Dissipative phase space correction (δf) models vs nonlinear kinetic theory

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with Mridula Damodaran, in preparation ...

Outline

I. The problem of dissipative (δf) corrections

II. A couple δf models

III. Test them against full kinetic theory

IV. Some lessons and refinements (positivity, memory)

V. Conclusions and future steps

Hydrodynamics

conservation laws:

$$\partial_\mu T^{\mu\nu}(x) = 0 \quad , \quad \partial_\mu N^\mu_B(x) = 0$$

additional EoM for dissipative fluids:

$$T^{\mu\nu}(x) = T^{\mu\nu}_{ideal}(x) + \delta T^{\mu\nu}(x)$$

$$N^\mu(x) = N^\mu_{ideal}(x) + \delta N^\mu(x)$$

$$\delta\dot{T}^{\mu\nu} = A^{\mu\nu}(\{T^{\alpha\beta}, N^\gamma\}) \quad , \quad \delta\dot{N}^\mu = B^\mu(\{T^{\alpha\beta}, N^\gamma\})$$

(e.g. Israel-Stewart '79)

utilizes equation of state ($p(e, n_B)$, $T(e, n_B)$), transport coeffs (η, ζ, κ_B), relaxation times (τ_η, τ_ζ, τ_κ)

some issues:
- **initial conditions needed $\leftrightarrow$ thermalization**
- **how to "turn" hydro off at late times**
- **very small systems - quantum effects?** DM, Wang, Greene, arXiv:1404.4119
- **is it really hydro?** He et al, arXiv:1502.05572

Hydro $\to$ particles

hydro gives N^μ & $T^{\mu\nu}$, but experiments measure particles

$$N^\mu(x) \equiv \sum_i \int \frac{d^3p}{E} p^\mu f_i(p,x)$$

$$T^{\mu\nu}(x) \equiv \sum_i \int \frac{d^3p}{E} p^\mu p^\nu f_i(p,x)$$

- **in local equilibrium (ideal hydro) - "one to one"**

$$T^{\mu\nu}_{LR}(x) = diag(e,p,p,p) \qquad \Leftrightarrow \qquad f_{eq,i}(x,p) = \frac{g_i}{(2\pi)^3} \frac{1}{e^{(p^\mu u_\mu - \mu_i)/T} + a}$$

- **near local equilibrium (viscous hydro) - "few to many"**

$$\begin{aligned} T^{\mu\nu}(x) &= T^{\mu\nu}_{ideal}(x) + \delta T^{\mu\nu}(x) \\ N^\mu(x) &= N^\mu_{ideal(x)} + \delta N^\mu(x) \end{aligned} \qquad \Leftarrow \qquad f_i(x,p) = f_{eq,i}(x,p) + \delta f_i(x,p)$$

$\Rightarrow$ **question of δf (even for single-species systems!)**

δf models, for shear

$$\delta T^{\mu\nu} = \pi^{\mu\nu} \qquad\qquad (\pi^{\mu\nu} u_\nu = 0,\ u_\nu \pi^{\mu\nu} = 0,\ \pi^\mu{}_\mu = 0)$$

purely spatial (in LR), symmetric, traceless

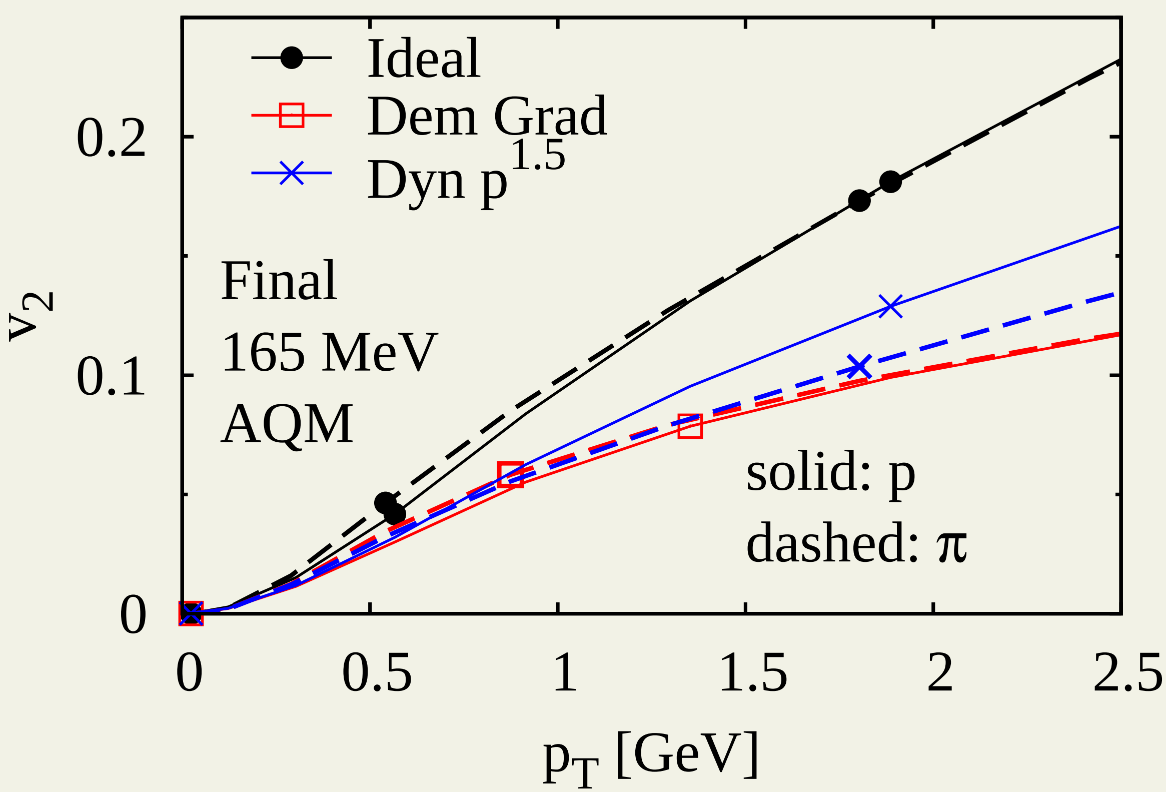
Navier-Stokes: $\pi^{\mu\nu} = \eta \underbrace{\left[\nabla^\mu u^\nu + \nabla^\nu u^\mu - \frac{2}{3}\Delta^{\mu\nu}(\partial\cdot u)\right]}_{2\sigma^{\mu\nu}}$

matter for: - hadron distributions (fluid$\to$ hadron gas conversion)
- E&M rates from QGP
- bound states in plasma ...

δf matters e.g., for v_2 in Au+Au at RHIC

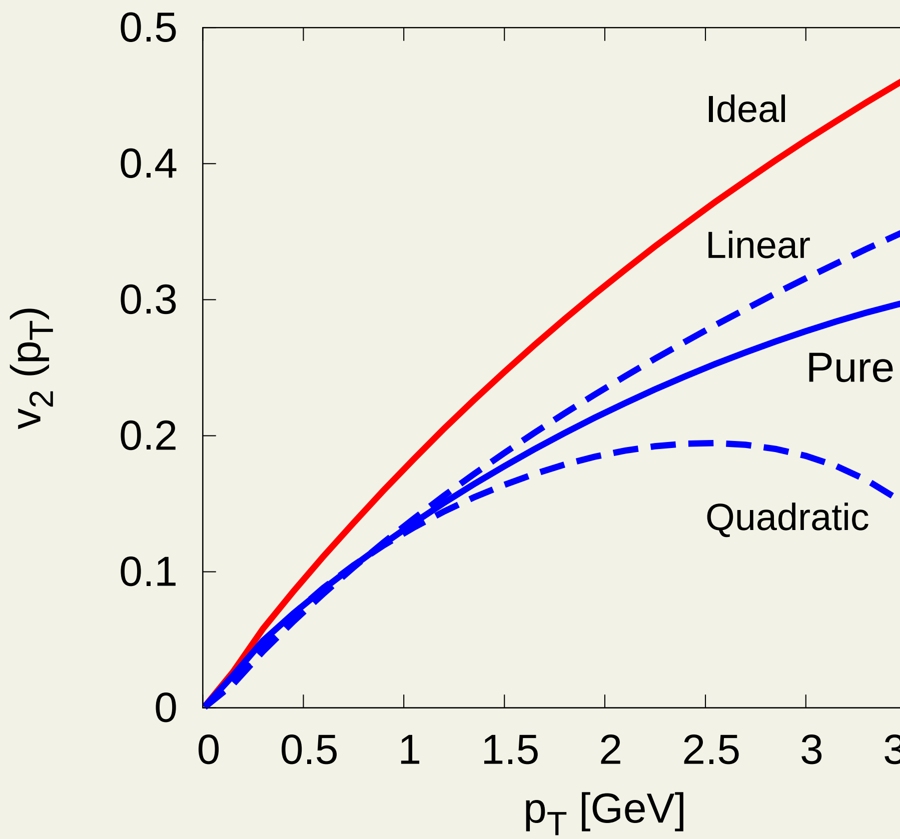
hadron gas ≈ 50 species

DM & Wolff, arXiv:1404.7850



massless quark-gluon gas

Dusling, Moore, Teaney, PRC ('09)



Grad / Israel-Stewart ansatz

From "14-moment" expansion in kinetic theory

$$\delta f^{Grad}(x,\vec{p}) = \frac{1}{2T(x)^2[e(x)+p(x)]} p^\mu p^\nu \pi_{\mu\nu}(x)\, f_{eq}(x,\vec{p})$$

Popular for hydro because for multicomponent system the same form

$$\delta f_i = \frac{1}{2T^2(e+p)} p^\mu p^\nu \pi_{\mu\nu}\, f_{eq,i}$$

only needs knowledge of the full $T^{\mu\nu}$ and automatically yields $\sum_i T_i^{\mu\nu} = T^{\mu\nu}$. Also used by Israel & Stewart (IS) to derive causal viscous hydrodynamics.

Can generalize to any power - "IS with p^α":

$$\delta f = \frac{C(\alpha)}{2T^2(e+p)} \left(\frac{p\cdot u}{T}\right)^{\alpha-2} p^\mu p^\nu \pi_{\mu\nu}\, f_{eq}$$

Weakness: **negative f at sufficiently high momenta ($\pi^{\mu\nu}$ not pos definite)**

Covariant transport

(on-shell) phase-space density $f(x,\vec{p}) \equiv \frac{dN(\vec{x},\vec{p},t)}{d^3x d^3p}$

transport equation:

$$p^\mu \partial_\mu f_i(x,p) = C^i_{2\to 2}[\{f_j\}](x,p) + C^i_{2\leftrightarrow 3}[\{f_j\}](x,p) + \cdots$$

with, e.g.,

$$C^i_{2\to 2} = \frac{1}{2}\sum_{jkl} \int_{234} (f_3^k f_4^l - f_1^i f_2^j) W^{ij\to kl}_{12\to 34} \qquad \left(\int_j \equiv \int \frac{d^3 p_j}{2E_j} \,, \quad f_a^k \equiv f^k(x,p_a) \right)$$

thermalizes (in box), fully causal and stable

near hydro limit transport coeffs & relaxation times:

$\eta \approx 1.2T/\sigma$, $\qquad \tau_\pi \approx 1.2\lambda_{tr}$

Linearized kinetic theory

late-time, near-equilibrium behavior universal, and can be studied via

de Groot, et al ('70s)... Arnold, Moore, Yaffe, JHEP 0011... Denicol et al PRD85 ('12)...

$$p \cdot \nabla f_{eq} = C[f_{eq}, \delta f] \qquad\qquad (\delta f = f - f_{eq})$$

integral eqn. relates δf to gradients in the system (one way to derive transport coeffs)

For shear, solution is of the form

$$\delta f(x, \vec{p}) = \chi(\frac{p \cdot u}{T})\, \hat{p}_\mu \hat{p}_\nu \frac{\sigma^{\mu\nu}}{T} f_{eq}$$

$\Rightarrow$ **nontrivial momentum dependence, but negativity problem remains**

theory preference for $\chi \sim p^{1.5}$

Relaxation time approx (RTA)

To simplify Boltzmann, try

$$p \cdot \partial f(x, \vec{p}) = (p \cdot u) \frac{f_{eq}(x, \vec{p}) - f(x, \vec{p})}{\tau_{REL}}$$

Conserved particle number, energy & momentum, if $f_{eq}(f)$ chosen well. τ_{REL} must be matched to some dynamical aspect, e.g., the viscosity.

In late-time near-equilibrium regime ($\partial_t \to 0$): $\quad p \cdot \nabla f_{eq} = -\tau_{REL}(p \cdot u)\delta f$

so with shear: $\delta f = \dfrac{\tau_{REL}}{2(p \cdot u)T} p_\mu p_\nu \sigma^{\mu\nu} f_{eq} = \dfrac{\tau_{REL}}{2} \dfrac{T}{(p \cdot u)} \dfrac{p_\mu p_\nu}{T^2} \dfrac{\pi^{\mu\nu}}{\eta} f_{eq}$

and from $\displaystyle\int \frac{d^3p}{E} p^\mu p^\nu \delta f = \pi^{\mu\nu} \quad \Rightarrow \quad \tau_{REL} = \frac{5\eta}{4nT}$

Notice that $\delta f / f_{eq} \sim p^1$ (linear) in this case.

SR ansatz

Strickland & Romatschke, PRD 71 ('05):

$$f(p_T, p_z) = N e^{-\sqrt{E^2+p_z^2}/\Lambda} = N \exp\left[-\frac{p_T}{\Lambda}\sqrt{\mathrm{ch}^2\xi + a\,\mathrm{sh}^2\xi}\right] \qquad (\xi \equiv \eta - y)$$

- N, Λ, a **fixed by** N^0**,** T^{00}**,** T^{zz}

- **matches free streaming solution in 0+1D** $[a = (\tau/\tau_0)^2]$

more generally Tinti & Florkowski, PRC89 ('14)

$$f = N \exp\left[-\frac{1}{\Lambda}\sqrt{p^\mu \Xi_{\mu\nu} p^\nu}\right]$$

- **always positive, also: linear for small momenta**

GOAL: test these models against full, nonlinear kinetic theory.

i) **obtain transport solutions with isotropic $2 \to 2$ interactions (MPC/Grid transport code)**

ii) **from f, determine $T^{\mu\nu}$**

iii) **study how well δf models reconstruct f from the $T^{\mu\nu}$ alone**

MPC/Grid

similar to BAMPS - not a cascade

- $2 \to 2$ **and** $3 \leftrightarrow 2$ **with test particles on a spatial grid**

  pair/triplet collisions with probability

$$P_{2\to X} = \frac{\sigma_{2\to X} v_{rel} \Delta t}{V_{cell}} , \qquad P_{3\to Y} = \frac{K_{3\to Y} \Delta t}{V_{cell}^2}$$

  5 numerical knobs: cell sizes (d_x, d_y, d_z/d_η), **timestep** Δt, **subdivision** ℓ

  still action at distance $\to$ **violates locality, causality, covariance**

  $\Rightarrow$ but more flexible than earlier MPC/Cascade

- **we also have a version for parallel computers (MPI)**

massless system, 0+1D Bjorken expansion (transversely uniform)

$\Rightarrow f(p_T, \xi \equiv \eta - y, \tau)$

$$u^\mu_{BJ} = (\mathrm{ch}\eta, 0, 0, \mathrm{sh}\eta), \qquad (p \cdot u) = p_T\, \mathrm{ch}\xi$$

$$T^{\mu\nu}_{ideal,LR} = p(\tau)\, \mathrm{diag}(3,1,1,1), \qquad \pi^{\mu\nu}_{LR} = \pi_L(\tau)\, \mathrm{diag}(0, -\tfrac{1}{2}, -\tfrac{1}{2}, 1)$$

dynamics governed by: $K(\tau) \equiv \dfrac{\tau_{exp}}{\tau_{sc}} = \dfrac{(\partial \cdot u)}{\lambda_{tr,MFP}} = \frac{2}{3}\tau n \sigma_{TOT} = \frac{2}{3}\tau_0 n_0 \sigma_{TOT}$

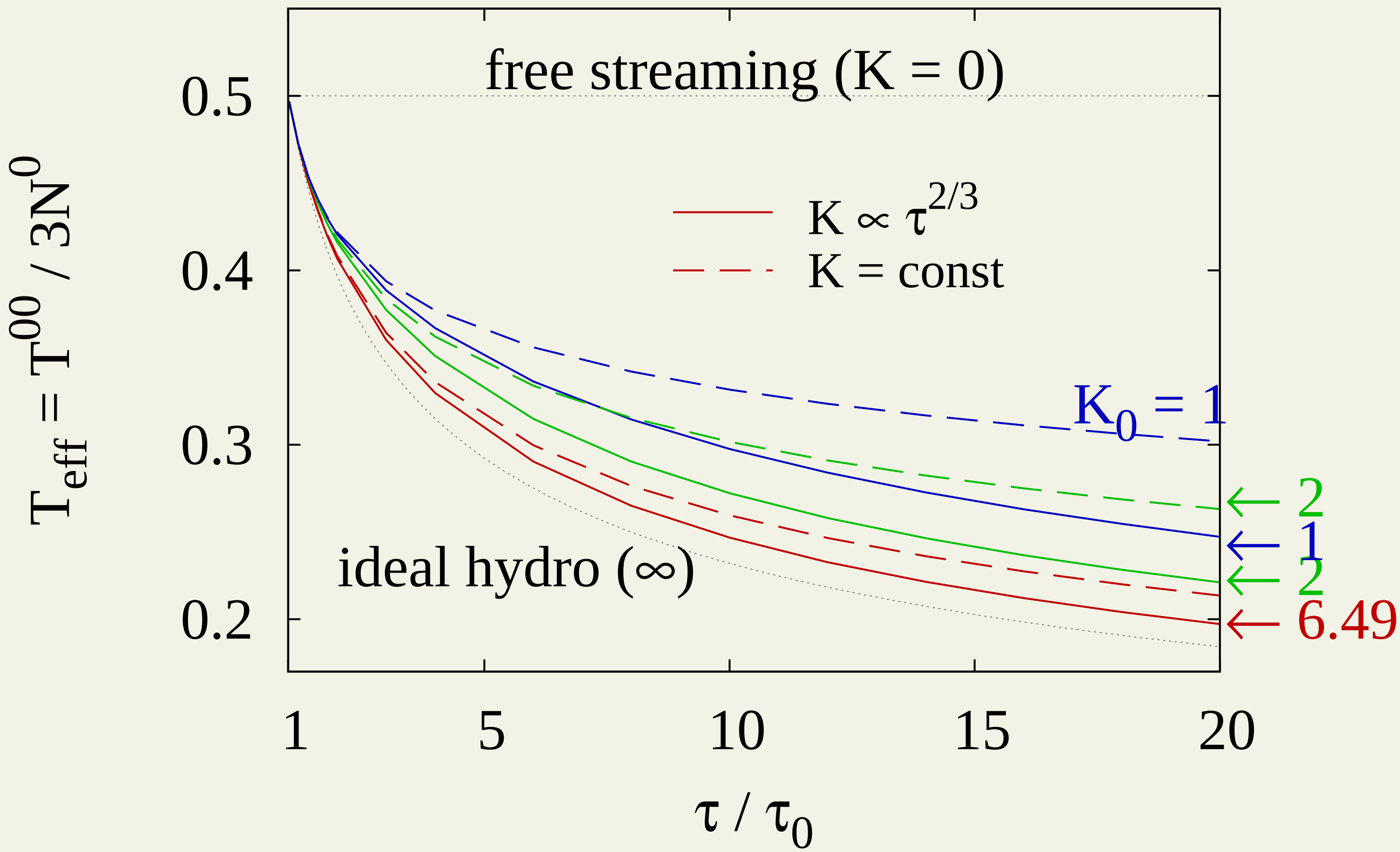
for $\boldsymbol{\eta/s \approx const}$**,** $\sigma_{TOT} \sim \tau^{2/3} \Rightarrow K(\tau) \propto \tau^{2/3}$ DM & Huovinen, PRC79 ('09)

for $\boldsymbol{\sigma = const}$**,** $K(\tau) = const$

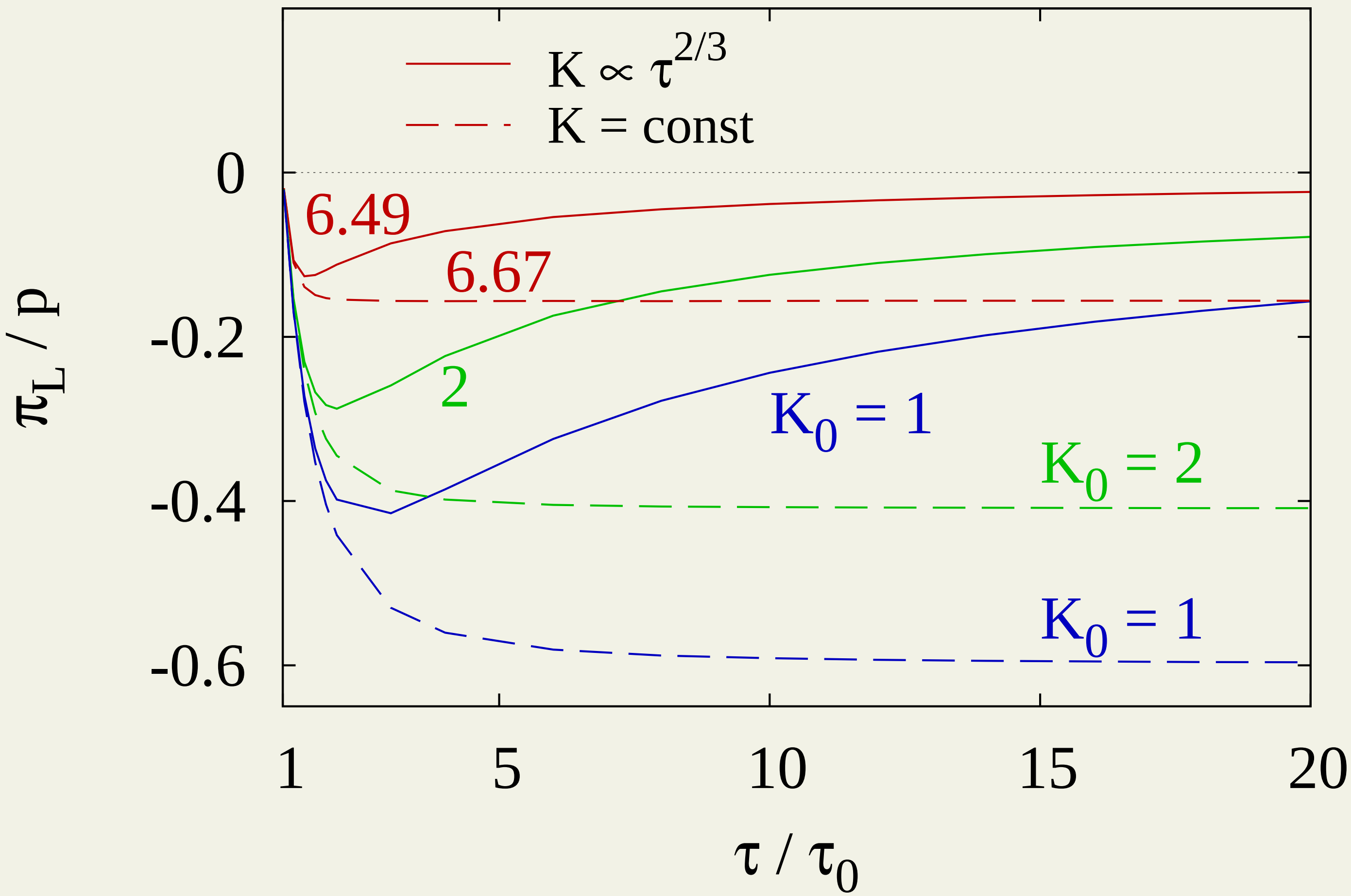
with $\eta = 1.26757..\dfrac{T}{\sigma_{TOT}}$**, we have** $K_0 \approx 0.2 T_0 \tau_0 \dfrac{s}{\eta}(\tau_0)$ **(used** $s = 4n$**)**

and typically $\tau_0 T_0 \sim 1$**, so** $\eta/s = 1/(4\pi)$ **corresponds to** $K_0 \sim 2$

T_{eff} vs τ - cooling due to $p\,dV$ work Gyulassy, Pang, Zhang, NPA626 ('97)



viscous pressure correction vs τ - just like DM & Huovinen, PRC79 ('09)



with $\eta/s \approx const$ **it first leaves equilibrium then returns**

First look at $f(p_T, \xi)$ with SR and Grad(IS) $\to$ plot $\dfrac{f_{rec}}{f_{trans}}$

Generic features illustrated for $\eta/s \sim 1/(4\pi)$
$[K_0 = 2,\ K(\tau) \propto \tau^{2/3}]$

Color coding:

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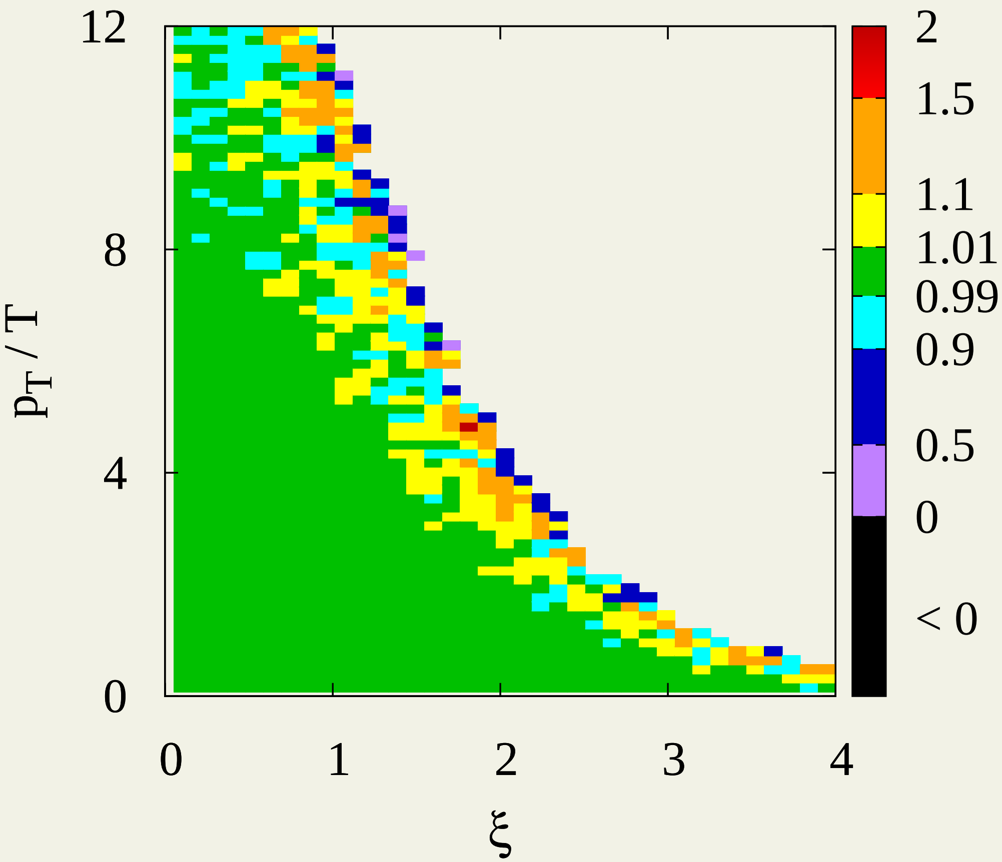
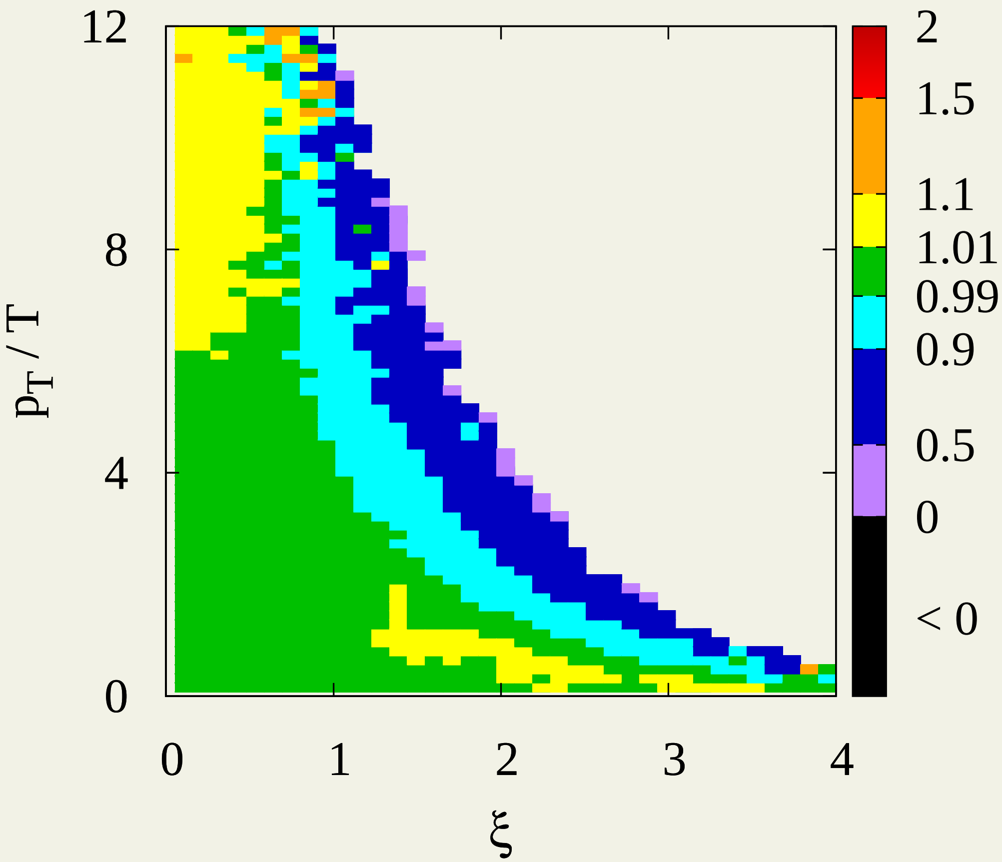
$$\frac{\tau}{\tau_0} = 1.02, \quad K = 2\left(\frac{\tau}{\tau_0}\right)^{2/3} \qquad \left(\frac{\tau_{REL}}{\tau_{exp}} = 0.52, \quad \frac{\pi_L}{p} = -0.02\right)$$

SR ansatz

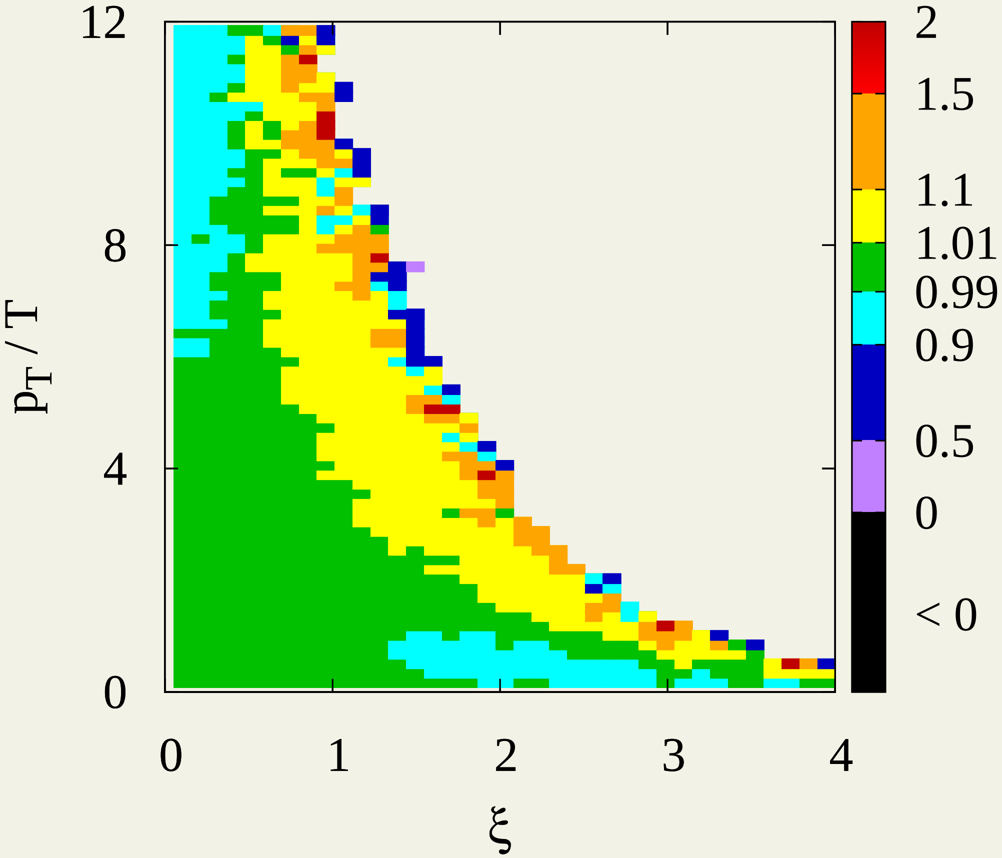


Grad (IS)

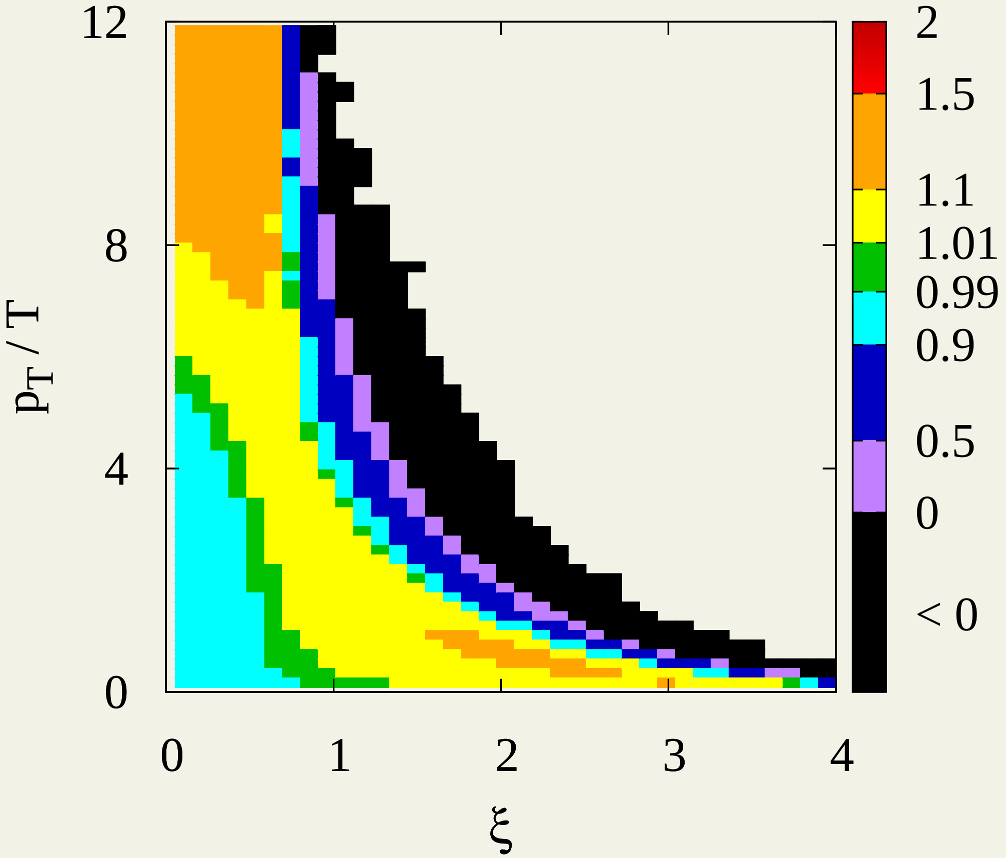


$$\frac{\tau}{\tau_0} = 1.2, \quad K = 2\left(\frac{\tau}{\tau_0}\right)^{2/3} \qquad \left(\frac{\tau_{REL}}{\tau_{exp}} = 0.47, \quad \frac{\pi_L}{p} = -0.15\right)$$

SR ansatz

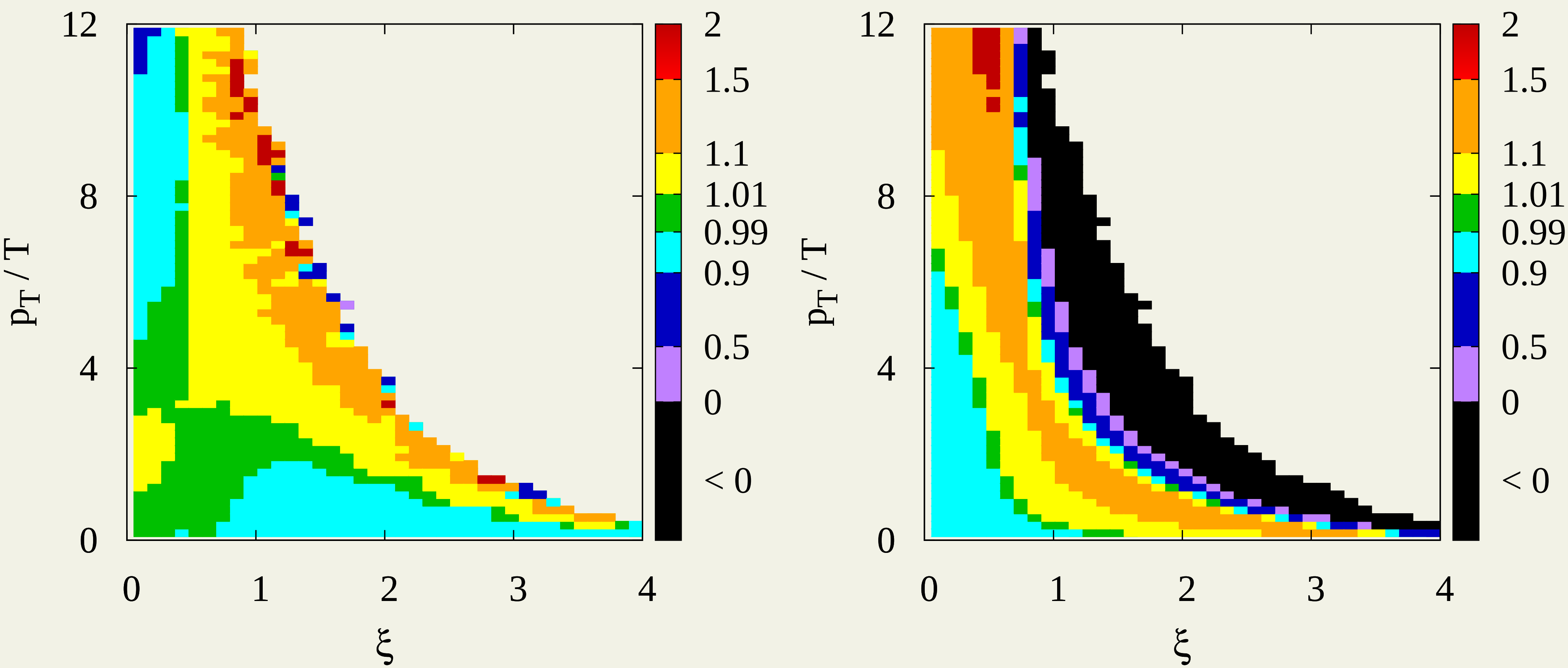


Grad (IS)



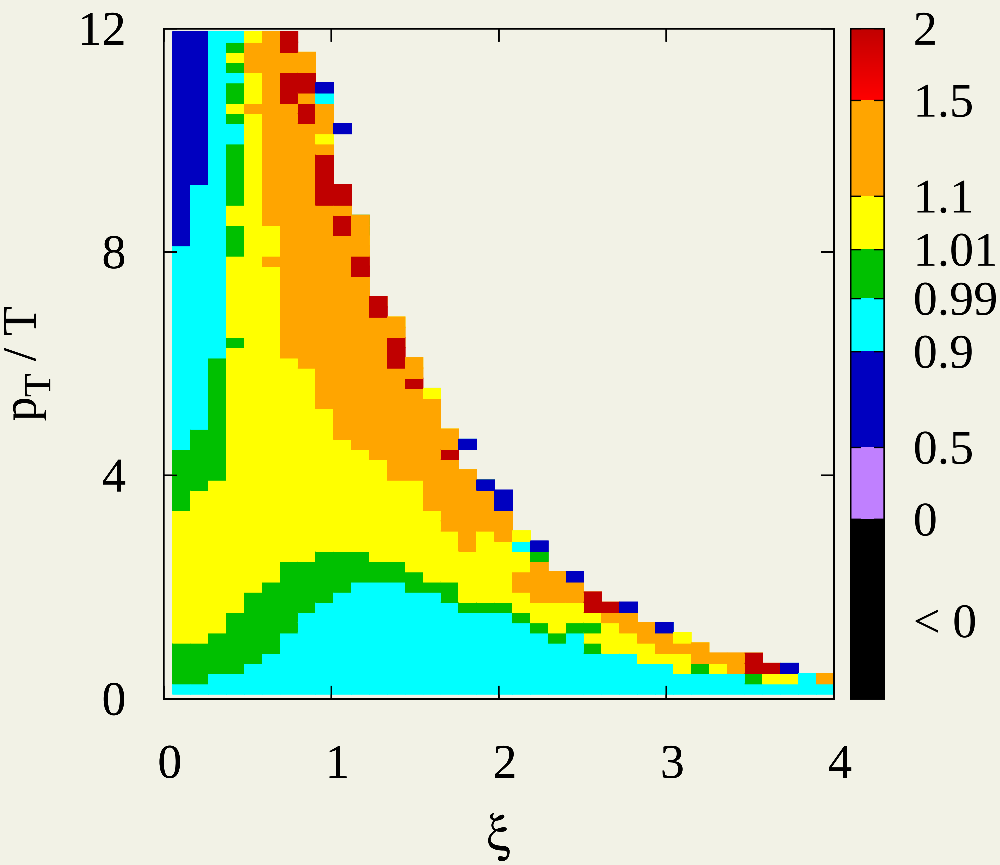
$$\frac{\tau}{\tau_0} = 1.4, \quad K = 2\left(\frac{\tau}{\tau_0}\right)^{2/3} \qquad \left(\frac{\tau_{REL}}{\tau_{exp}} = 0.42, \quad \frac{\pi_L}{p} = -0.23\right)$$

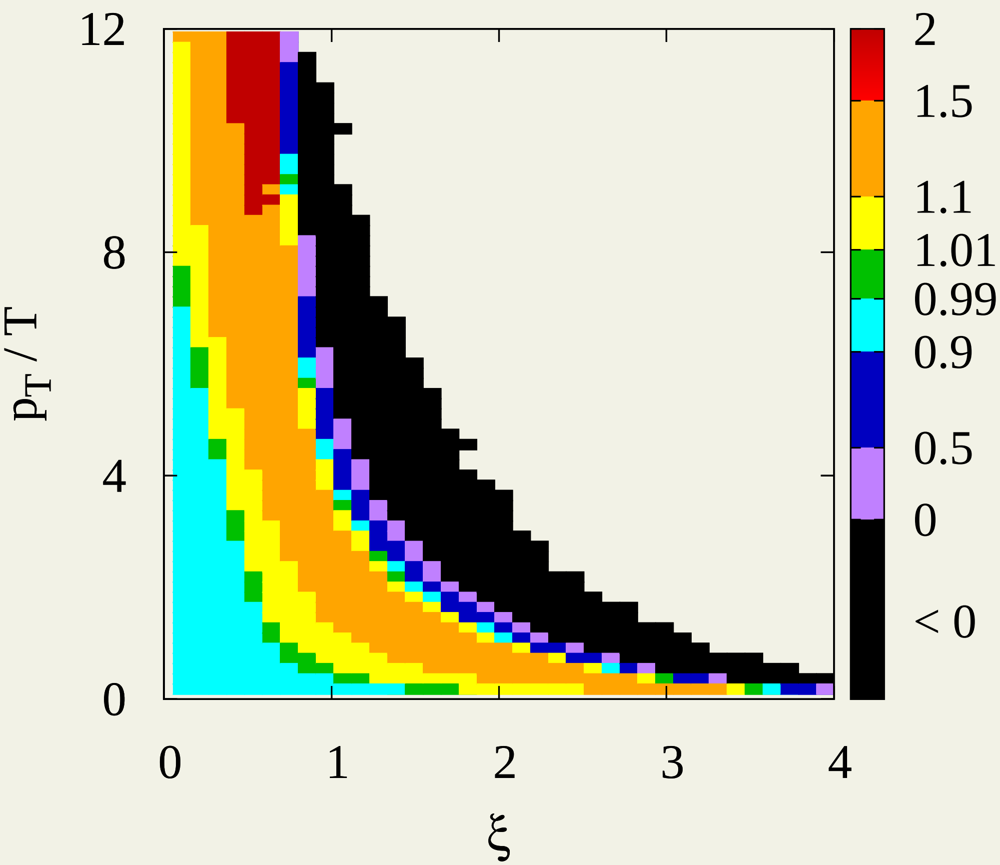
SR ansatz — Grad (IS)



$$\frac{\tau}{\tau_0} = 1.6, \quad K = 2\left(\frac{\tau}{\tau_0}\right)^{2/3} \qquad \left(\frac{\tau_{REL}}{\tau_{exp}} = 0.38, \quad \frac{\pi_L}{p} = -0.27\right)$$

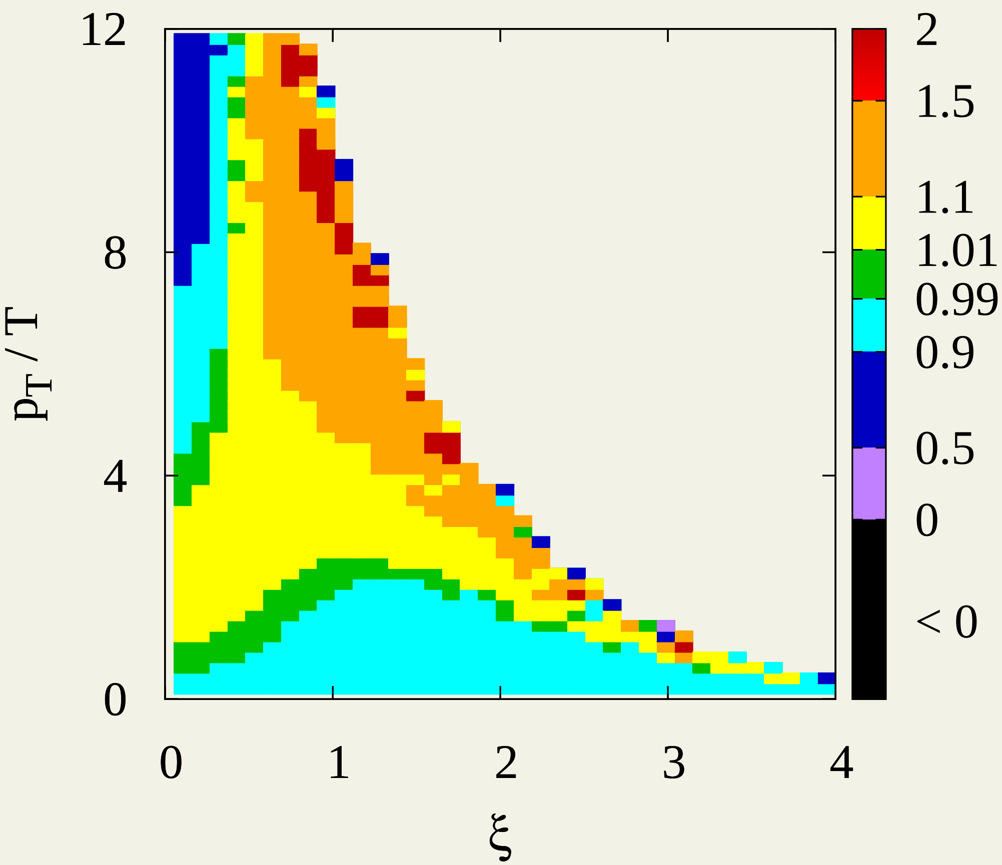
SR ansatz **Grad (IS)**



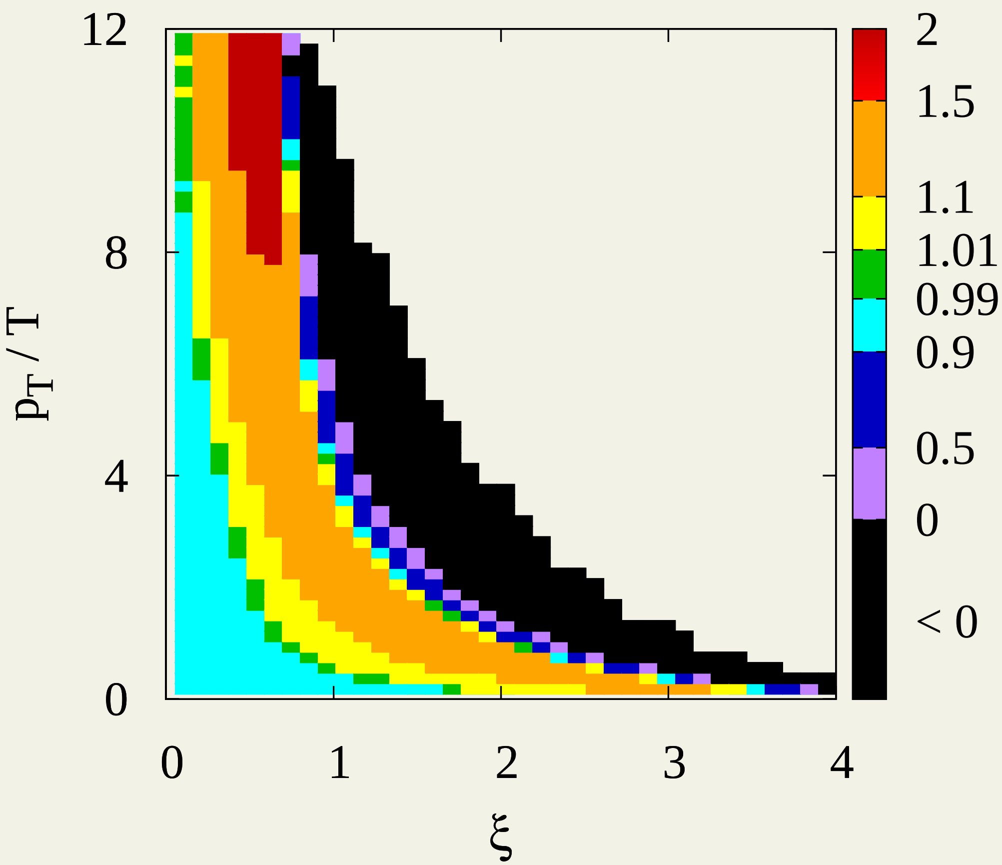


$$\frac{\tau}{\tau_0} = 1.8, \quad K = 2\left(\frac{\tau}{\tau_0}\right)^{2/3} \qquad \left(\frac{\tau_{REL}}{\tau_{exp}} = 0.36, \quad \frac{\pi_L}{p} = -0.28\right)$$

SR ansatz

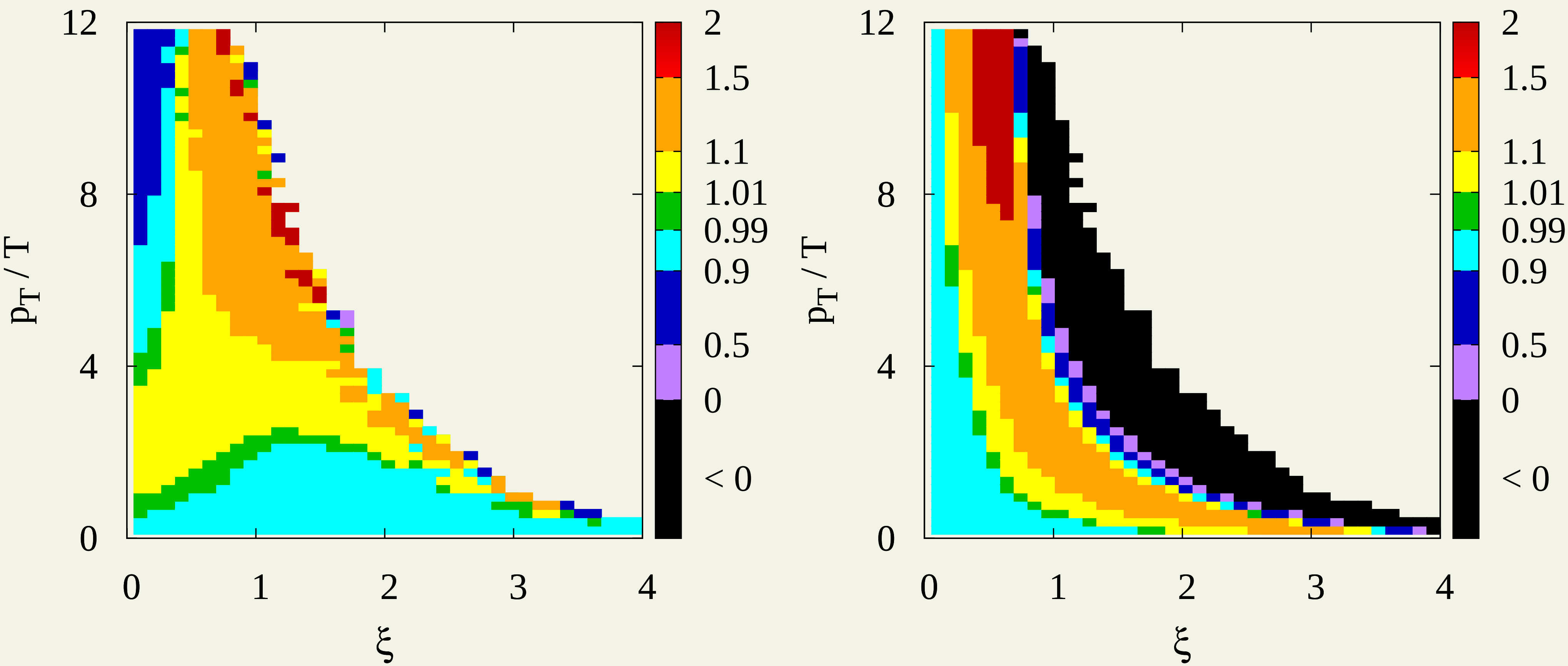


Grad (IS)



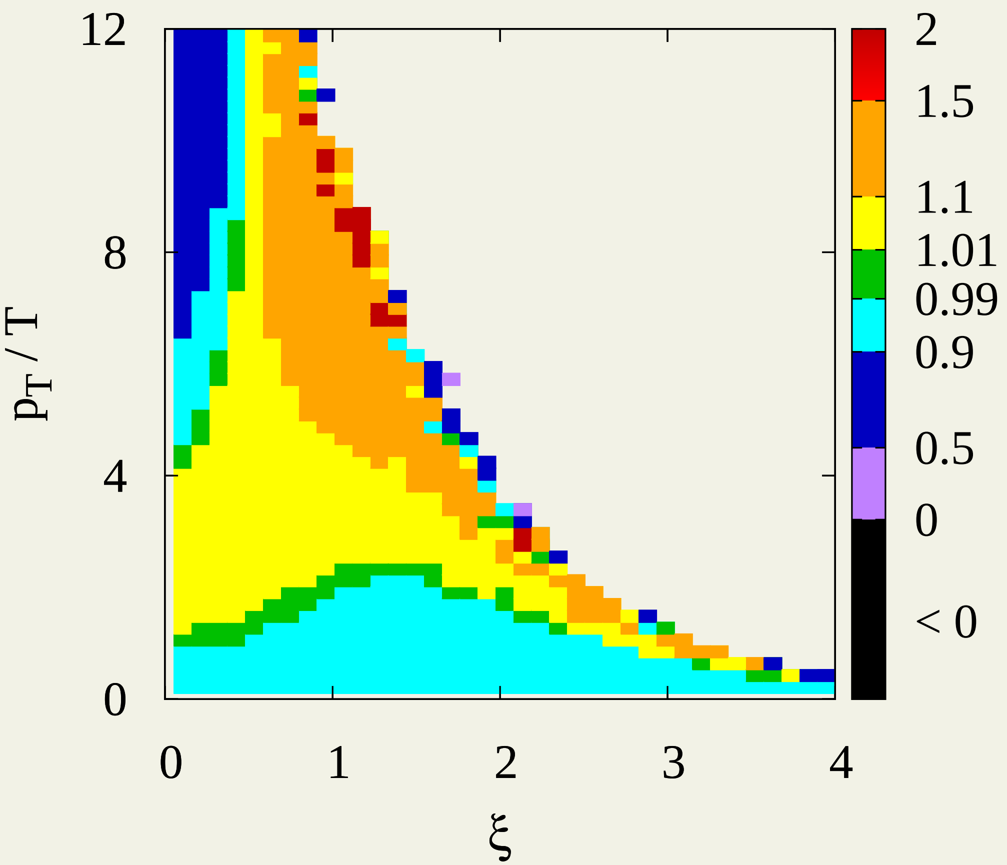
$$\frac{\tau}{\tau_0} = 2, \quad K = 2\left(\frac{\tau}{\tau_0}\right)^{2/3} \qquad \left(\frac{\tau_{REL}}{\tau_{exp}} = 0.33, \quad \frac{\pi_L}{p} = -0.29\right)$$

SR ansatz **Grad (IS)**

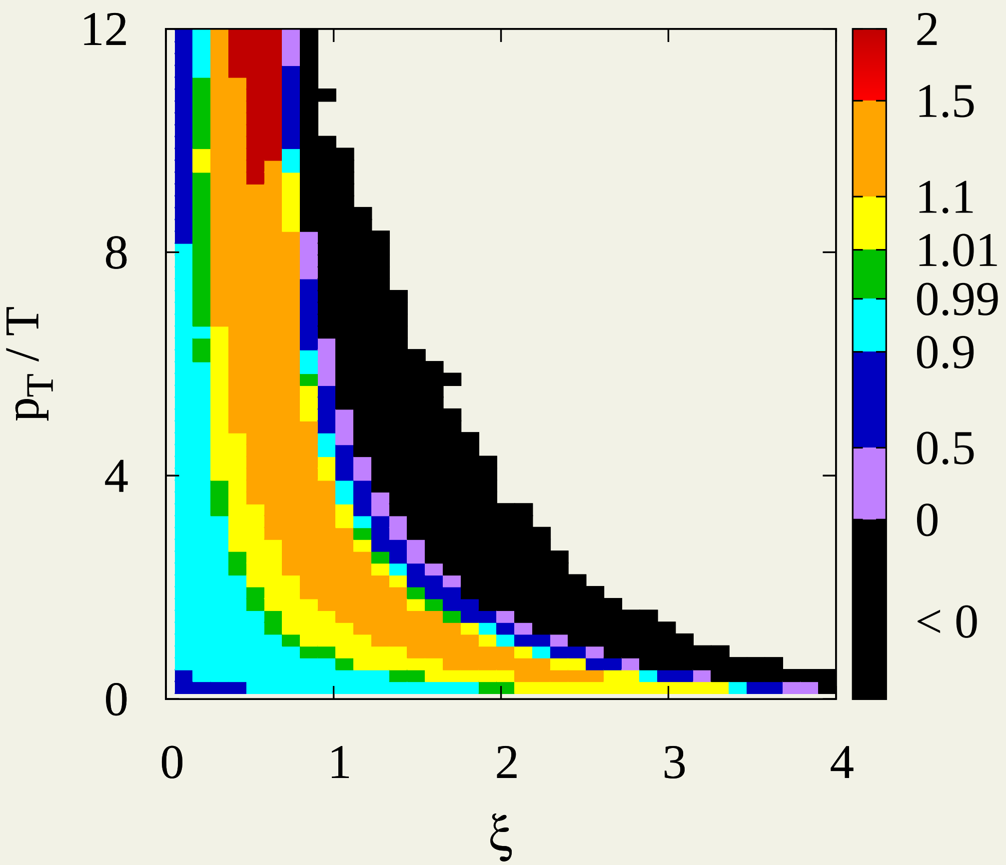


$$\frac{\tau}{\tau_0} = 3, \quad K = 2\left(\frac{\tau}{\tau_0}\right)^{2/3} \qquad \left(\frac{\tau_{REL}}{\tau_{exp}} = 0.25, \quad \frac{\pi_L}{p} = -0.26\right)$$

SR ansatz

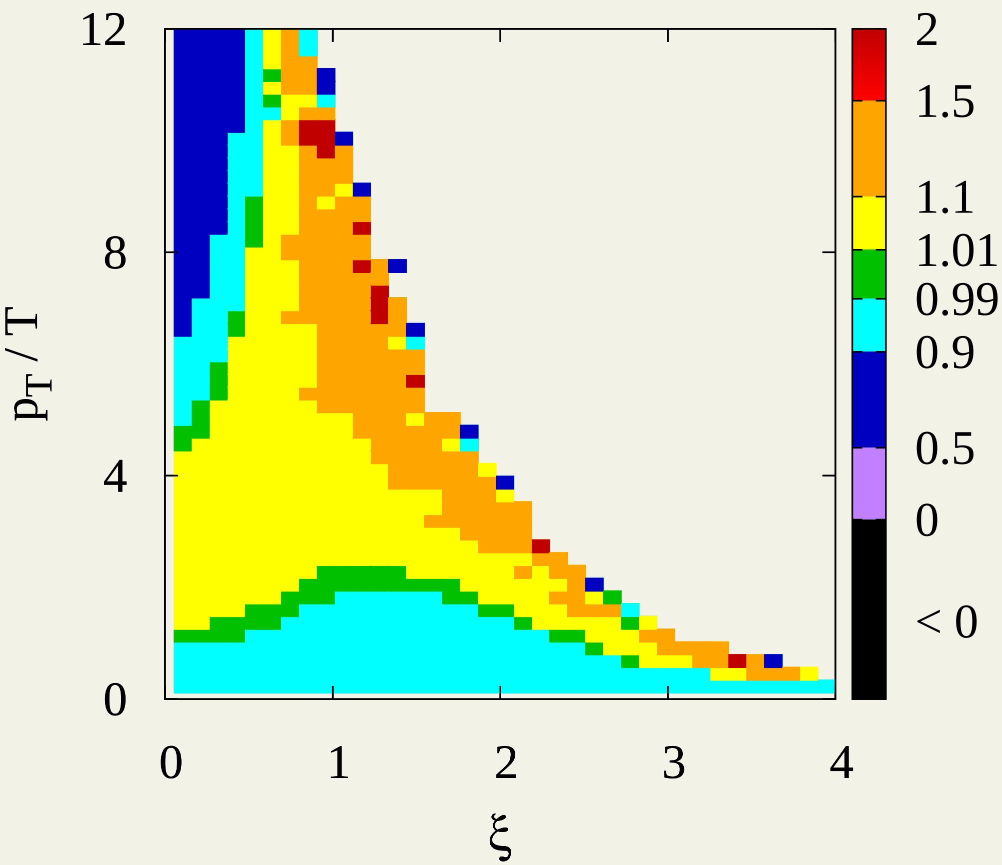


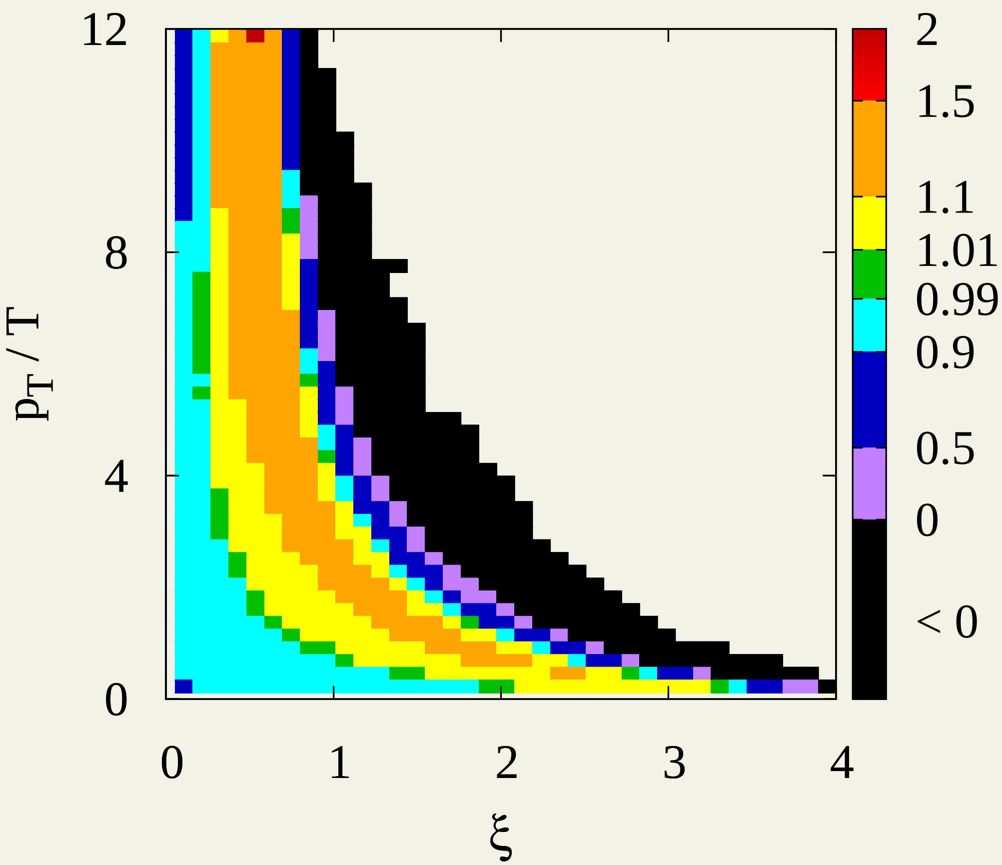
Grad (IS)



$$\frac{\tau}{\tau_0} = 4, \quad K = 2\left(\frac{\tau}{\tau_0}\right)^{2/3} \qquad \left(\frac{\tau_{REL}}{\tau_{exp}} = 0.21, \ \ \frac{\pi_L}{p} = -0.22\right)$$

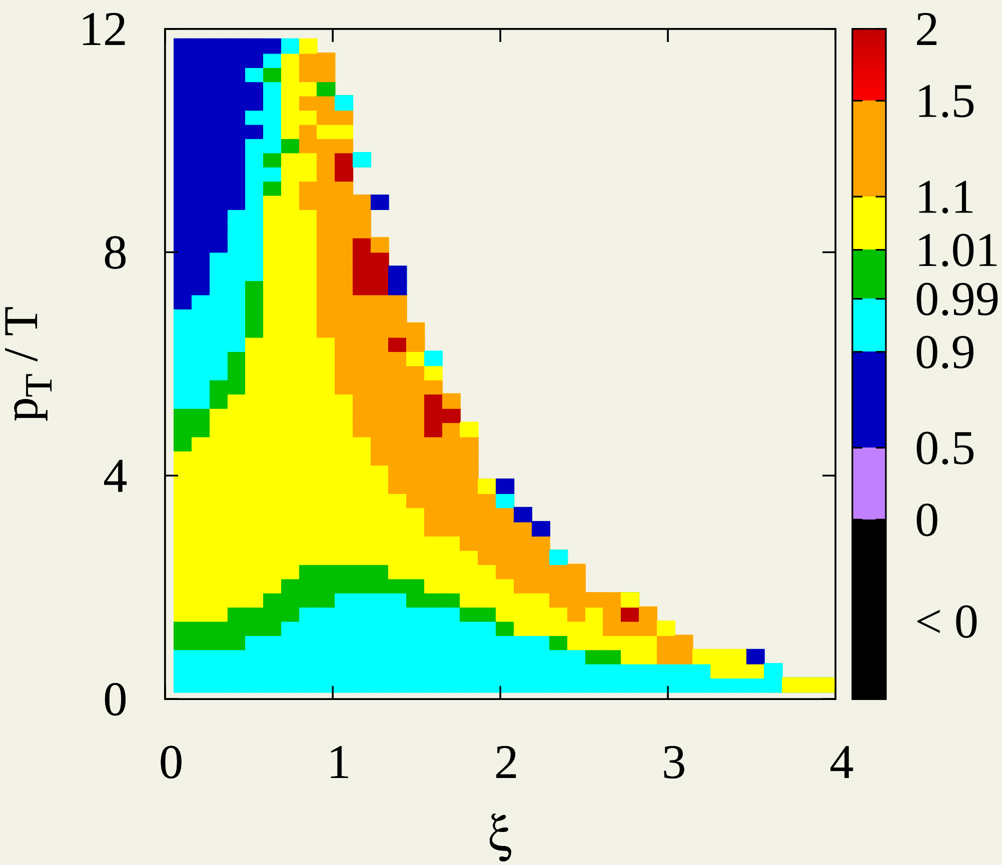
SR ansatz **Grad (IS)**



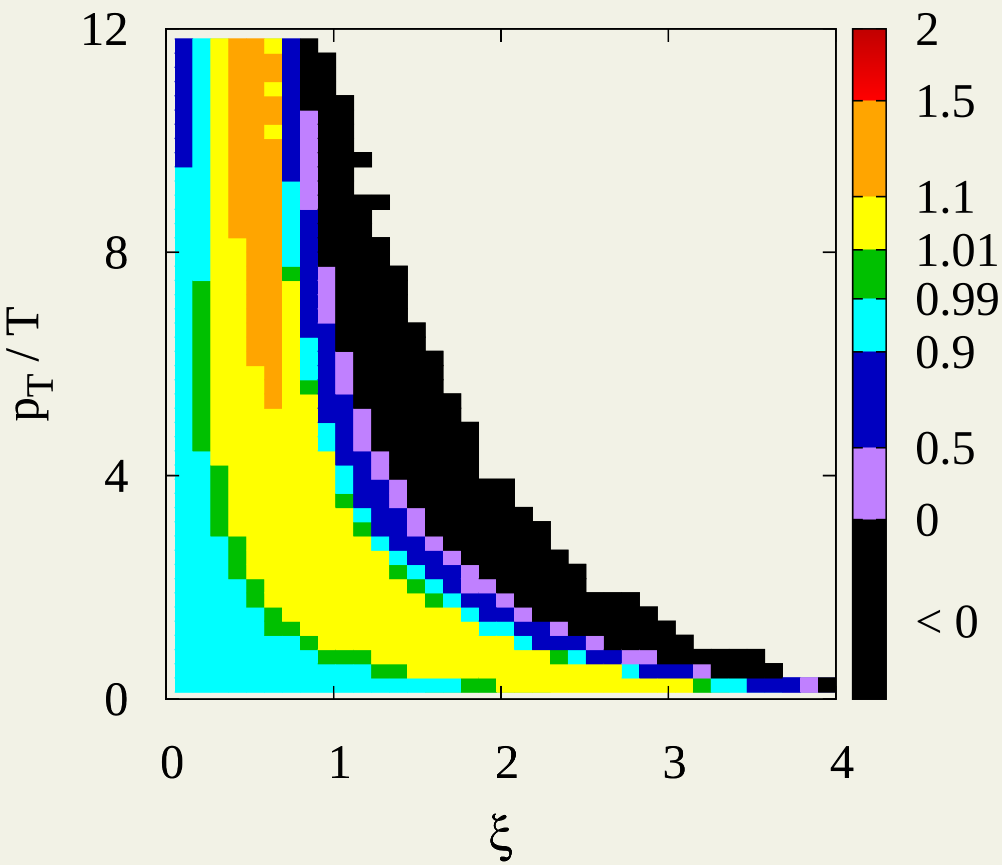


$$\frac{\tau}{\tau_0} = 6, \quad K = 2\left(\frac{\tau}{\tau_0}\right)^{2/3} \qquad \left(\frac{\tau_{REL}}{\tau_{exp}} = 0.16, \ \frac{\pi_L}{p} = -0.17\right)$$

SR ansatz

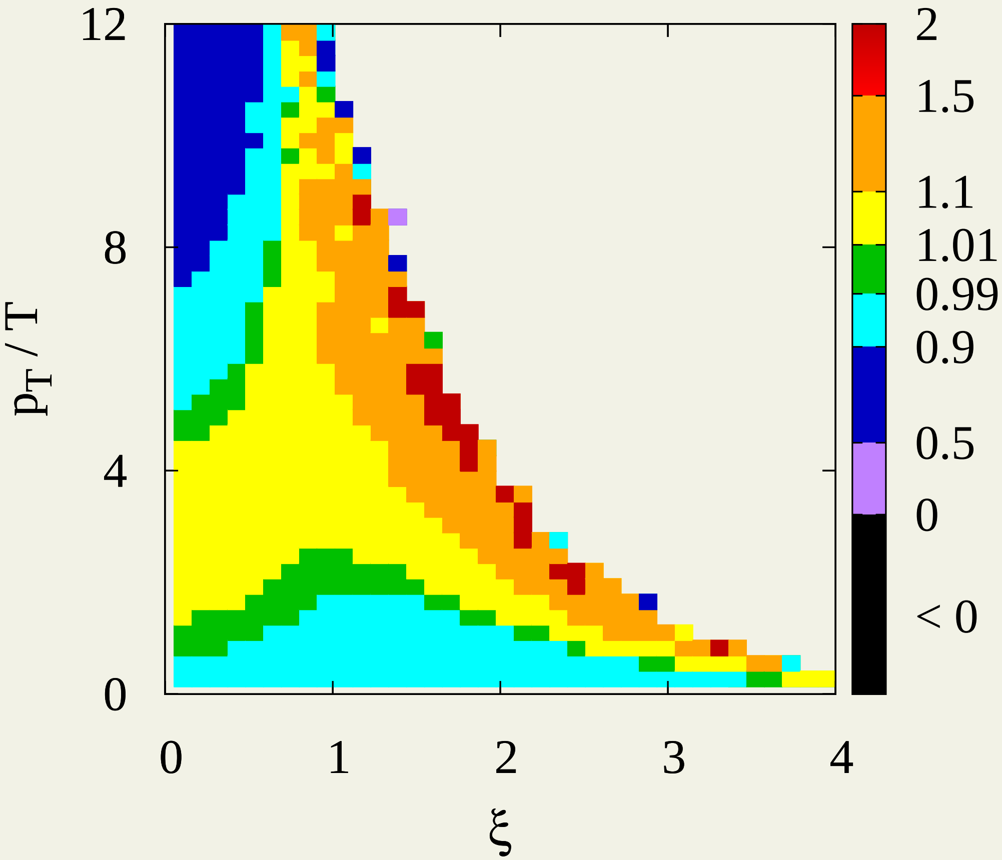


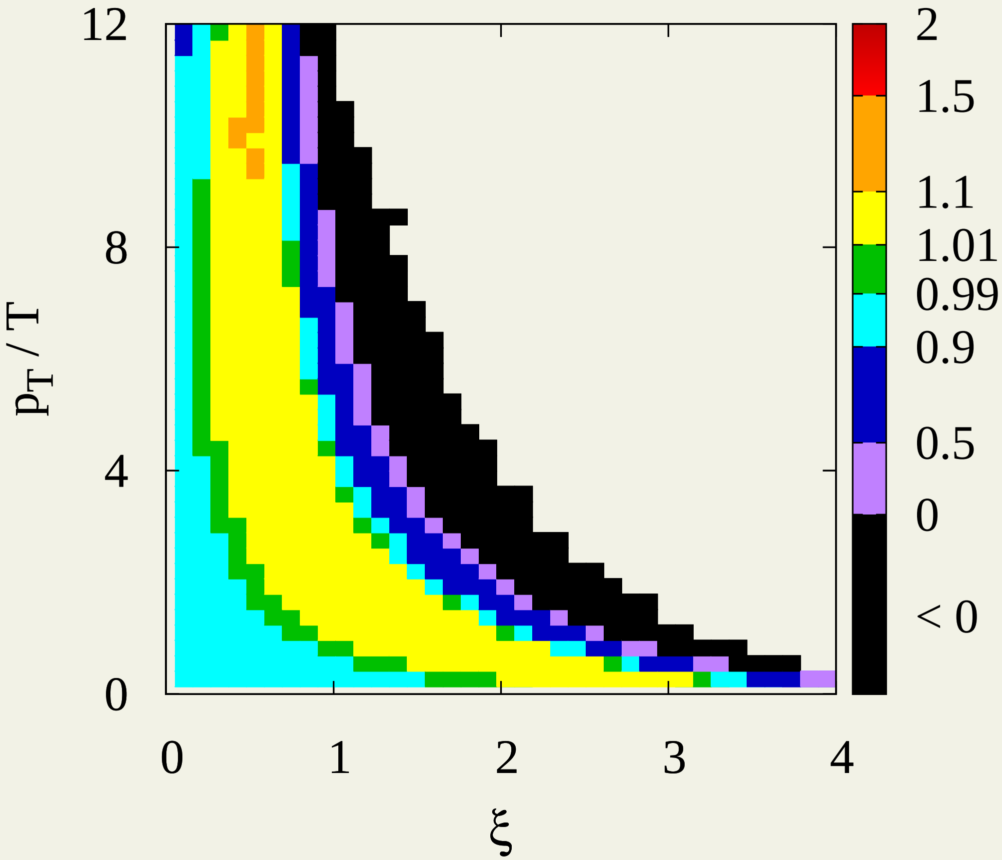
Grad (IS)



$$\frac{\tau}{\tau_0} = 8, \quad K = 2\left(\frac{\tau}{\tau_0}\right)^{2/3} \qquad \left(\frac{\tau_{REL}}{\tau_{exp}} = 0.13, \ \frac{\pi_L}{p} = -0.14\right)$$

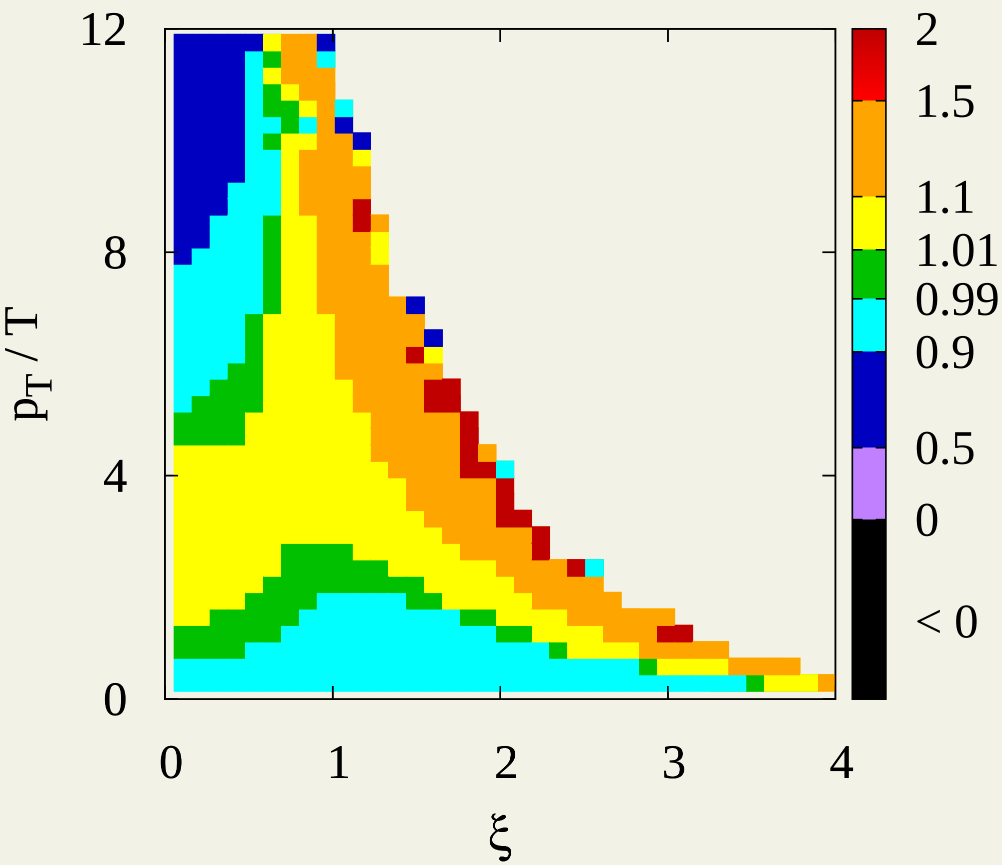
SR ansatz **Grad (IS)**



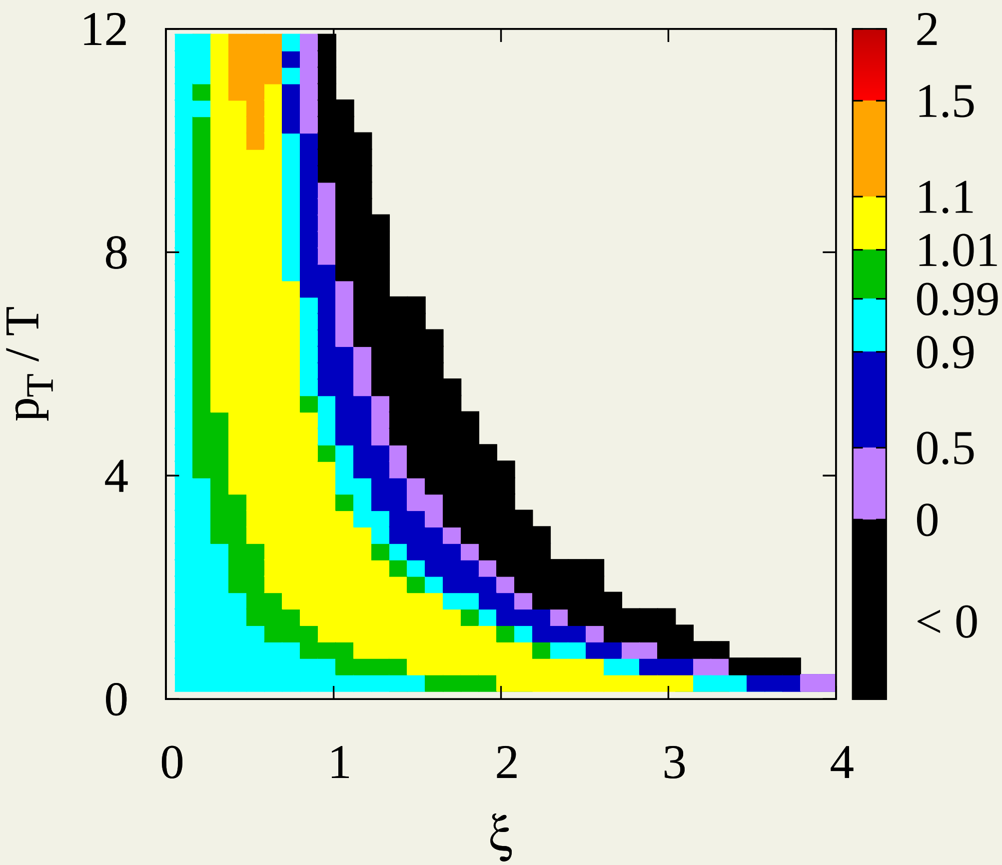


$$\frac{\tau}{\tau_0} = 10, \quad K = 2\left(\frac{\tau}{\tau_0}\right)^{2/3} \qquad \left(\frac{\tau_{REL}}{\tau_{exp}} = 0.11, \; \frac{\pi_L}{p} = -0.12\right)$$

SR ansatz

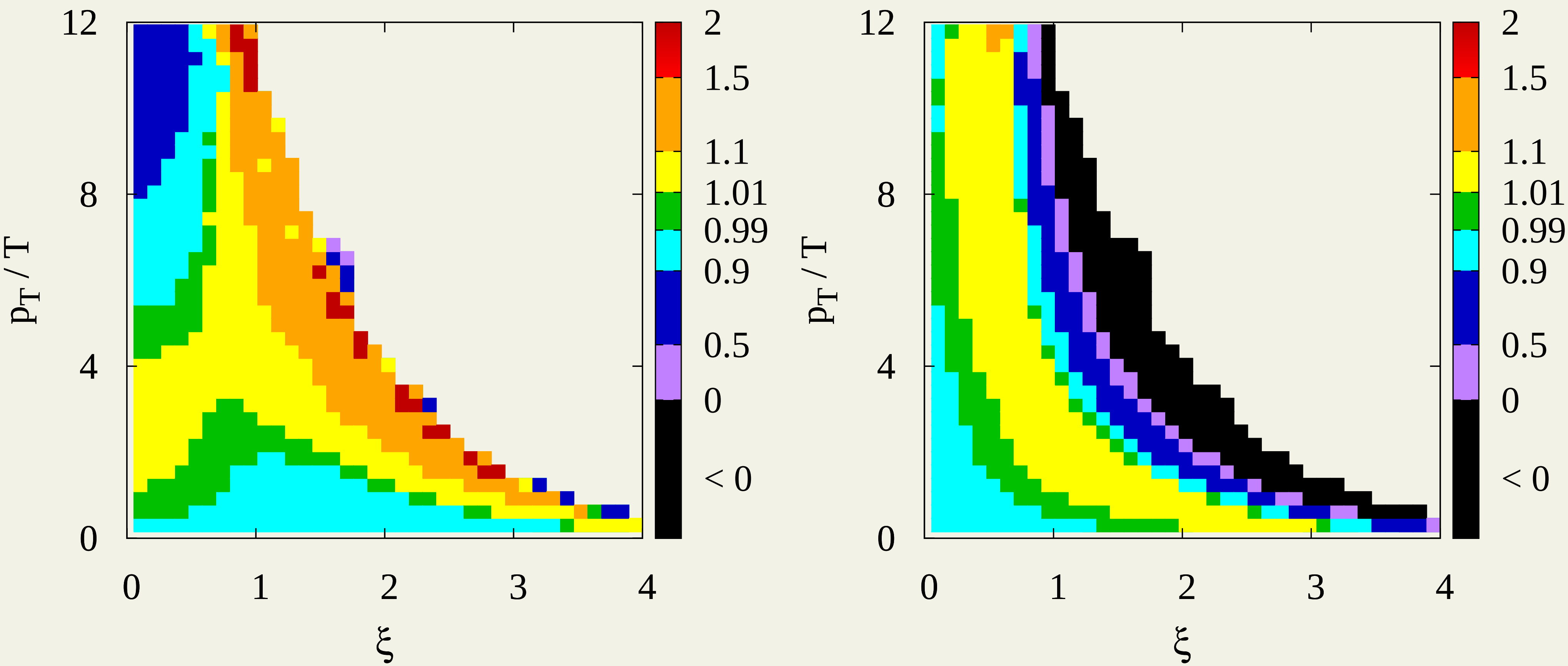


Grad (IS)



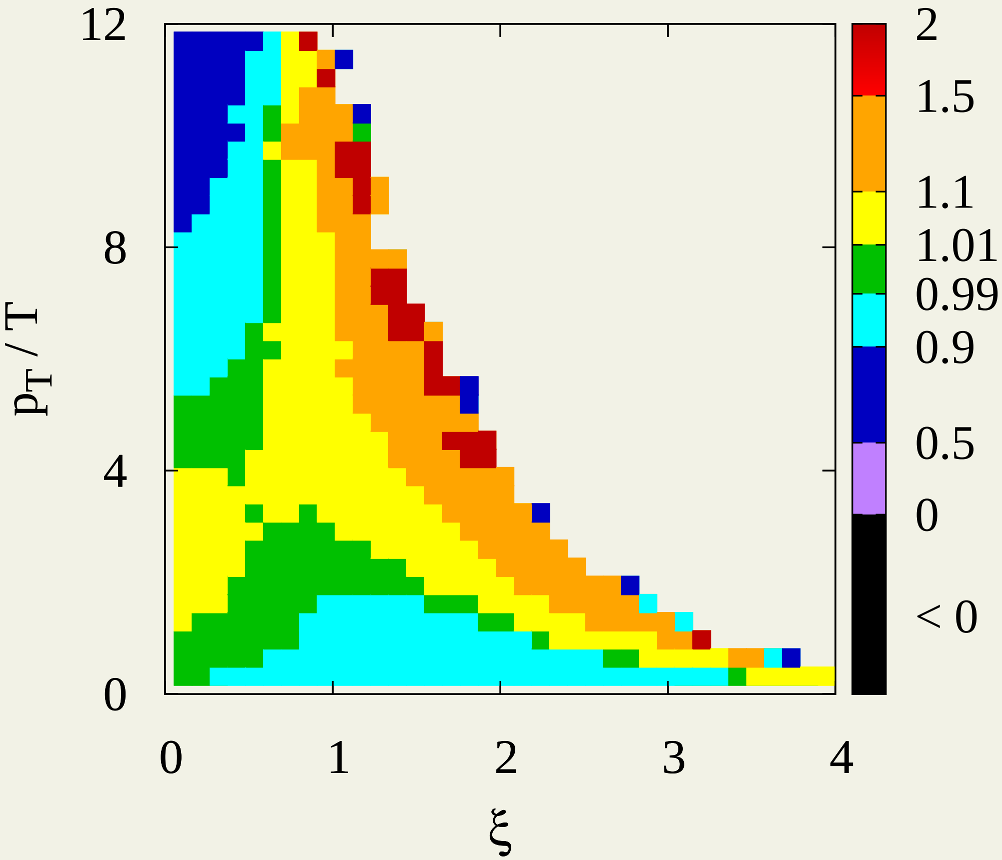
$$\frac{\tau}{\tau_0} = 12, \quad K = 2\left(\frac{\tau}{\tau_0}\right)^{2/3} \qquad \left(\frac{\tau_{REL}}{\tau_{exp}} = 0.10, \ \frac{\pi_L}{p} = -0.11\right)$$

SR ansatz **Grad (IS)**

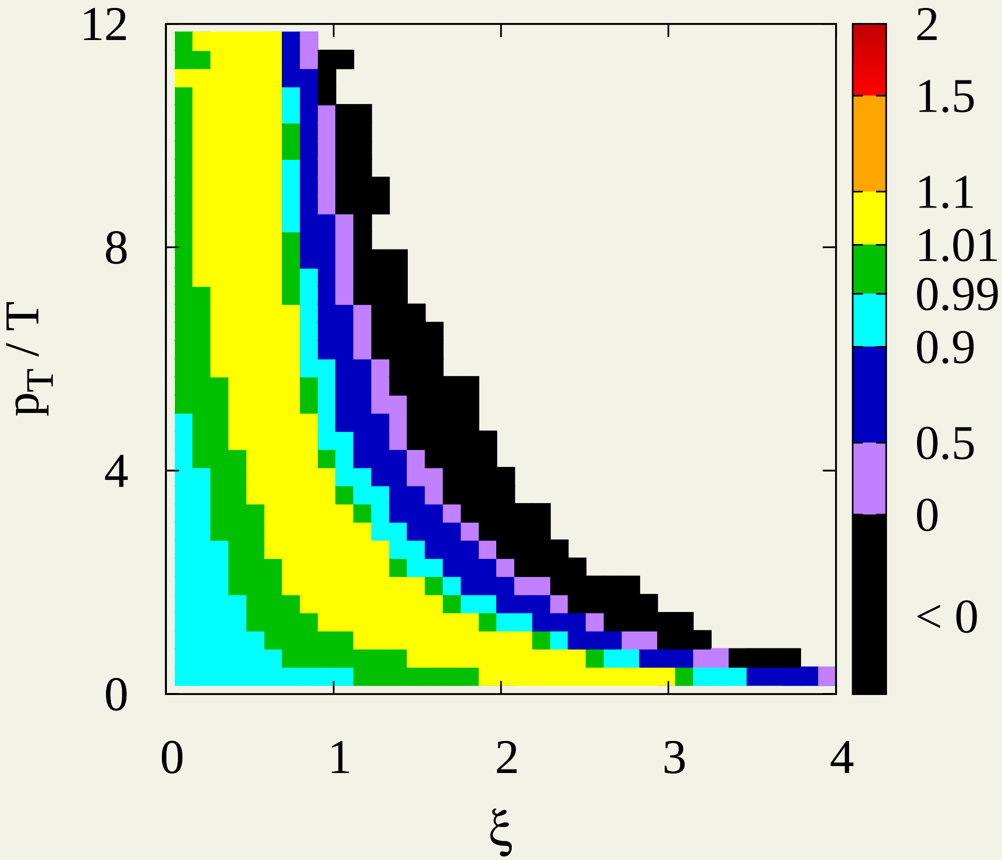


$$\frac{\tau}{\tau_0} = 14, \quad K = 2\left(\frac{\tau}{\tau_0}\right)^{2/3} \qquad \left(\frac{\tau_{REL}}{\tau_{exp}} = 0.09, \quad \frac{\pi_L}{p} = -0.10\right)$$

SR ansatz

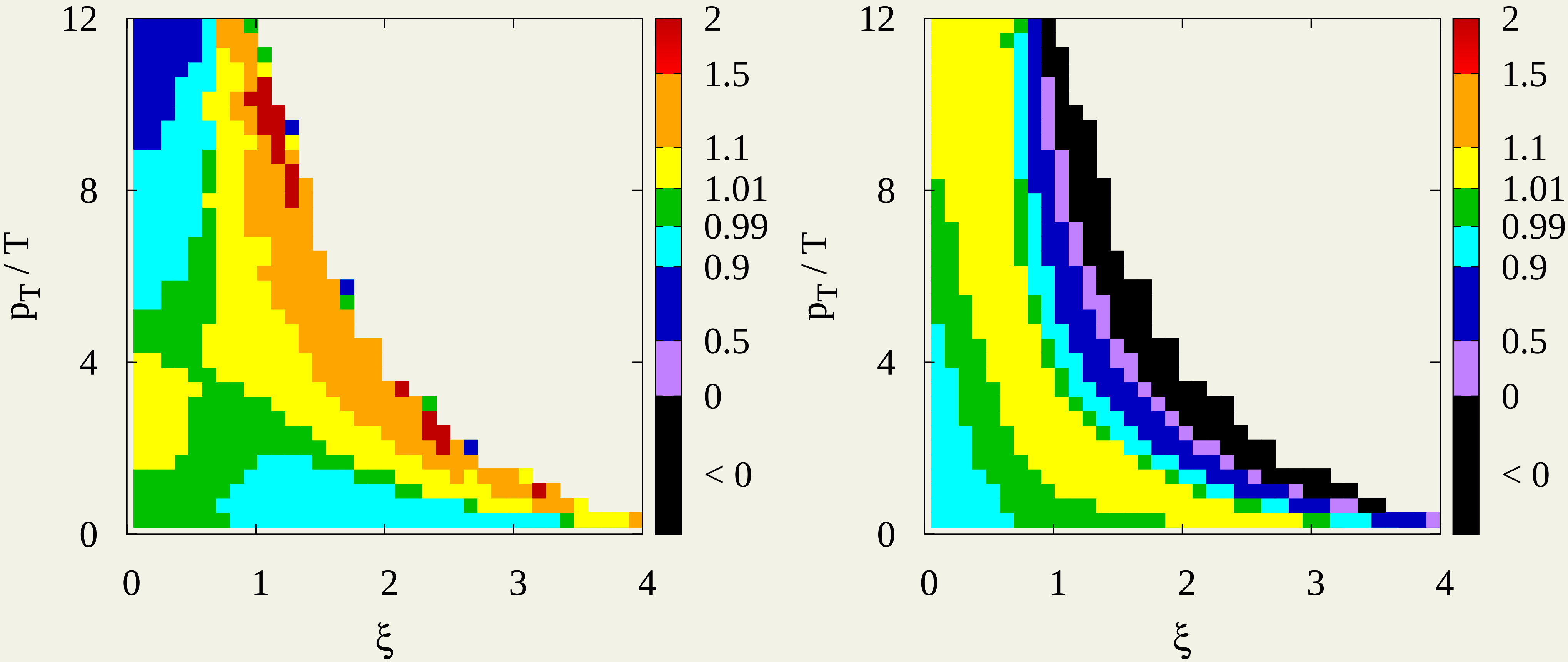


Grad (IS)



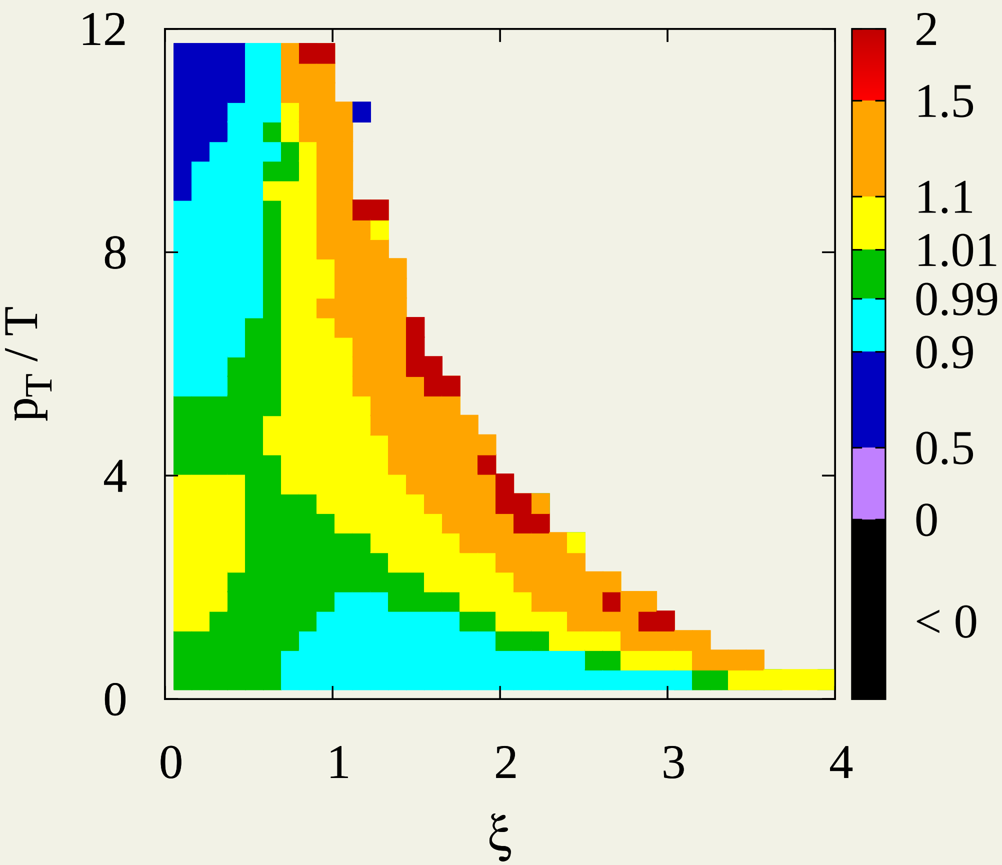
$$\frac{\tau}{\tau_0} = 16, \quad K = 2\left(\frac{\tau}{\tau_0}\right)^{2/3} \qquad \left(\frac{\tau_{REL}}{\tau_{exp}} = 0.08, \quad \frac{\pi_L}{p} = -0.09\right)$$

SR ansatz **Grad (IS)**

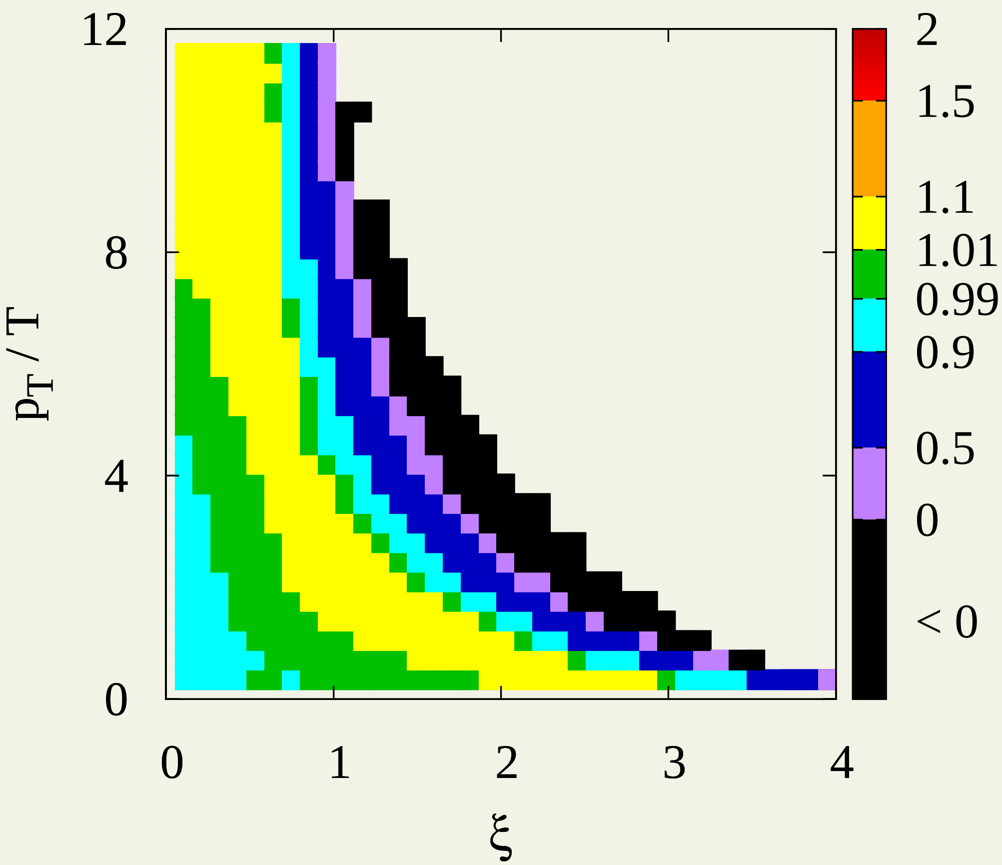


$$\frac{\tau}{\tau_0} = 18, \quad K = 2\left(\frac{\tau}{\tau_0}\right)^{2/3} \qquad \left(\frac{\tau_{REL}}{\tau_{exp}} = 0.08, \;\; \frac{\pi_L}{p} = -0.08\right)$$

SR ansatz

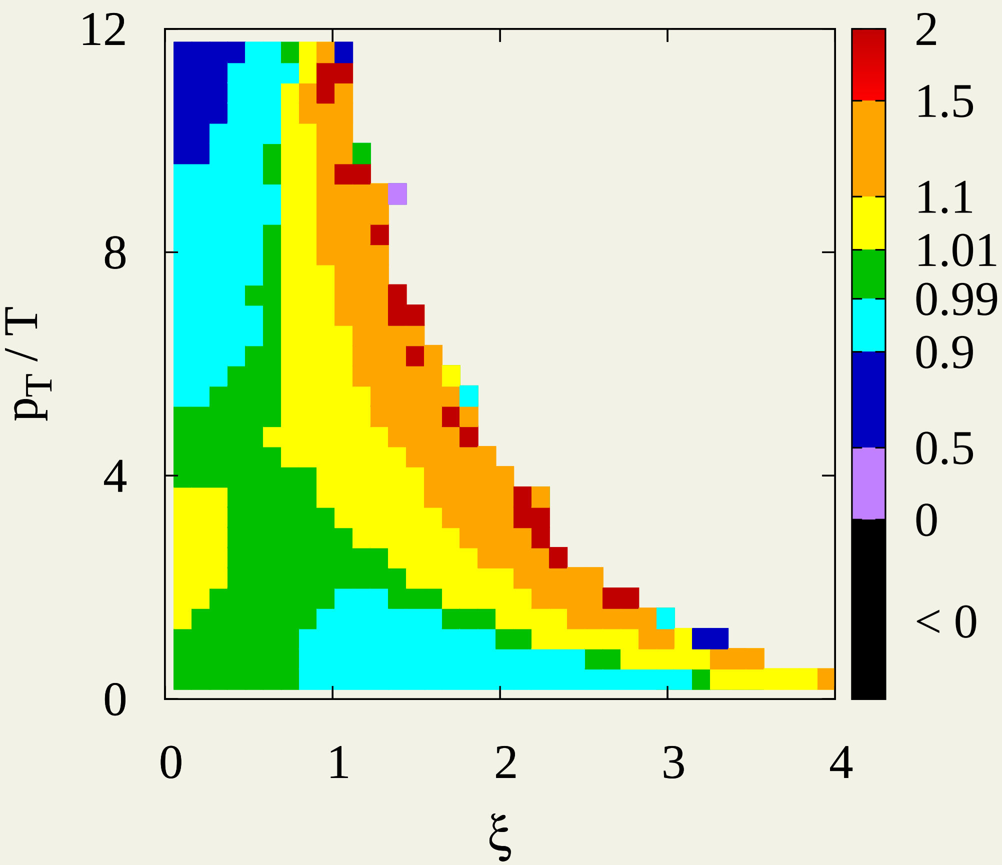


Grad (IS)

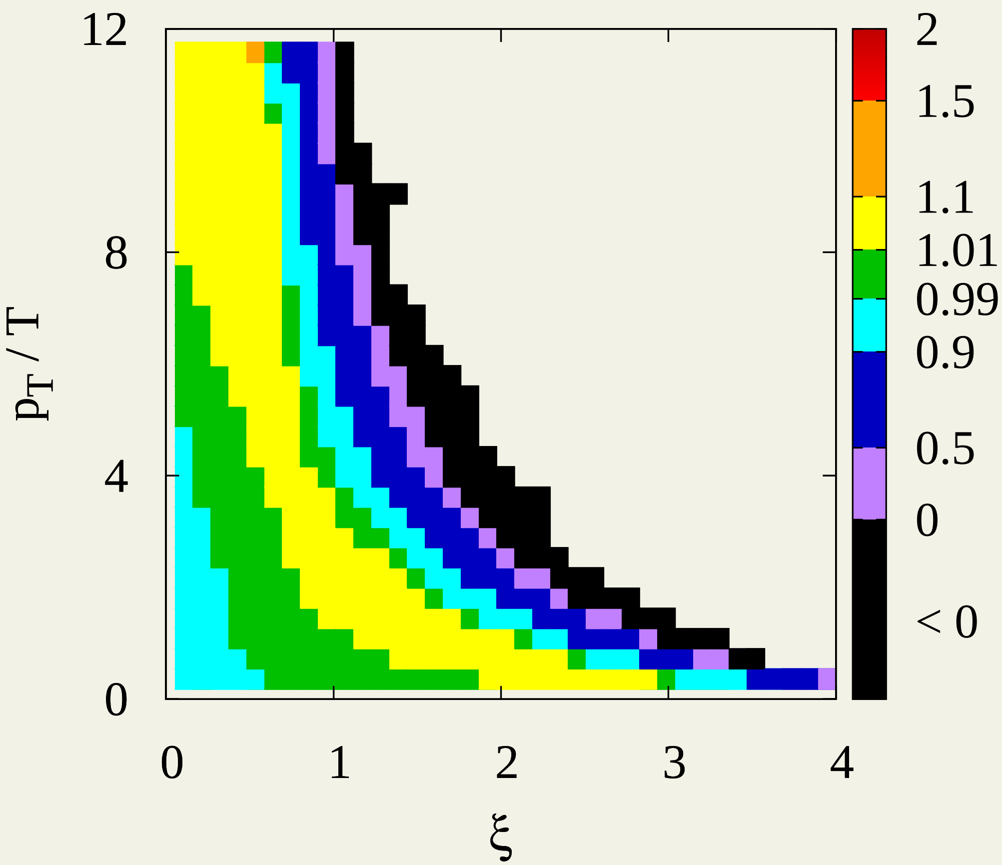


$$\frac{\tau}{\tau_0} = 20, \quad K = 2\left(\frac{\tau}{\tau_0}\right)^{2/3} \qquad \left(\frac{\tau_{REL}}{\tau_{exp}} = 0.07, \ \frac{\pi_L}{p} = -0.08\right)$$

SR ansatz



Grad (IS)



$\Rightarrow$

At early times or high ξ, SR looks better

At high pT and small ξ , Grad looks better.

Now compare four models:

a) SR, b) IS/Grad, c) IS with $p^{1.5}$, d) IS with p^1 (RTA)

study viscosity dependence $\frac{\eta}{s} \sim 0.03 - 0.2$ $[K_0 = 1, 2, 6.49]$

plot near three characteristic times:

i) early walk-away: $1.2\tau_0$

ii) maximum departure from equilibrium: $3\tau_0$

iii) late return towards equilibrium: $10\tau_0$

$$\frac{\boldsymbol{\eta}}{\boldsymbol{s}} \sim \mathbf{0.2} \qquad [K_0 = 1]$$

$$\frac{\tau}{\tau_0} = 1.2$$

$$K = 1\left(\frac{\tau}{\tau_0}\right)^{2/3}$$

$$\left(\frac{\tau_{REL}}{\tau_{exp}} = 0.93,\right.$$

$$\left.\frac{\pi_L}{p} = -0.17\right)$$

+100%

+50%

+10%

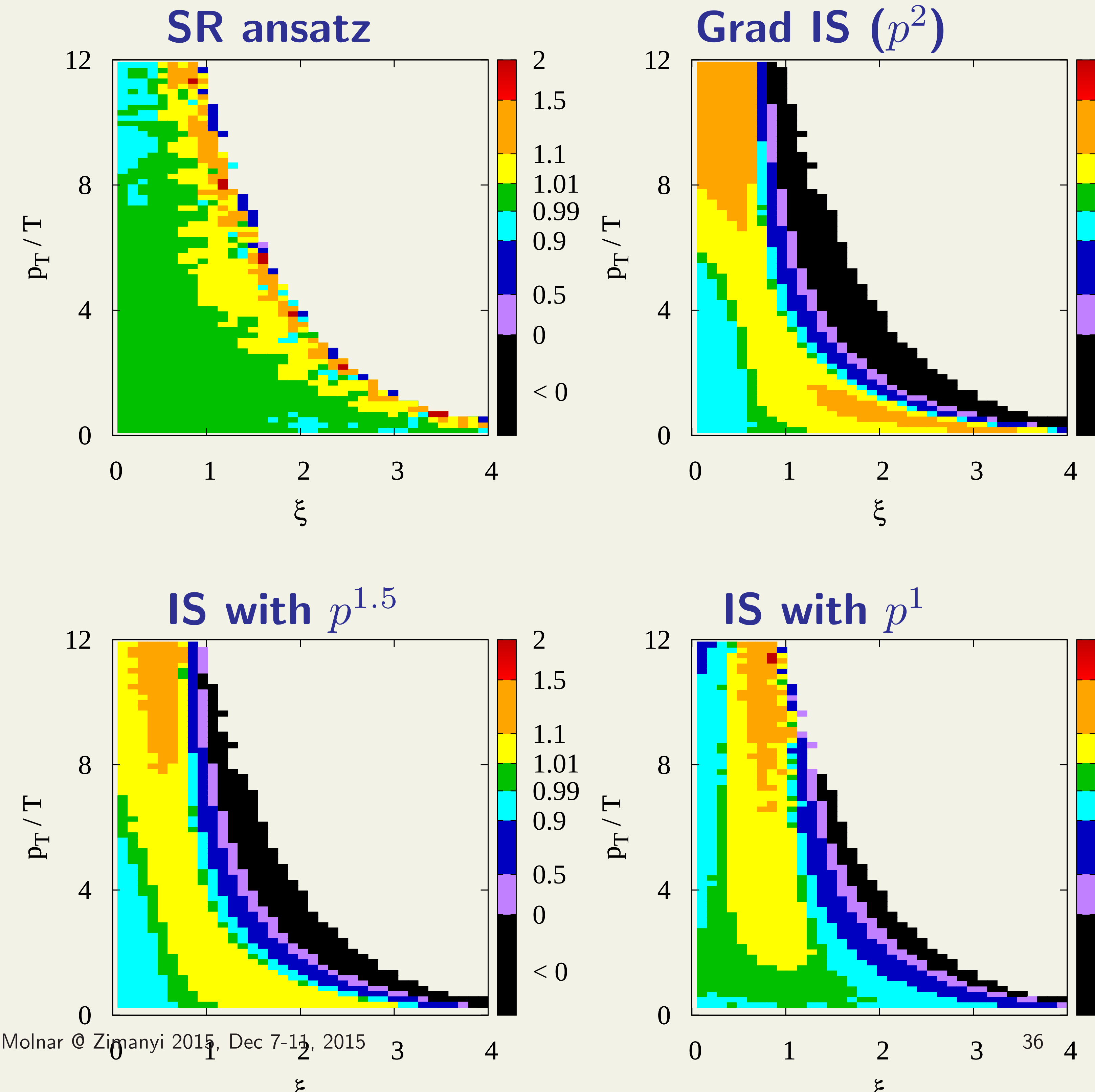
±1%

−10%

−50%

−100%

< 0



$$\frac{\tau}{\tau_0} = 3$$

$$K = 1\left(\frac{\tau}{\tau_0}\right)^{2/3}$$

$$\left(\frac{\tau_{REL}}{\tau_{exp}} = 0.51,\right.$$

$$\left.\frac{\pi_L}{p} = -0.41\right)$$

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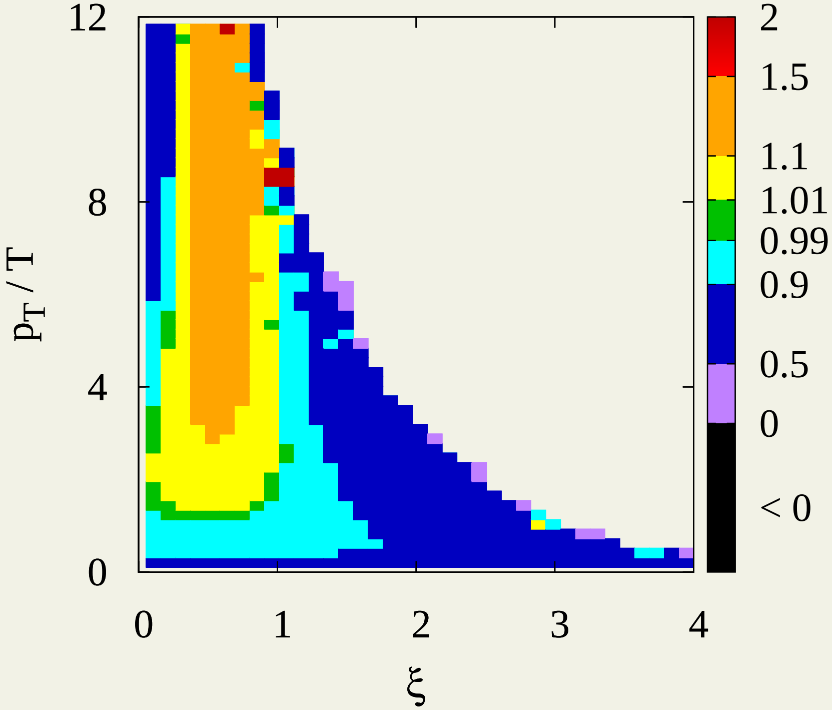
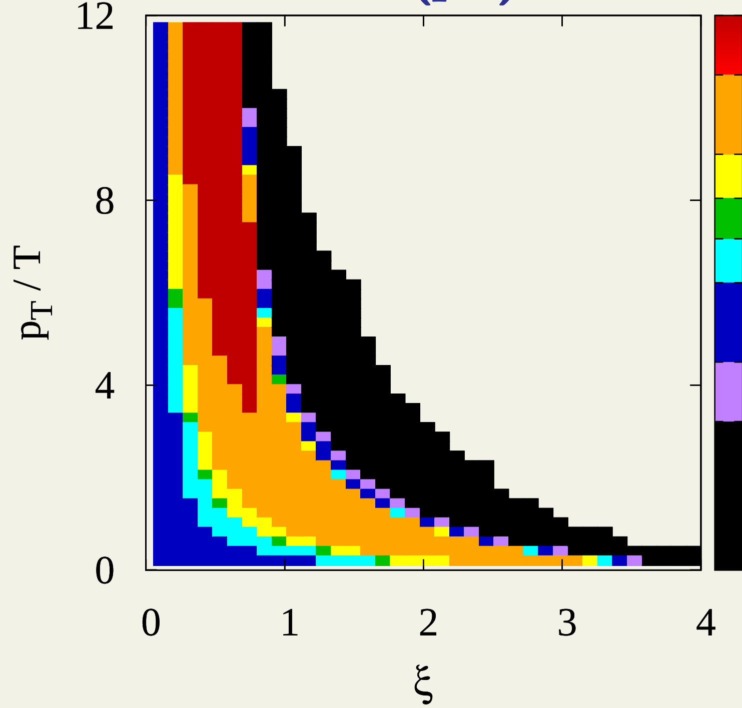
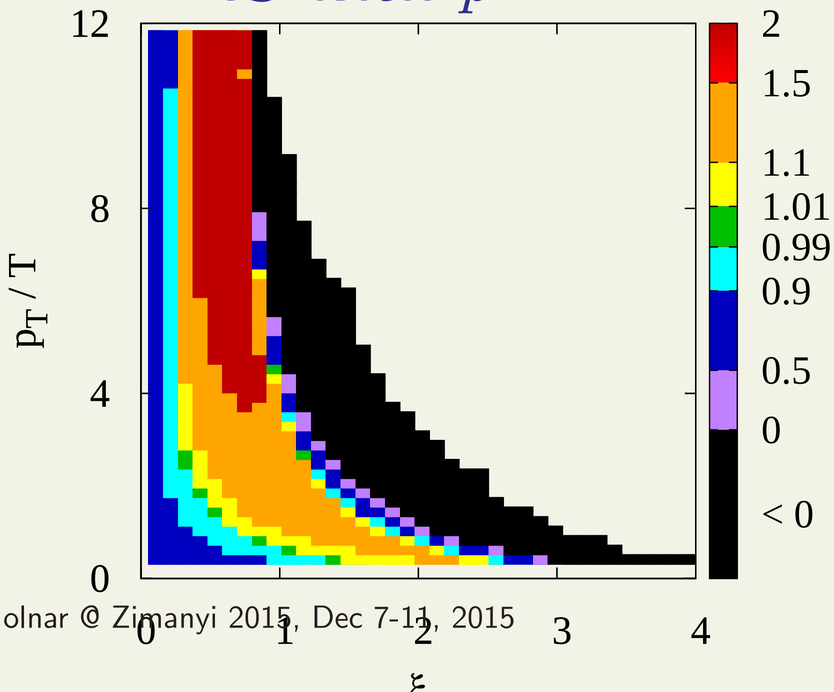
SR ansatz



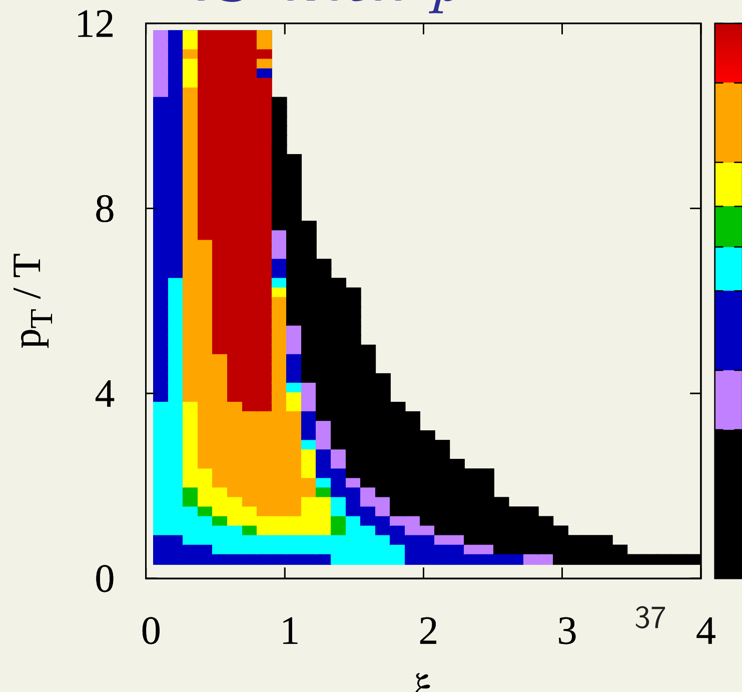
Grad IS (p^2)



IS with $p^{1.5}$



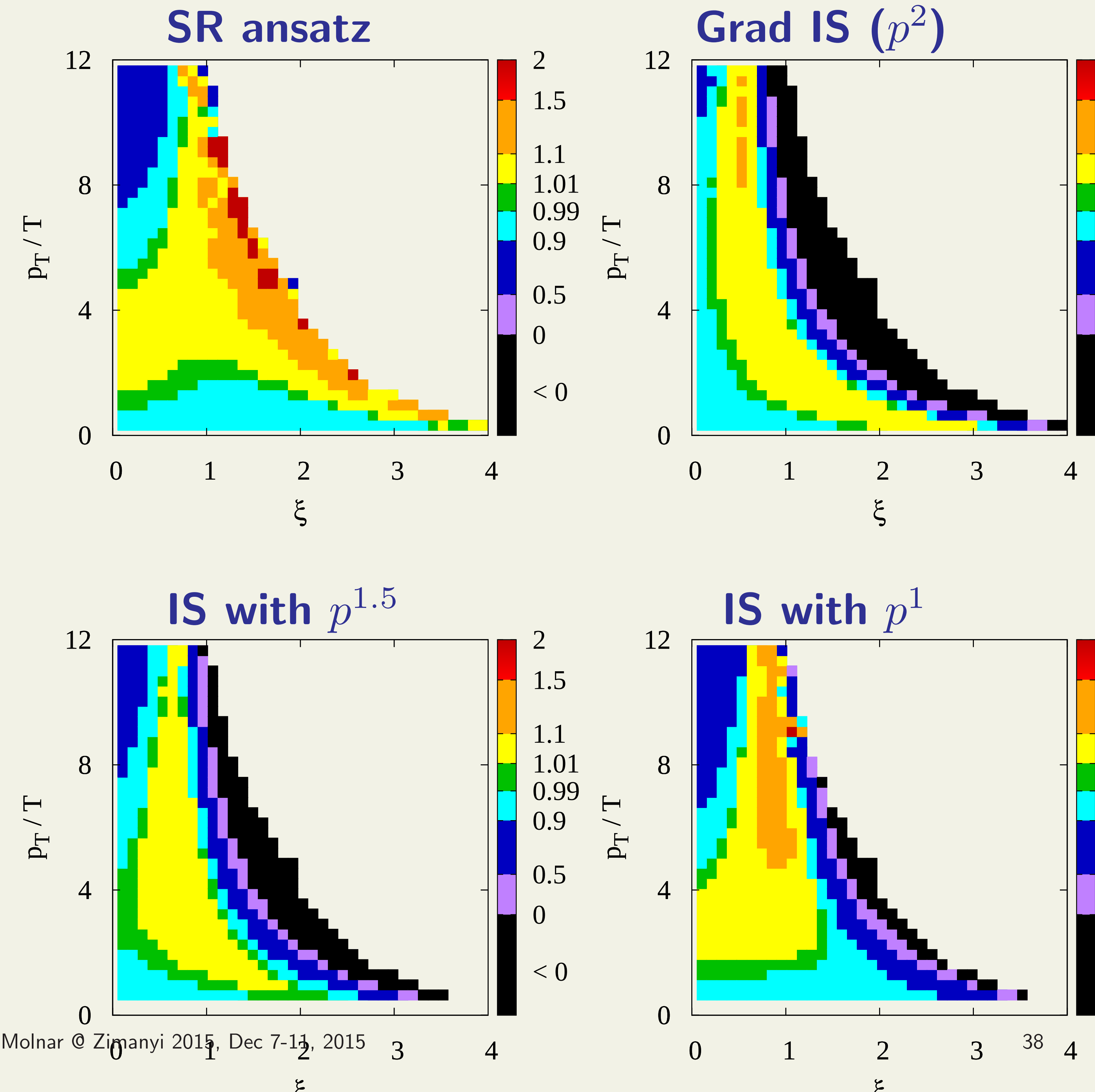
IS with p^1



$$\frac{\tau}{\tau_0} = 20$$

$$K = 1 \left(\frac{\tau}{\tau_0}\right)^{2/3}$$

$$\left(\frac{\tau_{REL}}{\tau_{exp}} = 14, \quad \frac{\pi_L}{p} = -0.16\right)$$



+100%

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−100%

< 0

$$\frac{\boldsymbol{\eta}}{\boldsymbol{s}} \sim \mathbf{0.1} \qquad [K_0 = 2]$$

$$\frac{\tau}{\tau_0} = 1.2$$

$$K = 2\left(\frac{\tau}{\tau_0}\right)^{2/3}$$

$$\left(\frac{\tau_{REL}}{\tau_{exp}} = 0.47, \quad \frac{\pi_L}{p} = -0.15\right)$$

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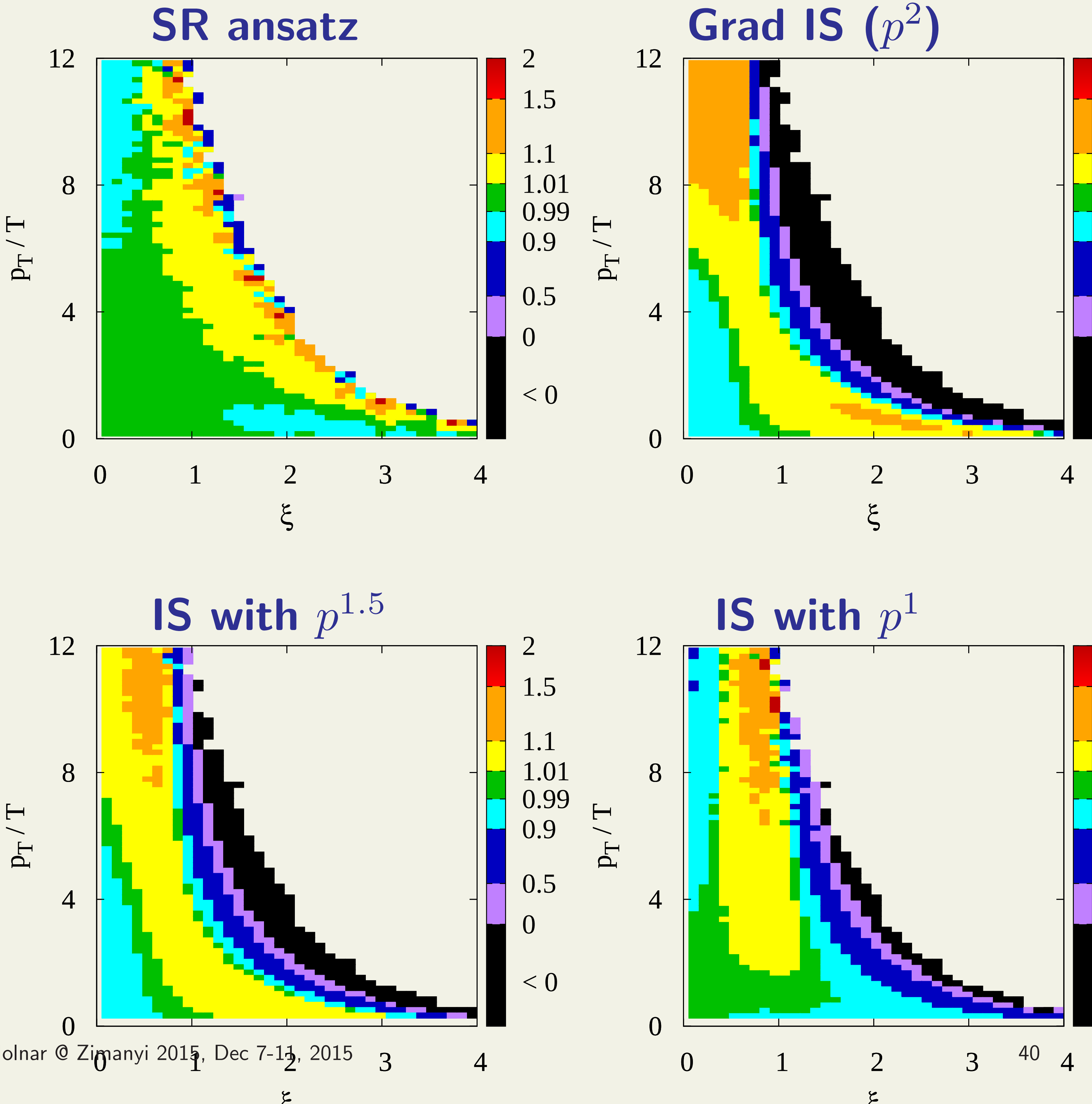
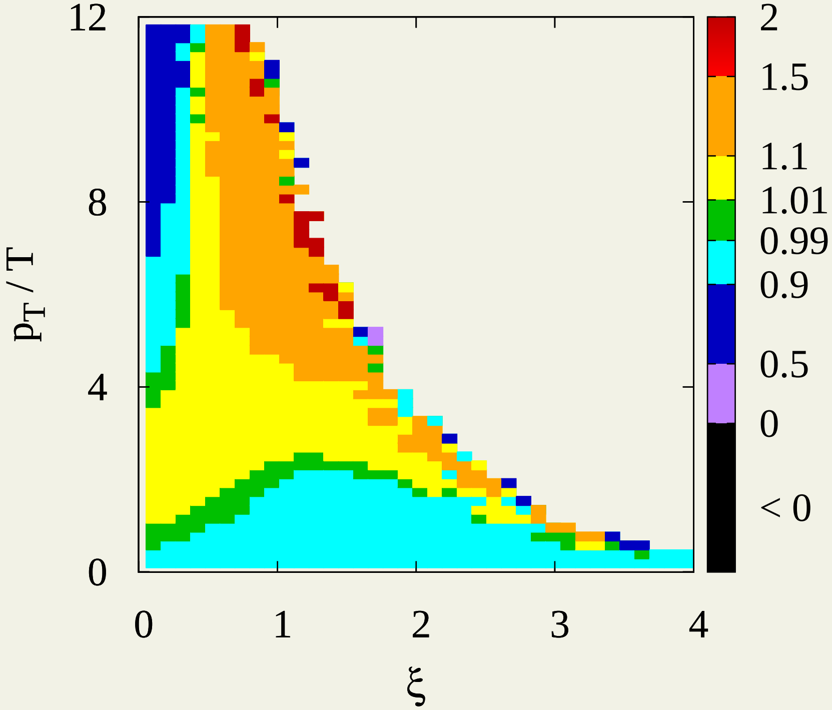


$$\frac{\tau}{\tau_0} = 2$$

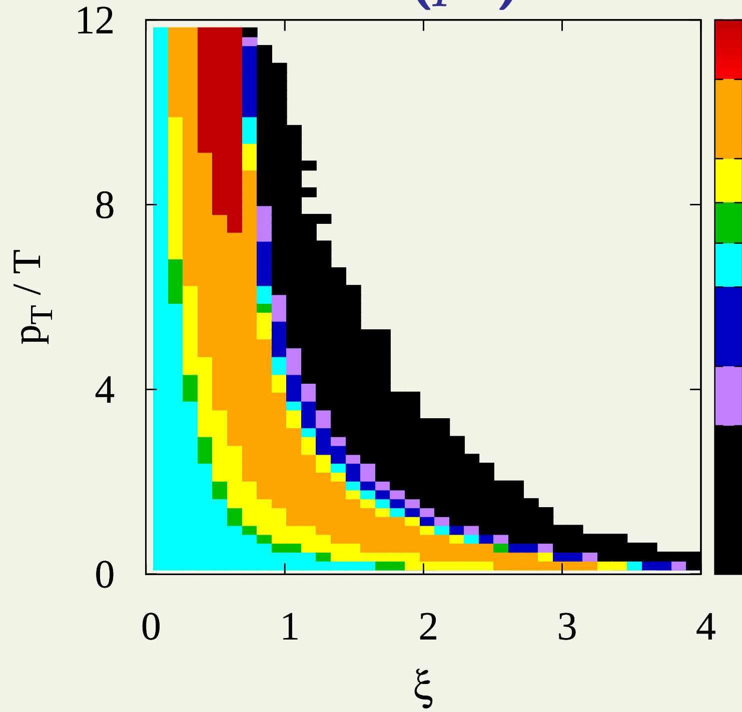
$$K = 2\left(\frac{\tau}{\tau_0}\right)^{2/3}$$

$$\left(\frac{\tau_{REL}}{\tau_{exp}} = 0.33, \quad \frac{\pi_L}{p} = -0.29\right)$$

SR ansatz



Grad IS (p^2)



+100%

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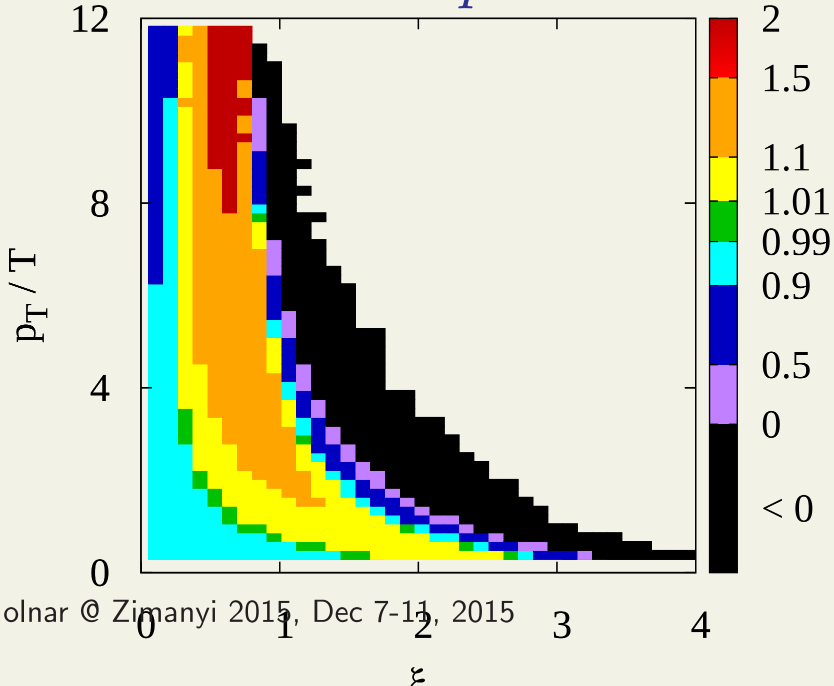
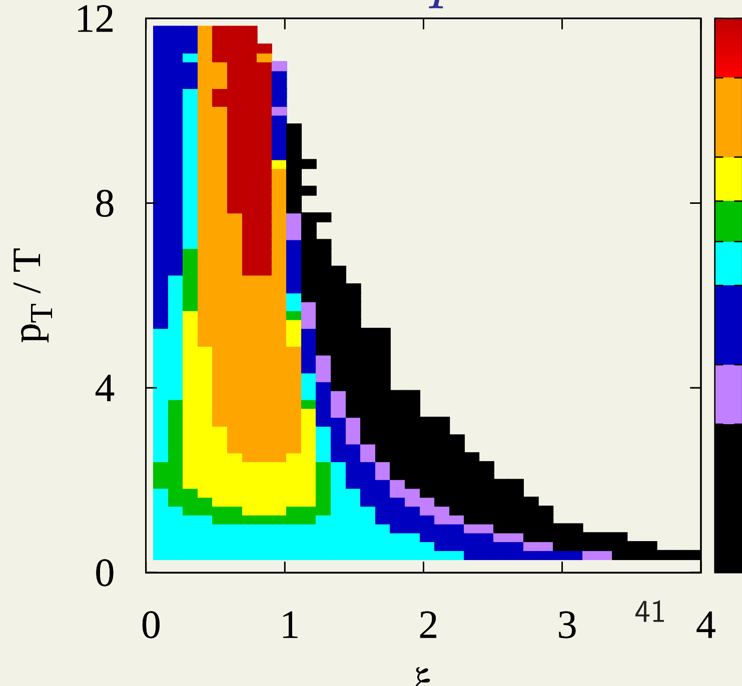
< 0

IS with $p^{1.5}$



IS with p^1



$$\frac{\tau}{\tau_0} = 20$$

$$K = 2\left(\frac{\tau}{\tau_0}\right)^{2/3}$$

$$\left(\frac{\tau_{REL}}{\tau_{exp}} = 0.07,\right.$$

$$\left.\frac{\pi_L}{p} = -0.08\right)$$

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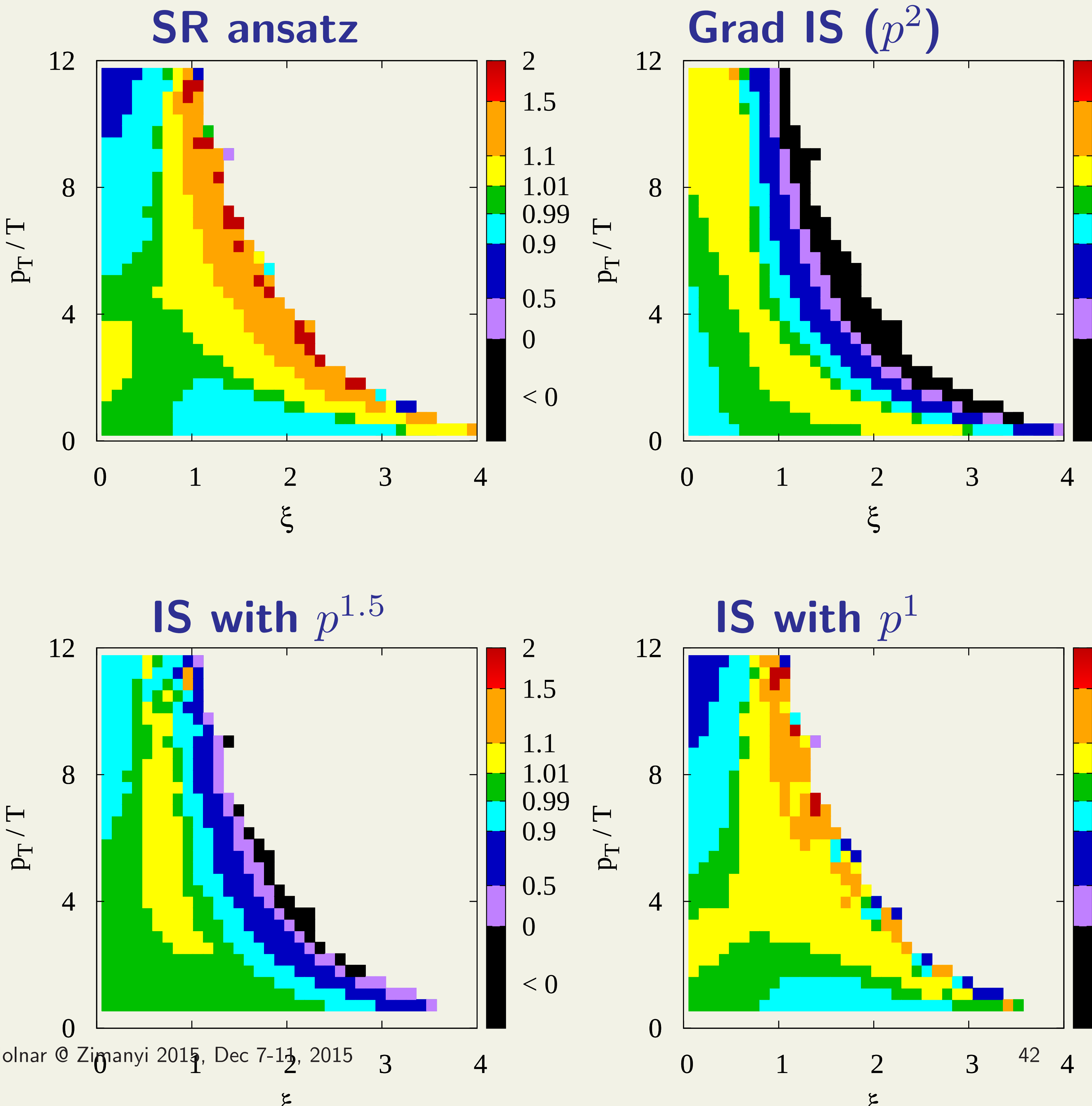

$$\frac{\boldsymbol{\eta}}{\boldsymbol{s}} \sim \mathbf{0.03} \qquad [K_0 = 6.49]$$

$$\frac{\tau}{\tau_0} = 1.2$$

$$K = 6.49 \left(\frac{\tau}{\tau_0}\right)^{2/3}$$

$$\left(\frac{\tau_{REL}}{\tau_{exp}} = 0.14,\right.$$

$$\left.\frac{\pi_L}{p} = -0.11\right)$$

+100%
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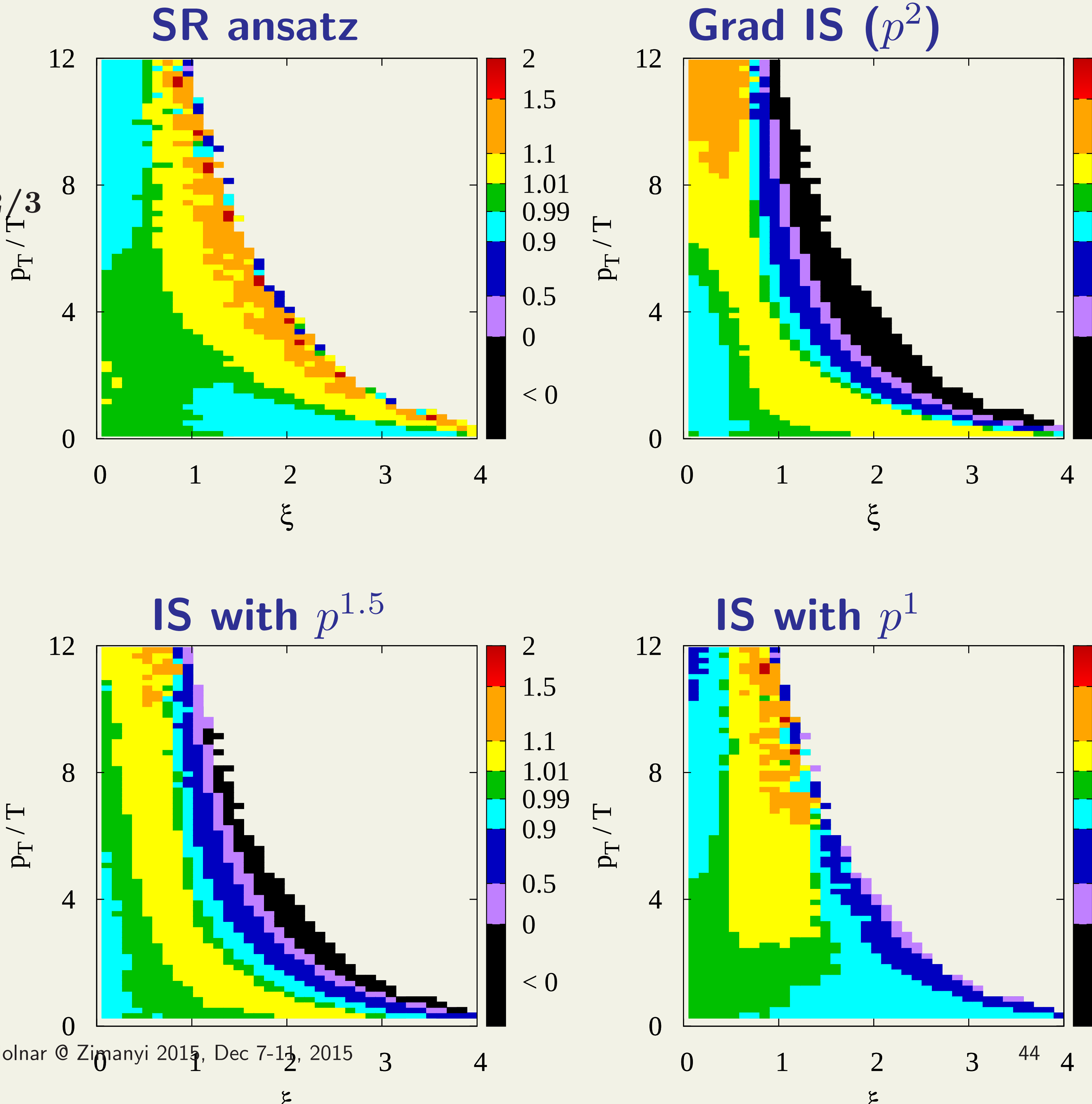


$$\frac{\tau}{\tau_0} = 1.6$$

$$K = 6.49\left(\frac{\tau}{\tau_0}\right)^{2/3}$$

$$\left(\frac{\tau_{REL}}{\tau_{exp}} = 0.12,\right.$$

$$\left.\frac{\pi_L}{p} = -0.12\right)$$

$+100\%$

$+50\%$

$+10\%$

$\pm1\%$

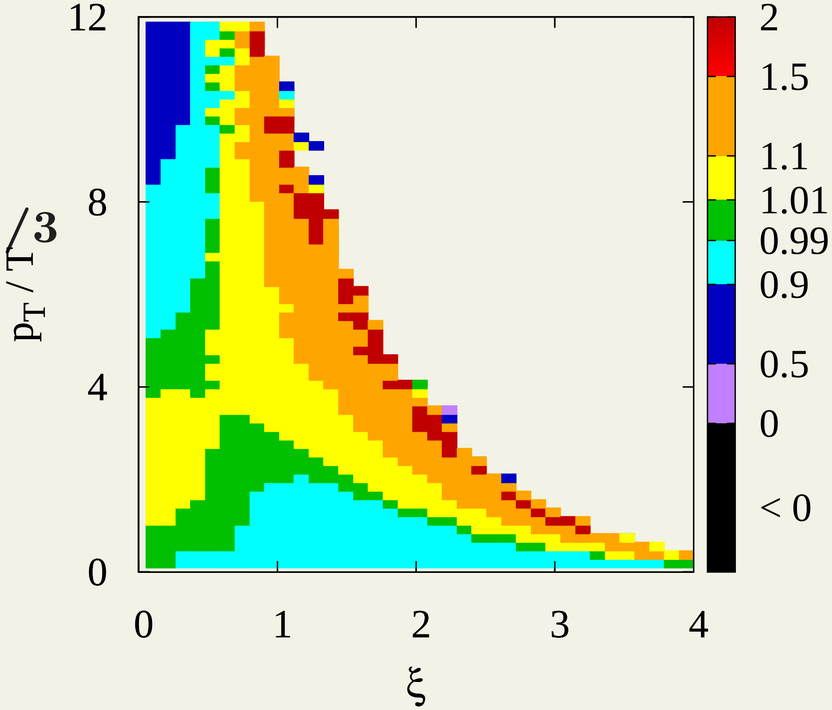
-10%

-50%

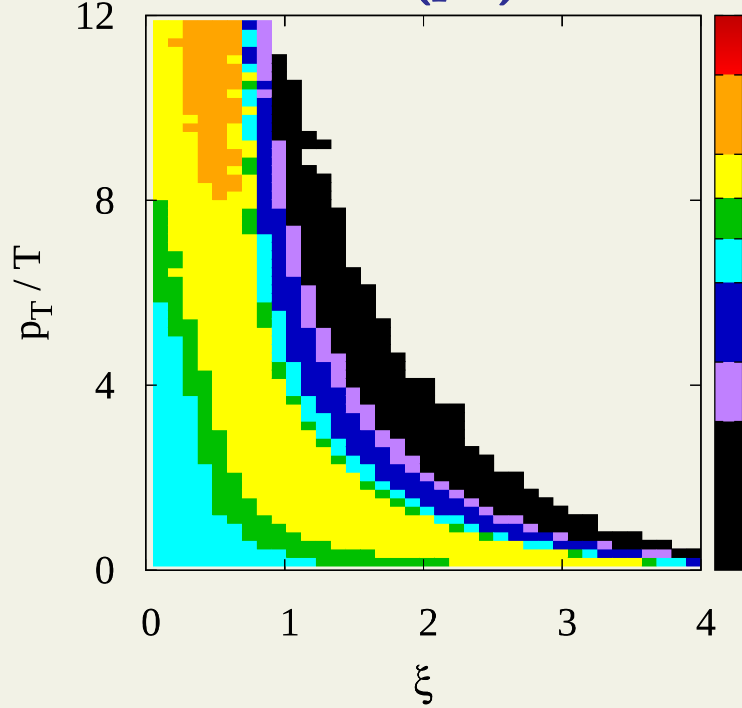
-100%

< 0

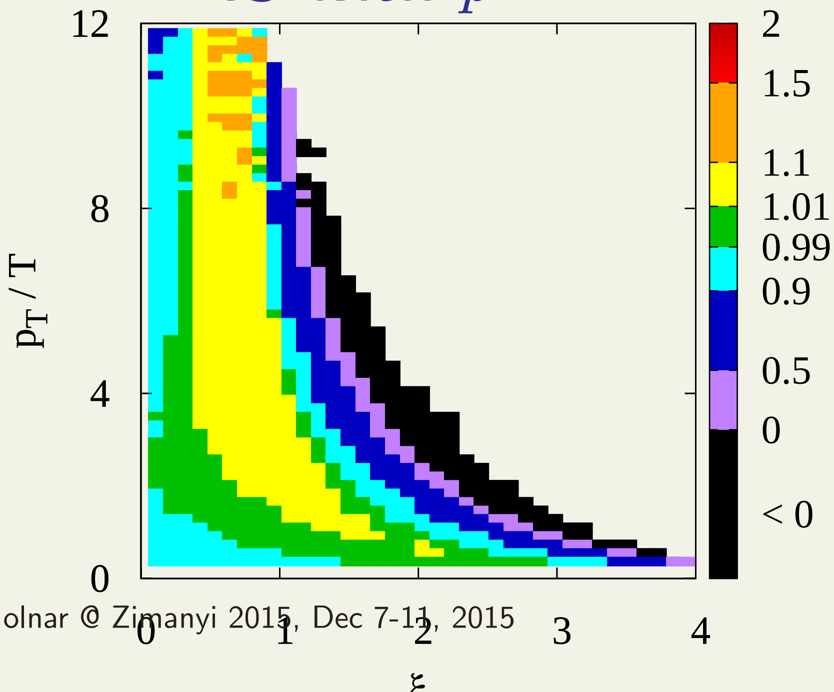
SR ansatz



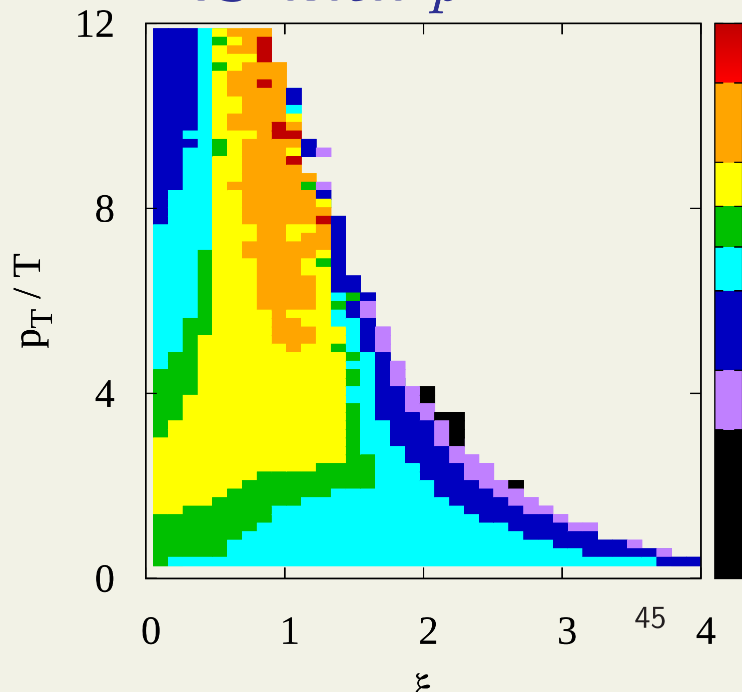
Grad IS (p^2)



IS with $p^{1.5}$



IS with p^1



$$\frac{\tau}{\tau_0} = 20$$

$$K = 6.49 \left(\frac{\tau}{\tau_0}\right)^{2/3}$$

$$\left(\frac{\tau_{REL}}{\tau_{exp}} = 0.02, \quad \frac{\pi_L}{p} = -0.02\right)$$

+100%

+50%

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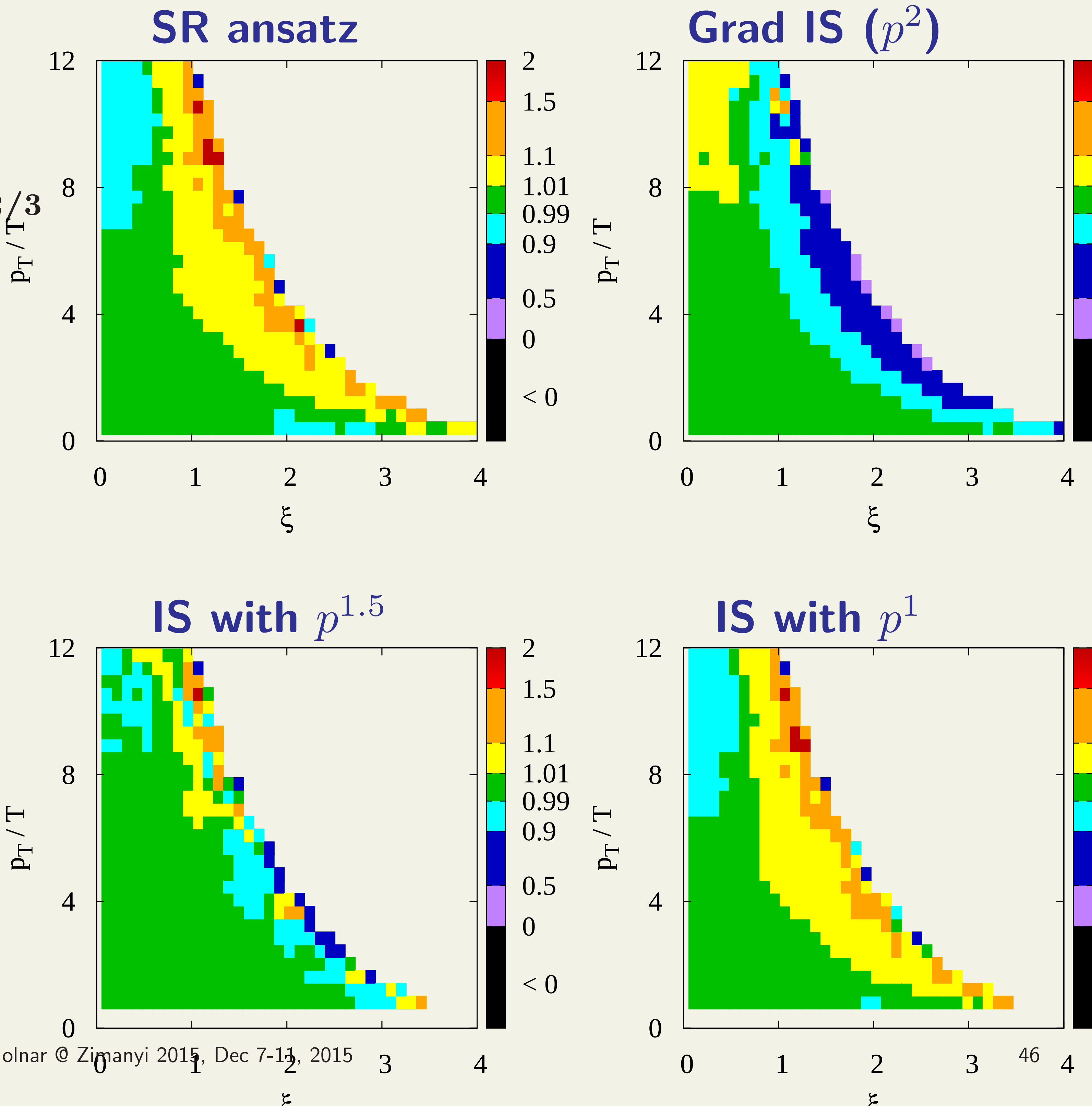


$\Rightarrow$

For high shear viscosity, SR looks more accurate

For small viscosity and late times, IS with $p^{1.5}$ works best

(linear response indeed works when $\tau_{REL} \ll \tau_{exp}$)

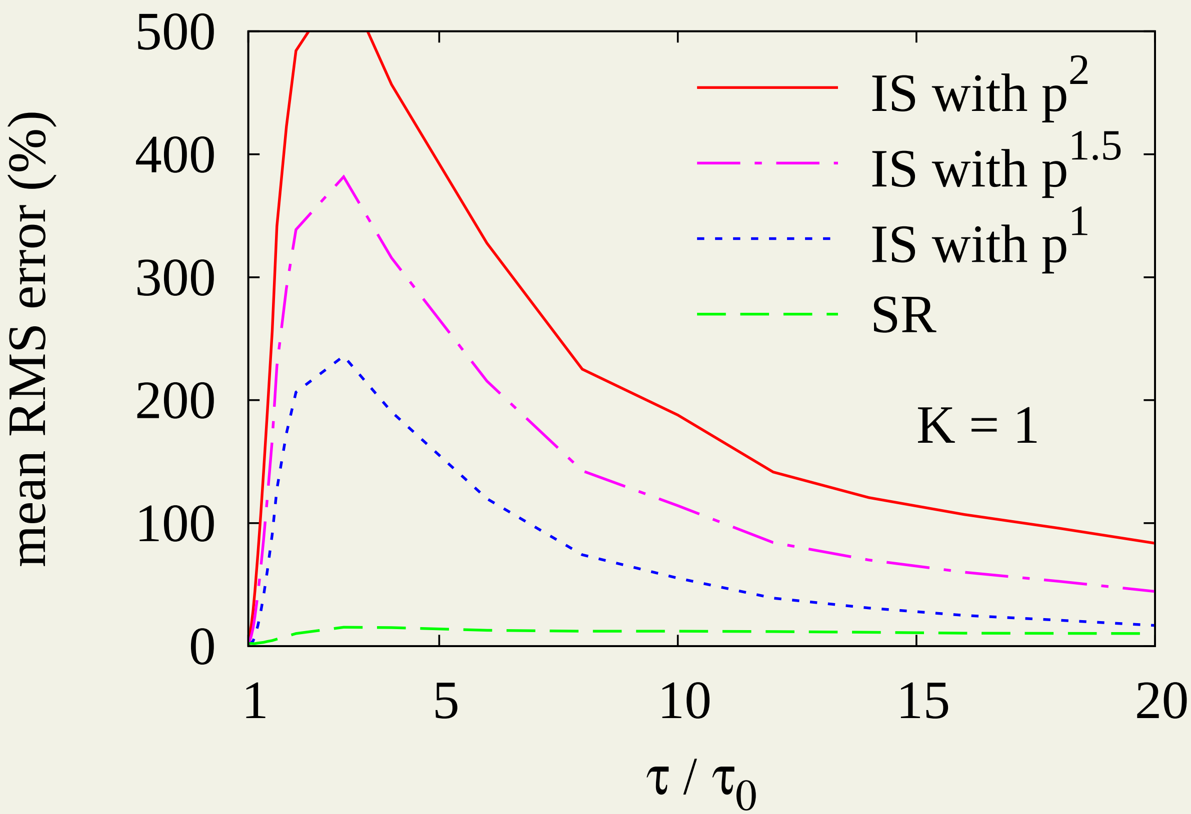
SR is generally very similar to IS with linear p^1

Mean RMS error

Cruder, single-number measure of accuracy

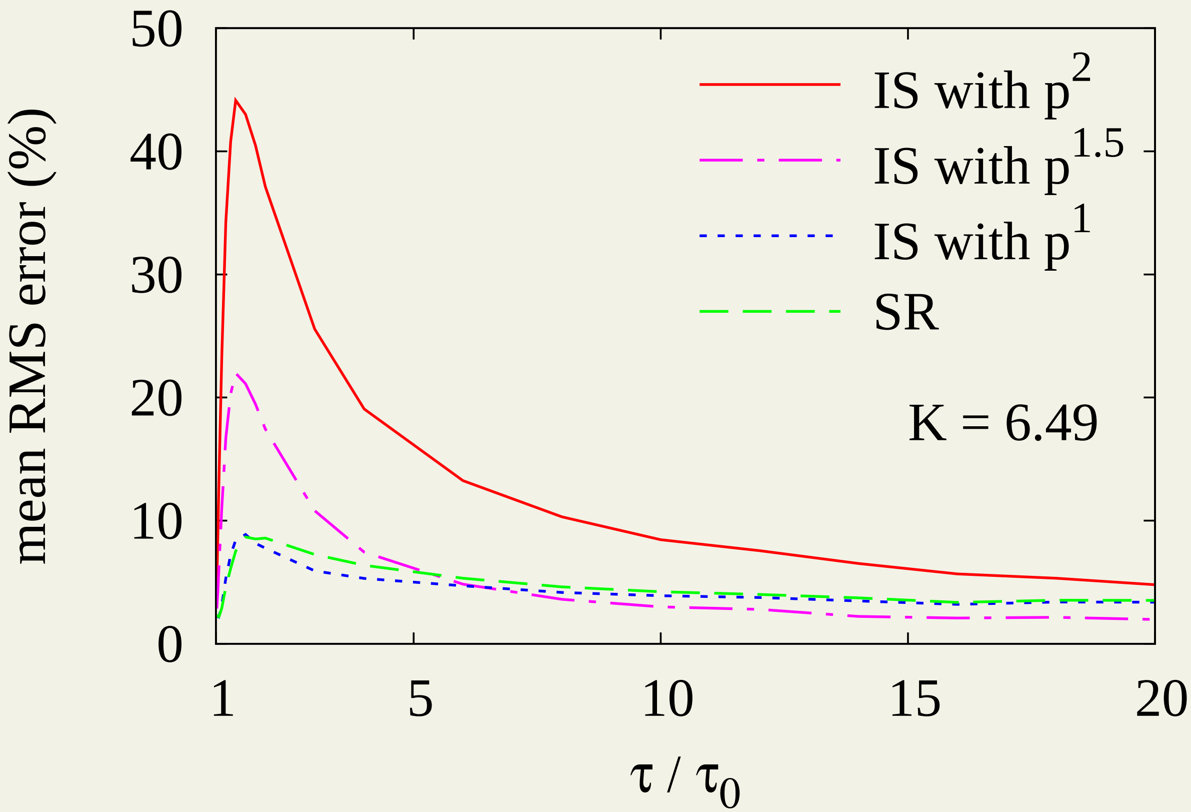
$$\varepsilon_{RMS}(\tau) \equiv \sqrt{\frac{1}{N}\sum_{ij}\left(\frac{f_{guess}(p_{T,i},\xi_j,\tau)}{f_{transp}(p_{T,i},\xi_j,\tau)}-1\right)^2}$$

DM & Damodaran ('15):



and for $K = 6.49$:

DM & Damodaran ('15):



IS gets <u>much</u> better if you ensure positivity for $f_{IS} = f_{eq}(1+\phi)$**:**

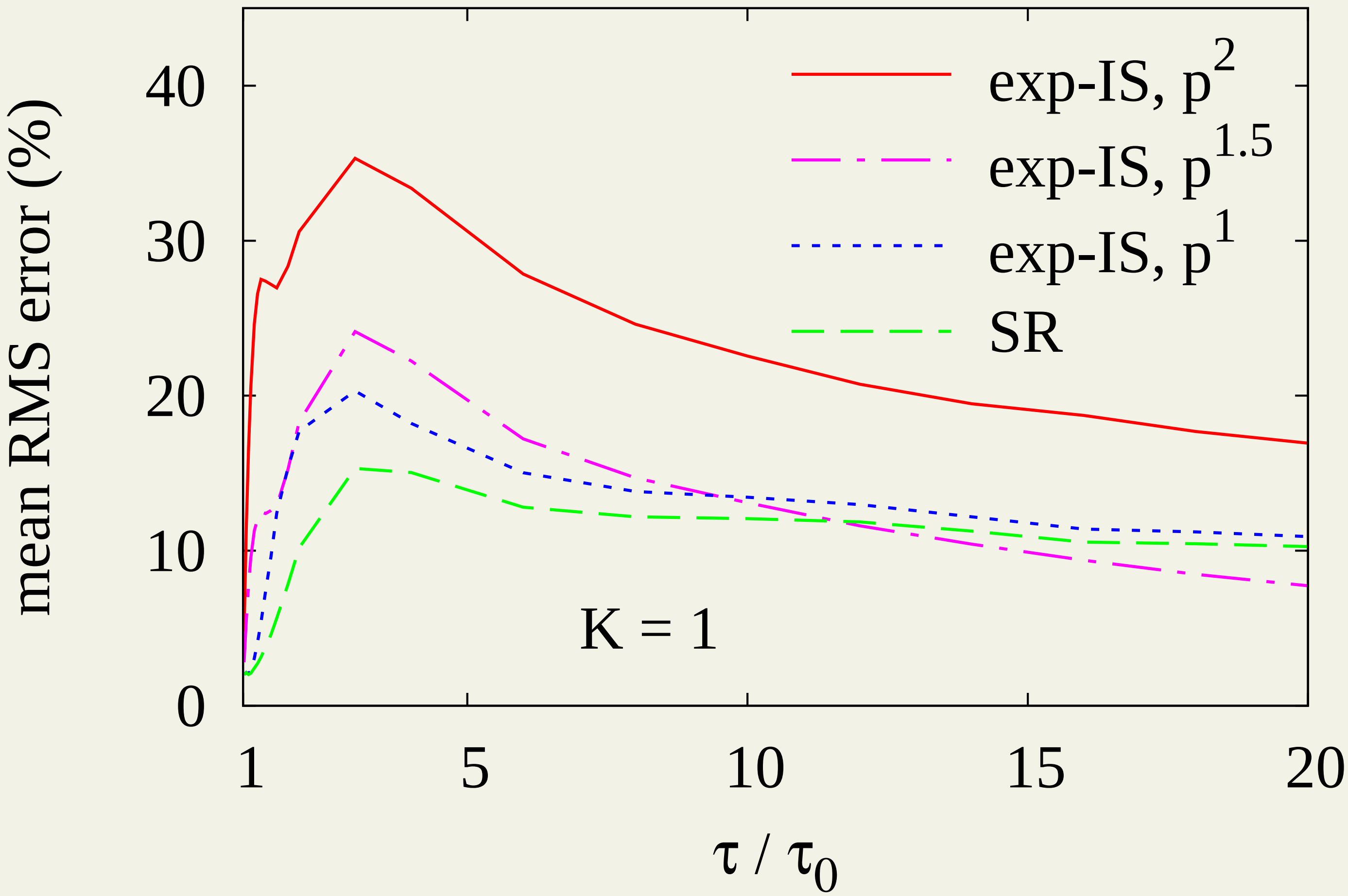
$$1 + \phi(p) \;\to\; e^{\phi(p)} \;\to\; e^{\tanh\phi(p)} \;\to$$

$$\to\; e^{\alpha\tanh\frac{\phi(p)}{\alpha}} \;\to\; e^{\alpha(p)\tanh\frac{\phi(p)}{\alpha(p)}} \;\to \ldots$$

lots of options, take $\alpha = const = 2$

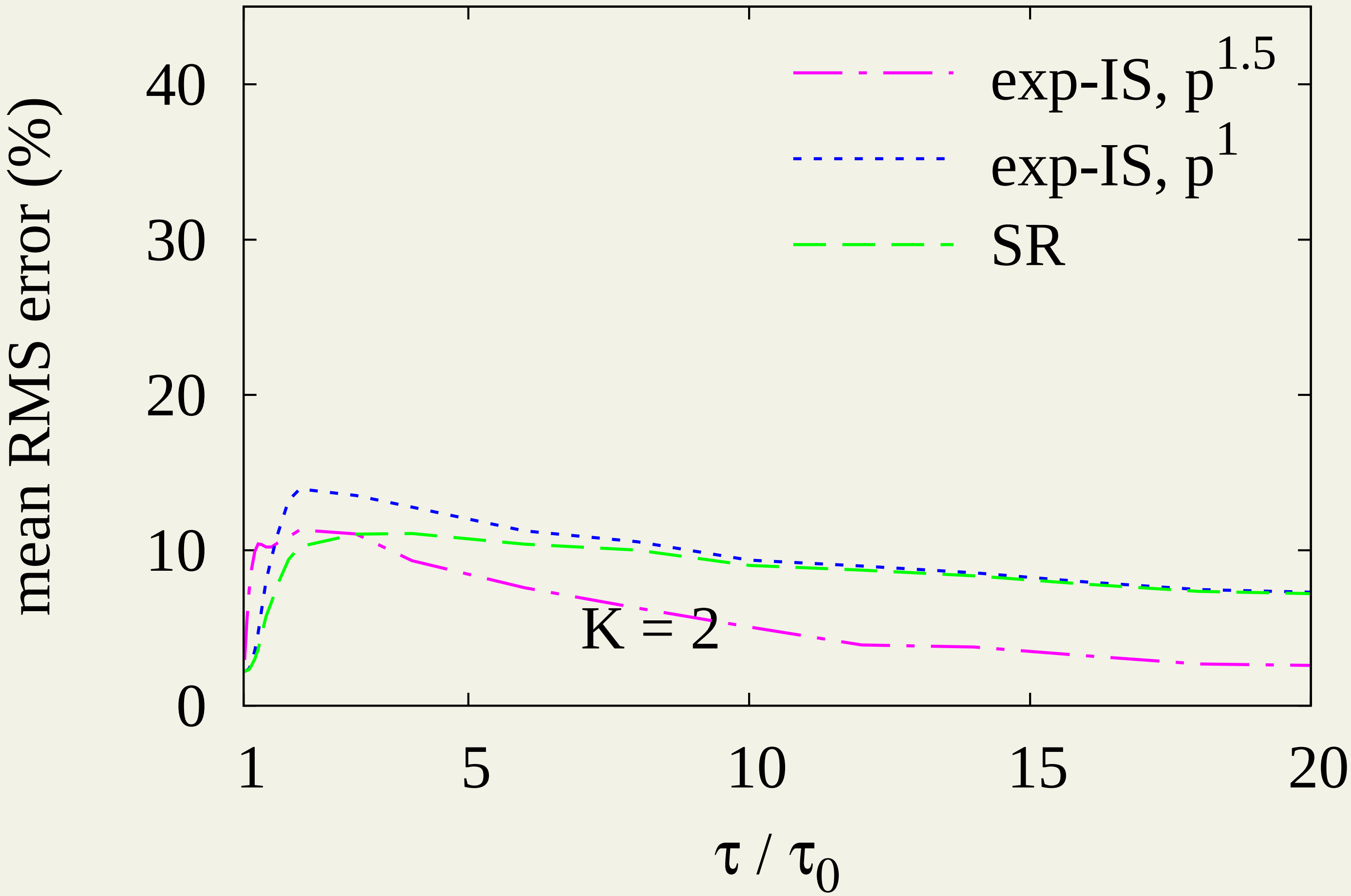
with positivity via "exponentiation" $(\alpha = 2)$:

DM & Damodaran ('15)



and for $K = 2$:

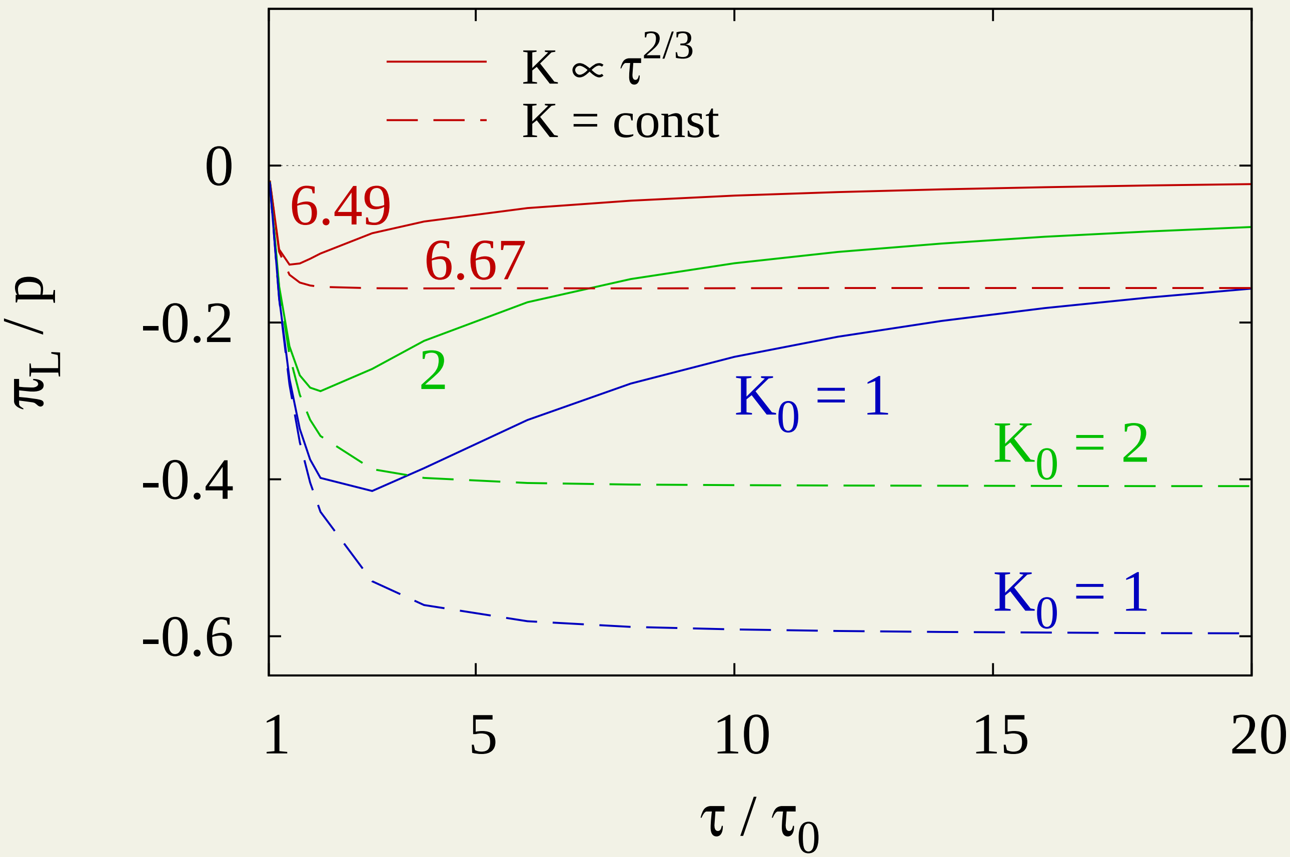
DM & Damodaran ('15)



Memory

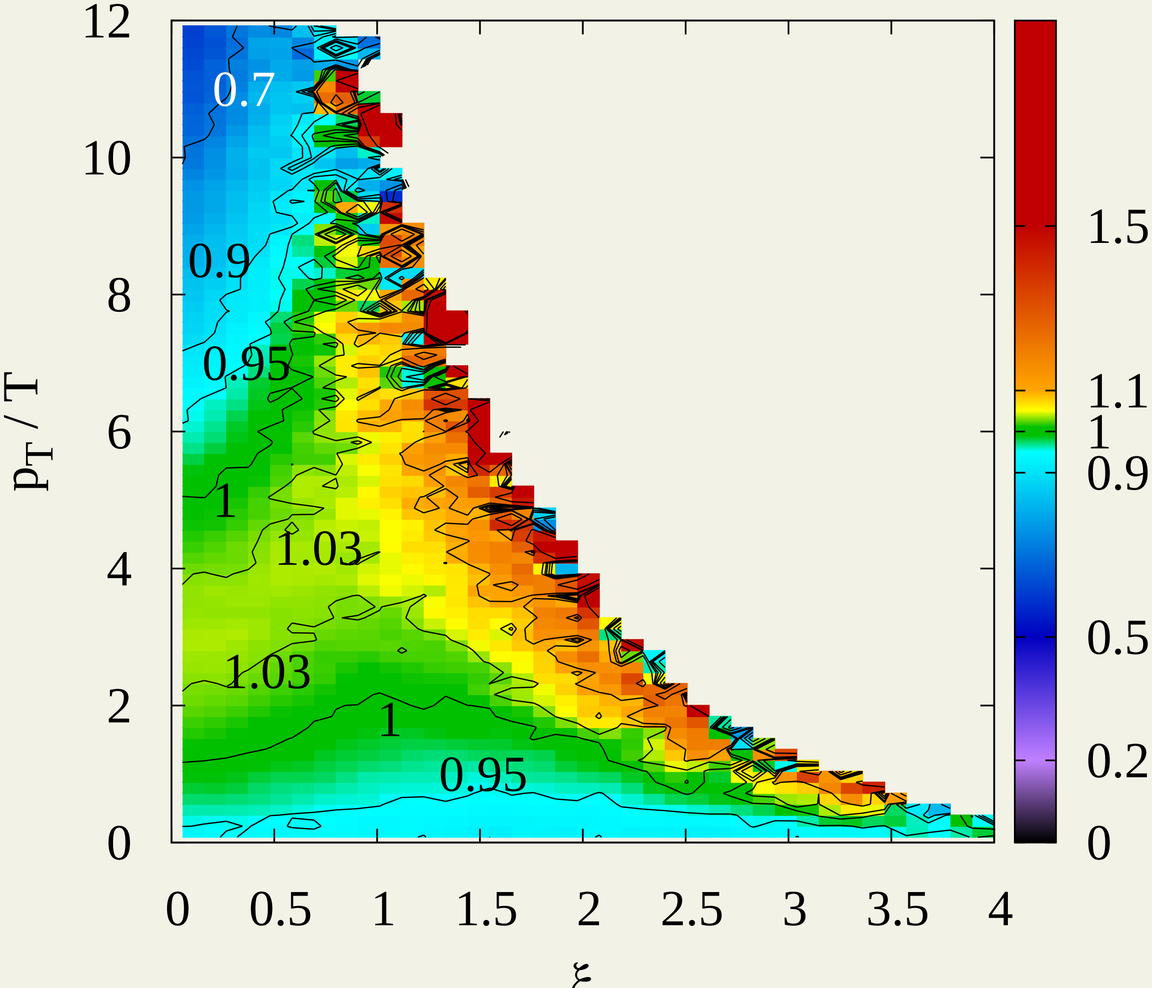
no ansatz universally accurate $\rightarrow$ fundamental limitations

- for massless system, only real hydro parameter is π_L/p



two points with identical π_L/p: e.g., $f(1.2\tau_0)/f(18\tau_0)$ $(K = 1)$

DM & Damodaran ('15):

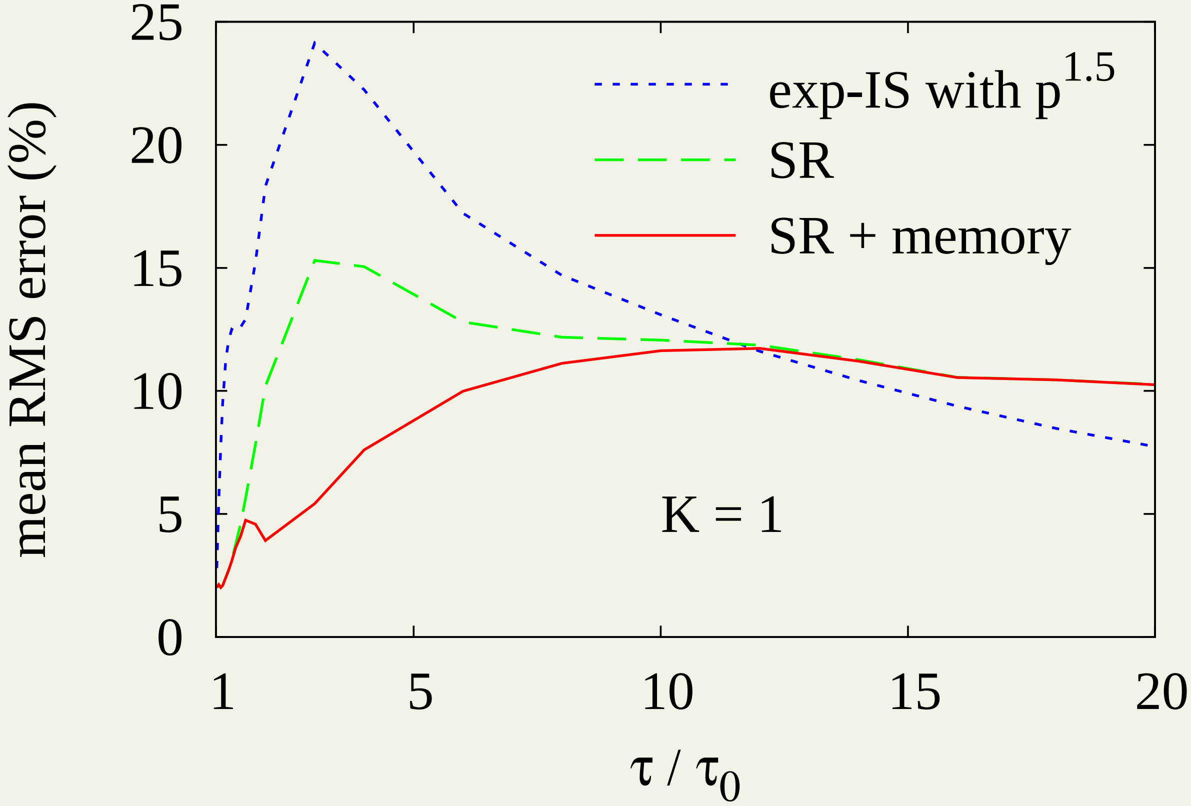


insufficient info in $T^{\mu\nu}$ and N^{μ} for better than ~ 10% accuracy (for $K = 1$)

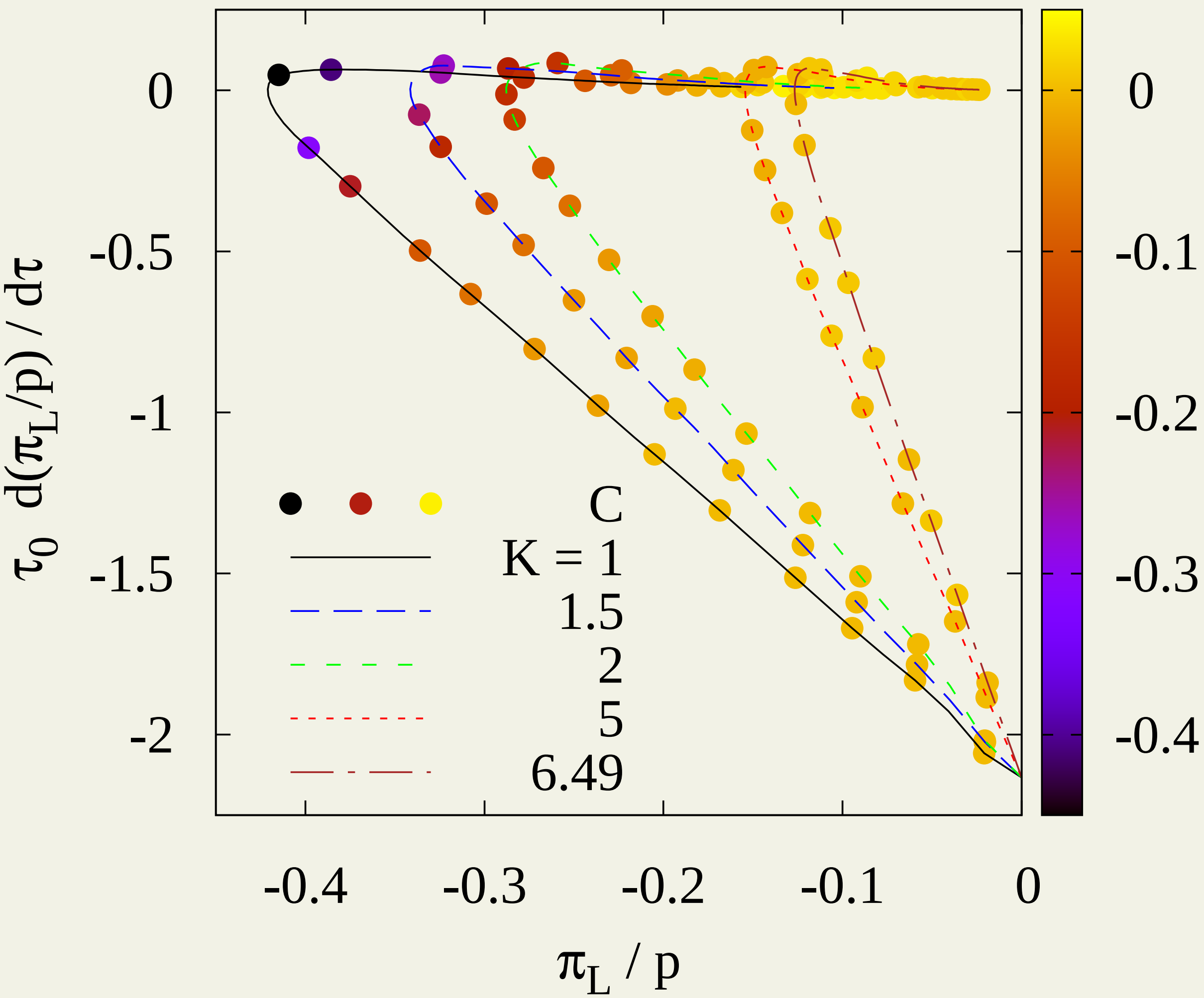
To improve: SR with 1 extra parameter, fit to minimize ε_{RMS}

$$f_{guess}(p_T, \xi, \tau) = A(\tau) \exp\left[-\frac{p_T \mathrm{ch}\xi}{B(\tau)} \sqrt{1 + C(\tau)\tanh^2\xi + D(\tau)\frac{p_T^{0.3}}{1+p_T^{0.3}}\frac{1}{\mathrm{ch}^7\xi}} \right]$$

DM & Damodaran ('15):



Extra parameter predictable from π_L/p AND $d(\pi_L/p)/d\tau$



⇒ slightly beyond std hydro but doable (dump derivs on HF)

Summary

- Getting particles from dissipative hydro requires some model of δf. Proposed δf models should be tested, e.g., against full kinetic theory.

- No free lunch: we compared Grad ($\sim p^2$), linearized Boltzmann, the Strickland-Romatschke ansatz, and relaxation time equation in the linear response regime in a simple 0+1D Bjorken scenario, and found none of these universally accurate.

  We do confirm good applicability of the linearized response approach (which does provide universal answers), but only at late times $\tau/\tau_0 \gtrsim 10$ and for fairly small $\eta/s \lesssim 0.05$.

  Positivity via exponentiation much improves the Isreal-Stewart ansatz.

- Very accurate viscous phase space distributions require additional information from hydro, such as the time derivative of π_L/p (memory).

Many comparisons can be done (e.g., 2+1D for elliptic flow, massive particles, more complex interactions, multicomponent case, ...). **Send us your δf's.**