Gábor Bíró[1,2]
Károly Ürmössy[1]
Gergely G. Barnaföldi[1]
Tamás S. Biró[1]
Shen Keming[3]

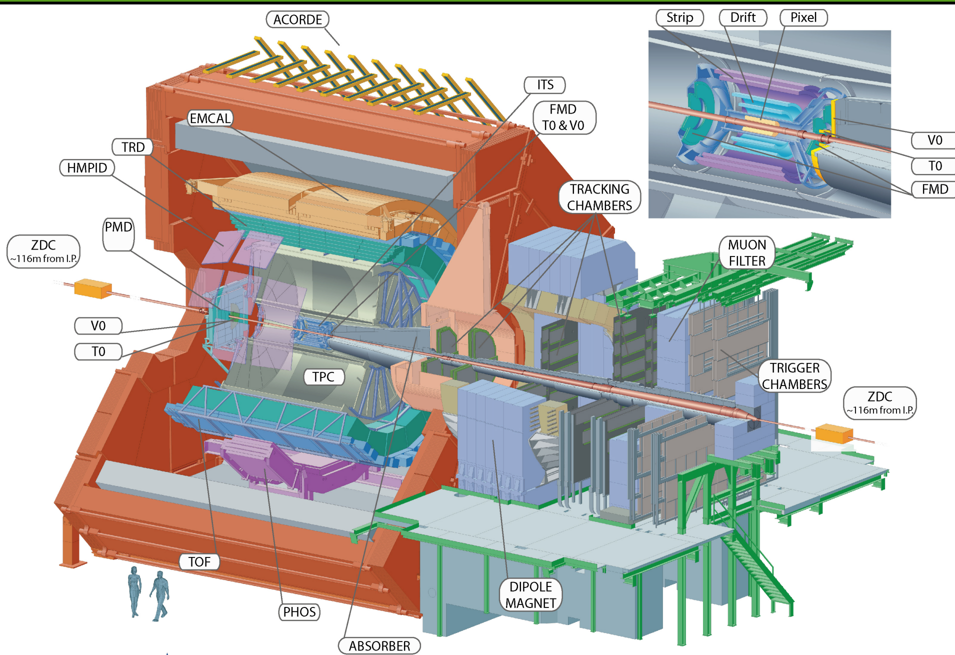
[1]Wigner RCP of the HAS
[2]Eötvös Loránd University
[3]Institute of Particle Physics Central China Normal University, Wuhan

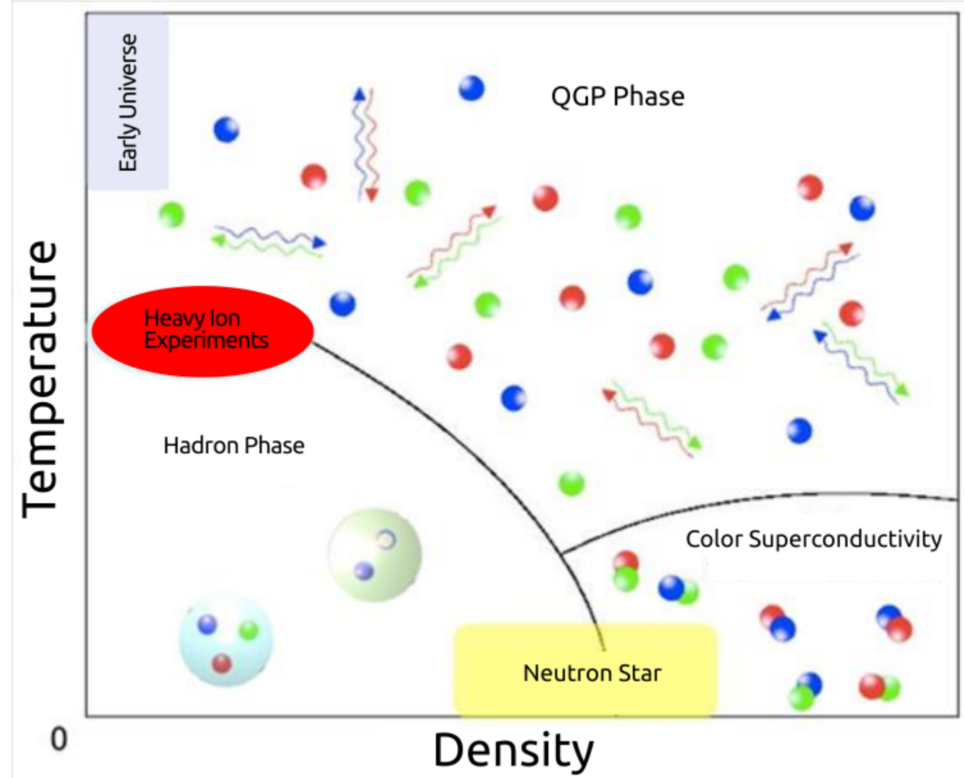
The latest results of the soft+hard model in heavy ion collisions

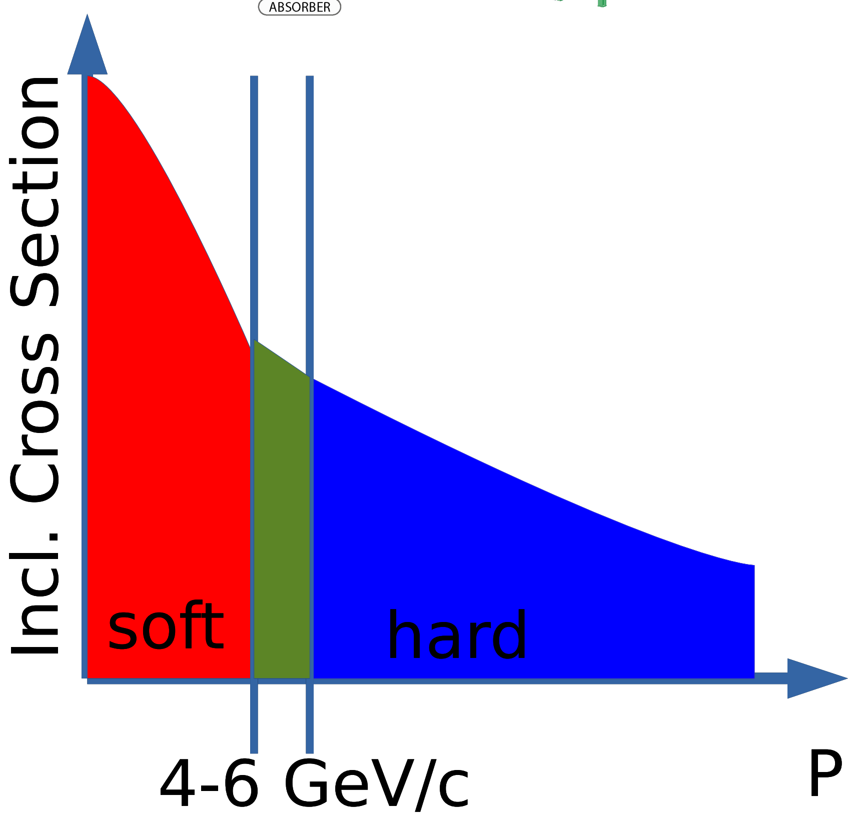
Outline

- **Motivation**
  - Non-extensive statistical approarch
  - Why to use Tsallis – Pareto-like distributions?
- **Fit to experimental data**
  - Can Tsallis – Pareto fit spectra of HIC?
  - The soft+hard model and its applications
  - Spectra fit and extraction of q and T
- **Connecting spectra and v_2**
- **Summary**

Motivation





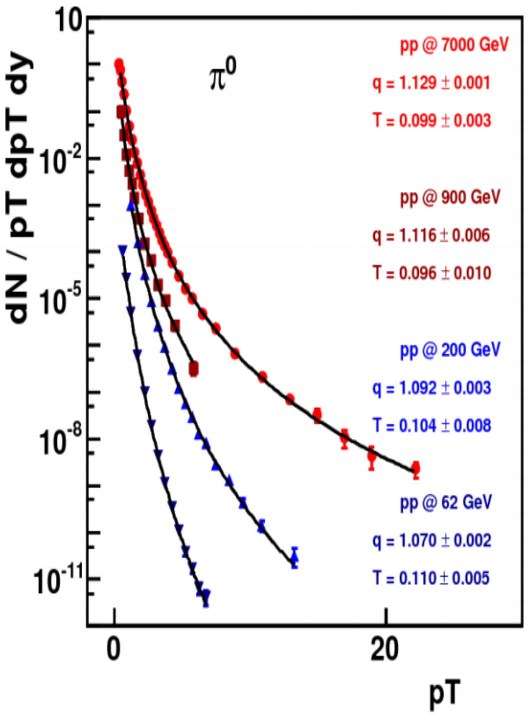


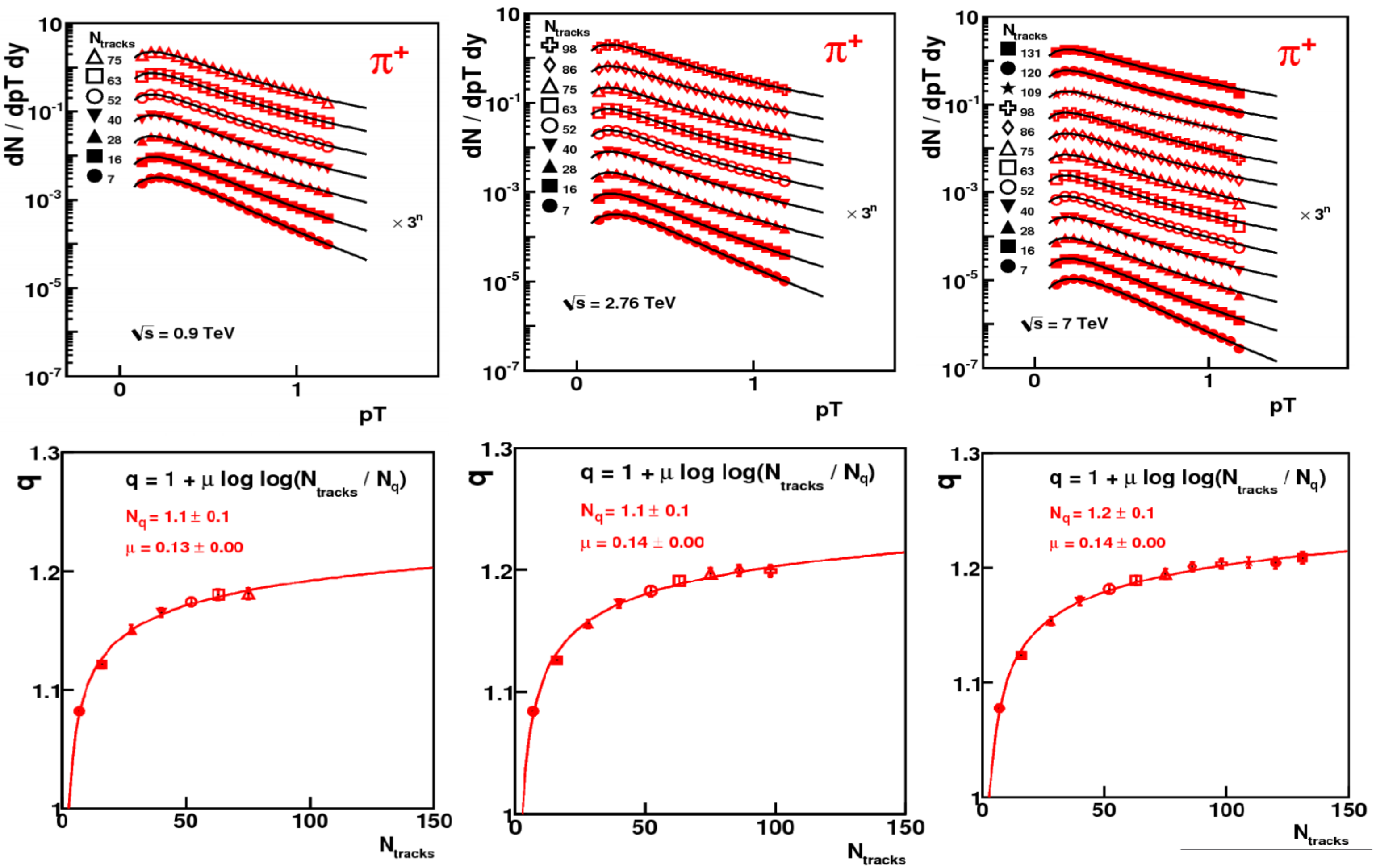
- ALICE: A Large Ion Colliding Experiment
- Analysing QGP: only through observable quantities
- Hadron spectra: a break around 4-6 GeV/c

Motivation

Hadron spectra in *pp* collisions can be described by the Tsallis distribution

π spectra in *pp* collisions depends similarly on $\sqrt{s}$ and on the multiplicity N





arXiv: 1405.3963, 1501.02352, 1501.05959, 1212.0260v2

- Extensive Boltzmann – Gibbs statistics

$$S_{12} = S_1 + S_2$$
$$E_{12} = E_1 + E_2$$
$$\rightarrow \quad S_B = -\sum_i p_i \ln p_i$$

- Non-extensivity ➞ generalized entropy

$$\hat{L}_{12} = \hat{L}_1(S_1) + \hat{L}_2(S_2)$$
$$L_{12} = L_1(E_1) + L_2(E_2)$$
$$\rightarrow \quad S_T = \frac{1}{1-q}\sum_i \left(p_i^q - p_i\right)$$

$$S_{12} = S_1 + S_2 + (q-1)S_1 S_2 \quad \rightarrow \quad \hat{L}(S) = \frac{1}{q-1}\ln\left(1 + (q-1)S\right)$$

- Tsallis entropy from here: Tsallis – Pareto distribution

$$f(\varepsilon) = \left[1 + (q-1)\frac{\varepsilon}{T}\right]^{-\frac{1}{q-1}}$$

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Physica A 392 (2013) 3132

- For more details see KeMings lecture!

Tsallis – Pareto distribution:

$$f(\varepsilon) = \left[1 + (q-1)\frac{\varepsilon}{T}\right]^{-\frac{1}{q-1}}$$

$$q = \frac{\langle S'(E)^2 + S''(E)\rangle}{\langle S'(E)\rangle^2}$$

$$\frac{1}{T} = \langle S'(E)\rangle$$

$$q = \frac{\langle n(n-1)\rangle}{\langle n\rangle^2}$$

$$q = 1 + \frac{\Delta T^2}{T^2} - \frac{1}{C}$$

$$\frac{E}{\langle n\rangle} = DT_{BG}$$

$$\frac{E}{\langle n\rangle} = \frac{\int \varepsilon f_{TS}(\varepsilon)}{\int f_{TS}(\varepsilon)} = \frac{DT}{1-(q-1)(D+1)}$$

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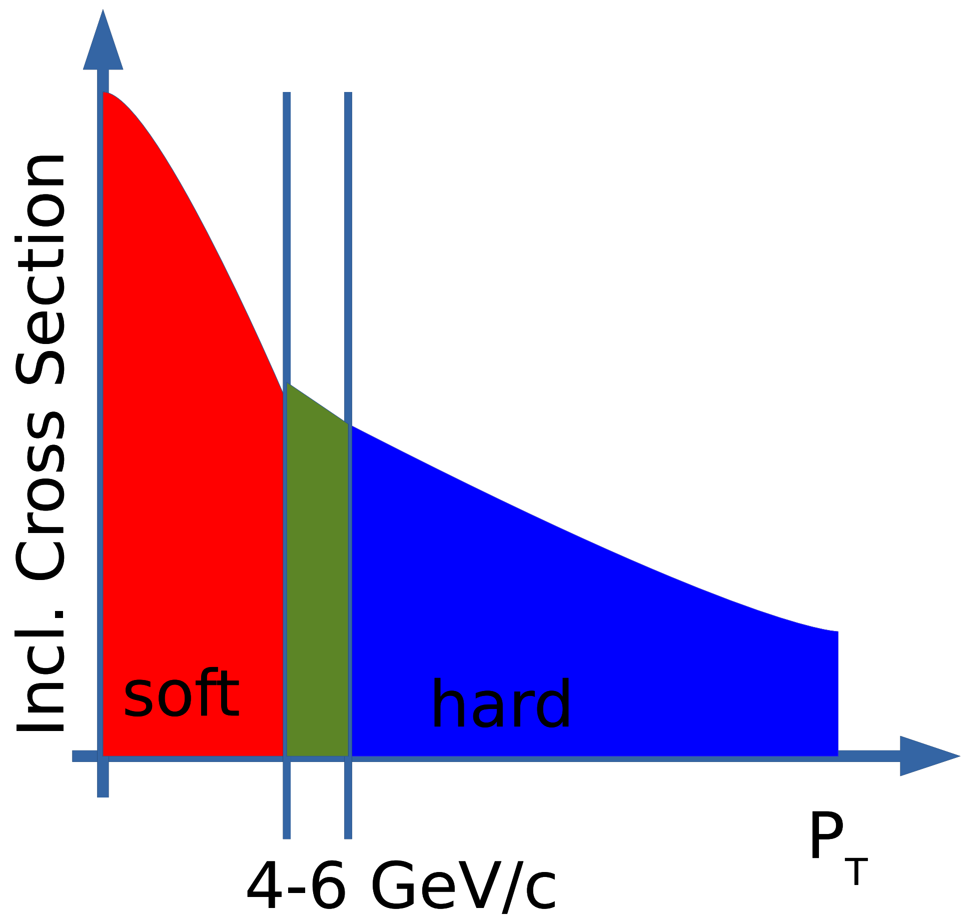
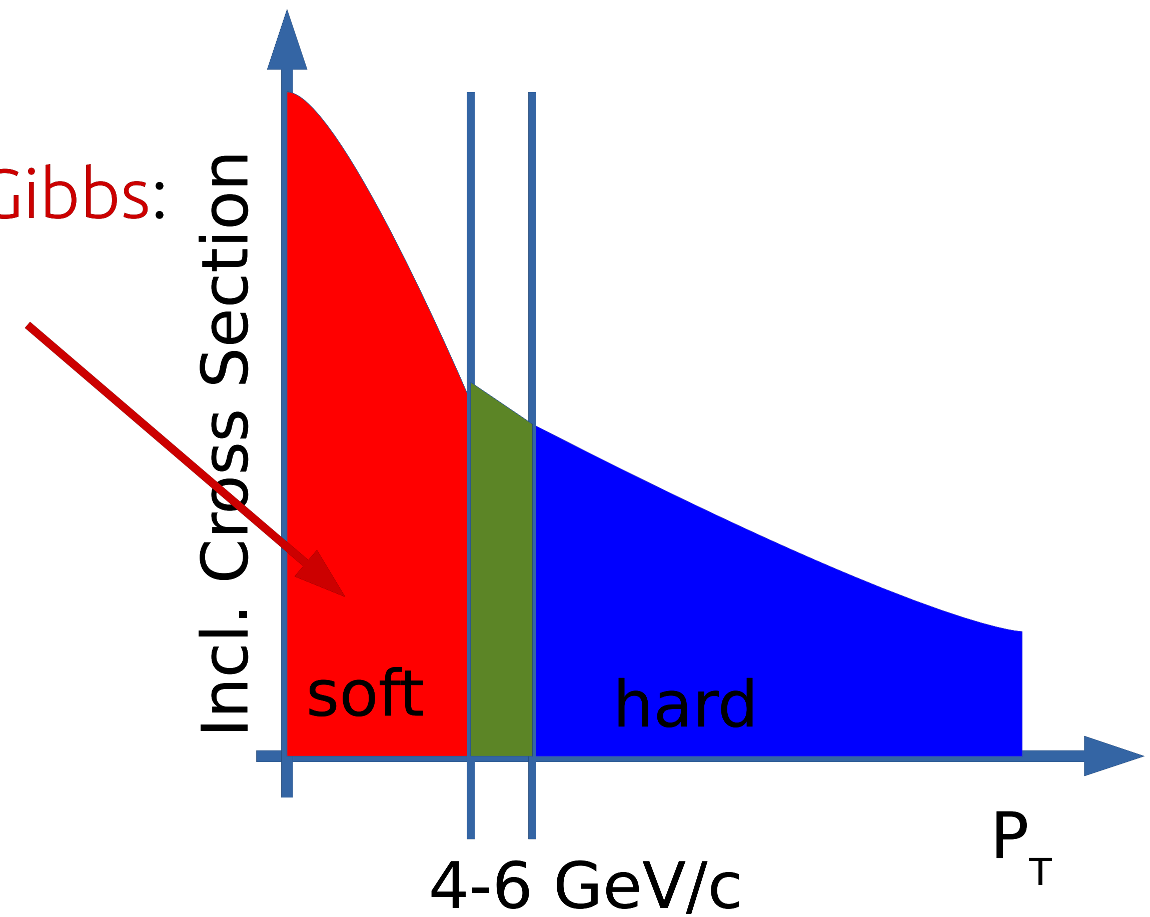
Why use Tsallis – Pareto distribution?



Why use Tsallis – Pareto distribution?

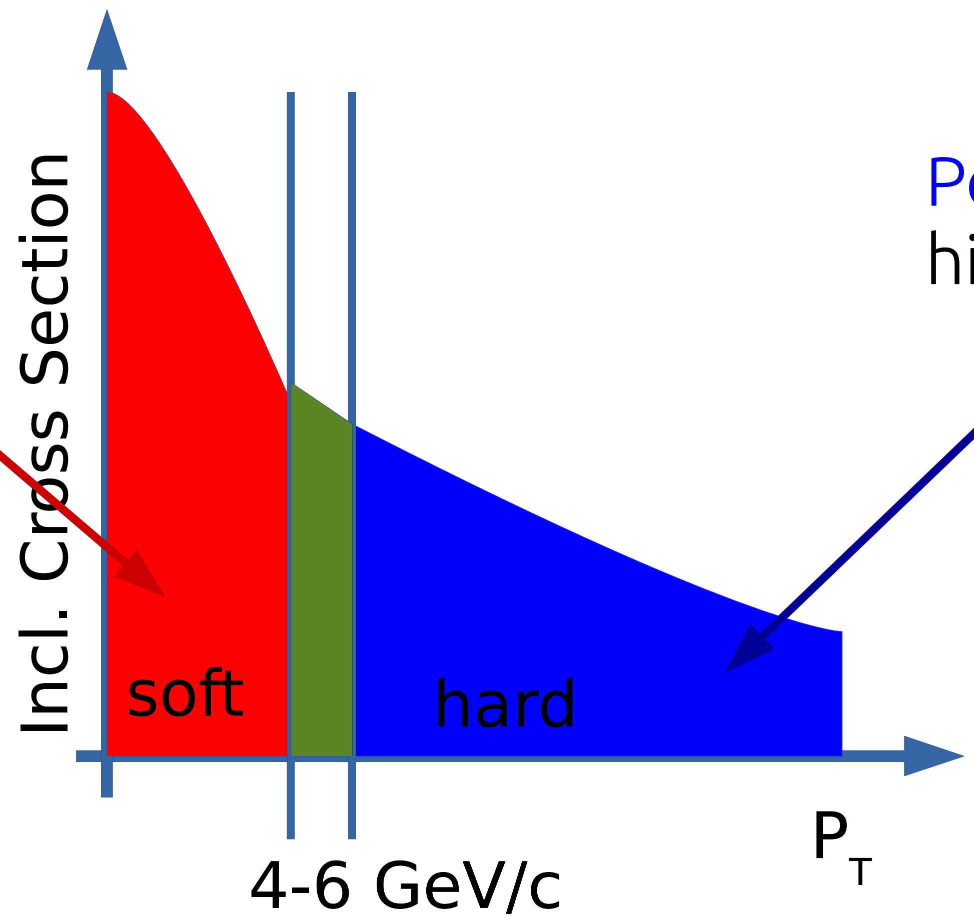
Boltzmann – Gibbs:
low momenta



Why use Tsallis – Pareto distribution?

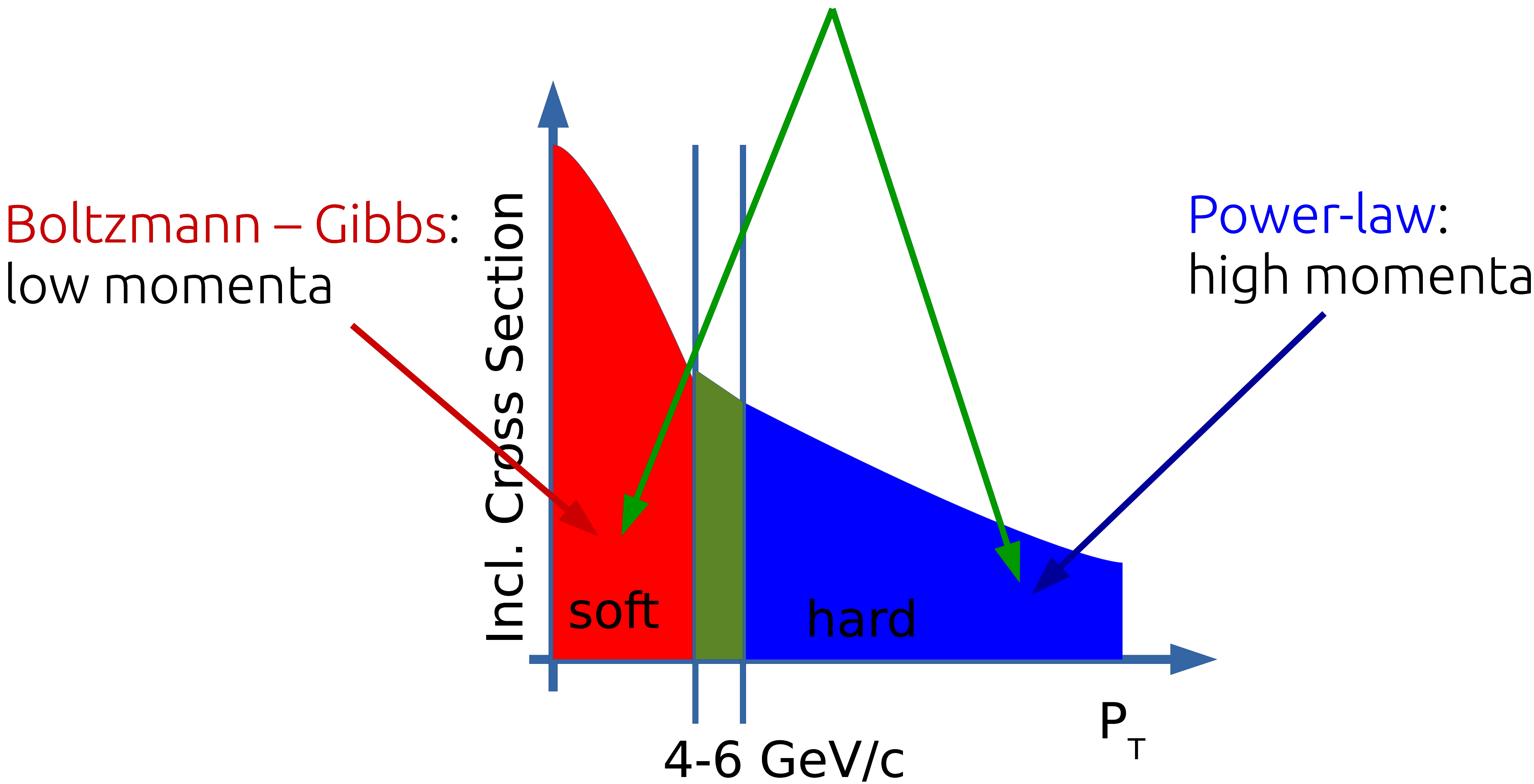
Boltzmann – Gibbs:
low momenta

Power-law:
high momenta



Why use Tsallis – Pareto distribution?

Tsallis – Pareto: whole mumentum range

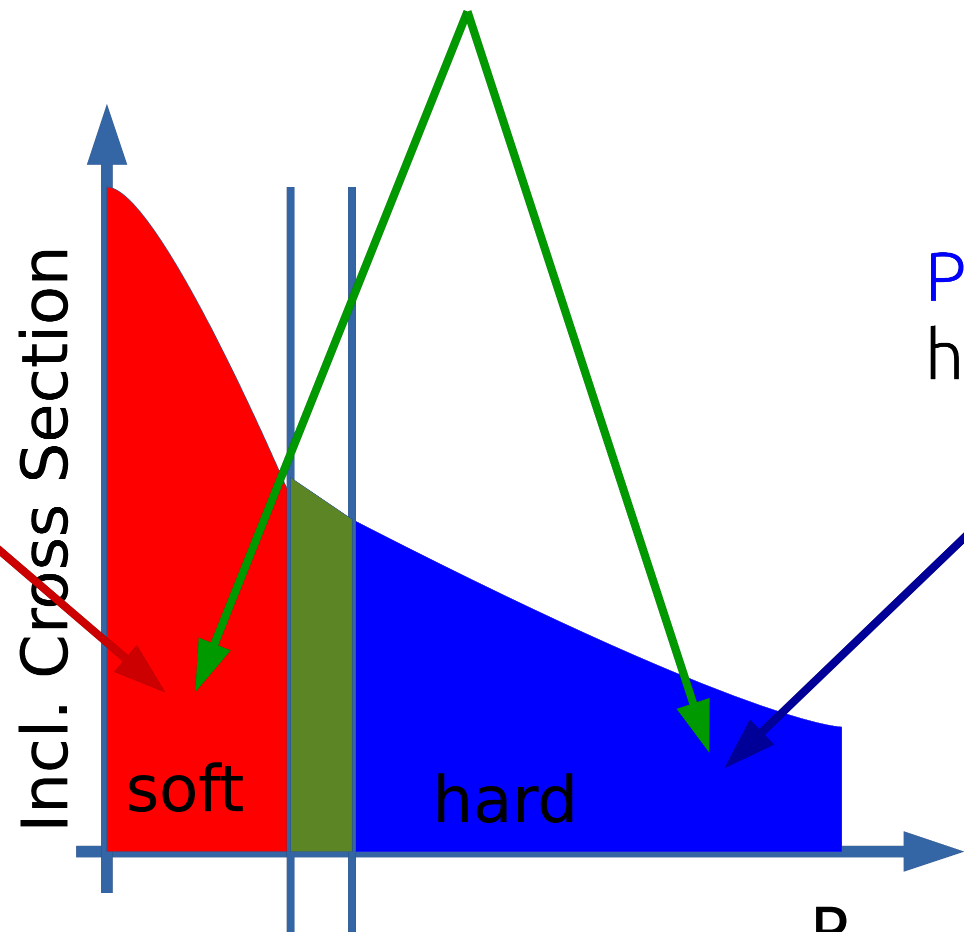


Why use Tsallis – Pareto distribution?

Tsallis – Pareto: ~~whole mumentum range~~

Boltzmann – Gibbs: low momenta

Power-law: high momenta

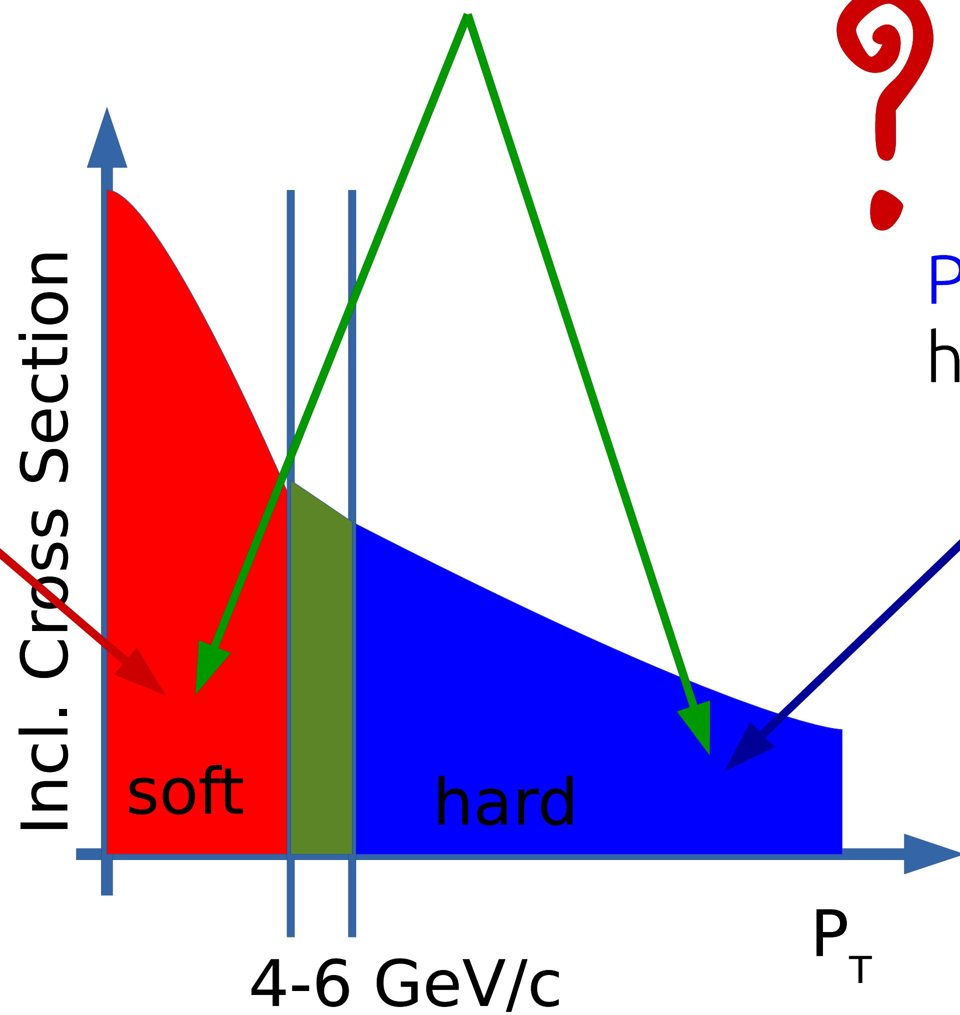


Why use Tsallis – Pareto distribution?

Tsallis – Pareto: ~~whole mumentum range~~

Boltzmann – Gibbs: low momenta

Power-law: high momenta

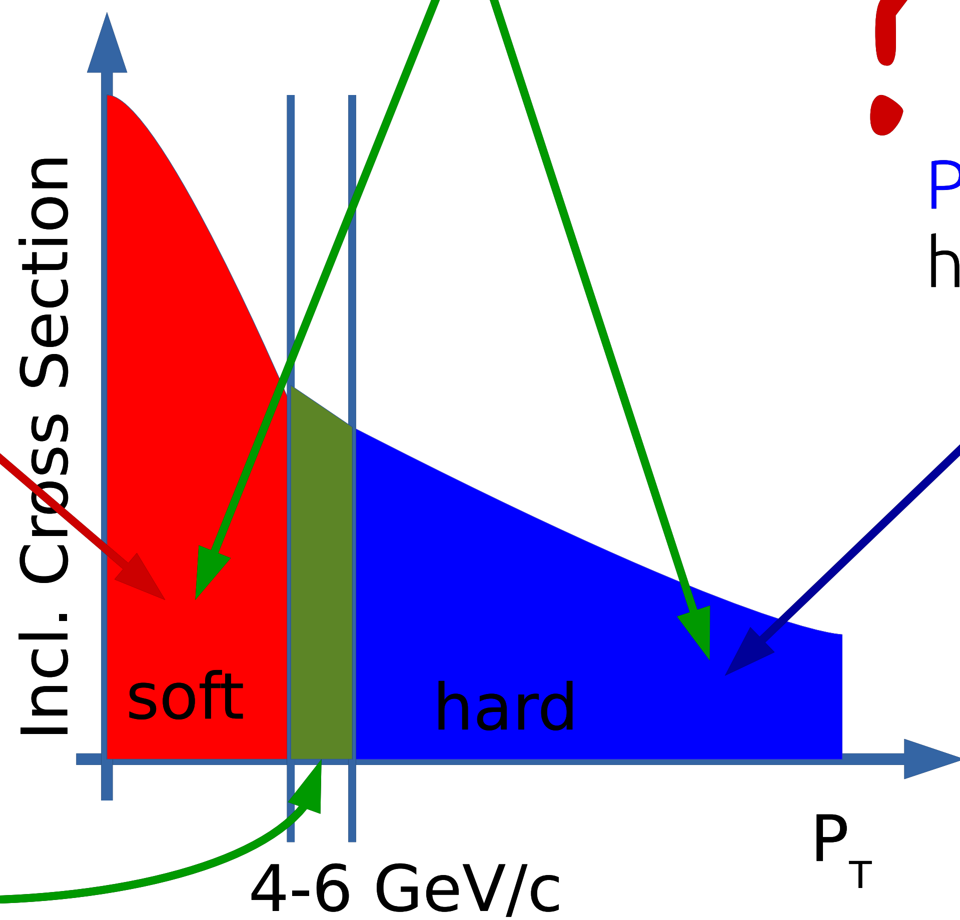


Why use Tsallis – Pareto distribution?

Tsallis – Pareto: ~~whole mumentum range~~

Boltzmann – Gibbs: low momenta

Power-law: high momenta



It was very good for **pp**, but in **AA** and **pA** collision there is a significant break!

- Simplest approximation: soft ('bulk') + hard ('jet') contribution

$$p^0 \frac{dN}{d^3p} = p^0 \frac{dN}{d^3p}^{hard} + p^0 \frac{dN}{d^3p}^{soft}$$

- Identified hadron spectra is given by double Tsallis–Pareto:

$$\left.\frac{dN}{2\pi p_T dp_T dy}\right|_{y=0} = f_{hard} + f_{soft} \qquad f_i = A_i \left[1 + \frac{(q_i - 1)}{T_i} \left[\gamma_i (m_T - v_i p_T) - m\right]\right]^{-1/(q_i - 1)}$$

in where parameters are given by

  - Lorentz factor $\gamma_i = 1/\sqrt{1 - v_i^2}$
  - Transverse mass $m_T = \sqrt{p_T^2 + m^2}$
  - Doppler temperature $T_i^{Dopp} = T_i \sqrt{\frac{1 + v_i}{1 - v_i}}$

ArXiv: 1405.3963, 1501.02352, 1501.05959

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1. **Two single Tsallis** fit:

   1) **1st Tsallis fit**: the range i$[p_0 - \varepsilon; p_{max}]$,

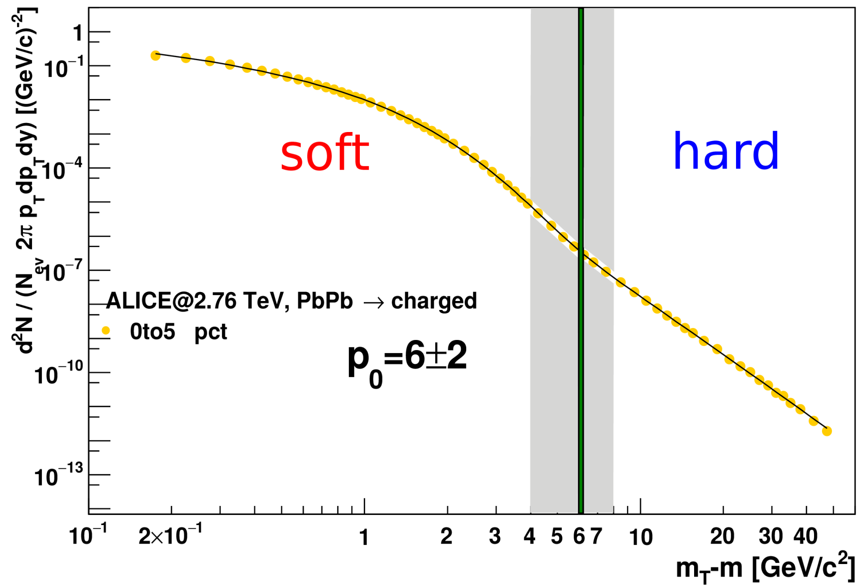
   the fitted parameters are:

   $q_{hard}, T_{hard}, A_{hard}, (v_{hard})$

   2) **2nd Tsallis fit**: the range is $[p_{min}; p_0 + \varepsilon]$

   the fitted parameters are:

   $q_{soft}, T_{soft}, A_{soft}, (v_{soft})$



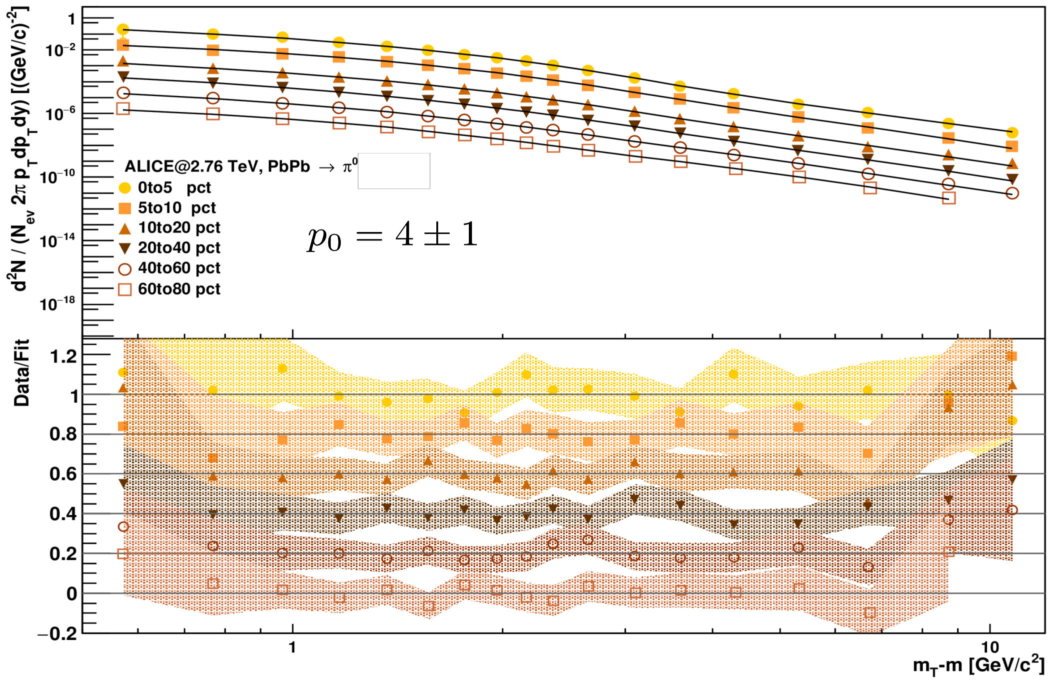
2. **Double Tsallis** fit, the range is $[p_0 - \varepsilon; p_{max}]$, the fitted parameters are the **hard** parameters

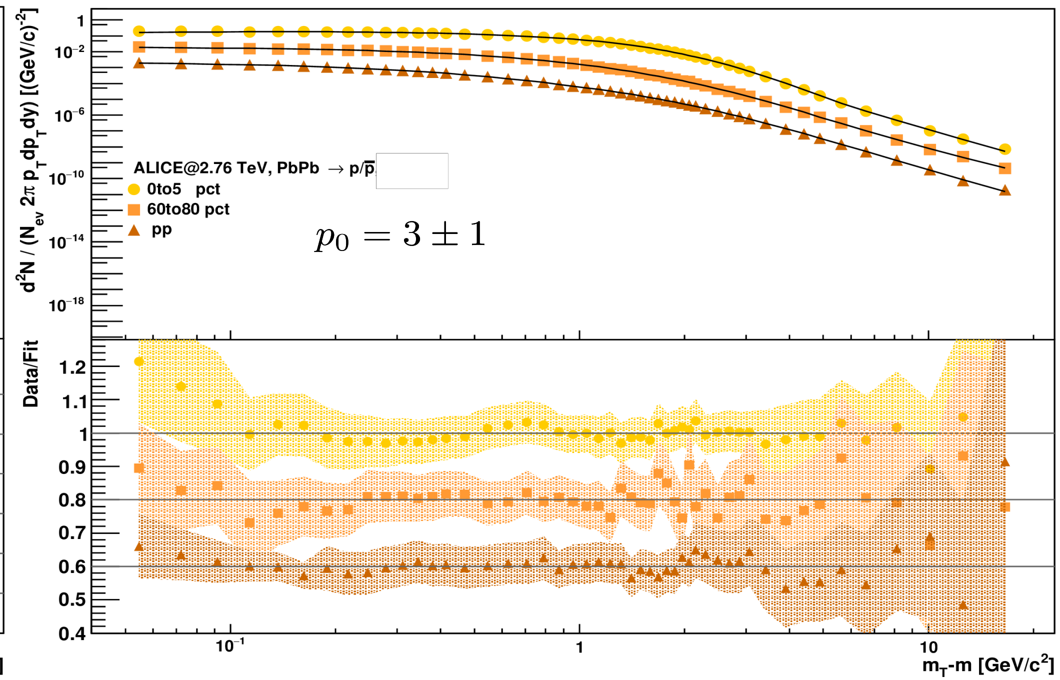
3. **Double Tsallis** fit, the range is $[p_{min}; p_0 + \varepsilon]$, the fitted parameters are the **soft** parameters

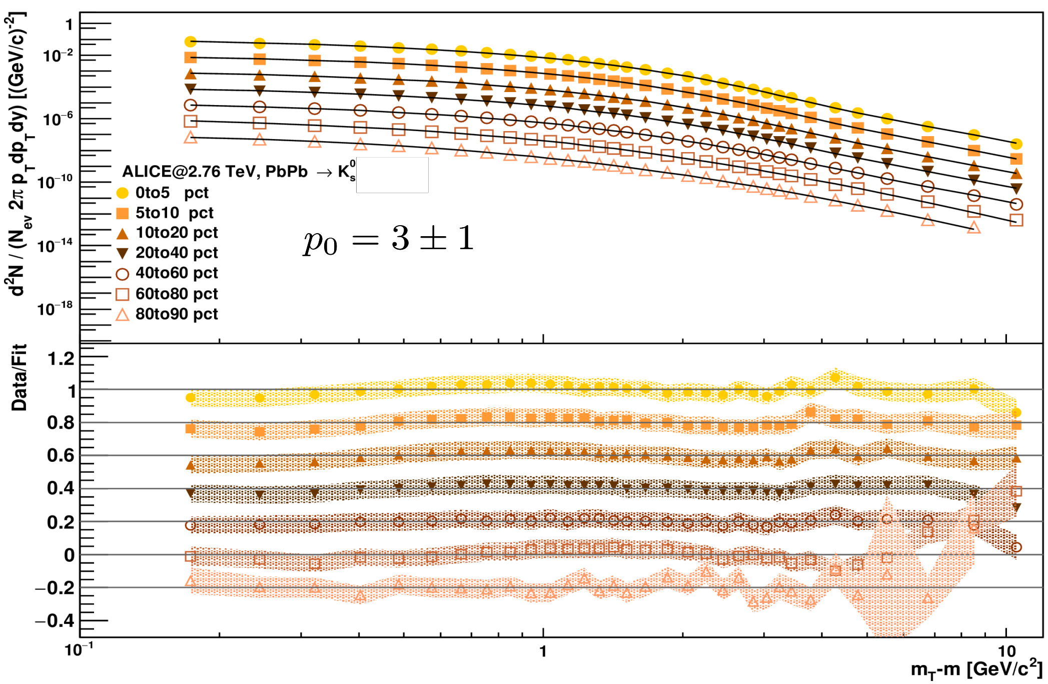
4. **Double Tsallis** fit, the range is $[p_{min}; p_{max}]$, **all** of the parameters are fitted

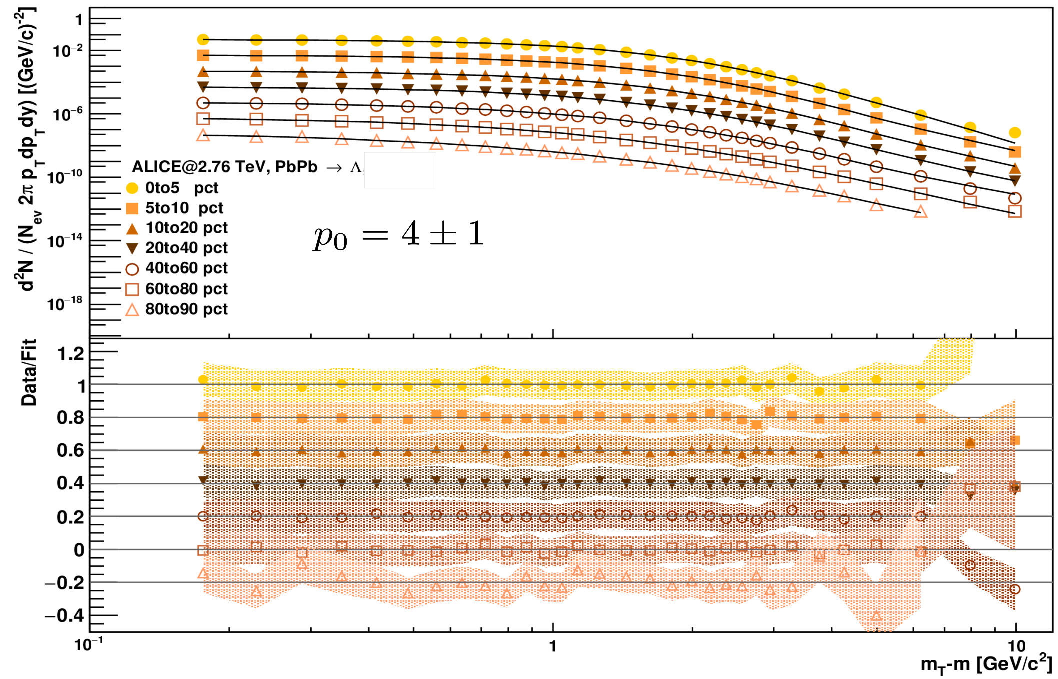
Results

Results – Id. particles







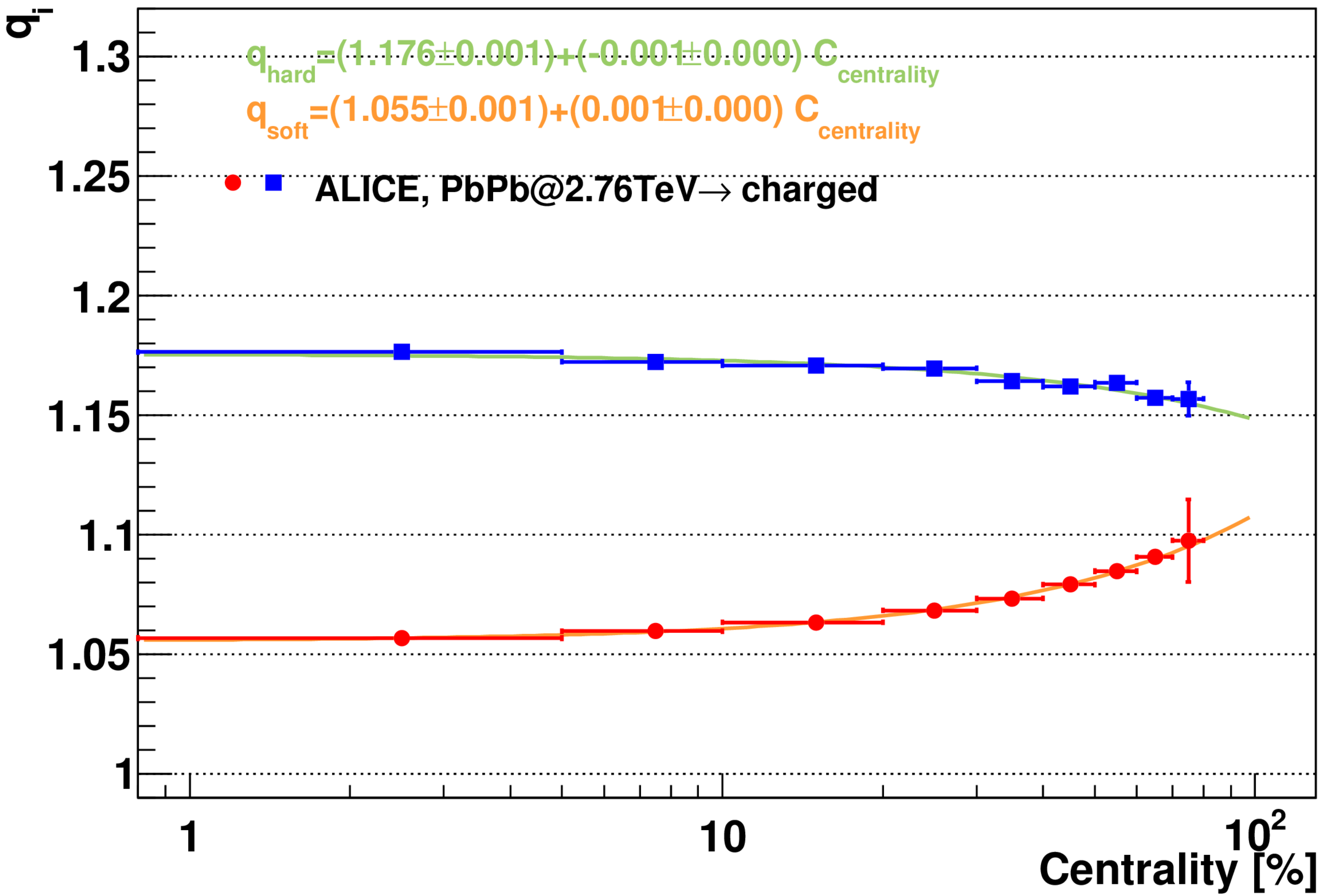


q

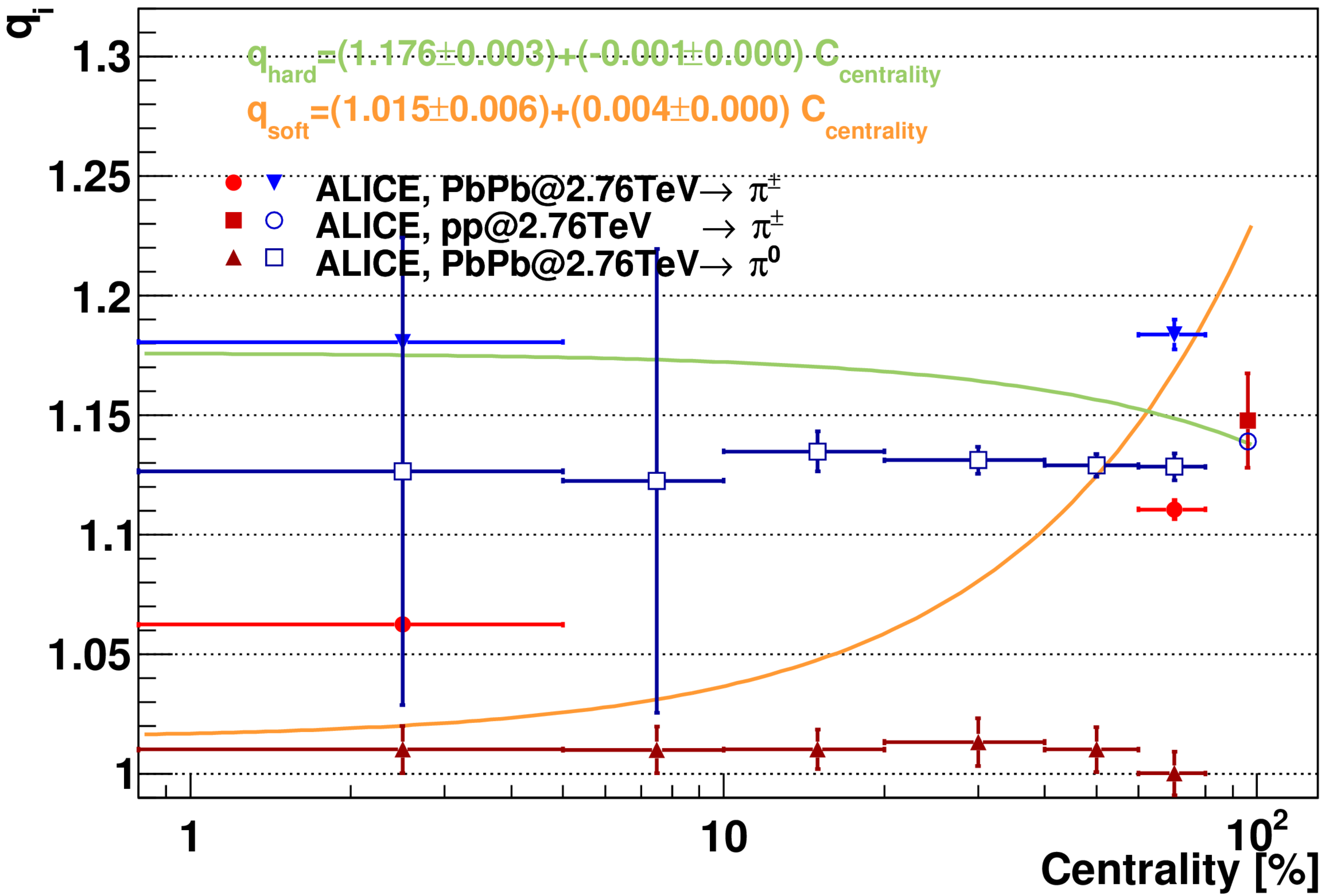
$$f_i = A_i \left[1 + \frac{(q_i - 1)}{T_i} \left[\gamma_i (m_T - v_i p_T) - m \right] \right]^{-1/(q_i - 1)}$$

Reminder: $$q = 1 + \frac{\Delta T^2}{T^2} - \frac{1}{C}$$

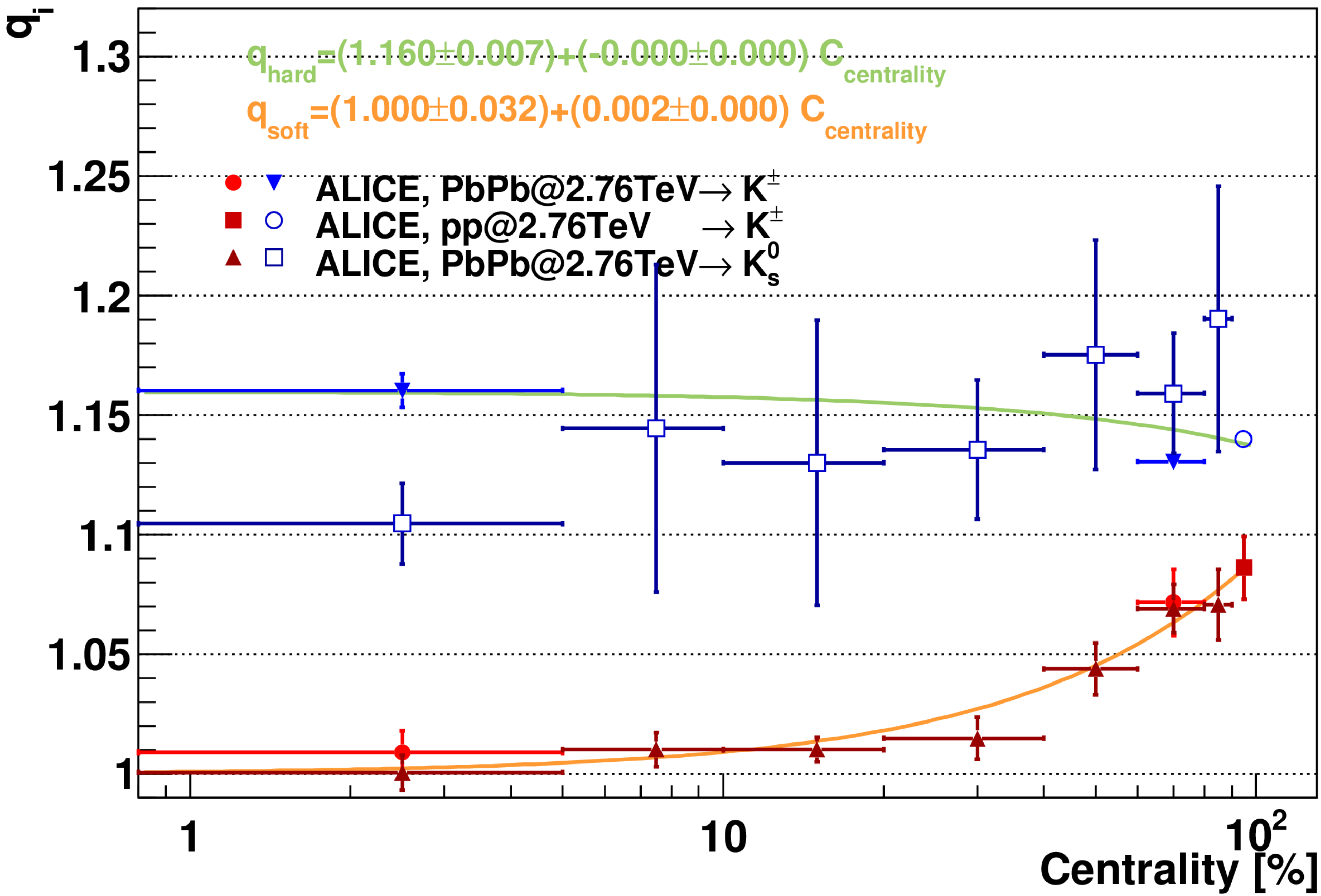
Results - charged



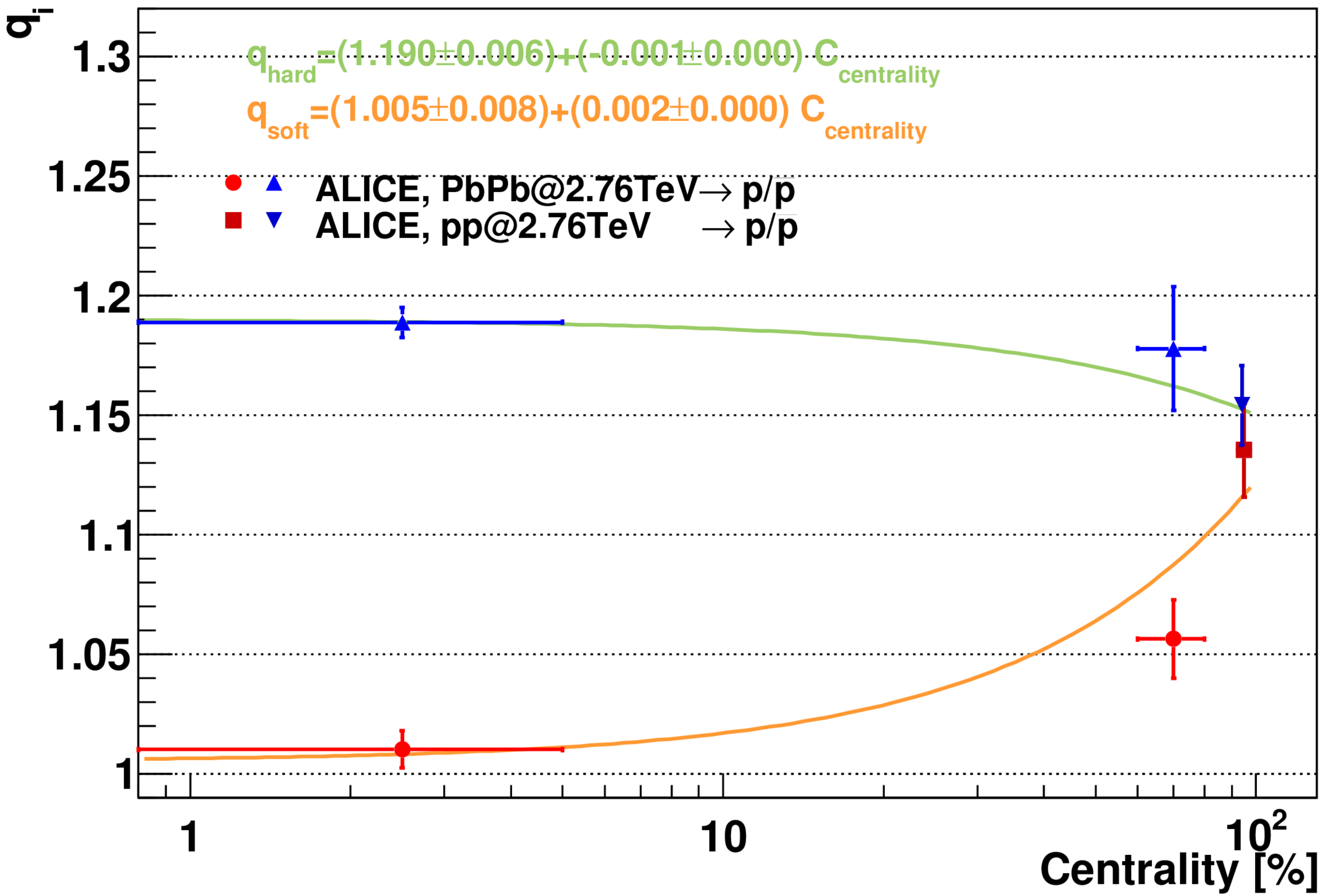
Results - Pion

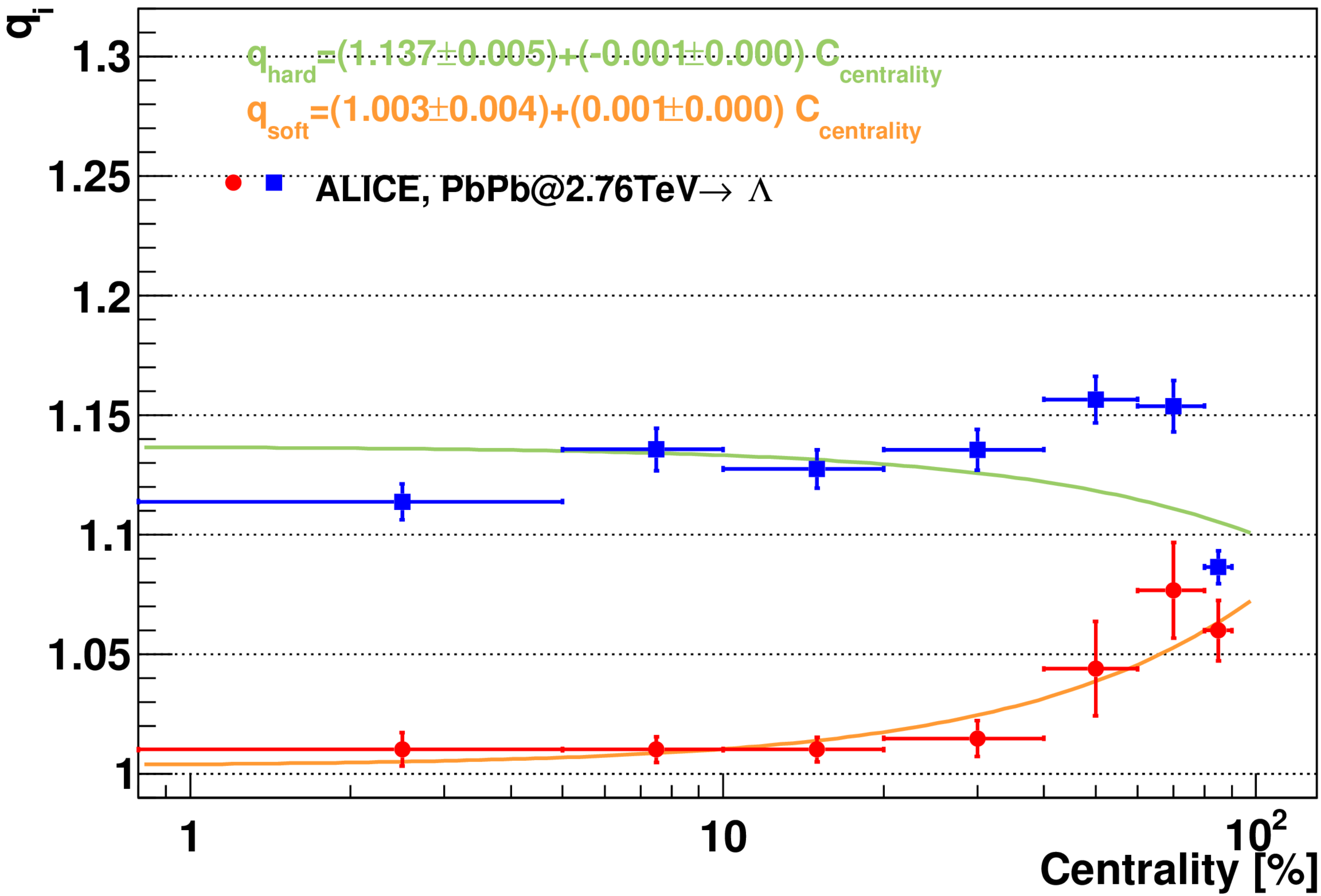


Results - Kaon



Results - (Anti)Proton



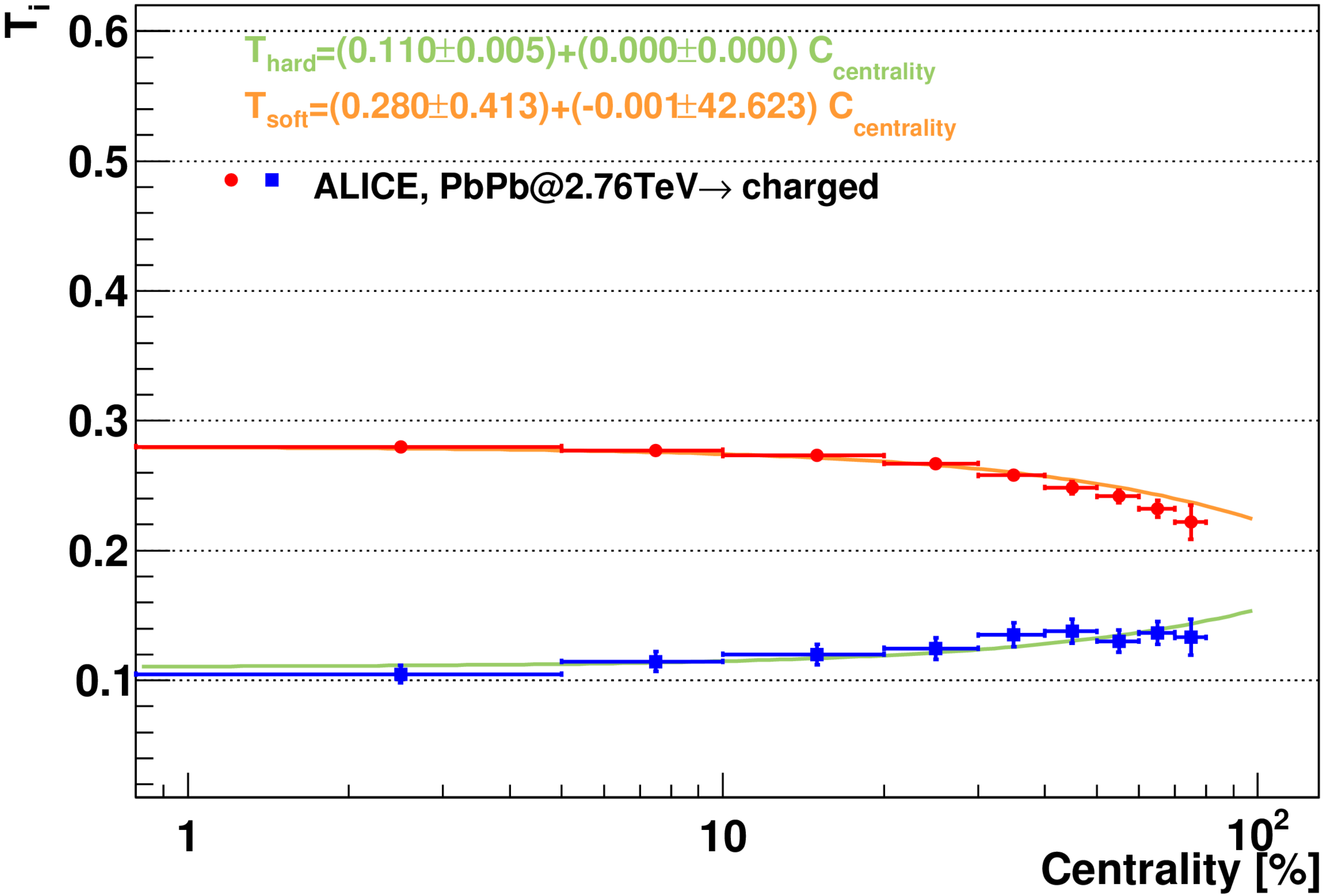

q_i
q_hard=(1.137±0.005)+(-0.001±0.000) C_centrality
q_soft=(1.003±0.004)+(0.001±0.000) C_centrality
ALICE, PbPb@2.76TeV→ Λ
1.3
1.25
1.2
1.15
1.1
1.05
1
1
10
10^2
Centrality [%]

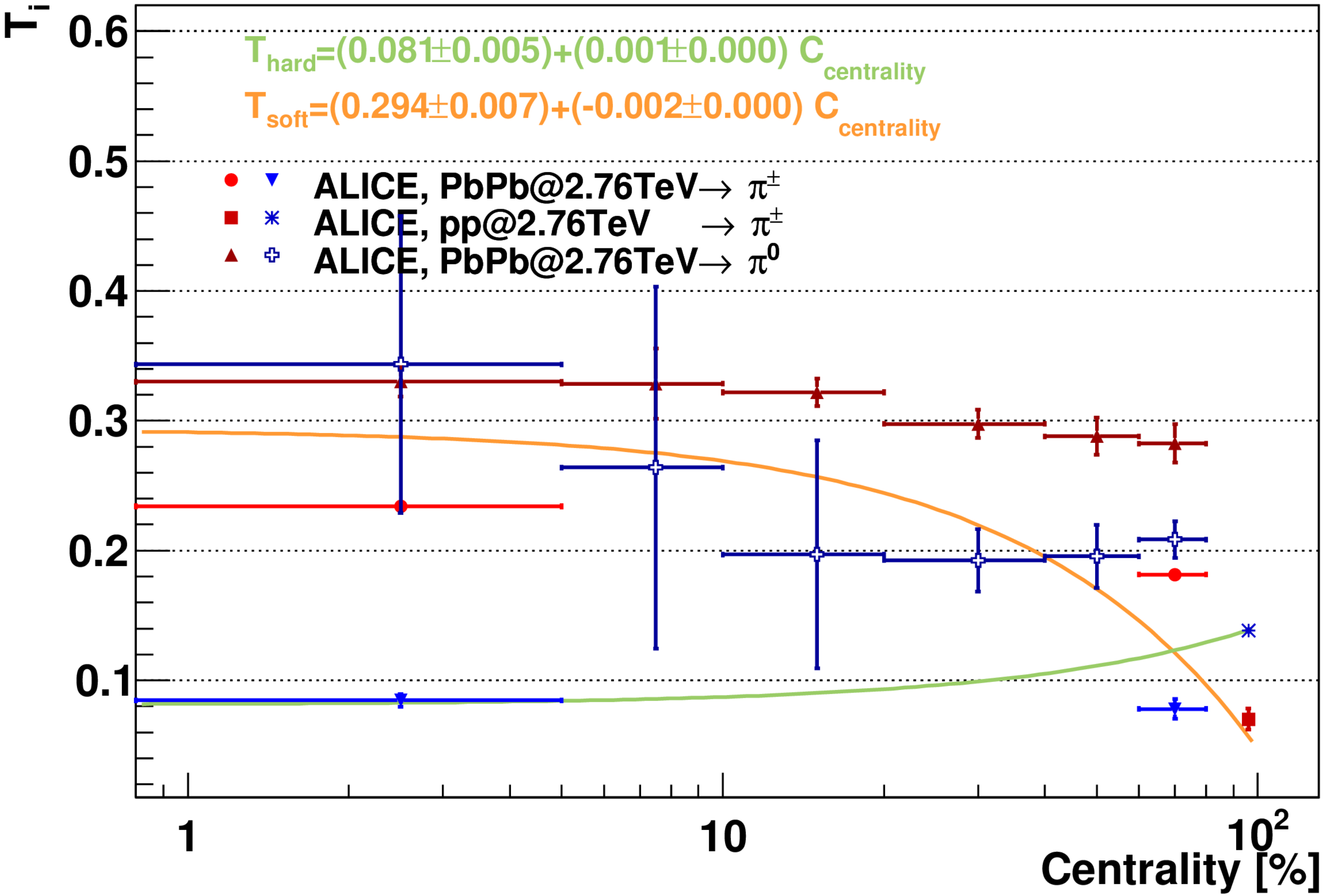
T

$$f_i = A_i \left[1 + \frac{(q_i - 1)}{T_i} \left[\gamma_i (m_T - v_i p_T) - m \right] \right]^{-1/(q_i - 1)}$$

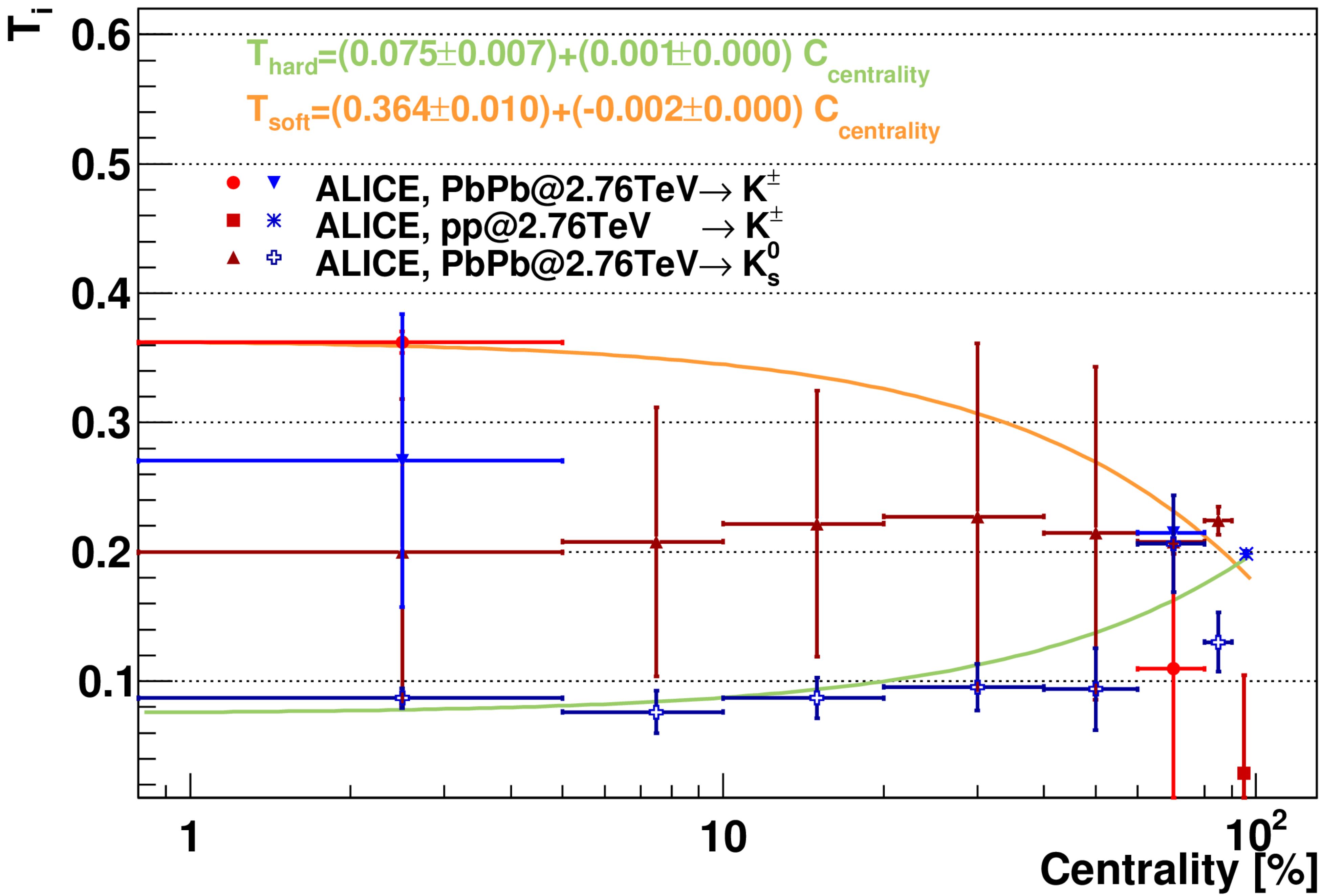
Reminder: $$\frac{E}{\langle n \rangle} = \frac{\int \varepsilon f_{TS}(\varepsilon)}{\int f_{TS}(\varepsilon)} = \frac{DT}{1 - (q-1)(D+1)}$$

Results - charged

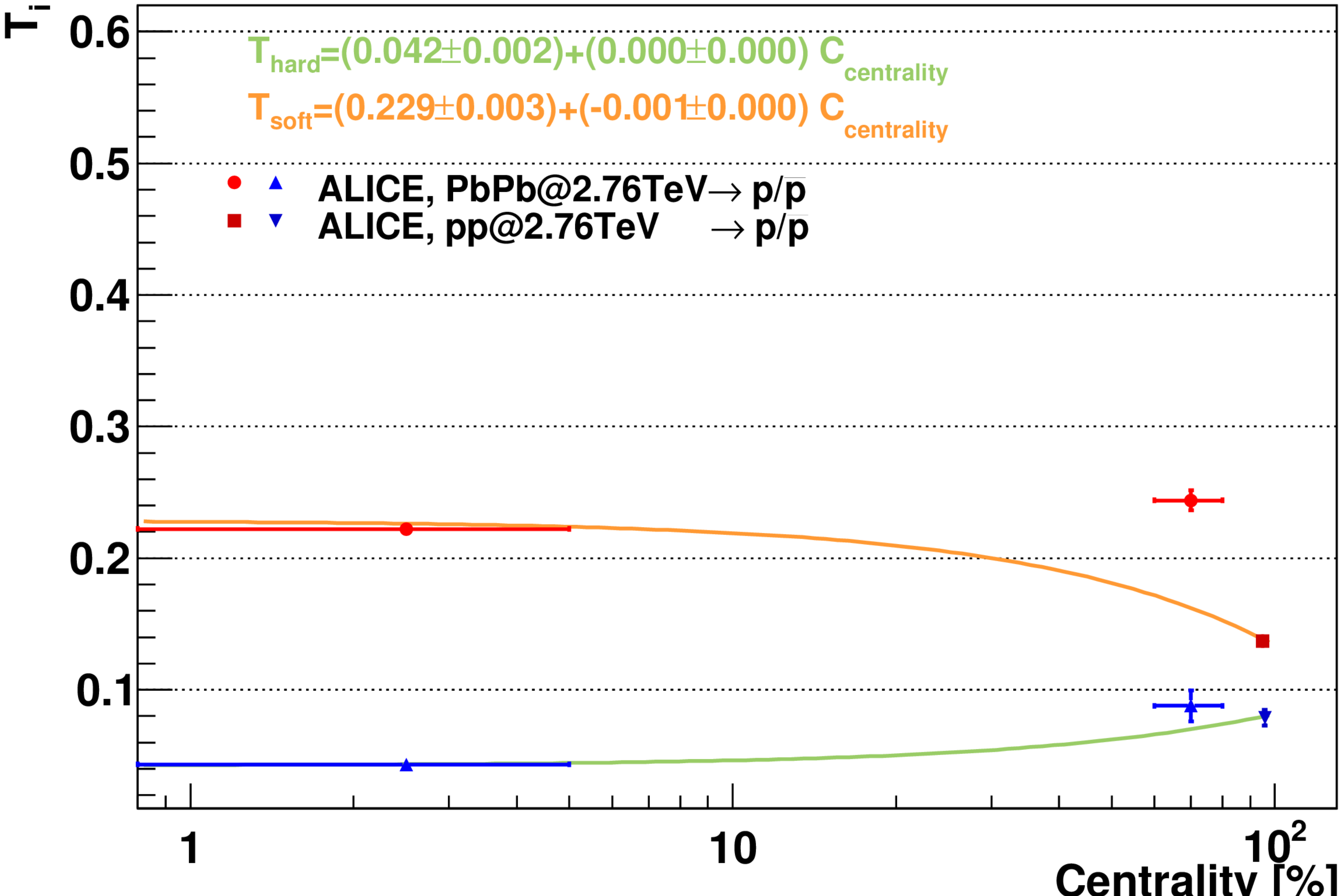


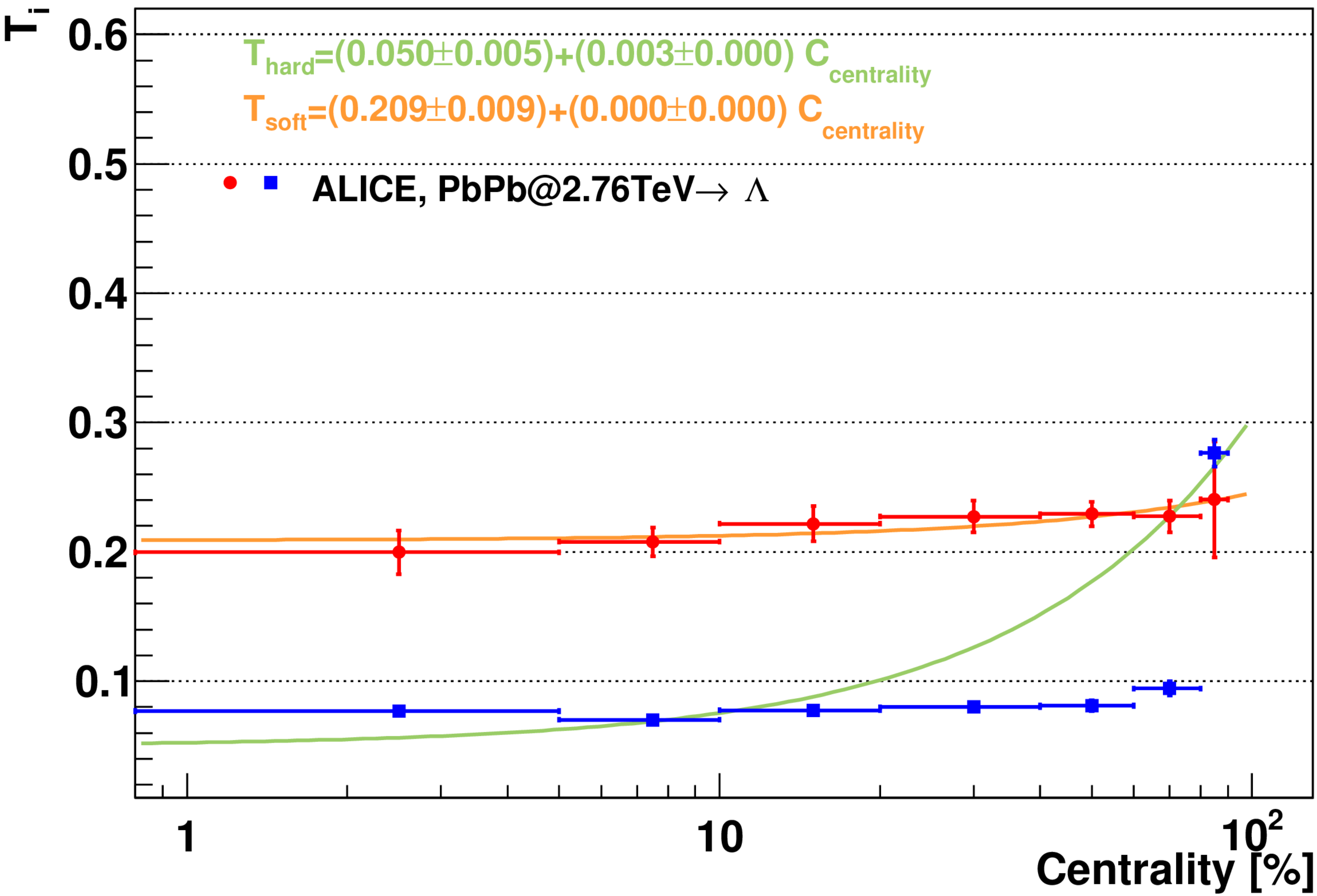

T_{hard}=(0.081±0.005)+(0.001±0.000) $C_{centrality}$
T_{soft}=(0.294±0.007)+(-0.002±0.000) $C_{centrality}$
ALICE, PbPb@2.76TeV → $\pi^{\pm}$
ALICE, pp@2.76TeV → $\pi^{\pm}$
ALICE, PbPb@2.76TeV → π^{0}
T_i
0.6
0.5
0.4
0.3
0.2
0.1
1
10
10^2
Centrality [%]

Results - Kaon



Results - (Anti)Proton

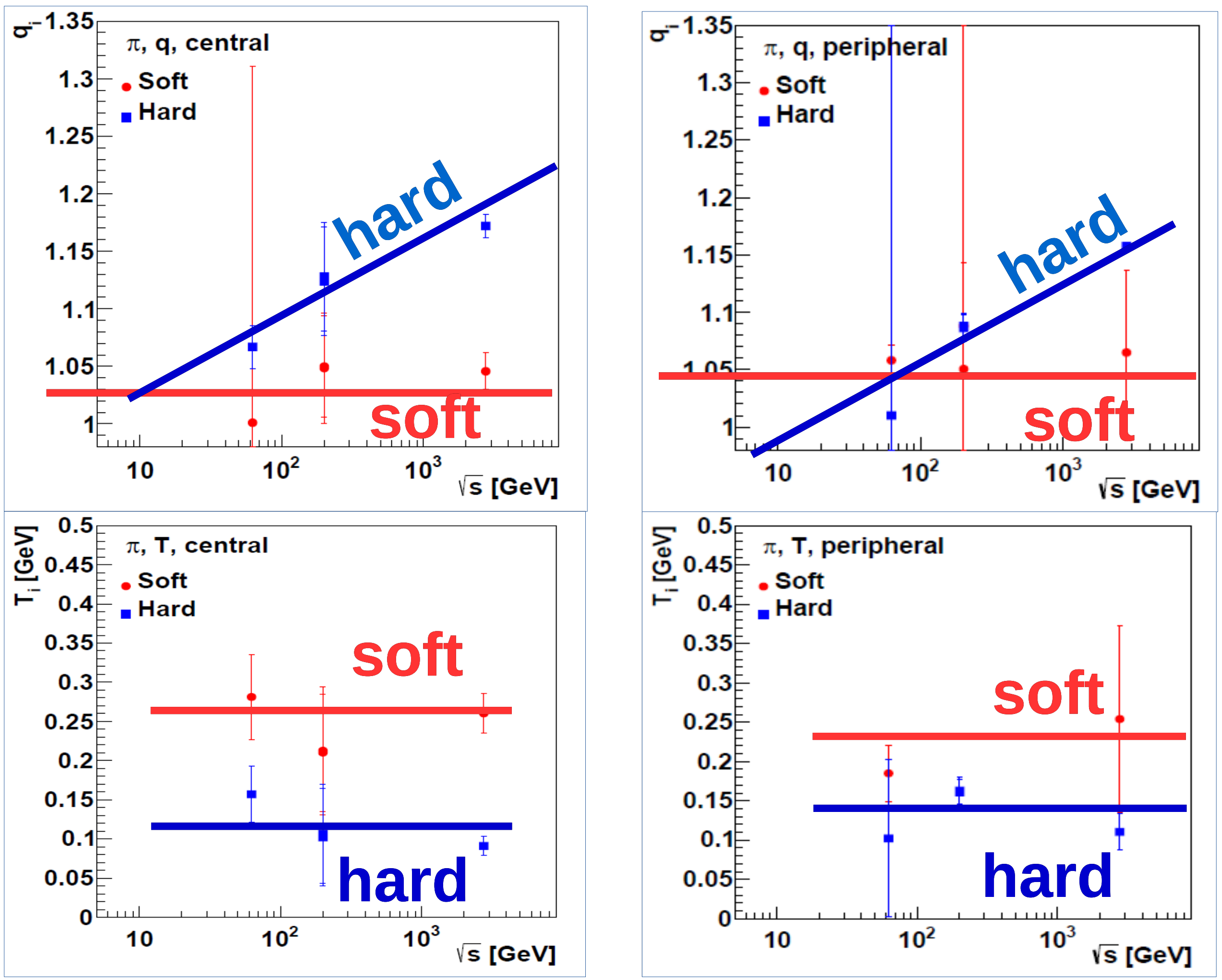




T_{hard}=(0.050±0.005)+(0.003±0.000) $C_{centrality}$
T_{soft}=(0.209±0.009)+(0.000±0.000) $C_{centrality}$
ALICE, PbPb@2.76TeV→ Λ
T_i
0.6
0.5
0.4
0.3
0.2
0.1
1
10
10^2
Centrality [%]

Summary

- Clear separation of the soft and hard components
- Strong centrality dependence
- There is a significant N_{part} and $\sqrt{s}$ dependence too



Connecting spectra and v_2

- Spectra originating from hadronic sources

$$p^0 \frac{dN}{d^3p}\bigg|_{y=0} = \int_{-\infty}^{+\infty} d\zeta \int_0^{2\pi} d\alpha\, f[u_\mu p^\mu] \quad \Rightarrow \quad \frac{dN}{2\pi p_T dp_T dy}\bigg|_{y=0} = \int_0^{2\pi} \frac{d\varphi}{2\pi} p^0 \frac{dN}{d^3p}\bigg|_{y=0}$$

where we used parameters and assumptions:

- Hadron momenum: $p^\mu = (m_T \cosh y, m_T \sinh y, p_T \cos\varphi, p_T \sin\varphi)$
- Cylindric symmetry: $u^\mu = (\gamma \cosh\zeta, \gamma \sinh\zeta, \gamma v \cos\alpha, \gamma v \sin\alpha)$
  - where $\zeta = \frac{1}{2}\ln\left[(t+z)/(t-z)\right]$ and $\gamma = 1/\sqrt{1-v^2}$
  - Co-moving energy: $u_\mu p^\mu|_{y=0} = \gamma\left[m_T \cosh\zeta - v p_T \cos(\varphi - \alpha)\right]$
  - Transverse flow: $v(\alpha) = v_0 + \sum_{m=1}^{\infty} \delta v_m \cos(m\alpha) \equiv v_0 + \delta v(\alpha)$
  - Taylor expansion: $f[u_\mu p^\mu]|_{y=0} = \sum_{m=0}^{\infty} \frac{[\delta v(\alpha)]^m}{m!} \frac{\partial^m}{\partial v_0^m} f[u_\mu p^\mu]|_{y=0}^{v(\alpha)=v_0}$

- Spectra originating from hadronic sources

$$\frac{dN}{2\pi p_T dp_T dy}\bigg|_{y=0} = \int\limits_0^{2\pi} \frac{d\varphi}{2\pi} p^0 \left.\frac{dN}{d^3p}\right|_{y=0} = \sum_{m=0}^{\infty} \frac{a_m}{m!}\frac{\partial^m}{\partial v_0^m} f[E(v_0)] \approx f[E(v_0)] + O(\delta v^2)$$

where $E(v_0) = \gamma_0(m_T - v_0 p_T)$ and $a_m = \int\limits_0^{2\pi} d\alpha[\delta v(\alpha)]^m$

- Azimuthal anisotropy:

$$v_n = \frac{\int\limits_0^{2\pi} d\varphi \cos(n\varphi) p^0 \left.\frac{dN}{d^3p}\right|_{y=0}}{\int\limits_0^{2\pi} d\varphi p^0 \left.\frac{dN}{d^3p}\right|_{y=0}} \approx \frac{\delta v_n \gamma_0^3}{2}\frac{(v_0 m_T - p_T) f'[E(v_0)]}{f[E(v_0)]} + O(\delta v^2)$$

  - Boltzmann–Gibbs: $f \sim \exp\left[-\beta E(v_0)\right]$ → $v_n^{BG} \approx \frac{\delta v_n \beta \gamma_0^3}{2}(p_T - v_0 m_T) + O(\delta v^2)$
  - Tsallis–Pareto: $f \sim [1 + (q-1)\beta E(v_0)]^{-1/(q-1)}$ → $v_n^{TS} \approx \frac{\delta v_n \beta \gamma_0^3}{2}\frac{p_T - v_0 m_T}{1 + (q-1)\beta\gamma_0(m_T - v_0 p_T)} + O(\delta v^2)$

- Spectra originating from hadronic sources

$$\left.\frac{dN}{2\pi p_T dp_T dy}\right|_{y=0} = \int_0^{2\pi} \frac{d\varphi}{2\pi} p^0 \left.\frac{dN}{d^3p}\right|_{y=0} = \sum_{m=0}^{\infty} \frac{a_m}{m!} \frac{\partial^m}{\partial v_0^m} f[E(v_0)] \approx f[E(v_0)] + O(\delta v^2)$$

where $E(v_0) = \gamma_0(m_T - v_0 p_T)$ and $a_m = \int_0^{2\pi} d\alpha [\delta v(\alpha)]^m$

- Azimuthal anisotropy:

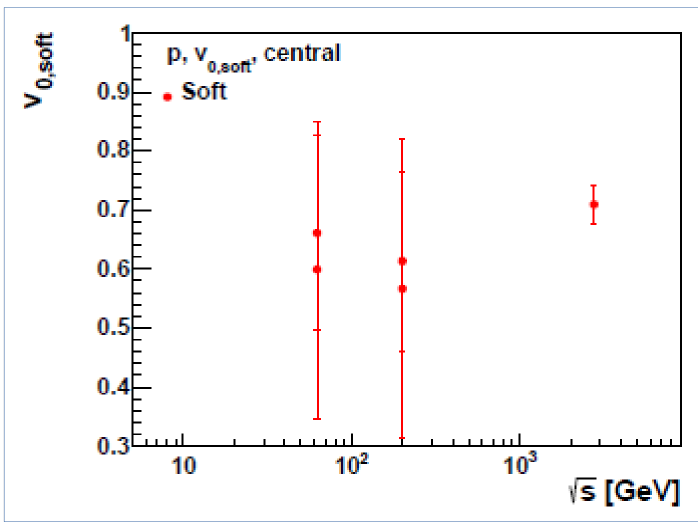
$$v_n = \frac{\int_0^{2\pi} d\varphi \cos(n\varphi) p^0 \left.\frac{dN}{d^3p}\right|_{y=0}}{\int_0^{2\pi} d\varphi p^0 \left.\frac{dN}{d^3p}\right|_{y=0}} \approx \frac{\delta v_n \gamma_0^3}{2} \frac{(v_0 m_T - p_T) f'[E(v_0)]}{f[E(v_0)]} + O(\delta v^2)$$

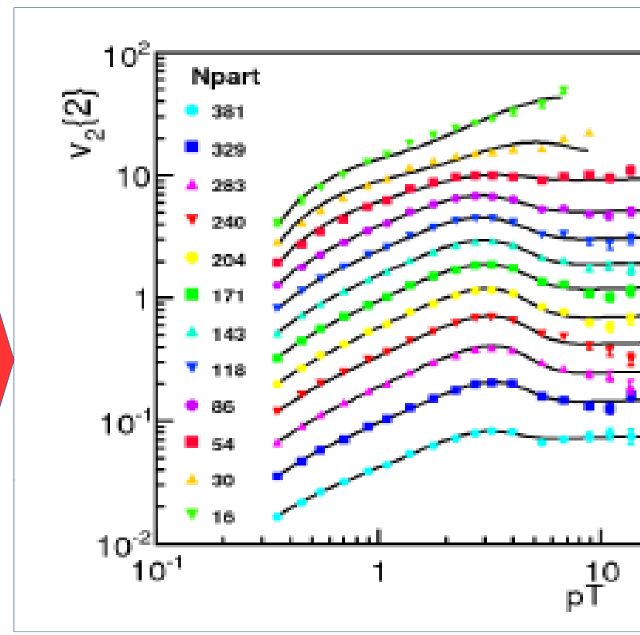
- Using the soft+hard model:

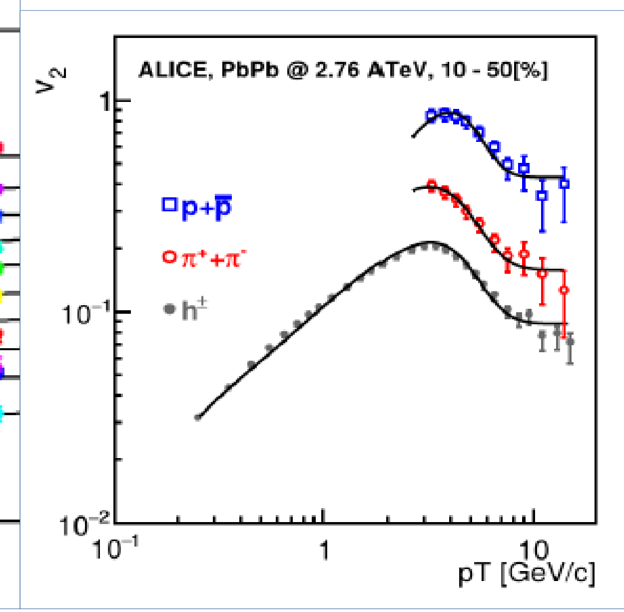
$$v_2 = \frac{w_{hard} f_{hard} + w_{soft} f_{soft}}{f_{hard} + f_{soft}}$$ with the coefficient $$w_i = \frac{\delta v_i \gamma_i^3}{2T_i} \frac{p_T - v_i m_T}{1 + \frac{q_i - 1}{T_i}[\gamma_i(m_T - v_i p_T) - m]}$$

- Using the soft+hard model:

$$v_2 = \frac{w_{hard} f_{hard} + w_{soft} f_{soft}}{f_{hard} + f_{soft}}$$ with the coefficient $$w_i = \frac{\delta v_i \gamma_i^3}{2T_i} \frac{p_T - v_i m_T}{1 + \frac{q_i - 1}{T_i}[\gamma_i(m_T - v_i p_T) - m]}$$







ArXiv: 1405.3963, 1501.02352, 1501.05959

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- **Non-extensive statistical approach HIC**
  - Providing phyiscal meaning of q: $q = 1 - \frac{1}{C} + \frac{\Delta T^2}{T^2}$
  - Boltzmann – Gibbs limit: $C \to \infty, \quad \frac{\Delta T^2}{T^2} \to 0 \quad (q \to 1)$
  - Tsallis – Pareto fits on spectra in e^+e^-, pp
  - Doesn't work for larger system, like pA, AA
- **Application of 'soft+hard' model in AA**
  - Double Tsallis – Pareto measures non-extensitivity
  - SOFT: q → 1, suggest Boltzmann – Gibbs (QGP)
  - HARD: q > 1.1, Tsallis – Pareto like
  - Asimuthal anisotropy can be obtained too
- **Future plans**
  - Investigation of multiplicity-, centrality- and energy dependency

Thank you for your attention!

References

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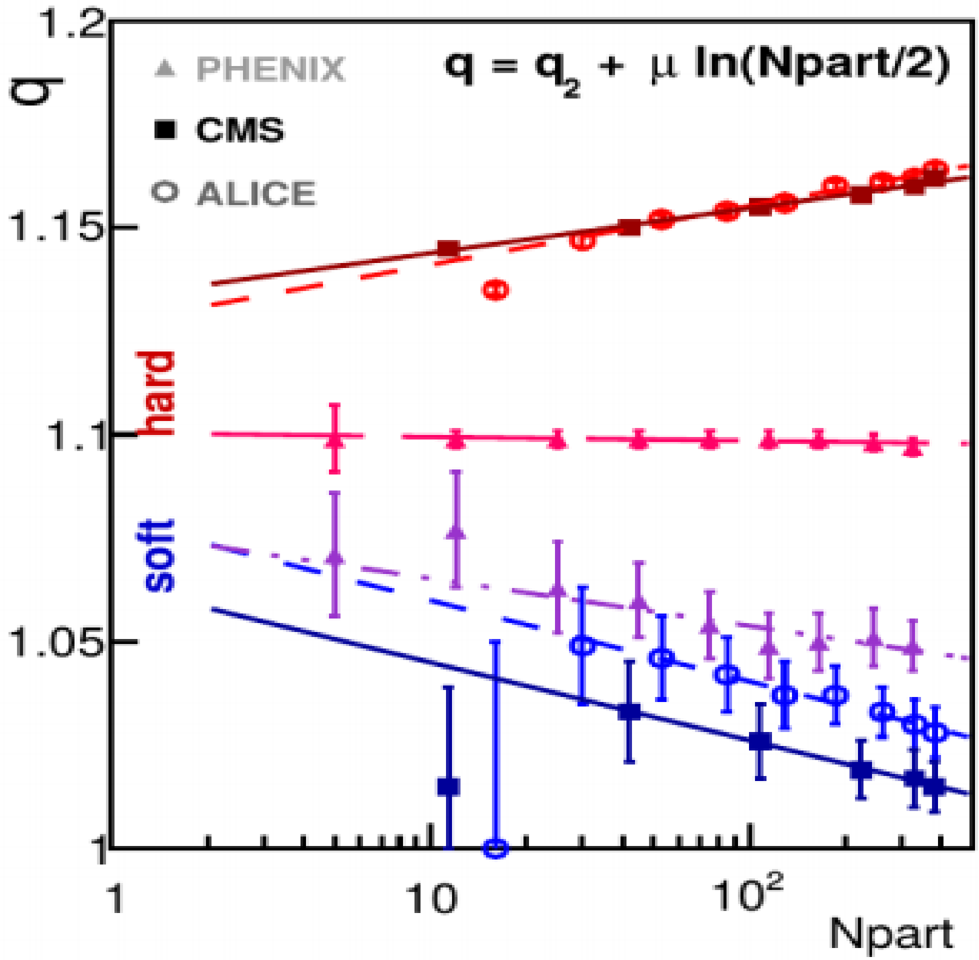
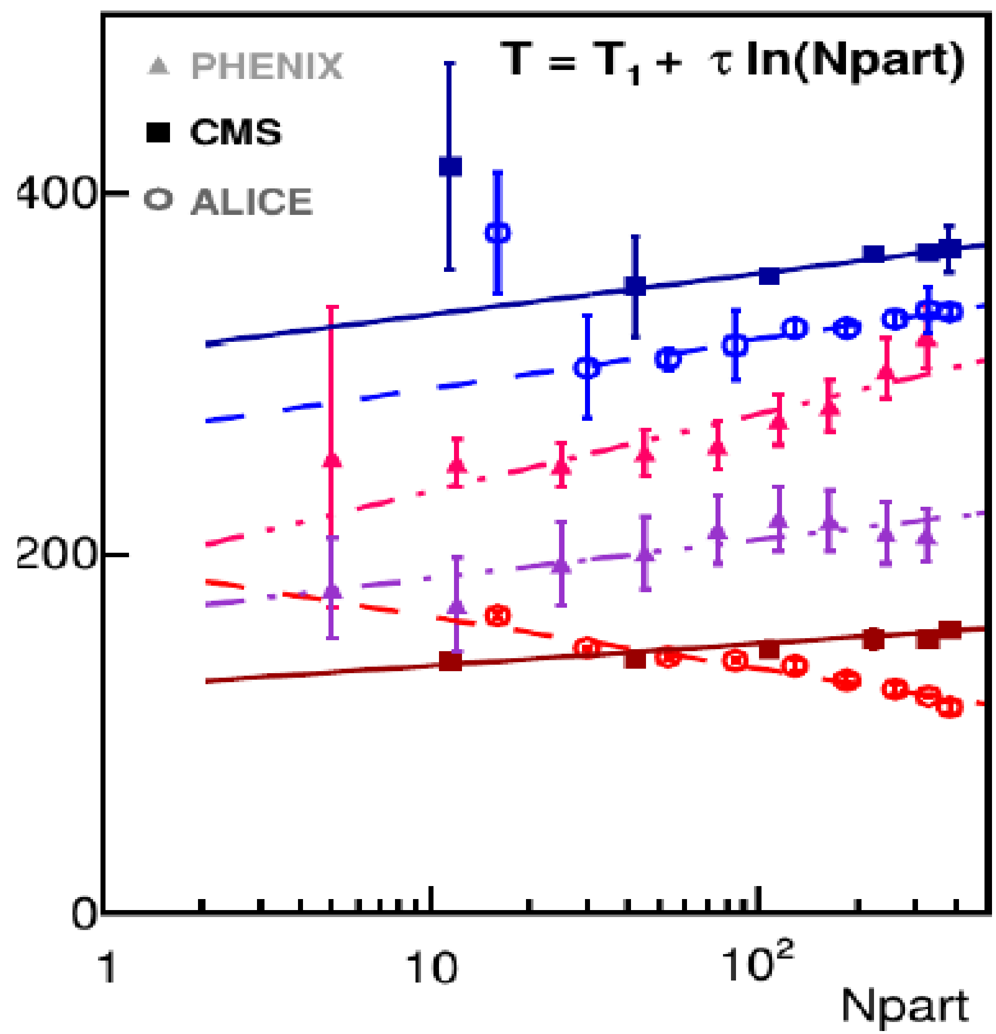
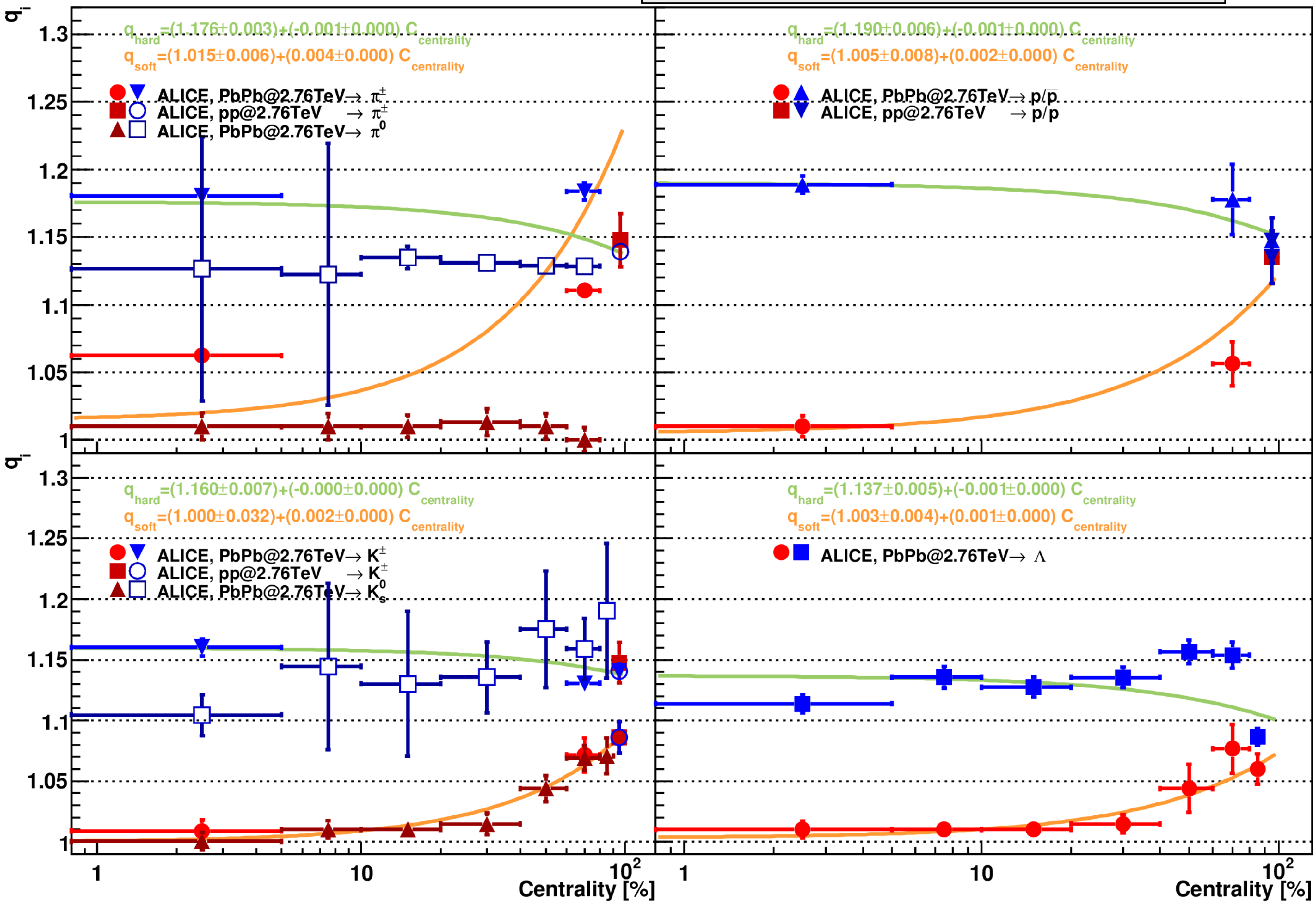
9. arXiv:1101.3023 Generalised Tsallis Statistics in Electron-Positron Collisions, Phys.Lett.B701:111-116,2011
10. arXiv:0802.0381 Pion and Kaon Spectra from Distributed Mass Quark Matter, J.Phys.G35:044012,2008

Backup

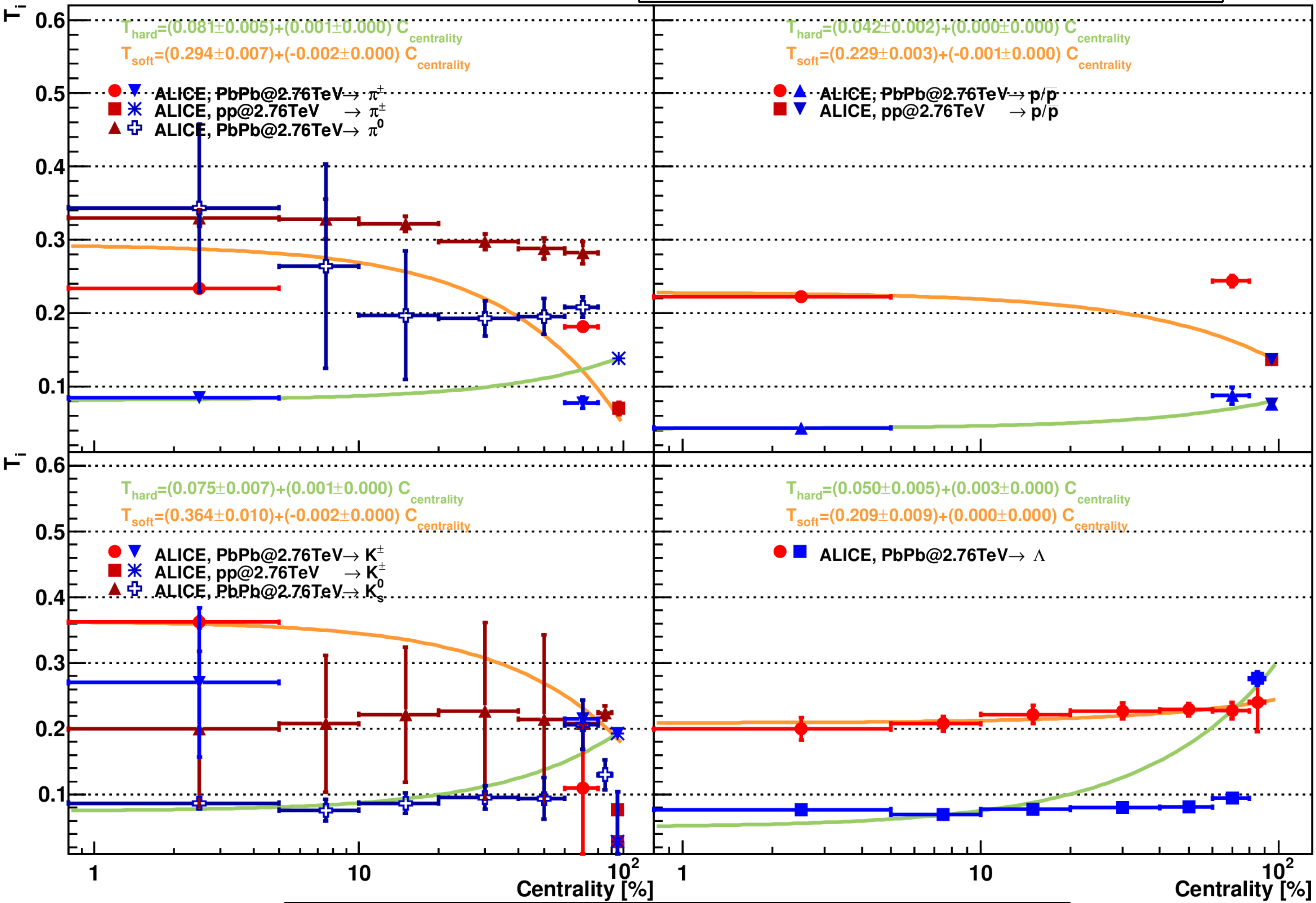

q
1.2
1.15
1.1
1.05
1
PHENIX
CMS
ALICE
q = q2 + μ ln(Npart/2)
hard
soft
1
10
10²
Npart

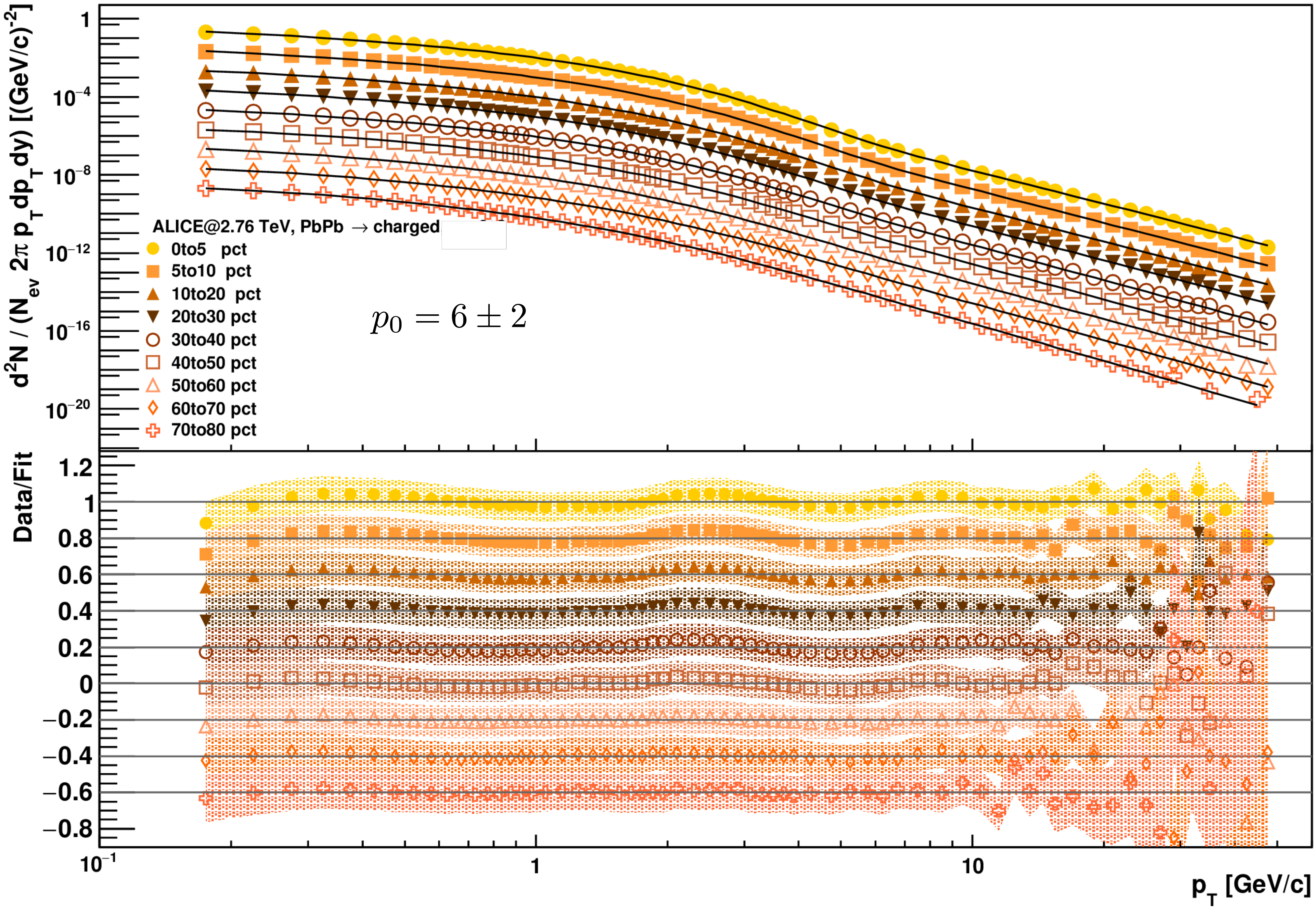

PHENIX
CMS
ALICE
T = T1 + τ ln(Npart)
400
200
0
1
10
10²
Npart

Results – q trends



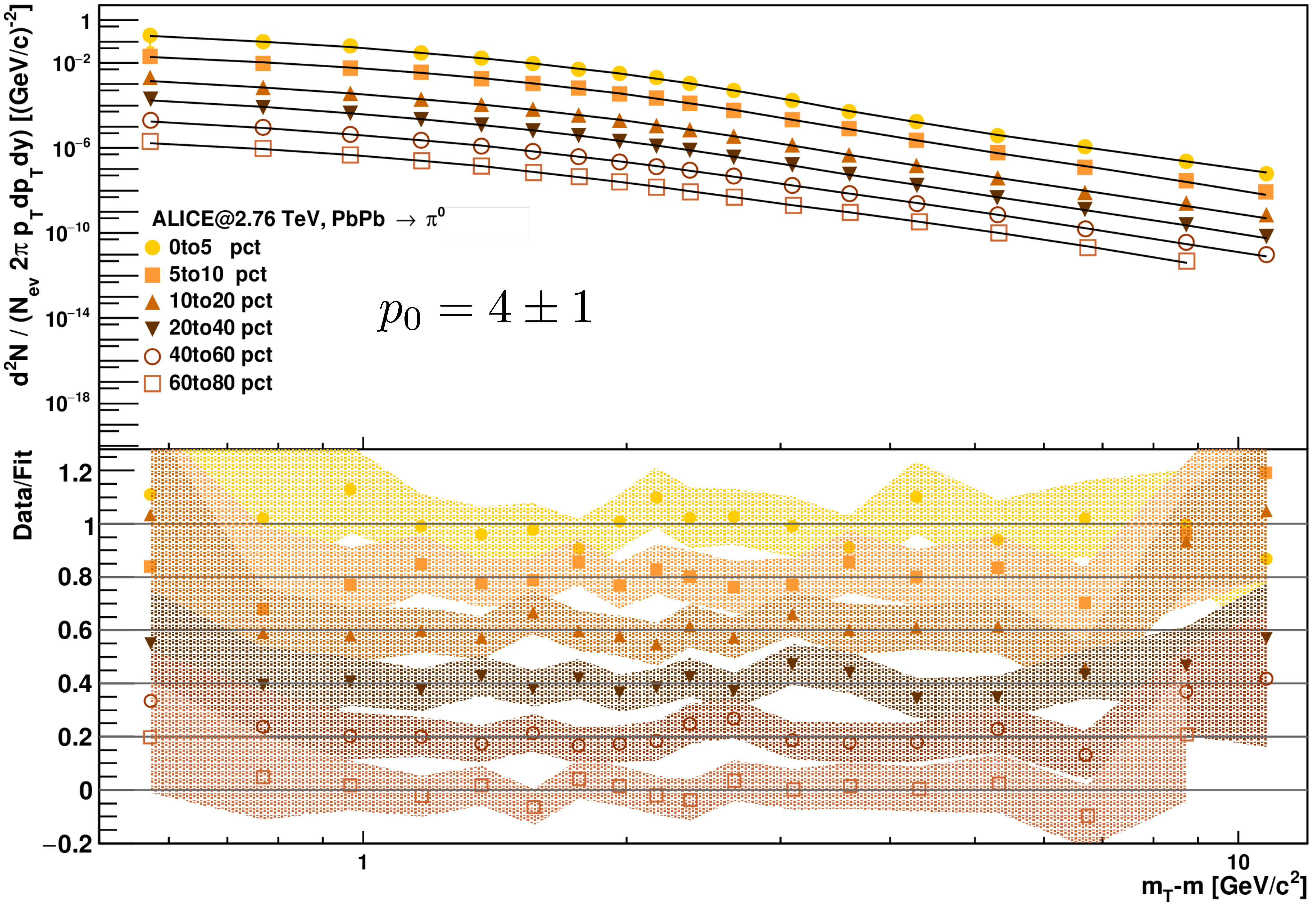
Results – T trend



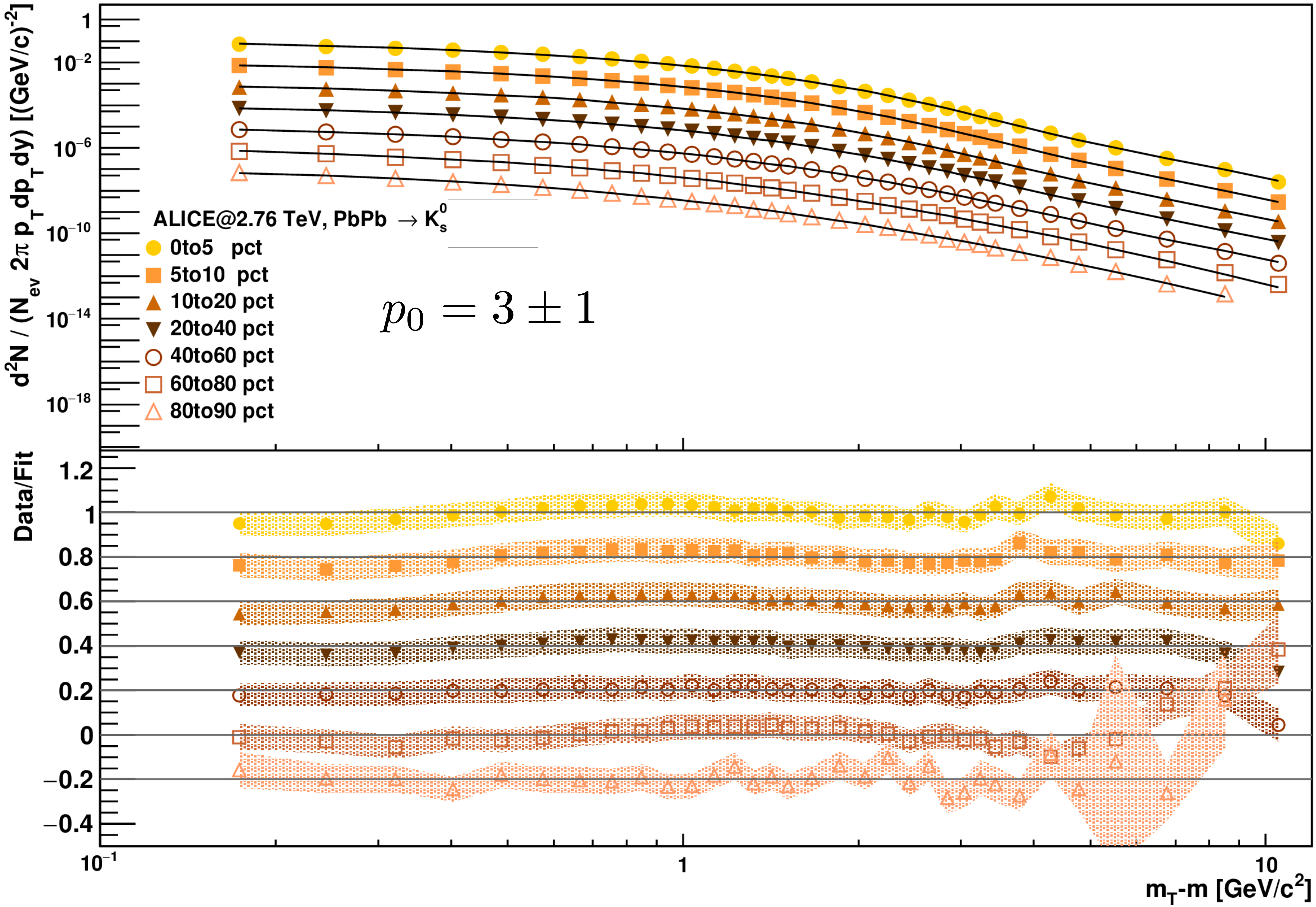


d²N / (N_ev 2π p_T dp_T dy) [(GeV/c)⁻²]
ALICE@2.76 TeV, PbPb → charged
0to5 pct
5to10 pct
10to20 pct
20to30 pct
30to40 pct
40to50 pct
50to60 pct
60to70 pct
70to80 pct
$p_0 = 6 \pm 2$
Data/Fit
p_T [GeV/c]

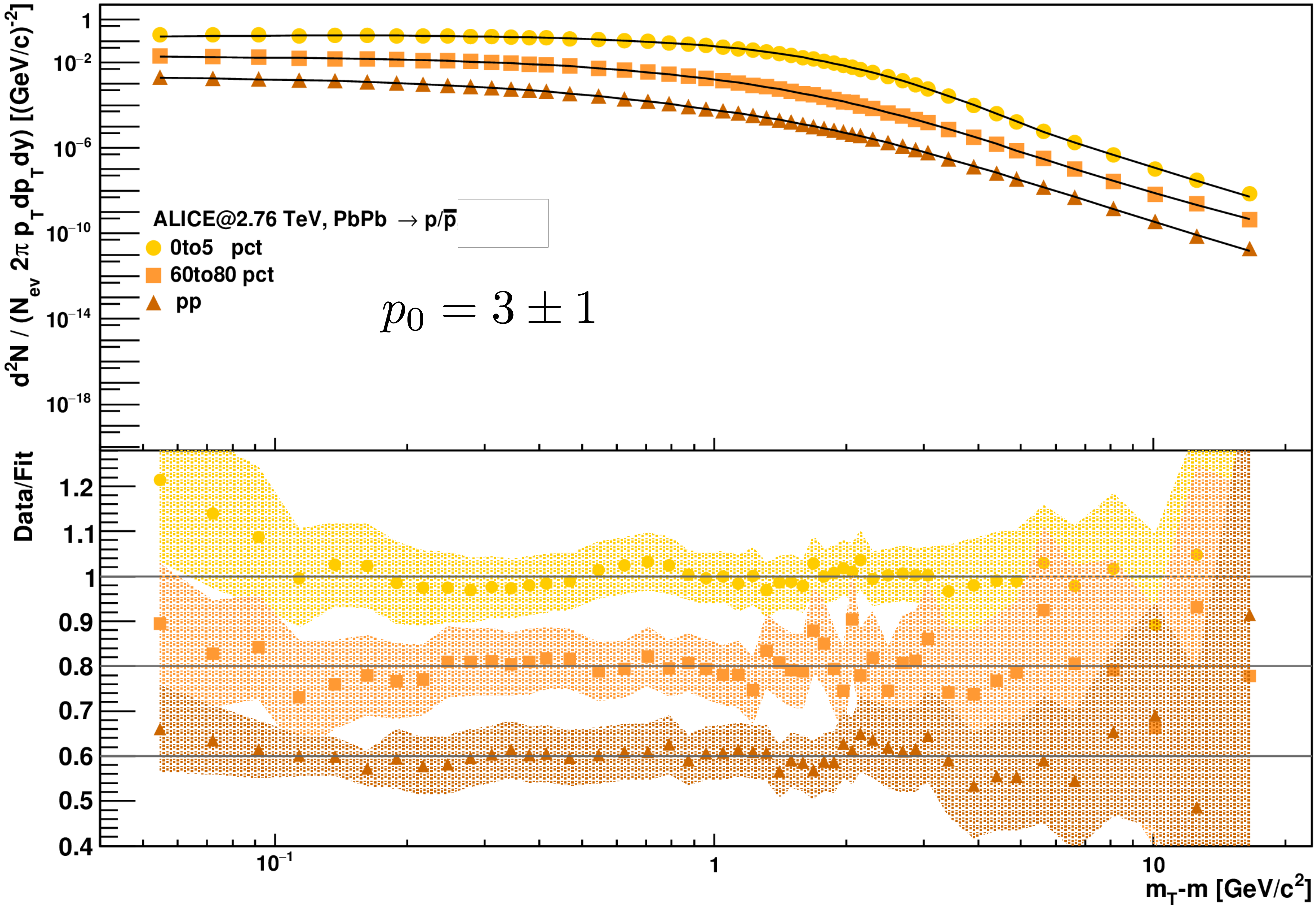
Results - Pion



Results - Kaon



Results - (Anti)Proton



Results - Lambda