heavy ion collision?

Wigner:It is nice to know that the computer understands the problem. But I would like to understand it too.

simplified model

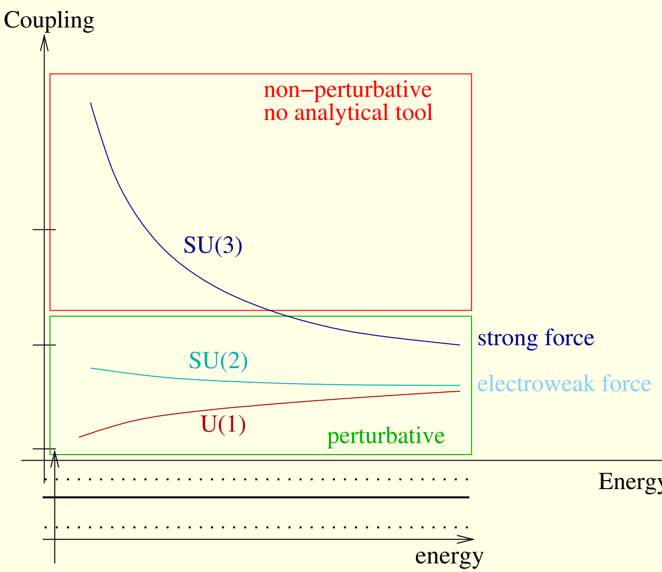
$\mathcal{N} = 4$ D=4 $SU(N)$ SYM



$$\frac{2}{g_{YM}^2}\int d^4x \text{Tr}\left[-\frac{1}{4}F^2 - \frac{1}{2}(D\Phi)^2 + i\overline{\Psi}D\!\!\!/\Psi + V\right]$$

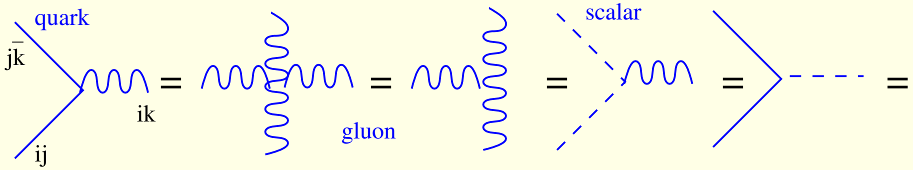
$$V(\Phi,\Psi) = \frac{1}{4}[\Phi,\Phi]^2 + \overline{\Psi}[\Phi,\Psi]$$

Maximally supersymmetric gauge theory



Elementary interactions

interaction	particles	gauge group
electromagnetic	photon+electron	$U(1)$
electroweak	$W^{\pm}, Z \quad \mu,\nu$+Higgs	$SU(2) \times U(1)$
strong	gluon+quark	$SU(3)$
$\mathcal{N} = 4$SYM	gluon A_μ+quark Ψ+scalar Φ	$SU(N)$



all fields $N^2 - 1$ component matrix (i, j)

$$Q_{1,2,3,4} \quad \Psi^{ij}_{1,2,3,4} \quad Q_{1,2,3,4}$$

$$\nearrow \quad \circlearrowleft \quad \searrow$$

$$A^{ij}_\mu \qquad su(4) = so(6) \quad \circlearrowleft \quad \Phi^{ij}_{1,2,3,4,5,6}$$

$$\searrow \quad \circlearrowleft \quad \nearrow$$

$$Q^\dagger_{1,2,3,4} \quad \overline{\Psi}^{ij}_{1,2,3,4} \quad Q^\dagger_{1,2,3,4}$$

$$\mathcal{L} = \frac{2}{g^2_{YM}} \int d^4x \mathrm{Tr} \left[-\frac{1}{4} F^2 - \frac{1}{2}(D\Phi)^2 + i\overline{\Psi} \not{D} \Psi + V \right]$$

$$V(\Phi, \Psi) = \frac{1}{4}[\Phi, \Phi]^2 + \overline{\Psi}[\Phi, \Psi]$$

$\beta = 0 \rightarrow$scaleinvariance $m = 0$

parameters: N, g_{YM}

superconformal CFT

Symmetries:

inner: $su(4) = so(6)$

space-time: conformal$\supset$Lorentz

$so(4, 2) = su(2, 2)$

super $psu(2, 2|4)$

$$\begin{pmatrix} su(2,2) & Q \\ Q^\dagger & su(4) \end{pmatrix}$$

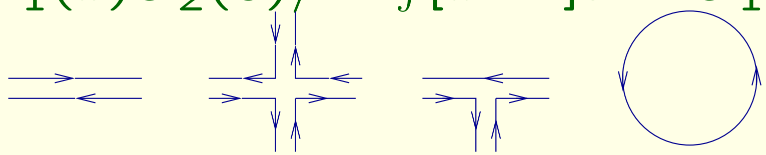
CFT: Observables

maximally supersymmetric gauge theory
$\Psi_{1,2,3,4}$
A $\quad\Phi_{1,2,3,4,5,6}$ fields $SU(N)$ matrices
$\overline{\Psi}_{1,2,3,4}$
$\mathcal{S} = \frac{2}{g_{YM}^2}\int d^4x \mathrm{Tr}\left[-\frac{1}{4}F^2 - \frac{1}{2}(D\Phi)^2 + i\overline{\Psi}\not{D}\Psi + V\right]$
$V(\Phi,\Psi) = \frac{1}{4}[\Phi,\Phi]^2 + \overline{\Psi}[\Phi,\Psi]$

observables (g_{YM}, N)
partition function
gaugeinvariant operators
$\mathcal{O}(x) = \mathrm{Tr}(A^{L_1}\Psi^{L_2}\Phi^{L_3}..)$
Wilson loops, $q - \bar{q}$ potentials
correlators: $\langle\mathcal{O}_1(x)\mathcal{O}_2(0)\rangle$

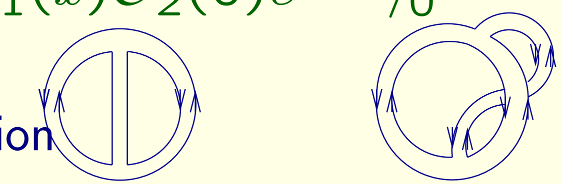
correlators: $\langle\mathcal{O}_1(x)\mathcal{O}_2(0)\rangle = \int[dA...]e^{-i\mathcal{S}}\mathcal{O}_1(x)\mathcal{O}_2(0) = \langle\mathcal{O}_1(x)\mathcal{O}_2(0)e^{-iV}\rangle_0$

perturbatively:



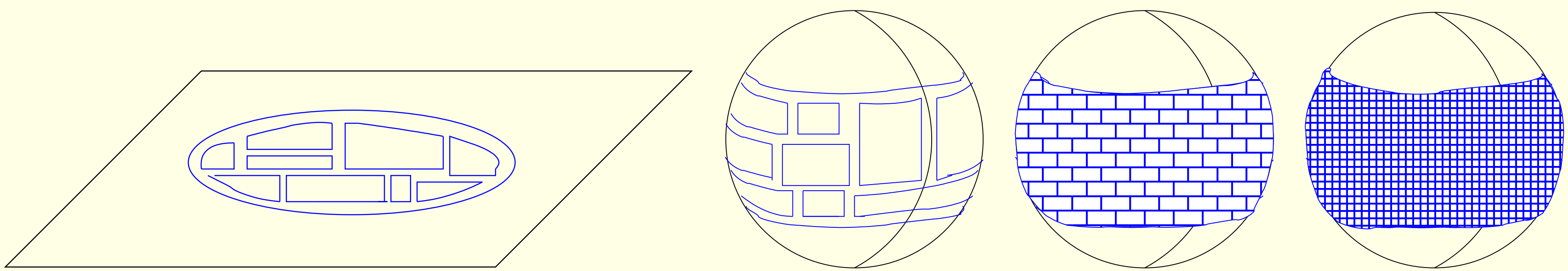
g_{YM}^2 $\quad g_{YM}^{-2}$ $\quad g_{YM}^{-2}$ $\quad N$

genus expansion

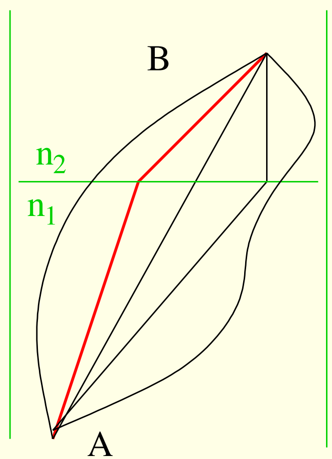
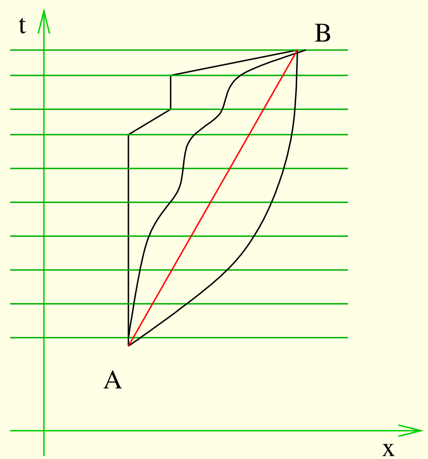
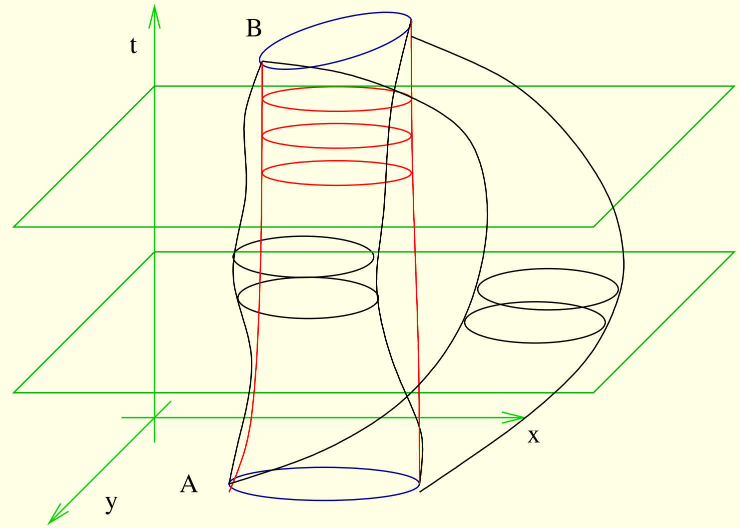


$g_{YM}^2 N^3 = N^2\lambda \quad \lambda = g_{YM}^2 N$

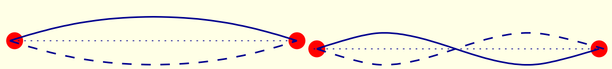
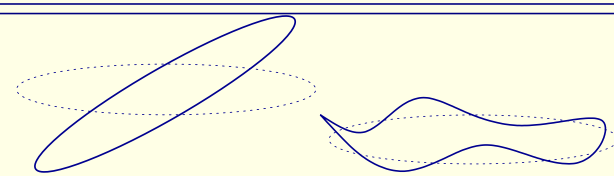
partition function. $Z(\lambda, \frac{1}{N}) = N^2 \sum_g (\frac{1}{N})^{2g} \sum_n \alpha(g,n)\lambda^n$ string theory? (t' Hooft)



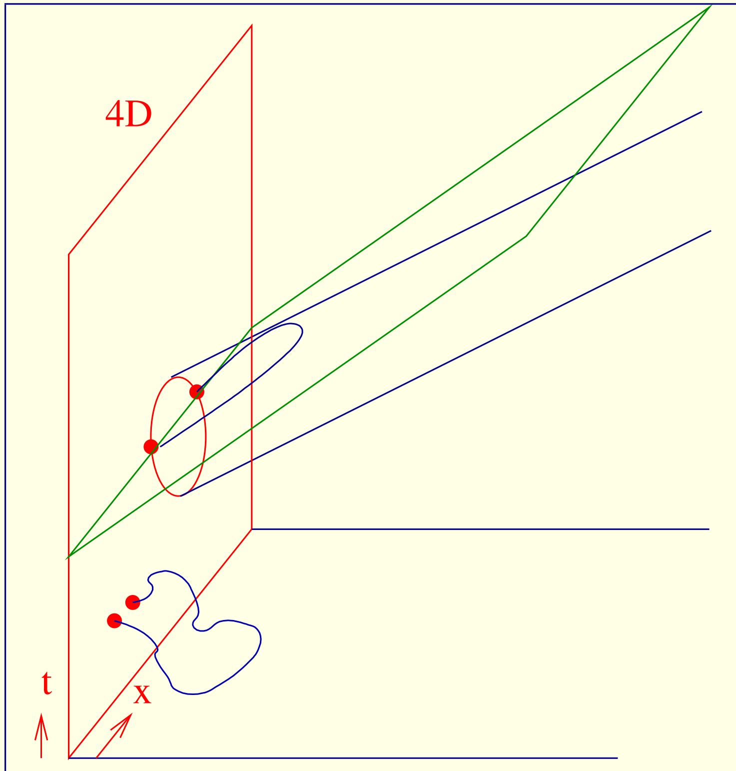
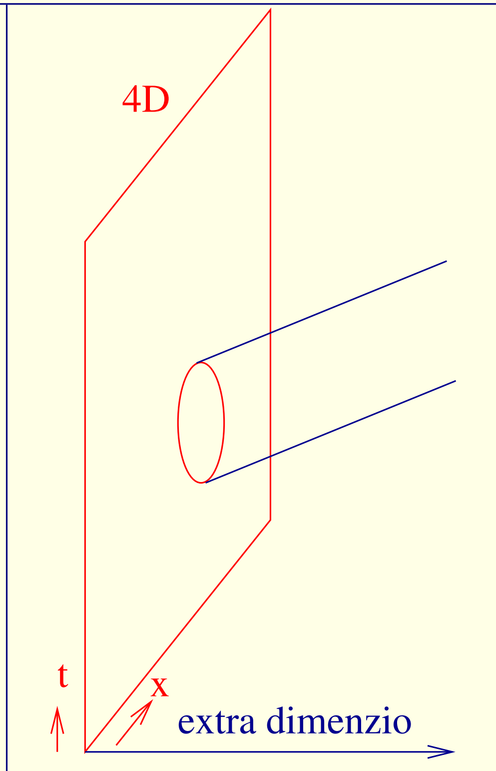
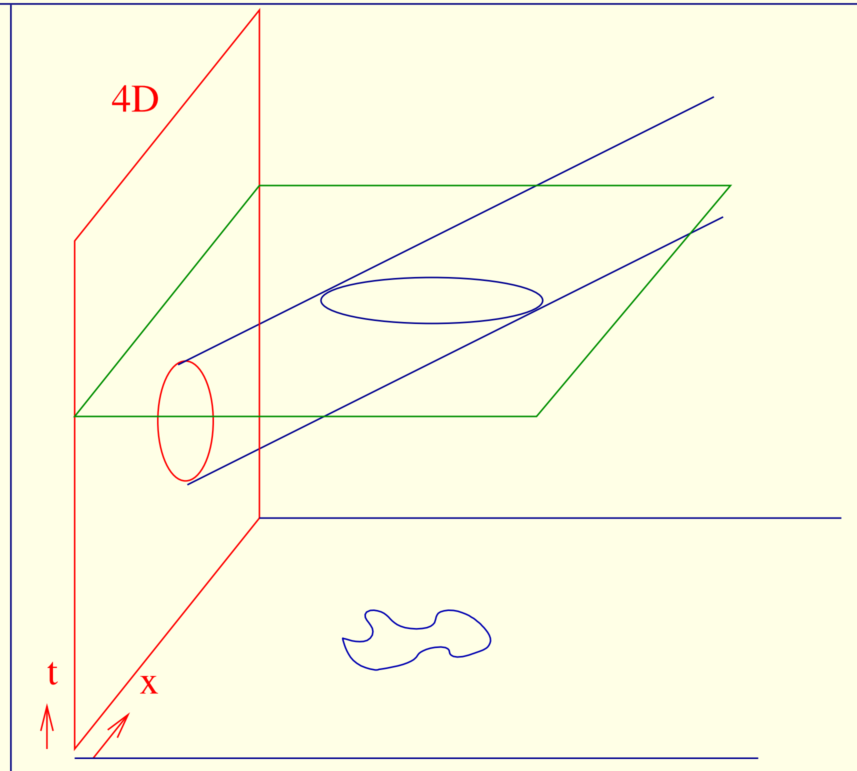
Dynamics of strings

light	pointparticle	string
Fermat principle	space-time (x, t)	space-time (x, y, t)
		
minimal time	minimal 'space-time'	minimal space-time surface

Spectrum of quantum strings

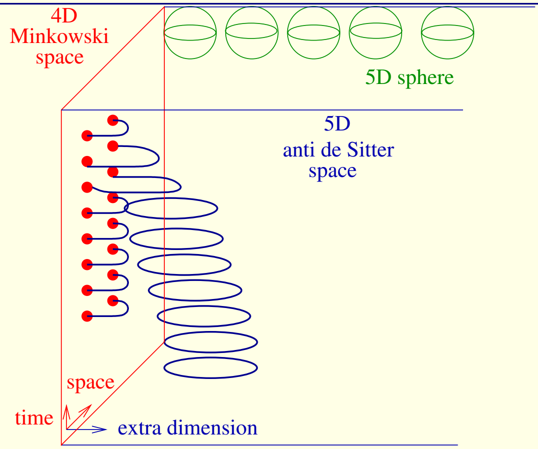
open strings		closed strings
		
photon, gaugeboson, electron, quarks+...		graviton+...
gaugetheory with matter		gravitation

Gauge/gravity duality

open string	space/time is relative	closed string
		
open string process		closed string process
gaugetheory	=	gravitation

AdS/CFT duality (Maldacena 1998)

II_B superstring on $AdS_5 \times S^5$



$\sum_1^6 Y_i^2 = R^2 \qquad -++++- = -R^2$

$\frac{R^2}{\alpha'} \int \frac{d\tau d\sigma}{4\pi} \left(\partial_a X^M \partial^a X_M + \partial_a Y^M \partial^a Y_M\right) + \dots$

$\equiv$

$\mathcal{N} = 4$ D=4 $SU(N)$ SYM

$$\frac{2}{g_{YM}^2} \int d^4x \mathrm{Tr} \left[-\frac{1}{4}F^2 - \frac{1}{2}(D\Phi)^2 + i\overline{\Psi}\not{D}\Psi + V\right]$$

$$V(\Phi, \Psi) = \frac{1}{4}[\Phi, \Phi]^2 + \overline{\Psi}[\Phi, \Psi]$$

$\beta = 0$ superconformal

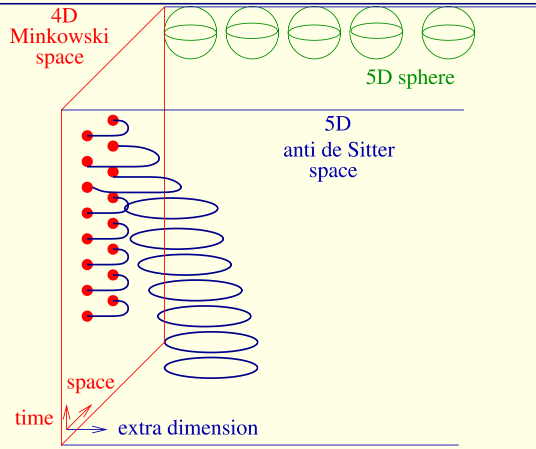
$$psu(2,2|4) \supset su(2,2) \otimes su(4)$$

$$su(2,2) = so(4,2) \quad su(4) = so(6)$$

gauge inv.:$\mathcal{O} = \mathrm{Tr}(\Phi^2), \det(\ ), \mathrm{Tr}(P e^{\int A})$

AdS/CFT duality (Maldacena 1998)

II_B superstring on $AdS_5 \times S^5$



$$\sum_1^6 Y_i^2 = R^2 \qquad -++++- = -R^2$$

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Dictionary

parameters: $\sqrt{\lambda} = \frac{R^2}{\alpha'}$, $g_s = \frac{\lambda}{N} \to 0$

2D QFT

string energies: $E(\lambda)$

$$E(\lambda) = E(\infty) + \frac{E_1}{\sqrt{\lambda}} + \frac{E_2}{\lambda} + \dots$$

strong$\leftrightarrow$weak

$\Downarrow$

$\lambda = g_{YM}^2 N$, $N \to \infty$ planar limit

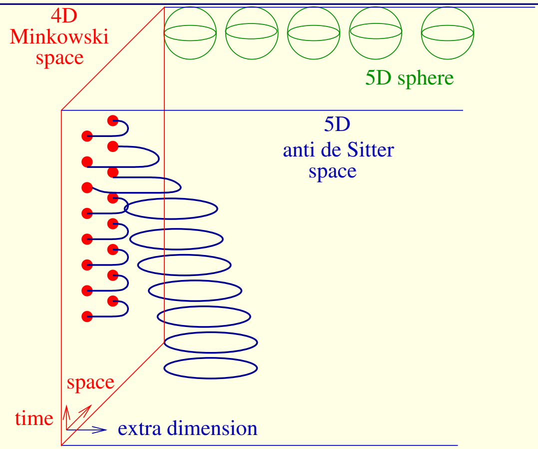
$$\langle \mathcal{O}_n(x) \mathcal{O}_m(0) \rangle = \frac{\delta_{nm}}{|x|^{2\Delta_n(\lambda)}}$$

scaling dimensions $\Delta(\lambda)$

$$\Delta(\lambda) = \Delta(0) + \lambda \Delta_1 + \lambda^2 \Delta_2 + \dots$$

AdS/CFT duality (Maldacena 1998)

II_B superstring on $AdS_5 \times S^5$



$$\sum_1^6 Y_i^2 = R^2 \qquad -++++- = -R^2$$
$$\frac{R^2}{\alpha'} \int \frac{d\tau d\sigma}{4\pi} \left(\partial_a X^M \partial^a X_M + \partial_a Y^M \partial^a Y_M\right) + \ldots$$

$\equiv$

$\mathcal{N} = 4$ D=4 $SU(N)$ SYM

$$\frac{2}{g_{YM}^2} \int d^4x \mathrm{Tr} \left[-\frac{1}{4}F^2 - \frac{1}{2}(D\Phi)^2 + i\overline{\Psi}\not{D}\Psi + V\right]$$
$$V(\Phi,\Psi) = \frac{1}{4}[\Phi,\Phi]^2 + \overline{\Psi}[\Phi,\Psi]$$
$\beta = 0$ superconformal
$$psu(2,2|4) \supset su(2,2) \otimes su(4)$$
$$su(2,2) = so(4,2) \quad su(4) = so(6)$$
gauge inv.:$\mathcal{O} = \mathrm{Tr}(\Phi^2), \det(\ ), \mathrm{Tr}(Pe^{\int A})$

Dictionary

parameters: $\sqrt{\lambda} = \frac{R^2}{\alpha'}$, $g_s = \frac{\lambda}{N} \to 0$

2D QFT

string energies: $E(\lambda)$

$$E(\lambda) = E(\infty) + \frac{E_1}{\sqrt{\lambda}} + \frac{E_2}{\lambda} + \ldots$$

strong$\leftrightarrow$weak

$\Downarrow$

$\lambda = g_{YM}^2 N$, $N \to \infty$ planar limit

$$\langle \mathcal{O}_n(x)\mathcal{O}_m(0)\rangle = \frac{\delta_{nm}}{|x|^{2\Delta_n(\lambda)}}$$

scaling dimensions $\Delta(\lambda)$

$$\Delta(\lambda) = \Delta(0) + \lambda\Delta_1 + \lambda^2\Delta_2 + \ldots$$

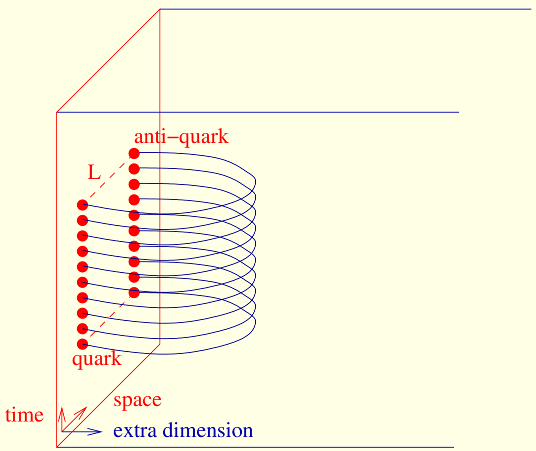
2D integrable QFT

spectrum: $Q = 1, 2, \ldots, \infty$ dispersion: $\epsilon_Q(p) = \sqrt{Q^2 + \frac{\lambda}{\pi^2}\sin^2\frac{p}{2}}$

Exact S-matrix: $S_{Q_1Q_2}(p_1, p_2, \lambda)$

AdS/CFT: $q-\bar{q}$ potential

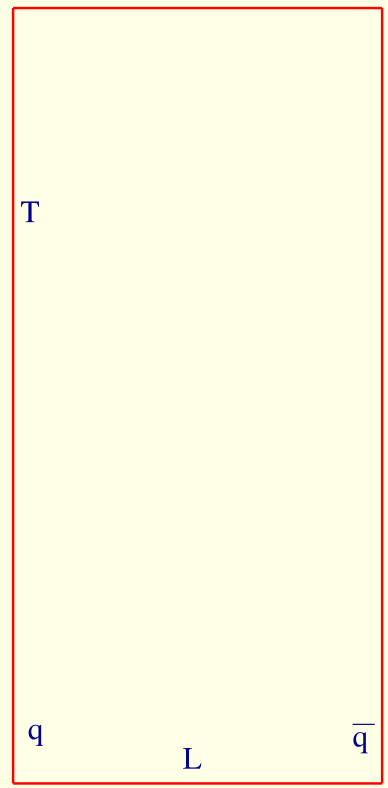
Strong coupling
minimal surface in AdS



$$V(r) = -\frac{4\pi^2\sqrt{2\lambda}}{\Gamma(\frac{1}{4})^4}\frac{1}{L}(1- -\frac{1.3359}{\sqrt{\lambda}} + \dots)$$

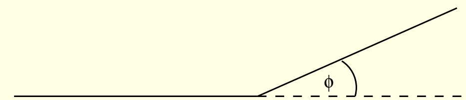
+fluctuations
[Maldacena]

$\equiv$



$$\langle W \rangle = e^{-TV_{q\bar{q}}(L,\lambda)}$$

Weak coupling
Wilson loops:



$$\langle \mathcal{P}e^{\oint_C A_\mu dx^\mu + \vec{\Phi}\vec{n}_0} Z^J \dots$$

$$\dots \mathcal{P}e^{\oint_C A_\mu dx^\mu + \vec{\Phi}\vec{n}_\theta}\rangle$$

$q-\bar{q}$ potential

$$V_{q\bar{q}}(\lambda,\phi,\theta) = \sum \Gamma_k \lambda^k$$

$$\Gamma_k = \sum_{n=1}^{k} \left(\frac{\cos\phi - \cosh\theta}{\sin\phi}\right)^n \gamma_k^{(n)}$$

$$\gamma_1^{(1)} = \frac{\phi}{2} \quad ; \gamma_2^{(1)} = \frac{\phi}{12}(\phi^2 - \pi^2)$$

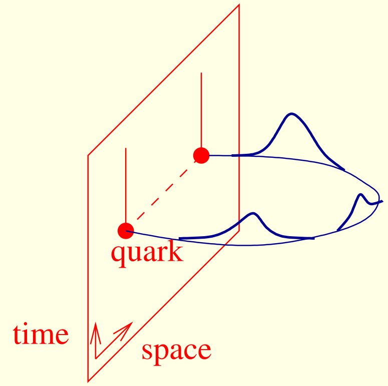
$$\gamma_2^{(2)}(0) = \gamma_2^{(2)\prime}(0) = 0$$

$$\gamma_2^{(2)\prime\prime}(\phi) = \frac{\phi}{2}\cot\phi$$

Bremsstrahlung

$q - \bar{q}$ potential: integrable description

Integrable system on the strip



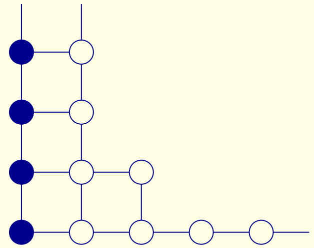
Boundary problem: particle reflections

$E_0(L)$, Casimir energy

$$E_0(L) = \int \frac{d\tilde{k}}{2\pi} \log(1 - R_-(\tilde{k}) R_+(-\tilde{k}) e^{-\tilde{\epsilon}(\tilde{k})})$$

$$\epsilon^j(\theta) = \delta^j_Q(\sigma_Q(\tilde{p}) + \tilde{E}_Q(\tilde{p}) L - \int K^j_i(\tilde{p}, \tilde{p}') \log(1 + e^{-\epsilon^i(\tilde{p}')}) d\tilde{p}'$$

Boundary TBA equation



[Correa,Maldacena,Sever '12][Drukker '12]

no systematic weak coupling expansion

change: $\sigma_Q(\tilde{p}) \to \tilde{\sigma}_Q(\tilde{p})$

2 loops [Bajnok et. al 13]: agrees with perturbation theory!

Regularized $q - \bar{q}$ BTBA equations

Singular boundary fugacity: $\sigma_Q(0) = \infty$, no-obvious weak coupling expansion

shifting countours $\to$ regularization (extra source terms, $\sim$ excited state TBA)

$$
\begin{aligned}
\log Y_Q &= -2(f+\Psi)Q - R\tilde{\epsilon}_Q + \log\sigma_Q + D^{Q'Q}(iu_{Q'}) + \log(1+Y_{Q'}) \star_\eta K^{Q'Q} \\
&\quad + \Big[2\log\big(1+Y_{v|1}\big) \star s \,\hat{\star} K_{yQ} + 2\log(1+Y_{v|Q-1}) \star s - 2\log\frac{1-Y_-}{1-Y_+} \hat{\star} s \star K^{1Q}_{vx} \\
&\qquad + \log\frac{1-\frac{1}{Y_-}}{1-\frac{1}{Y_+}} \hat{\star} K_Q + \log(1-\frac{1}{Y_-})(1-\frac{1}{Y_+}) \hat{\star} K_{yQ}\Big] \\
\log Y_- Y_+ &= 2D_{xvs}(iu_Q) - D_Q(iu_Q) - \log(1+Y_Q) \star_\eta K_Q + 2\log(1+Y_Q) \star K^{Q1}_{xv} \star s + 2\log\frac{1+Y_{v|1}}{1+Y_{w|1}} \star \\
\log\frac{Y_+}{Y_-} &= D_{Qy}(iu_Q) + \log(1+Y_Q) \star_\eta K_{Qy} \\
\log Y_{v|M} &= -D_s(iu_{M+1}) - \log(1+Y_{M+1}) \star_\eta s + I_{MN}\log(1+Y_{v|N}) \star s + \delta_{M1}\log\frac{1-Y_-}{1-Y_+} \hat{\star} s \\
\log Y_{w|M} &= I_{MN}\log(1+Y_{w|N}) \star s + \delta_{M1}\log\frac{1-\frac{1}{Y_-}}{1-\frac{1}{Y_+}} \hat{\star} s
\end{aligned}
$$

$$
f = i(\pi - \phi) \qquad ; \qquad \Psi = -i(\pi-\phi) \qquad ; \qquad R = 2L
$$

[ZB, Balog, Hegedus, Toth '13]