Thermodynamical properties of the extended linear sigma model with Polyakov loops

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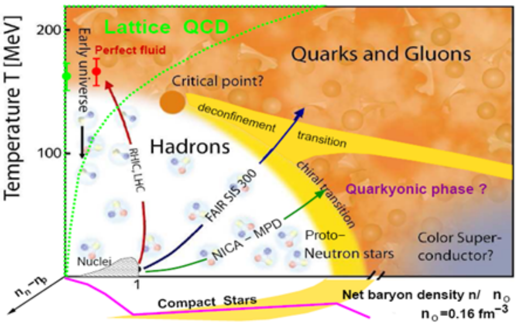
Overview

1. Introduction
   - Motivation
   - QCD's chiral symmetry, effective models
2. The model
   - Vector meson extended PQM model
   - Vector meson extended PQM model – Polyakov loop
3. eLSM at finite T/μ_B
   - Field equations
   - Meson masses
   - Parametrization at $T = 0$
4. Results
   - T or t dependence of order parameters and masses
   - Pressure and derived quantities
   - Critical endpoint
5. Summary

QCD phase diagram

Phase diagram in the $T - \mu_{\rm B} - \mu_{\rm I}$ space



- At $\mu_{\rm B} = 0$
  $T_c = 153(3)$ MeV
  Y. Aoki, *et al.*, PLB **643**, 46 (2006)
- Is there a CEP?
- At $T = 0$ in $\mu_{\rm B}$ where is the phase boundary?
- Behavior of thermodynamical quantities like pressure, interaction measure, quark density

Details of the phase diagram are heavily studied theoretically (Lattice, EFT), and experimentally (RHIC, LHC, FAIR, NICA)

Chiral symmetry, chiral models

If the quark masses are zero (chiral limit) $\Longrightarrow$ QCD invariant under the following global transformation (chiral symmetry):

$$U(3)_L \times U(3)_R \simeq U(3)_V \times U(3)_A = SU(3)_V \times SU(3)_A \times U(1)_V \times U(1)_A$$

$U(1)_V$ term $\longrightarrow$ baryon number conservation

$U(1)_A$ term $\longrightarrow$ broken through axial anomaly

$SU(3)_A$ term $\longrightarrow$ broken down by any quark mass

$SU(3)_V$ term $\longrightarrow$ broken down to $SU(2)_V$ if $m_u = m_d \neq m_s$
$\longrightarrow$ totally broken if $m_u \neq m_d \neq m_s$ (realized in nature)

Since QCD is very hard to solve $\longrightarrow$ low energy effective models $\longrightarrow$ reflecting the global symmetries of QCD $\longrightarrow$ degrees of freedom: observable particles instead of quarks and gluons

Linear realization of the symmetry $\longrightarrow$ linear sigma model

Lagrangian (1/2)

$$\mathcal{L} = \mathrm{Tr}[(D_\mu\Phi)^\dagger(D_\mu\Phi)] - m_0^2\mathrm{Tr}(\Phi^\dagger\Phi) - \lambda_1[\mathrm{Tr}(\Phi^\dagger\Phi)]^2 - \lambda_2\mathrm{Tr}(\Phi^\dagger\Phi)^2$$
$$+ c_1(\det\Phi + \det\Phi^\dagger) + \mathrm{Tr}[H(\Phi+\Phi^\dagger)] - \frac{1}{4}\mathrm{Tr}(L_{\mu\nu}^2 + R_{\mu\nu}^2)$$
$$+ \mathrm{Tr}\left[\left(\frac{m_1^2}{2}\mathbb{1} + \Delta\right)(L_\mu^2 + R_\mu^2)\right] + i\frac{g_2}{2}(\mathrm{Tr}\{L_{\mu\nu}[L^\mu, L^\nu]\} + \mathrm{Tr}\{R_{\mu\nu}[R^\mu, R^\nu]\})$$
$$+ \frac{h_1}{2}\mathrm{Tr}(\Phi^\dagger\Phi)\mathrm{Tr}(L_\mu^2 + R_\mu^2) + h_2\mathrm{Tr}[(L_\mu\Phi)^2 + (\Phi R_\mu)^2] + 2h_3\mathrm{Tr}(L_\mu\Phi R^\mu\Phi^\dagger)$$
$$+ \bar{\Psi}i\gamma_\mu D^\mu\Psi - g_F\bar{\Psi}(\Phi_S + i\gamma_5\Phi_{PS})\Psi,$$

$$\begin{aligned}
D^\mu\Phi &= \partial^\mu\Phi - ig_1(L^\mu\Phi - \Phi R^\mu) - ieA_e^\mu[T_3,\Phi],\\
L^{\mu\nu} &= \partial^\mu L^\nu - ieA_e^\mu[T_3, L^\nu] - \{\partial^\nu L^\mu - ieA_e^\nu[T_3, L^\mu]\},\\
R^{\mu\nu} &= \partial^\mu R^\nu - ieA_e^\mu[T_3, R^\nu] - \{\partial^\nu R^\mu - ieA_e^\nu[T_3, R^\mu]\},\\
D^\mu\Psi &= \partial^\mu\Psi - iG^\mu\Psi, \quad \text{with} \quad G^\mu = g_s G_a^\mu T_a.
\end{aligned}$$

+ Polyakov loop potential

Lagrangian (2/2)

the matter and external fields are

$$\Phi = \sum_{i=0}^{8}(\sigma_i + i\pi_i)T_i, \quad H = \sum_{i=0}^{8} h_i T_i \qquad T_i : U(3) \text{ generators}$$

$$R^\mu = \sum_{i=0}^{8}(\rho_i^\mu - b_i^\mu)T_i, \quad L^\mu = \sum_{i=0}^{8}(\rho_i^\mu + b_i^\mu)T_i, \quad \Delta = \sum_{i=0}^{8}\delta_i T_i$$

$$\Psi = (u, d, s)^{\mathrm{T}}$$

non strange – strange base:

$$\xi_N = \sqrt{2/3}\xi_0 + \sqrt{1/3}\xi_8,$$
$$\xi_S = \sqrt{1/3}\xi_0 - \sqrt{2/3}\xi_8, \qquad \xi \in (\sigma_i, \pi_i, \rho_i^\mu, b_i^\mu, h_i)$$

broken symmetry: non-zero condensates $\langle\sigma_{N/S}\rangle \equiv \bar{\sigma}_{N/S}$

Included fields - pseudoscalar and scalar meson nonets

$$\Phi_{PS} = \sum_{i=0}^{8} \pi_i T_i = \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{\eta_N+\pi^0}{\sqrt{2}} & \pi^+ & K^+ \\ \pi^- & \frac{\eta_N-\pi^0}{\sqrt{2}} & K^0 \\ K^- & \bar{K}^0 & \eta_S \end{pmatrix} \quad (\sim \bar{q}_i \gamma_5 q_j)$$

$$\Phi_{S} = \sum_{i=0}^{8} \sigma_i T_i = \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{\sigma_N+a_0^0}{\sqrt{2}} & a_0^+ & K_S^+ \\ a_0^- & \frac{\sigma_N-a_0^0}{\sqrt{2}} & K_S^0 \\ K_S^- & \bar{K}_S^0 & \sigma_S \end{pmatrix} \quad (\sim \bar{q}_i q_j)$$

Particle content:

Pseudoscalars: $\pi(138), K(495), \eta(548), \eta'(958)$

Scalars: $a_0(980 \text{ or } 1450), K_S(800 \text{ or } 1430)$,

2 of $f_0(500, 980, 1370, 1500, 1710)$

Structure of scalar mesons below 2 GeV

	Mass (MeV)	width (MeV)	decays
$a_0(980)$	980 ± 20	$50 - 100$	$\pi\pi$ dominant
$a_0(1450)$	1474 ± 19	265 ± 13	$\pi\eta$, $\pi\eta'$, $K\bar{K}$
$K_s(800) = \kappa$	682 ± 29	547 ± 24	$K\pi$
$K_s(1430)$	1425 ± 50	270 ± 80	$K\pi$ dominant
$f_0(500) = \sigma$	400–550	$400 - 700$	$\pi\pi$ dominant
$f_0(980)$	980 ± 20	$40 - 100$	$\pi\pi$ dominant
$f_0(1370)$	1200–1500	$200 - 500$	$\pi\pi \approx 250$, $K\bar{K} \approx 150$
$f_0(1500)$	1505 ± 6	109 ± 7	$\pi\pi \approx 38$, $K\bar{K} \approx 9.4$
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Possible scalar states: $\bar{q}q$, tetraquarks, glueballs

scalar $\bar{q}q$ nonet content: 1 a_0, 1 K_s, and 2 f_0: $a_0^{\bar{q}q} \to a_0(1450)$, $K_s^{\bar{q}q} \to K_s(1430)$, $f_0^{L,\bar{q}q} \to f_0(1370)$, $f_0^{H,\bar{q}q} \to f_0(1710)$

Parganlija et al., PRD87, 014011

tetraquarks: $f_0(500)$, $f_0(980)$, $a_0(980)$, $K_s(800)$?

glueballs: $f_0(1500)$?

Included fields - vector meson nonets

$$V^\mu = \sum_{i=0}^{8} \rho_i^\mu T_i = \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{\omega_N+\rho^0}{\sqrt{2}} & \rho^+ & K^{\star+} \\ \rho^- & \frac{\omega_N-\rho^0}{\sqrt{2}} & K^{\star 0} \\ K^{\star -} & \bar{K}^{\star 0} & \omega_S \end{pmatrix}^\mu$$

$$A^\mu = \sum_{i=0}^{8} b_i^\mu T_i = \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{f_{1N}+a_1^0}{\sqrt{2}} & a_1^+ & K_1^+ \\ a_1^- & \frac{f_{1N}-a_1^0}{\sqrt{2}} & K_1^0 \\ K_1^- & \bar{K}_1^0 & f_{1S} \end{pmatrix}^\mu$$

Particle content:

Vector mesons: $\rho(770), K^\star(894), \omega_N = \omega(782), \omega_S = \phi(1020)$

Axial vectors: $a_1(1230), K_1(1270), f_{1N}(1280), f_{1S}(1426)$

Spontaneous symmetry breaking

Interaction is approximately chiral symmetric, spectra is not $\longrightarrow$ SSB:

$$\sigma_{N/S} \to \sigma_{N/S} + \bar{\sigma}_{N/S} \qquad \bar{\sigma}_{N/S} \equiv <\sigma_{N/S}>$$

For tree level masses we have to select all terms quadratic in the new fields. Some of the terms include mixings arising from terms like $\mathrm{Tr}[(D_\mu\Phi)^\dagger(D_\mu\Phi)]$:

$$\begin{aligned}
\eta_N - f_{1N}^\mu &: \quad -g_1\bar{\sigma}_N f_{1N}^\mu \partial_\mu \eta_N, \\
\pi - a_1^\mu &: \quad -g_1\bar{\sigma}_N (a_1^{\mu +}\partial_\mu \pi^- + a_1^{\mu 0}\partial_\mu \pi^0) + \mathrm{h.c.}, \\
\eta_S - f_{1S}^\mu &: \quad -\sqrt{2}g_1\bar{\sigma}_S f_{1S}^\mu \partial_\mu \eta_S, \\
K_S - K_\mu^\star &: \quad \frac{ig_1}{2}(\sqrt{2}\bar{\sigma}_S - \bar{\sigma}_N)(\bar{K}_\mu^{\star 0}\partial^\mu K_S^0 + K_\mu^{\star -}\partial^\mu K_S^+) + \mathrm{h.c.}, \\
K - K_1^\mu &: \quad -\frac{g_1}{2}(\bar{\sigma}_N + \sqrt{2}\bar{\sigma}_S)(K_1^{\mu 0}\partial_\mu \bar{K}^0 + K_1^{\mu +}\partial_\mu K^-) + \mathrm{h.c.}.
\end{aligned} \tag{1}$$

Polyakov loops in Polyakov gauge

Polyakov loop variables: $\Phi(\vec{x}) = \frac{\mathrm{Tr}_c L(\vec{x})}{N_c}$ and $\bar{\Phi}(\vec{x}) = \frac{\mathrm{Tr}_c \bar{L}(\vec{x})}{N_c}$ with

$$L(x) = \mathcal{P} \exp\left[i \int_0^\beta d\tau \, G_4(\vec{x}, \tau)\right]$$

$\hookrightarrow$ signals center symmetry ($\mathbb{Z}_3$) breaking at the deconfinement transition

low T: confined phase, $\langle \Phi(\vec{x}) \rangle, \langle \bar{\Phi}(\vec{x}) \rangle = 0$
high T: deconfined phase, $\langle \Phi(\vec{x}) \rangle, \langle \bar{\Phi}(\vec{x}) \rangle \neq 0$

- Polyakov gauge: $G_4(\vec{x}, \tau) = G_4(\vec{x})$, plus gauge rotation to diagonal form in color space
- further simplification: $\vec{x}$-independence

$$\hookrightarrow \quad L = e^{i\beta G_4} = \mathrm{diag}(a, b, c) \left(\overset{!}{\in} SU(N_c) \right); \quad a, b, c \in \mathbb{Z}$$

$\hookrightarrow$ use this to calculate partition function of free quarks on constant gluon background

Effects of Polyakov loops on FD statistics

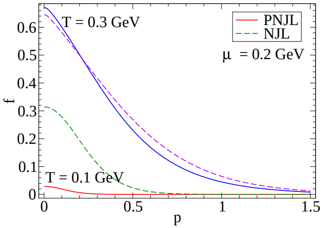
Inclusion of the Polyakov loop modifies the Fermi-Dirac distribution function

$$f(E_p-\mu_q)\longrightarrow f_\Phi^+(E_p) = \frac{\left(\bar\Phi+2\Phi e^{-\beta\left(E_p-\mu_q\right)}\right)e^{-\beta\left(E_p-\mu_q\right)}+e^{-3\beta\left(E_p-\mu_q\right)}}{1+3\left(\bar\Phi+\Phi e^{-\beta\left(E_p-\mu_q\right)}\right)e^{-\beta\left(E_p-\mu_q\right)}+e^{-3\beta\left(E_p-\mu_q\right)}}$$

$$f(E_p+\mu_q)\longrightarrow f_\Phi^-(E_p) = \frac{\left(\Phi+2\bar\Phi e^{-\beta\left(E_p+\mu_q\right)}\right)e^{-\beta\left(E_p+\mu_q\right)}+e^{-3\beta\left(E_p+\mu_q\right)}}{1+3\left(\Phi+\bar\Phi e^{-\beta\left(E_p+\mu_q\right)}\right)e^{-\beta\left(E_p+\mu_q\right)}+e^{-3\beta\left(E_p+\mu_q\right)}}$$

$\Phi,\bar\Phi\to 0 \Longrightarrow f_\Phi^\pm(E_p)\to f(3(E_p\pm\mu_q))$ $\quad\Phi,\bar\Phi\to 1 \Longrightarrow f_\Phi^\pm(E_p)\to f(E_p\pm\mu_q)$

three-particle state appears: mimics confinement of quarks within baryons



the effect of the Polyakov loop is more relevant for $T < T_c$

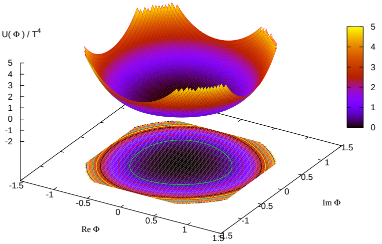
at $T = 0$ there is no difference between models with and without Polyakov loop: $\Theta(3(\mu_q - E_p)) \equiv \Theta((\mu_q - E_p))$
H. Hansen et al., PRD75, 065004

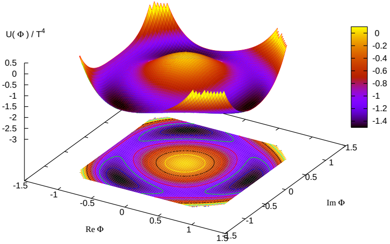
Polyakov loop potential

"Color confinement"
$\langle\Phi\rangle = 0 \longrightarrow$ no breaking of $\mathbb{Z}_3$

"Color deconfinement"
$\langle\Phi\rangle \neq 0 \longrightarrow$ spontaneous breaking of $\mathbb{Z}_3$

H. Hansen et al., PRD75, 065004 (2007)





Form of the potential:

- Polynomial: $U_{\mathrm{YM}}^{\mathrm{Poly}} \longrightarrow$ Not used (e.g.: negative susceptibilities)
- Logarithmic: U_{YM}
- Improved Polyakov loop potential (logarithmic): U_{glue}

Form of the potential

I.) Simple polynomial potential invariant under $\mathbb{Z}_3$ and charge conjugation: R.D.Pisarski, PRD 62, 111501

$$\frac{\mathcal{U}^{\text{YM}}_{\text{poly}}(\Phi,\bar\Phi)}{T^4} = -\frac{b_2(T)}{2}\bar\Phi\Phi - \frac{b_3}{6}\left(\Phi^3+\bar\Phi^3\right) + \frac{b_4}{4}\left(\bar\Phi\Phi\right)^2$$

with $$b_2(T) = a_0 + a_1\frac{T_0}{T} + a_2\frac{T_0^2}{T^2} + a_3\frac{T_0^3}{T^3}$$

II.) Logarithmic potential coming from the $SU(3)$ Haar measure of group integration K. Fukushima, Phys. Lett. **B591**, 277 (2004)

$$\frac{\mathcal{U}^{\text{YM}}_{\log}(\Phi,\bar\Phi)}{T^4} = -\frac{1}{2}a(T)\Phi\bar\Phi + b(T)\ln\left[1-6\Phi\bar\Phi+4\left(\Phi^3+\bar\Phi^3\right)-3\left(\Phi\bar\Phi\right)^2\right]$$

with $$a(T) = a_0 + a_1\frac{T_0}{T} + a_2\frac{T_0^2}{T^2}, \qquad b(T) = b_3\frac{T_0^3}{T^3}$$

$\mathcal{U}^{\text{YM}}\left(\Phi,\bar\Phi\right)$ models the free energy of a pure gauge theory

$\longrightarrow$ the parameters are fitted to the pure gauge lattice data

Improved Polyakov loop potential

Previous potentials describe successfully the first order phase transition of the pure $SU(3)$ Yang – Mills

$\hookrightarrow$ taking into account the gluon dynamics (quark polarization of gluon propagator) $\longrightarrow$ QCD glue potential

$\hookrightarrow$ can be implemented by changing the reduced temperature

$$t_{\text{glue}} \equiv \frac{T - T_c^{\text{glue}}}{T_c^{\text{glue}}}, \quad t_{\text{YM}} \equiv \frac{T^{\text{YM}} - T_c^{\text{YM}}}{T_c^{\text{YM}}}$$

$$t_{\text{YM}}(t_{\text{glue}}) \approx 0.57 t_{\text{glue}}$$

$$\frac{\mathcal{U}^{\text{glue}}}{T^4}(\Phi, \bar{\Phi}, t_{\text{glue}}) = \frac{\mathcal{U}^{\text{YM}}}{(T^{\text{YM}})^4}(\Phi, \bar{\Phi}, t_{\text{YM}}(t_{\text{glue}}))$$

L. M. Haas *et al.*, PRD 87, 076004 (2013)

Field equations for the order parameters

Hybrid approach: fermions at one-loop, mesons at tree-level
$\longrightarrow$ calculate Ω the grand canonical potential

$$\Omega(T,\mu_q) = U^{\text{tree}}_{\text{meson}}(\langle\phi\rangle) + \Omega^{vac}_{\bar{q}q} + \Omega^{T}_{\bar{q}q}(T,\mu_q) + \mathcal{U}^{\text{glue}}(\Phi,\bar{\Phi},t_{\text{glue}}(T))$$

$$\imath.) \qquad \frac{\partial\Omega}{\partial\bar{\sigma}_N} = \frac{\partial\Omega}{\partial\bar{\sigma}_S}\bigg|_{\bar{\sigma}_N=\phi_N,\bar{\sigma}_S=\phi_S} = 0$$

$$m_0^2\phi_N + \left(\lambda_1 + \frac{1}{2}\lambda_2\right)\phi_N^3 + \lambda_1\phi_N\phi_S^2 - h_N + \frac{g_F}{2}N_c\left(\langle u\bar{u}\rangle_T + \langle d\bar{d}\rangle_T\right) = 0$$

$$m_0^2\phi_S + (\lambda_1+\lambda_2)\phi_S^3 + \lambda_1\phi_N^2\phi_S - h_S + \frac{g_F}{\sqrt{2}}N_c\langle s\bar{s}\rangle_T = 0$$

$$\langle q\bar{q}\rangle_T = -4m_q\int\frac{d^3\mathbf{p}}{(2\pi)^3}\frac{1}{2E_q(p)}\left(1 - f_\Phi^-(E_q(p)) - f_\Phi^+(E_q(p))\right)$$

Field equations for the Polyakov-loop variables

$$ii.)\qquad \frac{\partial\Omega}{\partial\Phi} = \frac{\partial\Omega}{\partial\bar{\Phi}}\bigg|_{\bar{\sigma}_N=\phi_N,\bar{\sigma}_S=\phi_S} = 0,$$

$$-\frac{d}{d\Phi}\left(\frac{U(\Phi,\bar{\Phi})}{T^4}\right) + \frac{2N_c}{T^3}\sum_{q=u,d,s}\int\frac{d^3\mathbf{p}}{(2\pi)^3}\left(\frac{e^{-\beta E_q^-(p)}}{g_q^-(p)} + \frac{e^{-2\beta E_q^+(p)}}{g_q^+(p)}\right) = 0$$

$$-\frac{d}{d\bar{\Phi}}\left(\frac{U(\Phi,\bar{\Phi})}{T^4}\right) + \frac{2N_c}{T^3}\sum_{q=u,d,s}\int\frac{d^3\mathbf{p}}{(2\pi)^3}\left(\frac{e^{-\beta E_q^+(p)}}{g_q^+(p)} + \frac{e^{-2\beta E_q^-(p)}}{g_q^-(p)}\right) = 0$$

$$g_q^+(p) = 1 + 3\left(\bar{\Phi} + \Phi e^{-\beta E_q^+(p)}\right)e^{-\beta E_q^+(p)} + e^{-3\beta E_q^+(p)}$$

$$g_q^-(p) = 1 + 3\left(\Phi + \bar{\Phi} e^{-\beta E_q^-(p)}\right)e^{-\beta E_q^-(p)} + e^{-3\beta E_q^-(p)}$$

$$E_q^{\pm}(p) = E_q(p) \mp \mu_B/3,\quad E_{u/d}(p) = \sqrt{p^2 + m_{u/d}^2},\ E_s(p) = \sqrt{p^2 + m_s^2}$$

Curvature masses

$$\mathcal{M}^2_{i,ab} = \frac{\partial^2\Omega(T,\mu_f)}{\partial\varphi_{i,a}\partial\varphi_{i,b}}\Big|_{\min} = m^2_{i,ab} + \Delta_0 m^2_{i,ab} + \Delta_T m^2_{i,ab},$$

$m^2_{i,ab} \longrightarrow$ tree-level mass matrix,

$\Delta_{0/T} m^2_{i,ab} \longrightarrow$ fermion vacuum/thermal fluctuation,

$$\Delta_0 m^2_{i,ab} = \frac{\partial^2\Omega^{\text{vac}}_{q\bar{q}}}{\partial\varphi_{i,a}\partial\varphi_{i,b}}\Big|_{\min} = -\frac{3}{8\pi^2}\sum_{f=u,d,s}\left[\left(\frac{3}{2}+\log\frac{m_f^2}{M^2}\right)m^{2(i)}_{f,a}m^{2(i)}_{f,b} + m_f^2\left(\frac{1}{2}+\log\frac{m_f^2}{M^2}\right)m^{2(i)}_{f,ab}\right],$$

$$\Delta_T m^2_{i,ab} = \frac{\partial^2\Omega^{\text{th}}_{q\bar{q}}}{\partial\varphi_{i,a}\partial\varphi_{i,b}}\Big|_{\min} = 6\sum_{f=u,d,s}\int\frac{d^3p}{(2\pi)^3}\frac{1}{2E_f(p)}\left[\left(f_f^+(p)+f_f^-(p)\right)\left(m^{2(i)}_{f,ab} - \frac{m^{2(i)}_{f,a}m^{2(i)}_{f,b}}{2E_f^2(p)}\right) + \left(B_f^+(p)+B_f^-(p)\right)\frac{m^{2(i)}_{f,a}m^{2(i)}_{f,b}}{2TE_f(p)}\right],$$

where $m^{2(i)}_{f,a} \equiv \partial m_f^2/\partial\varphi_{i,a}$, $m^{2(i)}_{f,ab} \equiv \partial^2 m_f^2/\partial\varphi_{i,a}\partial\varphi_{i,b}$

Determination of the parameters

14 unknown parameters $(m_0, \lambda_1, \lambda_2, c_1, m_1, g_1, g_2, h_1, h_2, h_3, \delta_S, \Phi_N, \Phi_S, g_F) \longrightarrow$ determined by the min. of χ^2:

$$\chi^2(x_1,\ldots,x_N) = \sum_{i=1}^{M}\left[\frac{Q_i(x_1,\ldots,x_N) - Q_i^{\text{exp}}}{\delta Q_i}\right]^2,$$

$(x_1,\ldots,x_N) = (m_0, \lambda_1, \lambda_2, \ldots)$, $Q_i(x_1,\ldots,x_N) \longrightarrow$ from the model, $Q_i^{\text{exp}} \longrightarrow$ PDG value, $\delta Q_i = \max\{5\%, \text{PDG value}\}$
multiparametric minimalization $\longrightarrow$ MINUIT

- PCAC $\rightarrow$ 2 physical quantities: f_π, f_K
- Curvature masses $\rightarrow$ 16 physical quantities:
  $m_{u/d}, m_s, m_\pi, m_\eta, m_{\eta'}, m_K, m_\rho, m_\Phi, m_{K^\star}, m_{a_1}, m_{f_1^H}, m_{K_1}, m_{a_0}, m_{K_s}, m_{f_0^L}, m_{f_0^H}$
- Decay widths $\rightarrow$ 12 physical quantities:
  $\Gamma_{\rho\to\pi\pi}, \Gamma_{\Phi\to KK}, \Gamma_{K^\star\to K\pi}, \Gamma_{a_1\to\pi\gamma}, \Gamma_{a_1\to\rho\pi}, \Gamma_{f_1\to KK^\star}, \Gamma_{a_0}, \Gamma_{K_S\to K\pi}, \Gamma_{f_0^L\to\pi\pi}, \Gamma_{f_0^L\to KK}, \Gamma_{f_0^H\to\pi\pi}, \Gamma_{f_0^H\to KK}$
- Pseudocritical temperature T_c at $mu_B = 0$

Features of our approach

- D.O.F's: scalar, pseudoscalar, vector, axial vector nonets,
- Polyakov loop variables with $\mathcal{U}^{\mathrm{YM}}$ or $\mathcal{U}^{\mathrm{glue}}$
- constituent quarks
- Four order parameters $(\phi_N, \phi_S, \Phi, \bar{\Phi}) \longrightarrow$ four coupled T/μ_B-dependent equations
- Fermion vacuum fluctuations
- Fermion thermal fluctuations
- Fermion contributions to the tree-level meson masses $\longrightarrow$ curvature masses
- $+$ Thermal π, K, f_0^L fluctuations for the pressure and other thermodynamical quantities

Consequence of scalar mesons sector (below 2 GeV)

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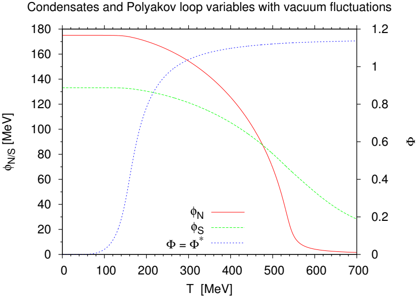
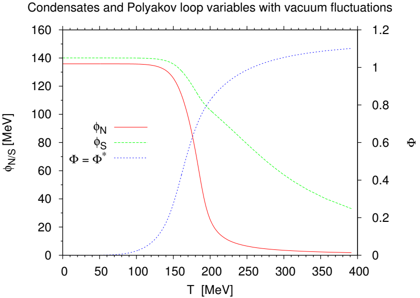
$\hookrightarrow$ We have 40 assignment possibilities!

Different parameterizations can give different thermodynamical behavior

Condensates with high (1326 MeV) and low (402 MeV) scalar masses





Calculation of thermodynamical quantities

pressure: $p = \frac{\partial(T \ln Z)}{\partial V} = -\Omega$

entropy density: $s = \frac{\partial p}{\partial T}$, quark number density: $\rho_q = \frac{\partial p}{\partial \mu_q}$

energy density: $\epsilon = -p + Ts + \mu_q \rho_q$, speed of sound: $c_s^2 = \frac{\partial p}{\partial \epsilon}$,

mesonic thermal 1-loop contribution to the pressure:

$$p_{\text{meson}} = -\Omega^{\text{1-loop},T}_{meson} = -NT \int \frac{d^3p}{(2\pi)^3} \ln\left(1 - e^{-\beta\omega(p)}\right)$$

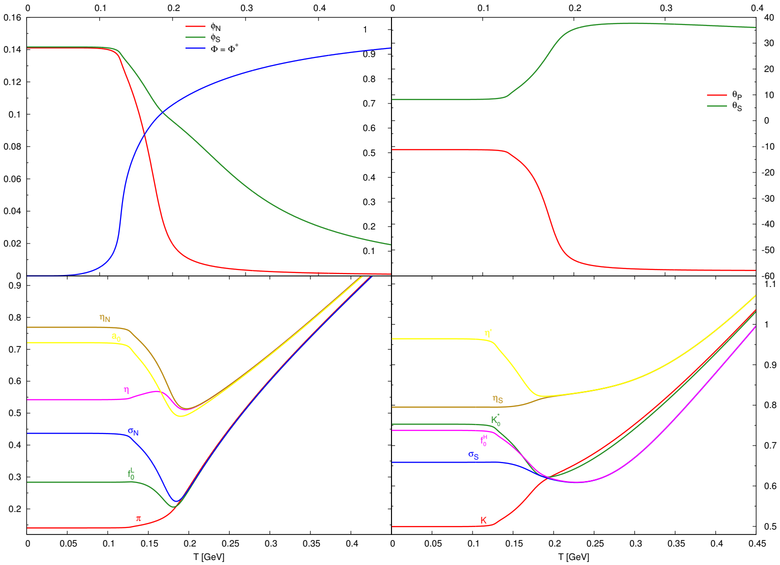
where, $\omega(p) = \sqrt{p^2 + m^2}$

to compare with the lattice $\longrightarrow$

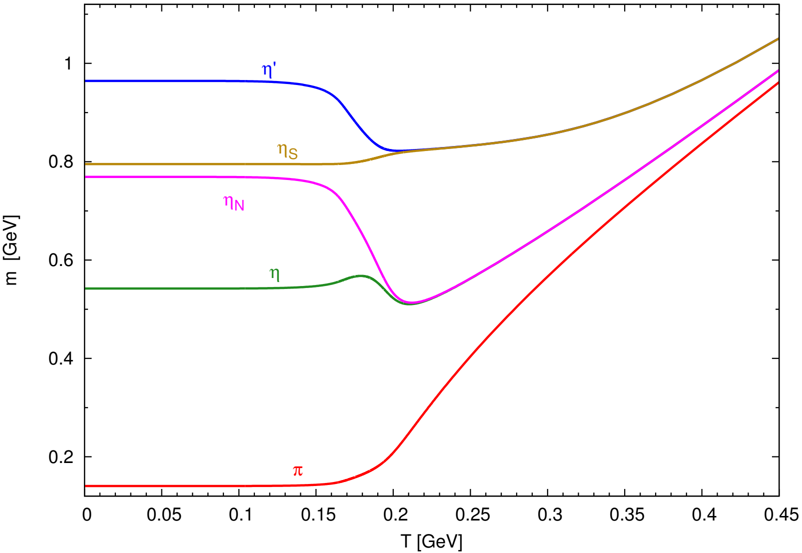
subtracted condensate: $\Delta_{l,s} = \frac{\Phi_N - \frac{h_N}{h_S} \cdot \Phi_S|_T}{\Phi_N - \frac{h_N}{h_S} \cdot \Phi_S|_{T=0}}$

scaled interaction measure: $I/T^4 = (\epsilon - 3p)/T^4$

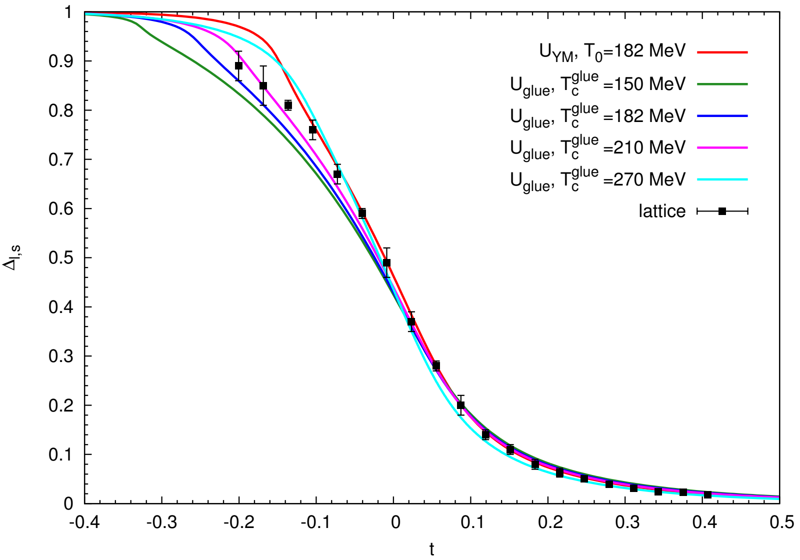
T dependence of masses, condensates, mixing angles



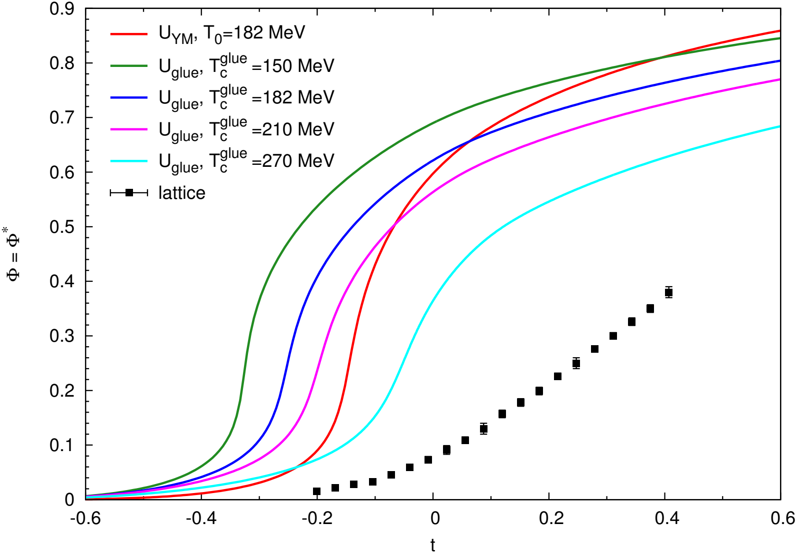
Temperature dependence of the η, η' sector



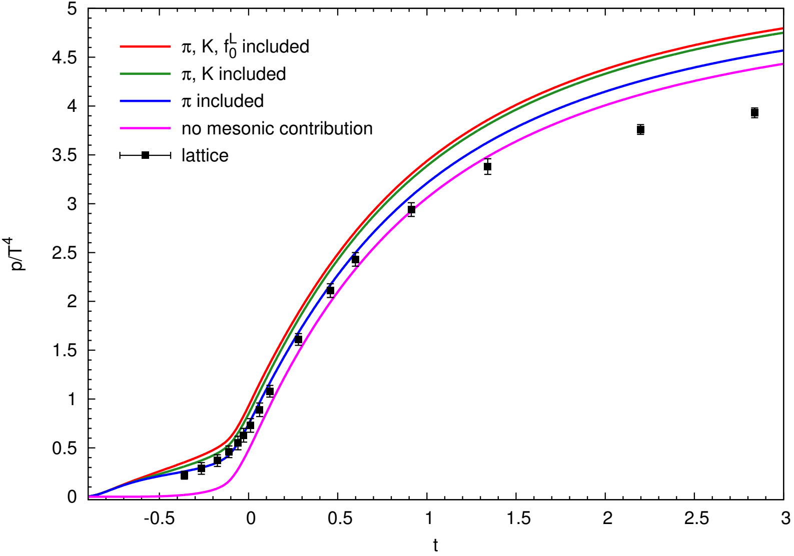
The subtracted condensate for different U's



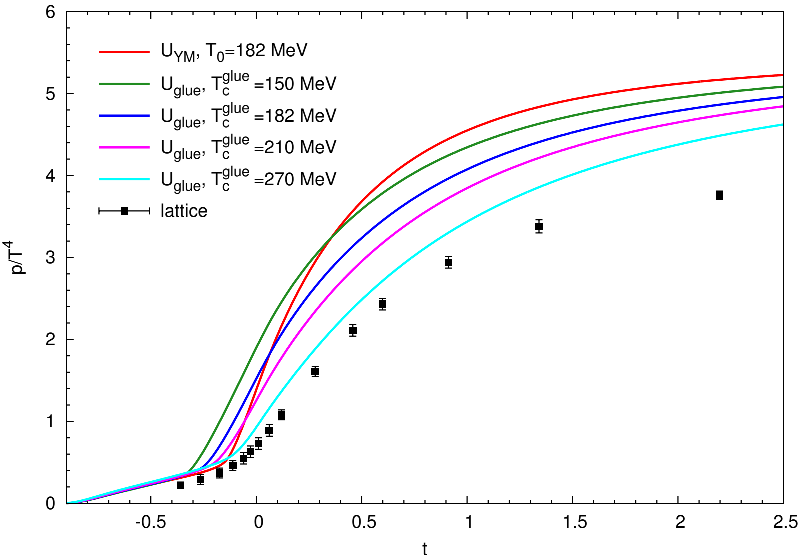
Polyakov loop variables for different U's



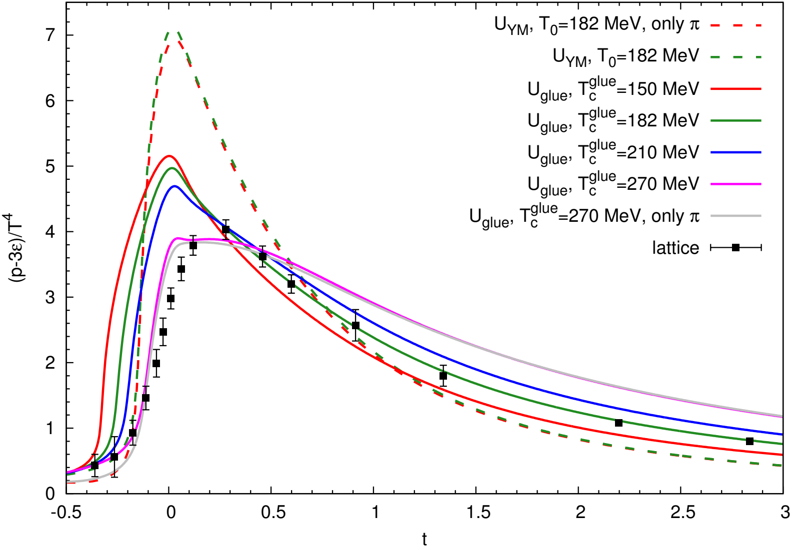
Normalized pressure and the effects of meson contributions ($T_{glue} = 270 MeV$)



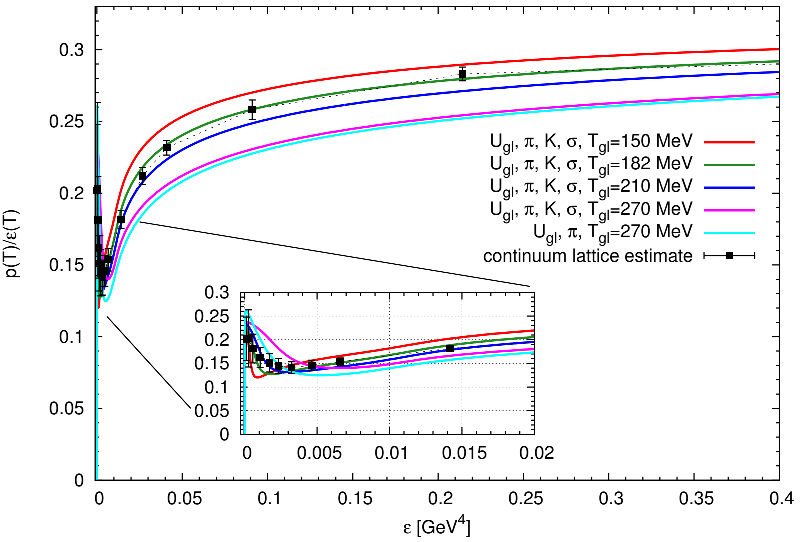
Normalized pressure and the effects of different U's



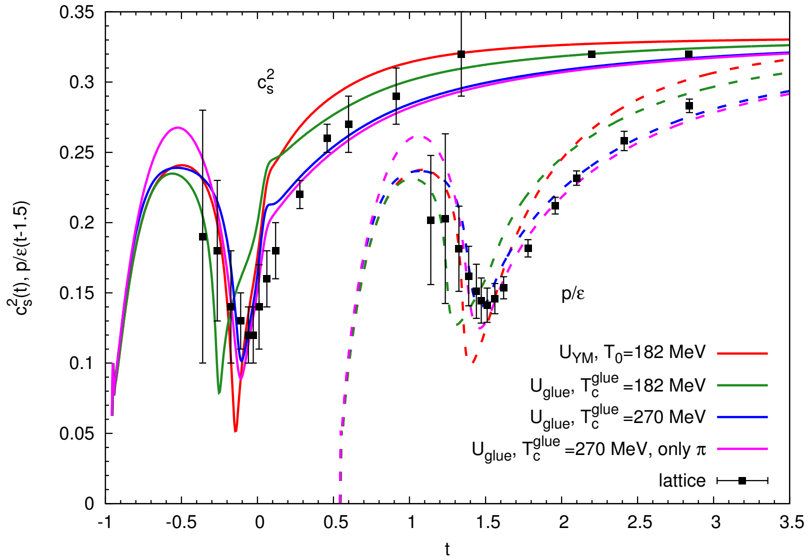
Scaled interaction measure



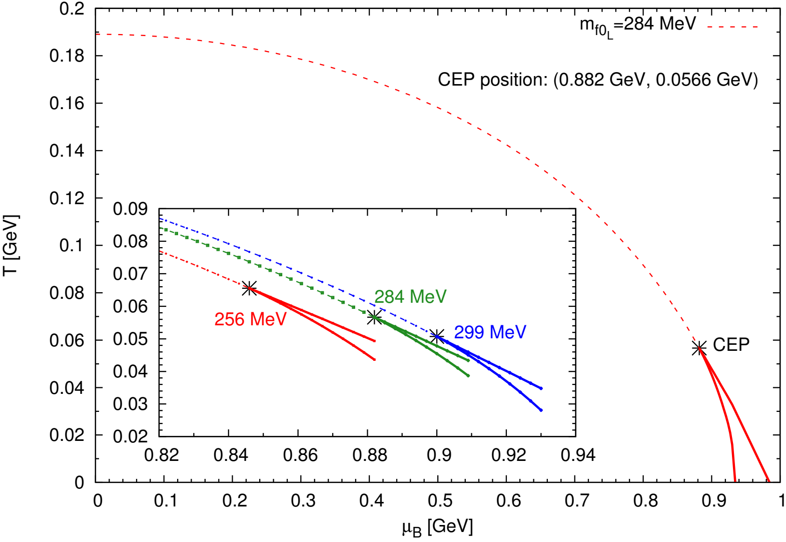
Equation of state



Speed of sound and p/ϵ



CEP and its variation with the f_0^L mass



Summary

- Thermodynamics of a vector meson extended Polyakov quark meson was investigated
- We used a hybrid approach: fermion vacuum/thermal fluctuations (assumption: gives the largest contribution), bosons at tree–level (except in pressure and such)
- We investigated the 40 possible scalar parameterization scenarios
- At finite T/μ_B there was 4 coupled equations for the 4 order parameters
- Various thermodynamical quantities were calculated, which show quiet good agreement with lattice results, if we use the improved Polyakov potential
- Model can be used to give predictions at finite densities (masses, decay widths, thermodynamical quantities)

Thank you for your attention!