An Application of the Non-extensive Phenomena: Fragmentation Functions for High-energy Collisions

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O U T L I N E

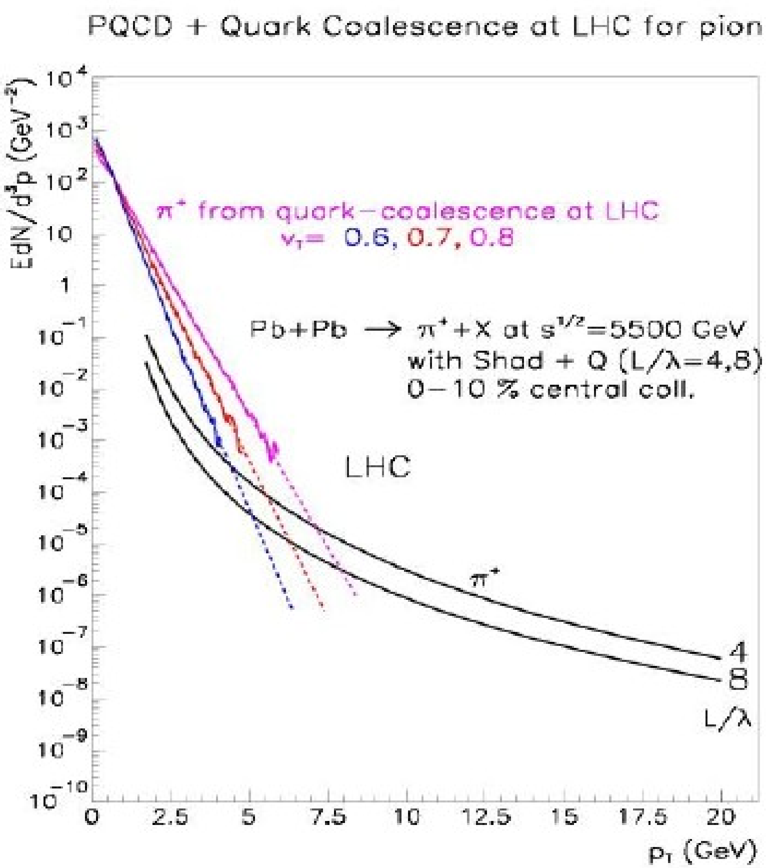
- Motivation...
  - Is there physics behind the parameters of FFs?
  - How about the p_T power of the tail?
  - Can we understand an experimental parameter, T, which we use to fit to low the p_T spectra?

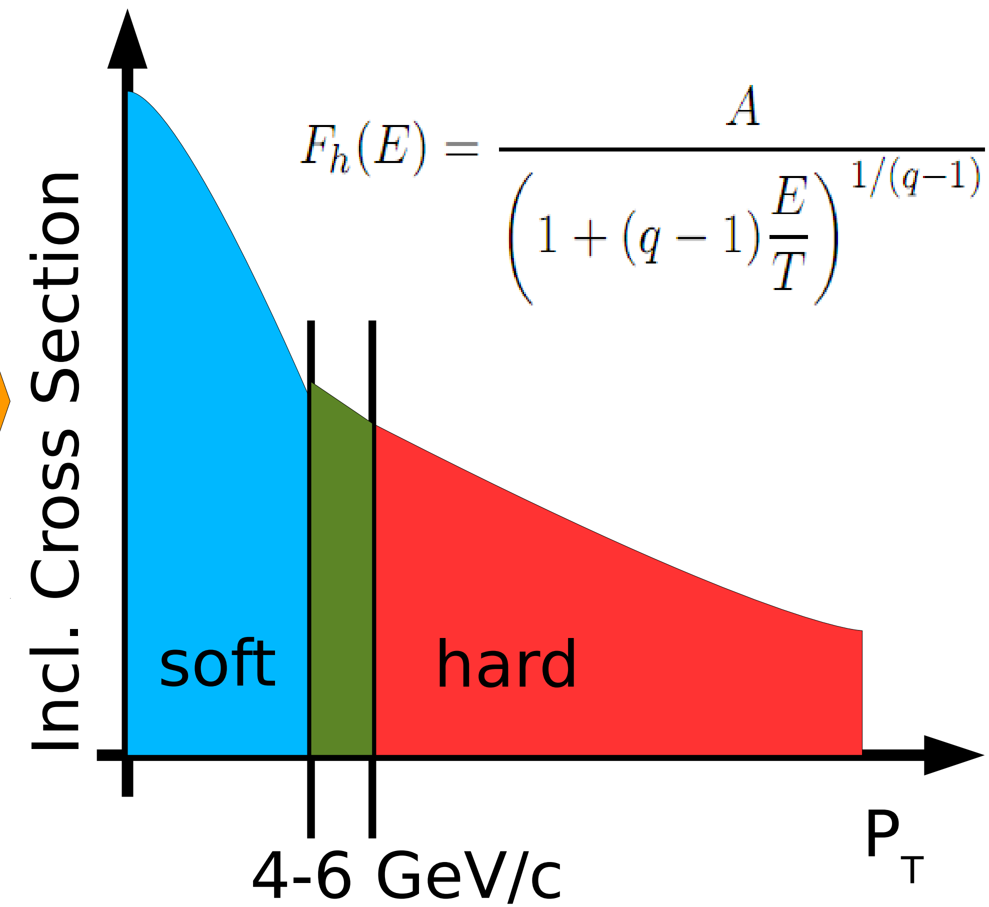
- Fragmentation Function parametrization
  - Test of the polynomial FFs
  - Parameters of the Tsallis-Pareto FF

- Test of the parameterized FF?
  - An application in kTpQCDv2.0

Motivation

- Simplest and best fit to hadron spectra at low-p_T & high-p_T



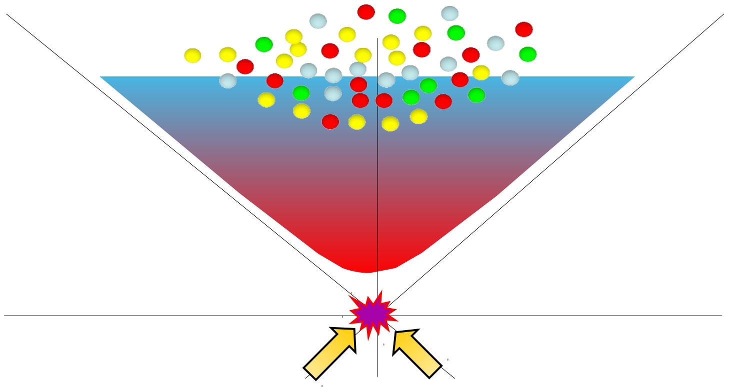


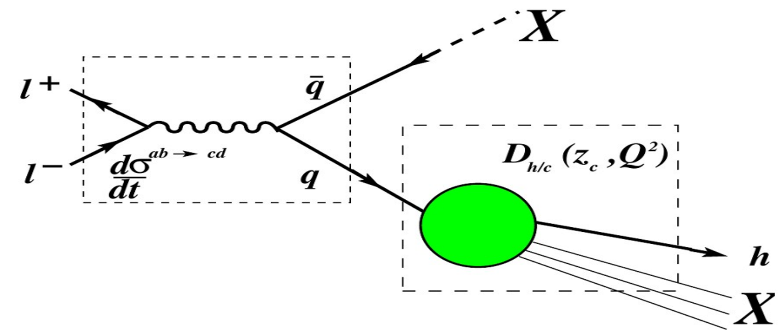
$$F_h(E) = \frac{A}{\left(1 + (q-1)\frac{E}{T}\right)^{1/(q-1)}}$$

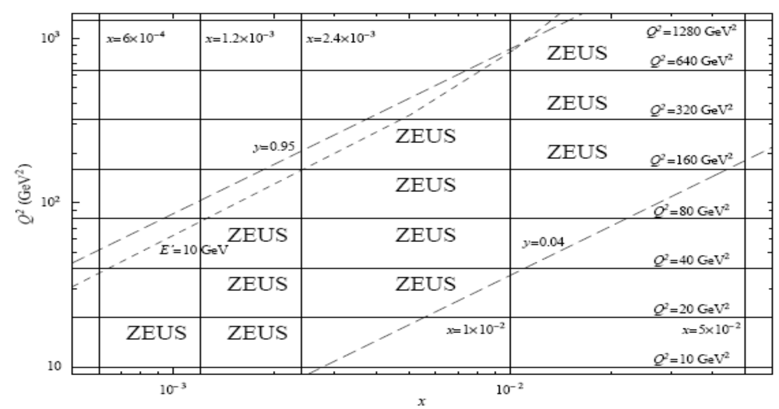
P. Lévai, GGB, G. Fai: JPG35, 104111 (2008)

Hadronization Processes & Fragmentation

- Hadronization: requires a model, based on local parton-hadron duality (kvantum numbers & momenta connected to a cone around or to the leading particle.)
- Parton/hadron shower evolution comes from statistical processes (step-by-step MC evolution).
- Fragmentation function (FF) carries integrated (phenomenological) information on how parton fragment into hadron.
- Measurement lepton-antilepton annihilation



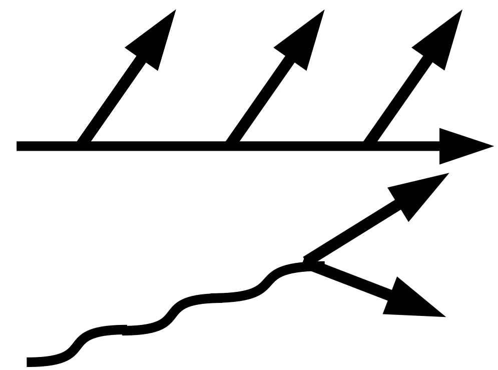




Models for Fragmentation

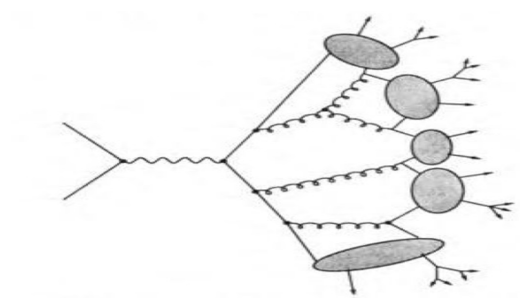
- Independent fragmentation model (Feynman - Field)

  Simplest model for fragmentation by
  Field & Feynman : q & g channels



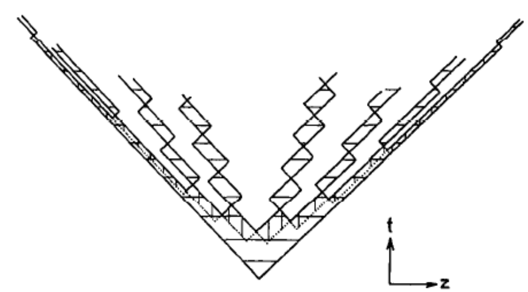
- (Quark) string model

  color strings between $q\bar{q}$, breaking into
  quark-antiquark pair → mesons



- Cluster model (Lund model)

  Lund model: phase-space separation,
  froming clusters: $q\bar{q}$ → M, qqq → B

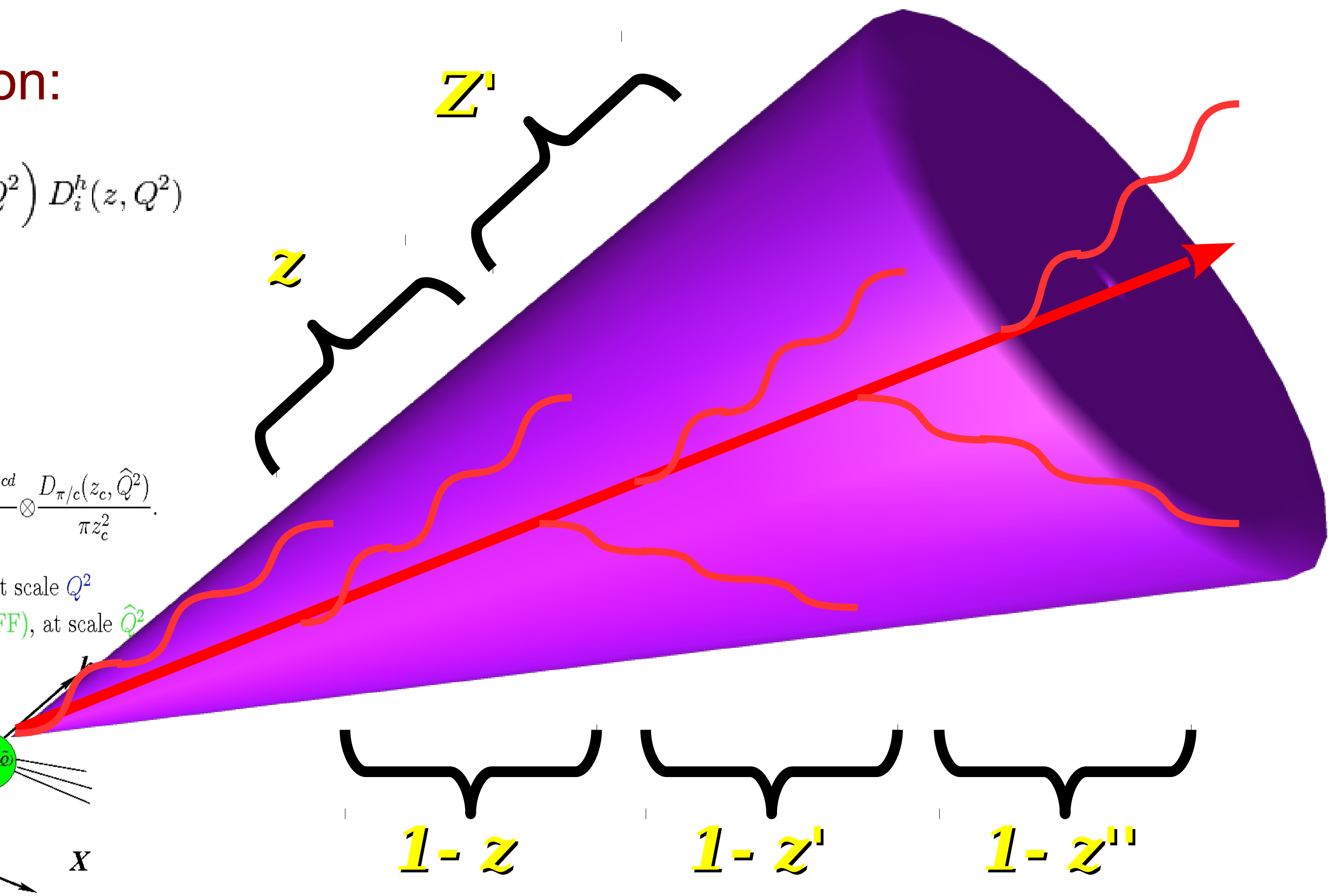


Fragmentation in Parton Model

In a pQCD based parton model, fragmentation functions (FF) gives how parton (*a*) fragment into a hadron (*h*), $D_{h/a}(z,Q^2)$.

DGLAP scale evolution:

$$\frac{\partial}{\partial \ln Q^2} D_i^h(x,Q^2) = \sum_j \int_x^1 \frac{dz}{z} \frac{\alpha_S}{4\pi} P_{ji}\left(\frac{x}{z},Q^2\right) D_i^h(z,Q^2)$$

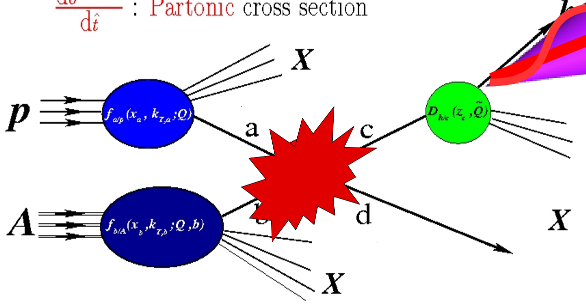


$$E_\pi \frac{d\sigma_\pi^{pA}}{d^3p_\pi} \sim f_{a/p}(x_a,Q^2;k_T) \otimes f_{b/A}(x_b,Q^2;k_T,b) \otimes \frac{d\sigma^{ab\to cd}}{d\hat{t}} \otimes \frac{D_{\pi/c}(z_c,\hat{Q}^2)}{\pi z_c^2}.$$

$f_{b/A}(x_a,Q^2;k_T,b)$: Parton Dist. Function (PDF), at scale Q^2

$D_{\pi/c}(z_c,\hat{Q}^2)$: Fragmentation Function for π (FF), at scale $\hat{Q}^2$

$\frac{d\sigma^{ab\to cd}}{d\hat{t}}$: Partonic cross section



Connection to the non-extensive phenomena

Non-extensive thermodynamics: associative composition rule, (non-additive) T.S. Biró: EPL84, 56003,2008:

$$h(h(x,y),z) = h(x,h(y,z))$$

Then should exist a strict monotonic function, X(x) 'generalised logarithm' (an entropy-like quantity), for which:

$$h(x,y) = X^{-1}\left(X(x) + X(y)\right) \qquad X(h(x,y)) = X(x) + X(y).$$

Example: (i) Classical thermodynamics:

$$f(E) = e^{-\beta E}/Z \qquad h(x,y) = x + y.$$

(ii) Tsallis – Pareto distribution

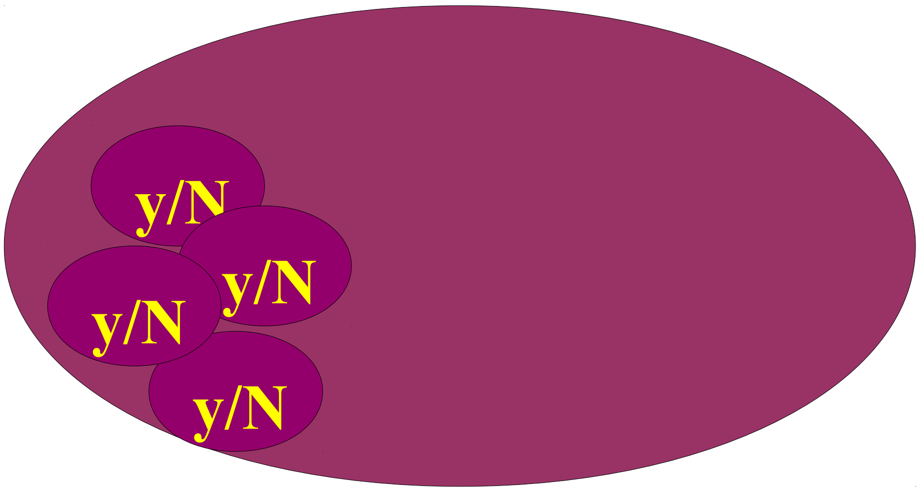
$$h(x,y) = x + y + axy \qquad a = q - 1$$

$$f(E) = \frac{1}{Z} e^{-\frac{\beta}{a}\ln(1+aE)} = \frac{1}{Z}(1+aE)^{-\beta/a} \qquad S = \int f \, \frac{e^{-a\ln(f)} - 1}{a} = \frac{1}{a}\int (f^{1-a} - f).$$

Associative composition

Non-extensive Gibbs, generalised

logarithm: $f(x) = \frac{1}{Z} e^{-\beta X(x)}.$



Compositin rule for sub-systems:

$$x_N(y) := \underbrace{h \circ \ldots \circ h}_{N-1} \left(\frac{y}{N}, \ldots, \frac{y}{N} \right)$$

Meanwhile satisfy: $\lim_{N \to \infty} x_N(y) < \infty.$

Assimptotically, if $N_1, N_2 \to \infty.$

$$x_{N_1+N_2} = \varphi(x_{N_1}, x_{N_2})$$

recursive equation can be given:

$$x_n = h\left(x_{n-1}, \frac{y}{N}\right)$$ here $h(x,0) = x.$

$$x_n - x_{n-1} = h\left(x_{n-1}, \frac{y}{N}\right) - h(x_{n-1}, 0).$$

Evolution equation can carry out:

$$\frac{dx}{dt} = \frac{y}{t_f} \, h_2'(x, 0^+)$$

$$L(x) = \int_0^x \frac{dz}{h_2'(z, 0^+)} = y \frac{t}{t_f}.$$

Fragmentation via associative composition

Program:

1) Search and fit Tsallis-Pareto distribution to data.

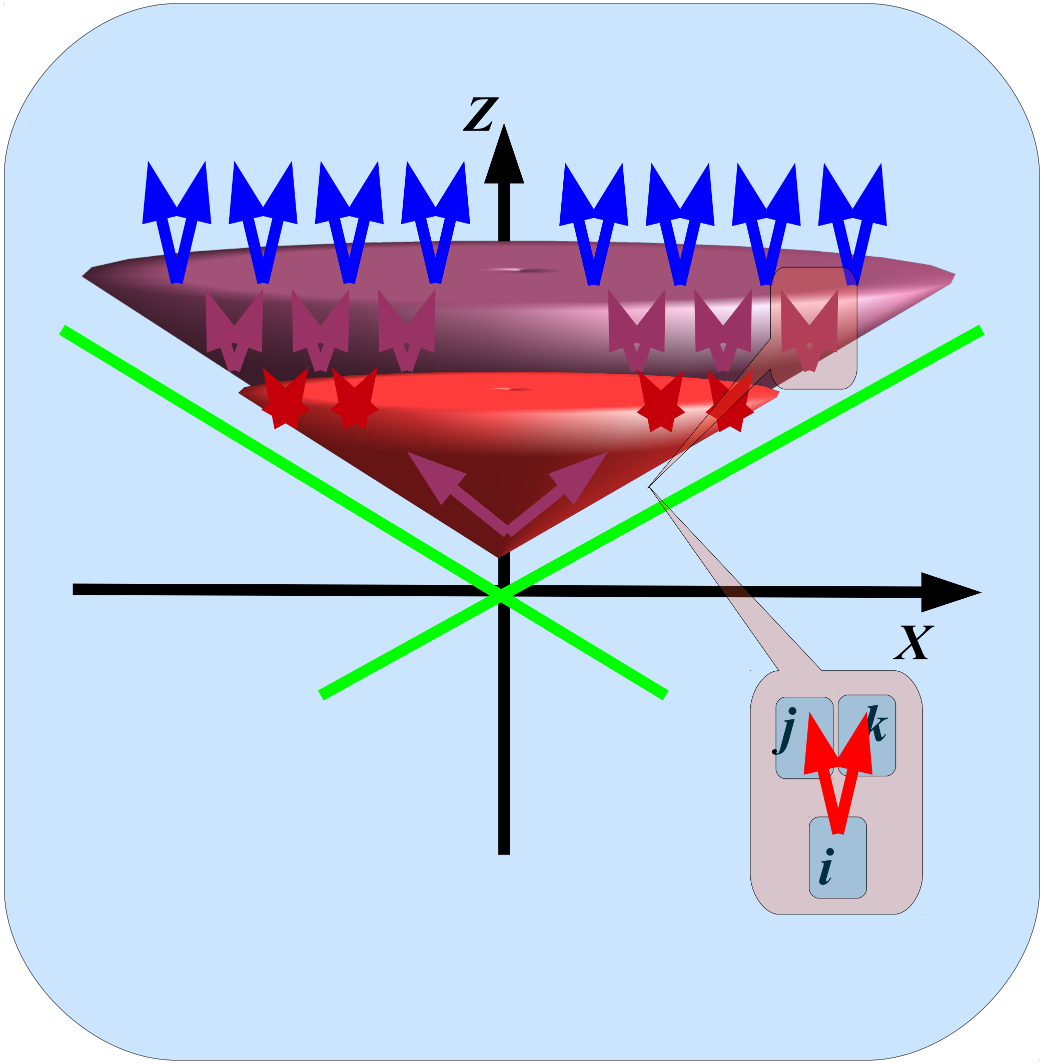
2) Serach for physical meaning of T and q parameters.

3) Components of the sub-systems are e.g. 'splitting functions' P_{qg}, P_{gg}

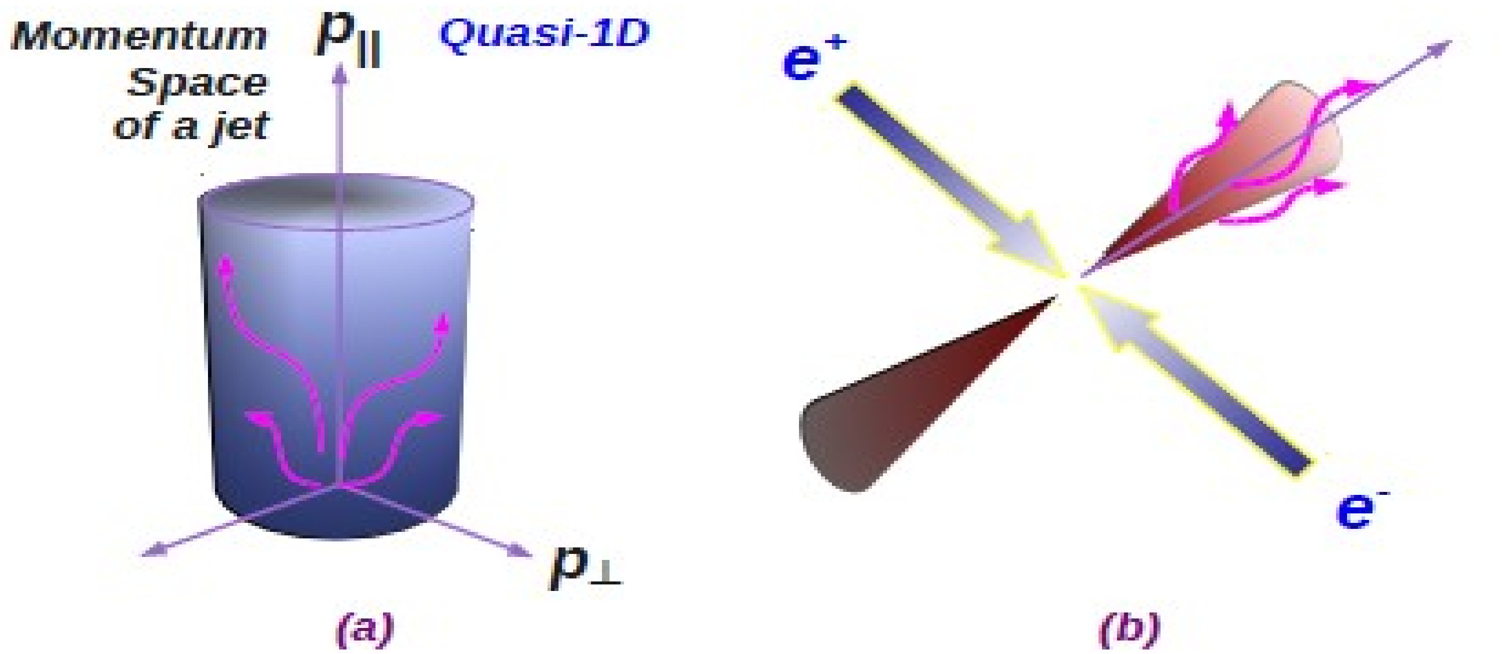
4) Test: can a DGLAP-like evolution equation be obtained?

$$D(x,Q^2) \sim f(E,T,q) * f(\ln(Q^2))$$

$$D(x,Q^2) \sim f(E,T(\ln(Q^2)),q(\ln(Q^2)))$$



The 'Thermodynamics of Jets'



K. Ürmössy, G.G. Barnaföldi, T.S. Bíró:

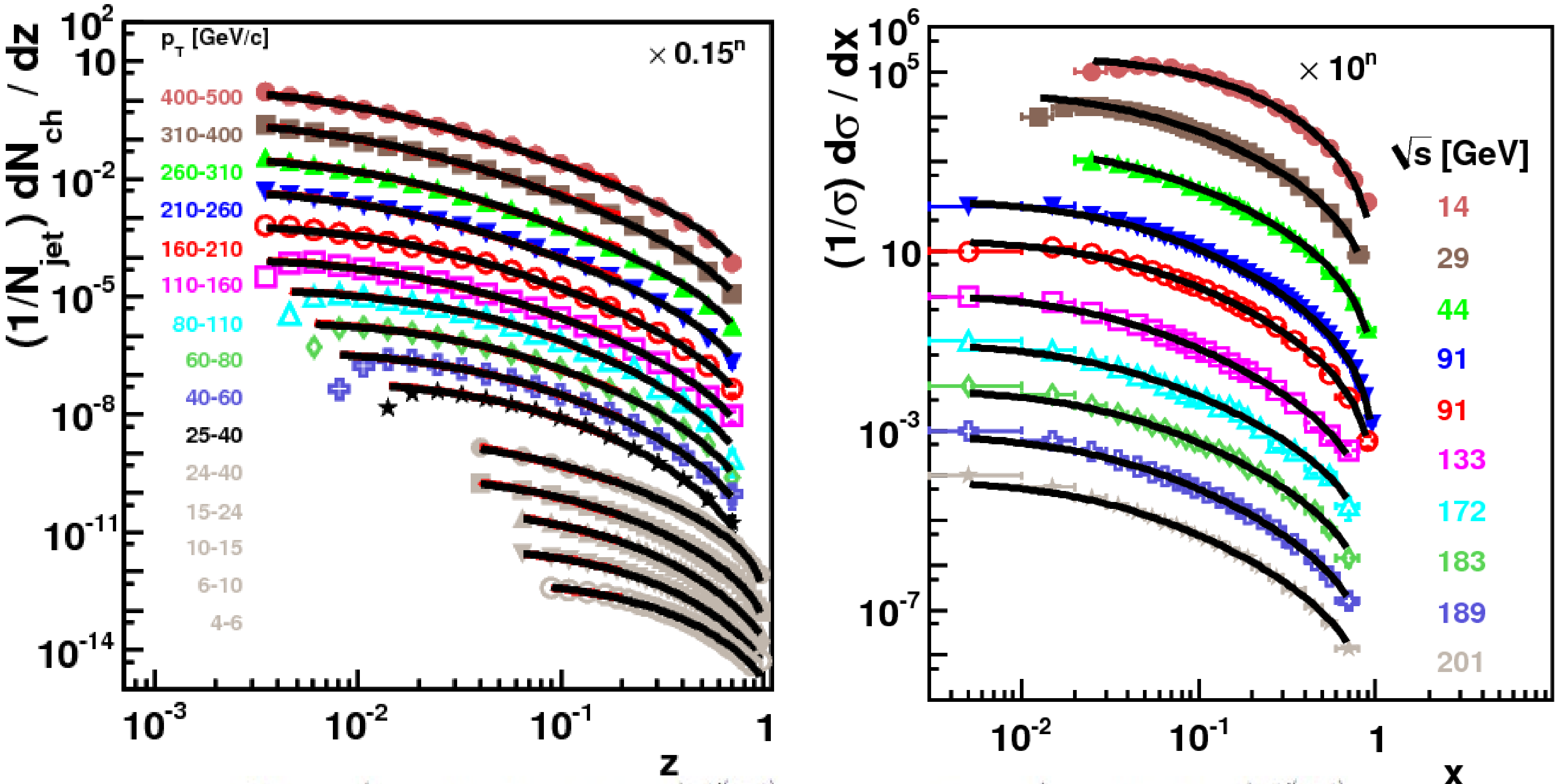
- Microcanonical Jet-Fragmentation in pp at LHC energies:

  Phys. Lett. B701 (2011) 111

- Generalized Tsallis distribution in e^+e^- collisons

  Phys. Lett. B718 (2012) 125

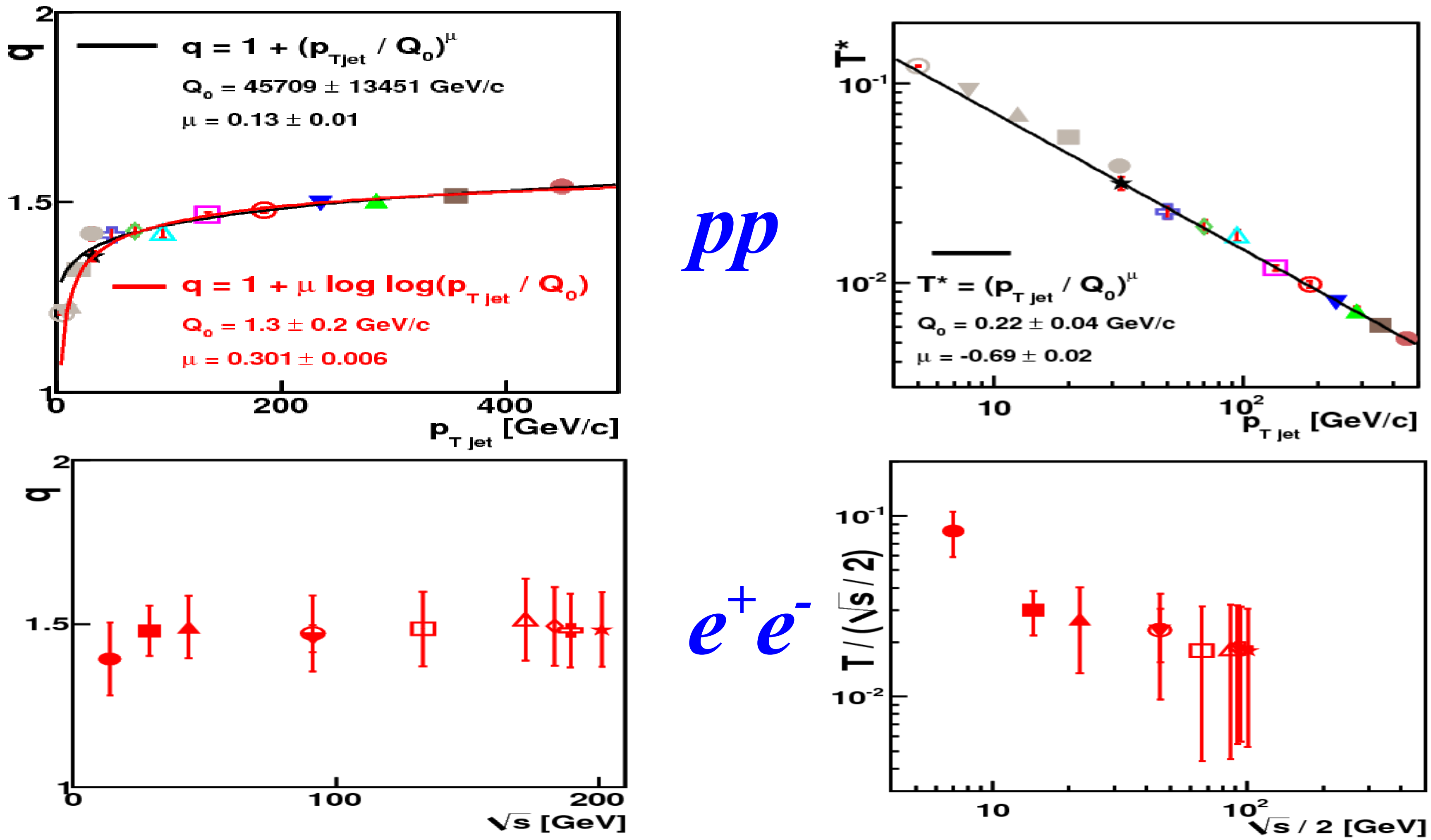
Fits for jet spectra in pp (left) and e^+e^- (right)



$$\frac{d\sigma}{dx} \propto \left(1 - \frac{q-1}{T/(\sqrt{s}/2)} \ln(1-x)\right)^{-1/(q-1)} \quad \Longrightarrow \quad \left(1 + \frac{q-1}{T/(\sqrt{s}/2)} x\right)^{-1/(q-1)}$$

Ref: K Ürmössy, GGB, TS Biró, PLB 710 (2011) 111, PLB 718 (2012) 125.

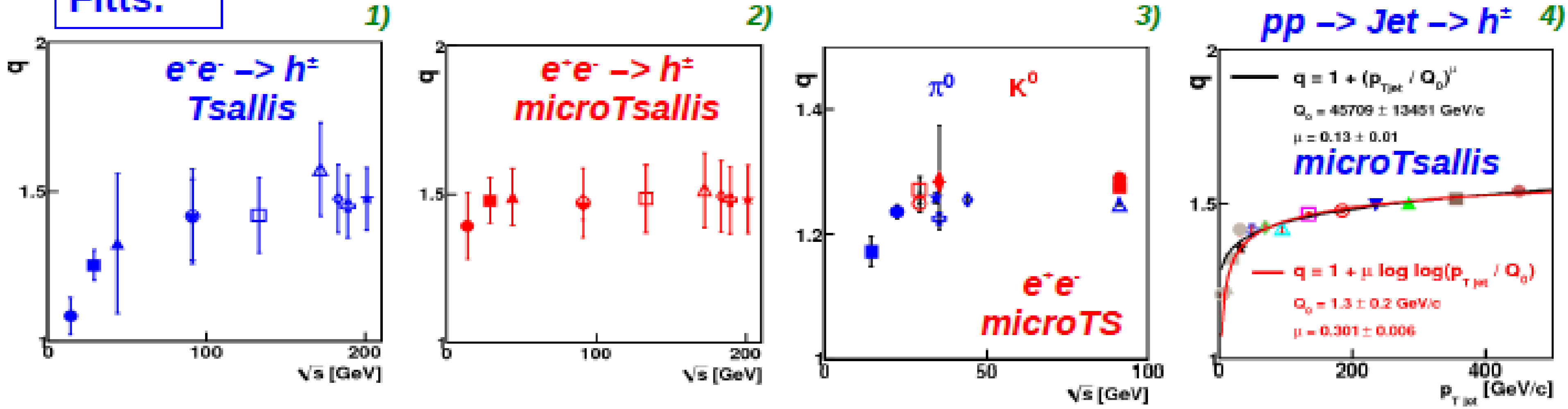
Scale evolution in pp and ee data



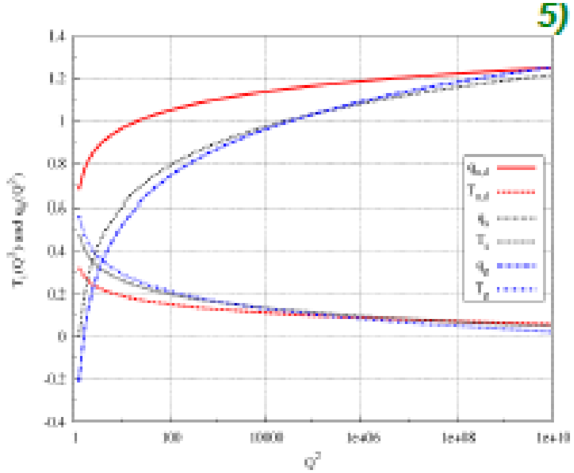
Ref: K Ürmössy, GGB, TS Biró, PLB 710 (2011) 111, PLB 718 (2012) 125.

Scale Evolution of the parameter q

Fitts:



Theory: ***Scale evolution of q, T from fits to AKK Frag. Funcs:***



$$D_{p_i}^{\pi^+}(z) \sim \left(1+(q_i-1)\,z/T_i\right)^{-1/(q_i-1)}$$

$$q_i = q_{0i} + q_{1i}\ln(\ln(Q^2))$$

1-2) U.K. etal., *Phys.Lett. B*, **701** (2011) 111-116

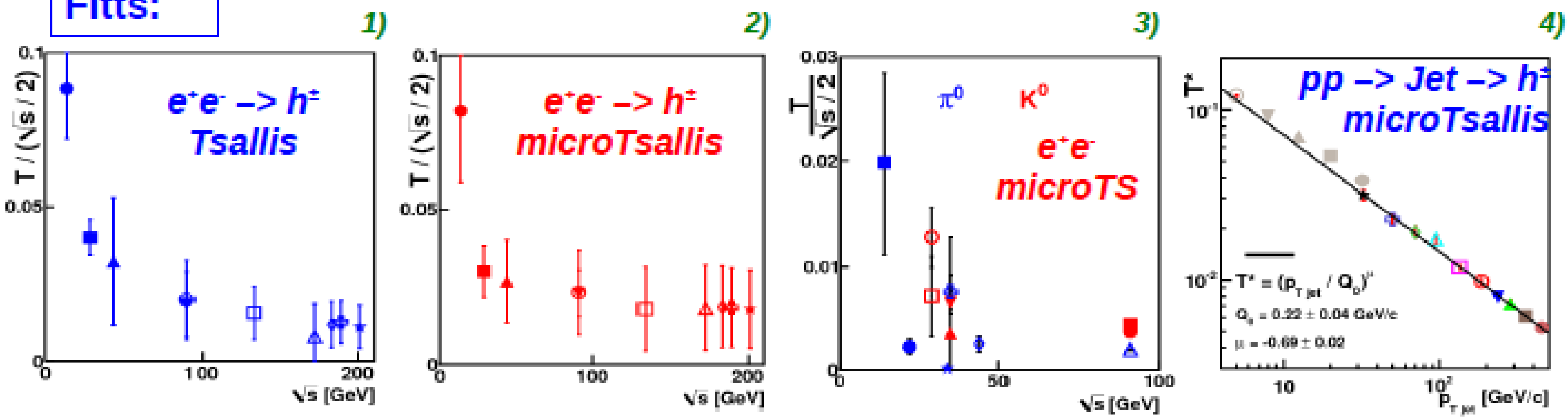
3) T. S. Biró etal., *Acta Phys.Polon. B*, **43** (2012) 811-820

4) U.K. etal., *Phys.Lett. B*, **718** (2012) 125-129

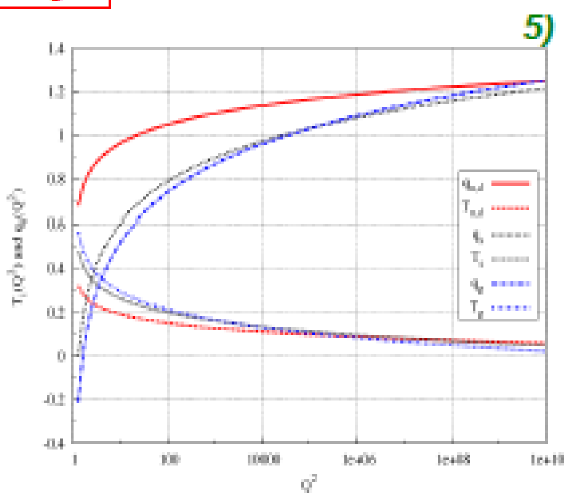
5) Barnaföldi etal., *Gribov-80 Conf.* C10-05-26.1, p.357-363

Scale Evolution of the parameter T

Fitts:



Theory: Scale evolution of *q, T from fits to AKK Frag. Funcs:*



$$D_{p_i}^{\pi^+}(z) \sim (1+(q_i-1)z/T_i)^{-1/(q_i-1)}$$

$$T_i = T_{0i} + T_{1i} \ln(\ln(Q^2))$$

1-2) U.K. etal., *Phys.Lett. B*, **701** (2011) 111-116

3) T. S. Biró etal., *Acta Phys.Polon. B*, **43** (2012) 811-820

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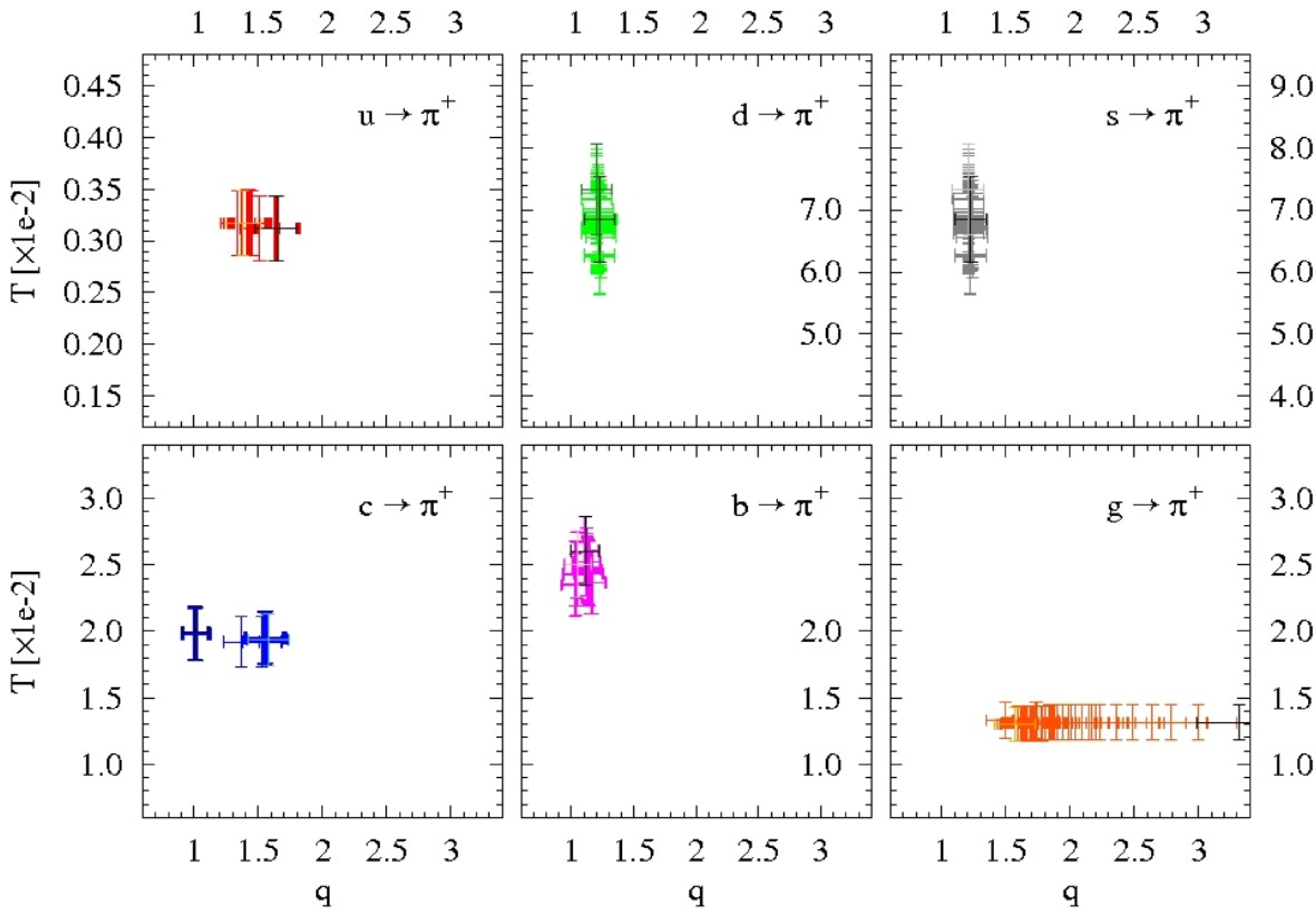
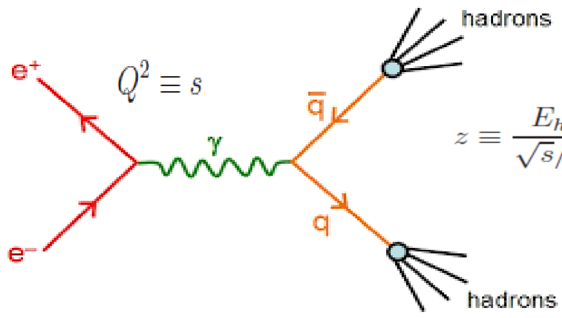
Full calculation of fitted FFs with DGLAP (Q^2)



The Tsallis–Pareto like Fragmentation Function parametriztion

Fragmentation Functions from e⁺e⁻ annihilation

- Inclusive spectra in electron-positron annihilation



$$z \equiv \frac{E_h}{\sqrt{s}/2} = \frac{2E_h}{Q}$$

$$F^h(z,Q^2) = \frac{1}{\sigma_0}\frac{\mathrm{d}\sigma(e^+ + e^- \to h + X)}{\mathrm{d}z}$$

$$\sigma_0(Q^2) = \sum_q \sigma_0^q(Q^2)\left[1 + \frac{\alpha_s(Q^2)}{\pi}\right], \quad q \in \{u,d,s,c,b,t\}$$

$$\frac{\mathrm{d}\sigma^h}{\mathrm{d}z} = \sum_i C_i(z,Q^2) \otimes D_i^h(z,Q^2) = \sum_i \int_z^1 \frac{1}{x}\, C_i(x,Q^2)\, D_i^h(z/x,Q^2)\,\mathrm{d}x,$$

$$C_i(z,Q^2) = \begin{cases} [\delta(1-z) + \mathcal{O}(\alpha_s(Q^2))]\,\sigma_0^q(Q^2) & i \in \{q,\bar{q}\} \\ 0 & i \in \{g\} \end{cases}$$

$$\frac{\mathrm{d}\sigma^h}{\mathrm{d}z} = \sum_i \int_z^1 \frac{1}{x}\,\delta(1-x)\,\sigma_0^q(Q^2)\cdot D_i^h\left(z/x,Q^2\right)\mathrm{d}x = \sum_q \sigma_0^q(Q^2)\cdot D_q^h(z,Q^2)\,\mathrm{d}z$$

Fragmentation Functions from e^+e^- annihilation

- Inclusive spectra in electron-positron annihilation

$$\frac{\mathrm{d}\sigma^h}{\mathrm{d}z} = \sum_q \sigma_0^q D_q^h(z) + \frac{\alpha_s}{2\pi} C_F \sum_q \left[\frac{2}{3}\pi^2 - \frac{9}{2}\right] \sigma_0^q D_q^h(z) +$$

$$+ \frac{\alpha_s}{2\pi} C_F \int_z^1 \left\{ \sum_q \left[\frac{1+x^2}{x} \sigma_0^q D_q^h(z/x) - 2\,\sigma_0^q D_q^h(z)\right] \left[\frac{\ln(1-x)}{1-x}\right] - \right.$$

$$- \sum_q \left[\frac{3}{2x} \sigma_0^q D_q^h(z/x) - \frac{3}{2} \sigma_0^q D_q^h(z)\right] \left[\frac{1}{1-x}\right] +$$

$$+ \sum_q \left[\frac{2\,(1+x^2)\ln x}{(1-x)\,x} + \frac{3}{2}\frac{1-x}{x} + \frac{1}{x}\right] \sigma_0^q D_q^h(z/x) +$$

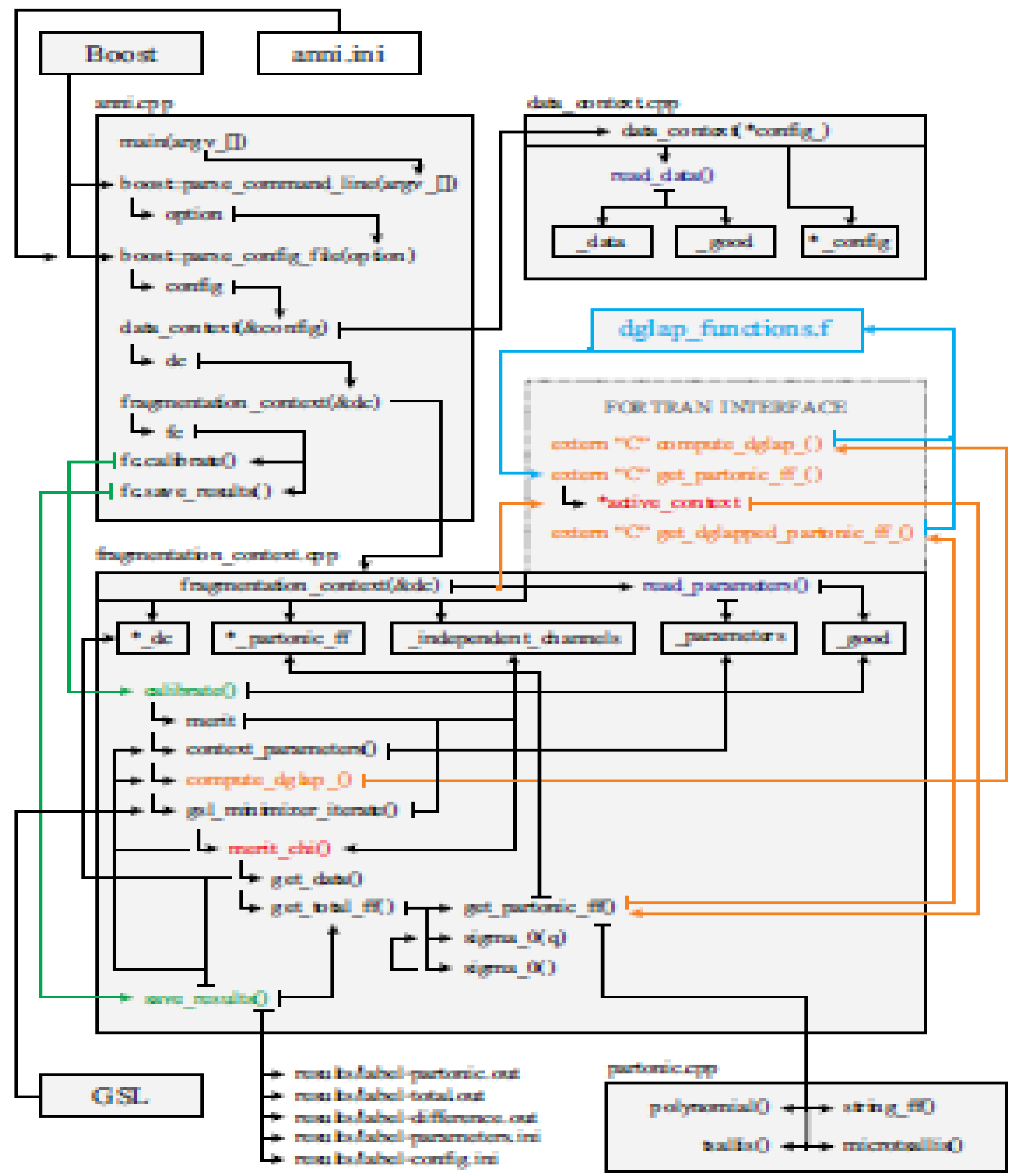
$$\left. + \left[\frac{2 + 2\,(1-x)^2}{x}\right] \left[\frac{\ln(1-x)}{x} + \frac{2\ln x}{x}\right] \sigma_0 D_g^h(z/x) \right\} \mathrm{d}x.$$

- DGLAP evolution

$$\frac{\partial}{\partial \ln Q^2} \begin{bmatrix} D_S^h(z,Q^2) \\ D_g^h(z,Q^2) \end{bmatrix} = \frac{\alpha_s(Q^2)}{2\pi} \begin{bmatrix} P_{qq}(z) & 2N_f P_{gq}(z) \\ P_{qg}(z) & P_{gg}(z) \end{bmatrix} \otimes \begin{bmatrix} D_S^h(z,Q^2) \\ D_g^h(z,Q^2) \end{bmatrix}$$

Fit Program for FF Parametrizations

- Calculation of the identified hadron spectra in electron-positron annihilation.
- Comparison to the available data
- Fine-tuning the parameters for all q and g channels.
- Test ot the FF parametrizaions.



Fragmentation Functions based on HKNS07

- Fit ansatz for the polynomial fragmentation function

$$D_i^h(z,Q_0^2) = N_i^h z^{\alpha_i^h}(1-z)^{\beta_i^h}$$

- Parameters for all q and g channels:

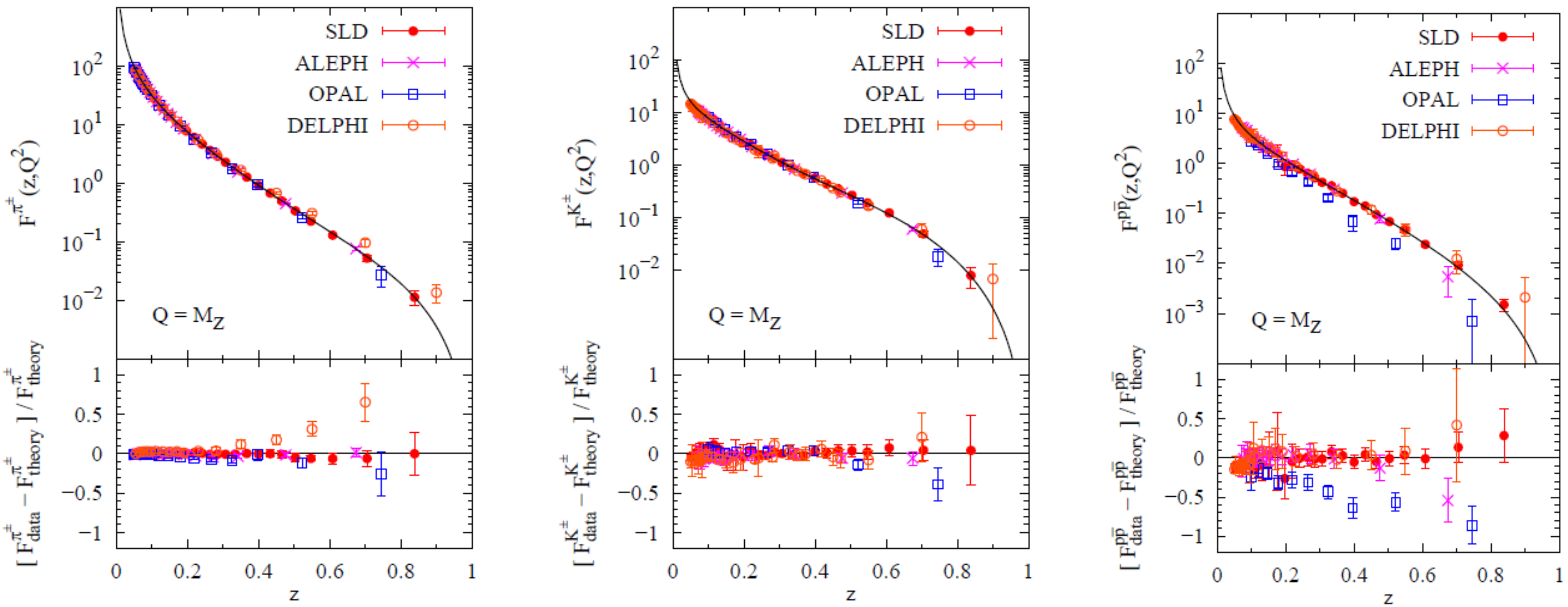
i	$N_i^{\pi^+}$	$\alpha_i^{\pi^+}$	$\beta_i^{\pi^+}$	$N_i^{K^+}$	$\alpha_i^{K^+}$	$\beta_i^{K^+}$	N_i^p	α_i^p	β_i^p
u	1,054	−1,100	1,282	1,493	0,588	1,632	0,298	−0,814	1,628
$\bar{u}$	4,609	−0,500	5,197	12,294	0,565	7,093	2,323	0,866	5,078
c	1,476	−1,007	3,918	4,690	0,230	4,549	6,073	0,683	7,375
b	1,275	−1,176	5,805	7,633	0,017	8,808	2,194	0,071	8,802
g	102,876	1,405	8,000	0,407	5,000	0,247	30,508	5,000	2,927
$\bar{s}$	$N_{\bar{u}}^{\pi^+}$	$\alpha_{\bar{u}}^{\pi^+}$	$\beta_{\bar{u}}^{\pi^+}$	23,235	2,190	2,829	$N_{\bar{u}}^p$	$\alpha_{\bar{u}}^p$	$\beta_{\bar{u}}^p$

Fragmentation Functions based on HKNS07

- Fit ansatz for the polynomial fragmentation function

$$D_i^h(z,Q_0^2) = N_i^h z^{\alpha_i^h}(1-z)^{\beta_i^h}$$

- Pion, kaon and proton spectra:



Fragmentation Functions based onTsallis-Pareto

- Fit ansatz for the TP-like fragmentation function

$$D_i^h(x) = N_i^h \cdot (1-x) \cdot \left[1 - \hat{q}_i^h \, \hat{T}_i^h \ln(1-x)\right]^{-1/\hat{q}_i^h}$$

- Parameters for all q and g channels:

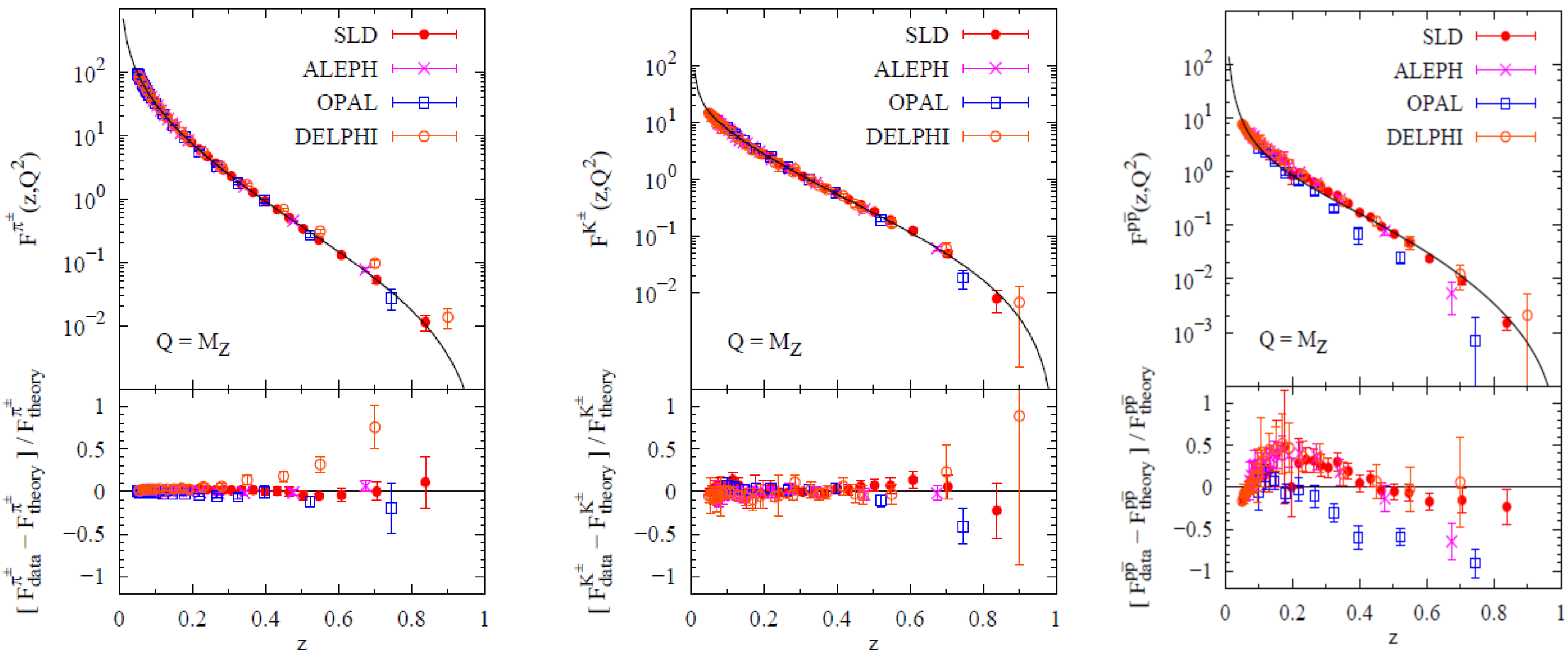
i	$N_i^{\pi^+}$	$q_i^{\pi^+}$	$1/T_i^{\pi^+}$	$N_i^{K^+}$	$q_i^{K^+}$	$1/T_i^{K^+}$	N_i^p	q_i^p	$1/T_i^p$
u	869,225	1,400	304,943	0,789	2,530	7,367	28,954	1,648	88,868
$\bar{u}$	75,242	1,305	19,357	4,926	1,373	8,696	6,712	1,579	64,709
c	164,239	1,558	53,258	4,367	1,357	8,022	2,642	1,230	11,956
b	201,927	1,040	55,531	9,093	1,262	11,644	0,939	1,000	2,740
g	133,967	1,524	54,880	213,641	1,742	213,641	199,973	1,375	100,334
$\bar{s}$	$N_{\bar{u}}^{\pi^+}$	$q_{\bar{u}}^{\pi^+}$	$T_{\bar{u}}^{\pi^+}$	0,520	1,000	1,315	$N_{\bar{u}}^p$	$q_{\bar{u}}^p$	$T_{\bar{u}}^p$

Fragmentation Functions based on Tsallis-Pareto

- Fit ansatz for the TP fragmentation function

$$D_i^h(x) = N_i^h \cdot (1-x) \cdot \left[1 - \hat{q}_i^h \, \hat{T}_i^h \ln(1-x)\right]^{-1/\hat{q}_i^h}$$

- Pion, kaon and proton spectra:



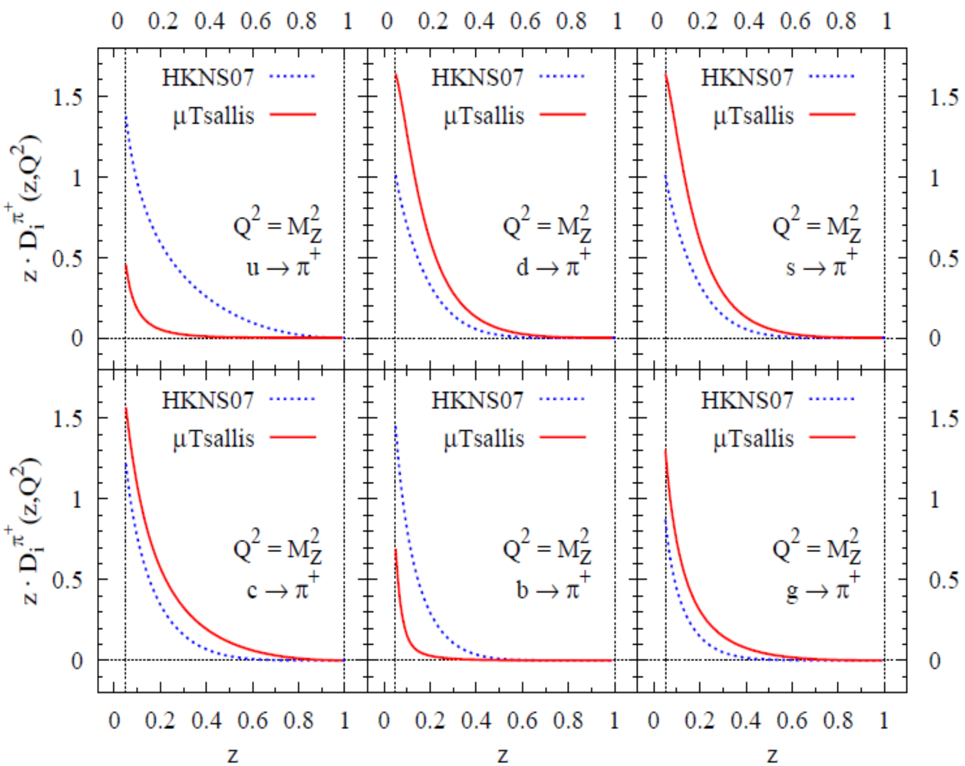
Fragmentation Functions: Polynomial vs. Tsallis-Pareto

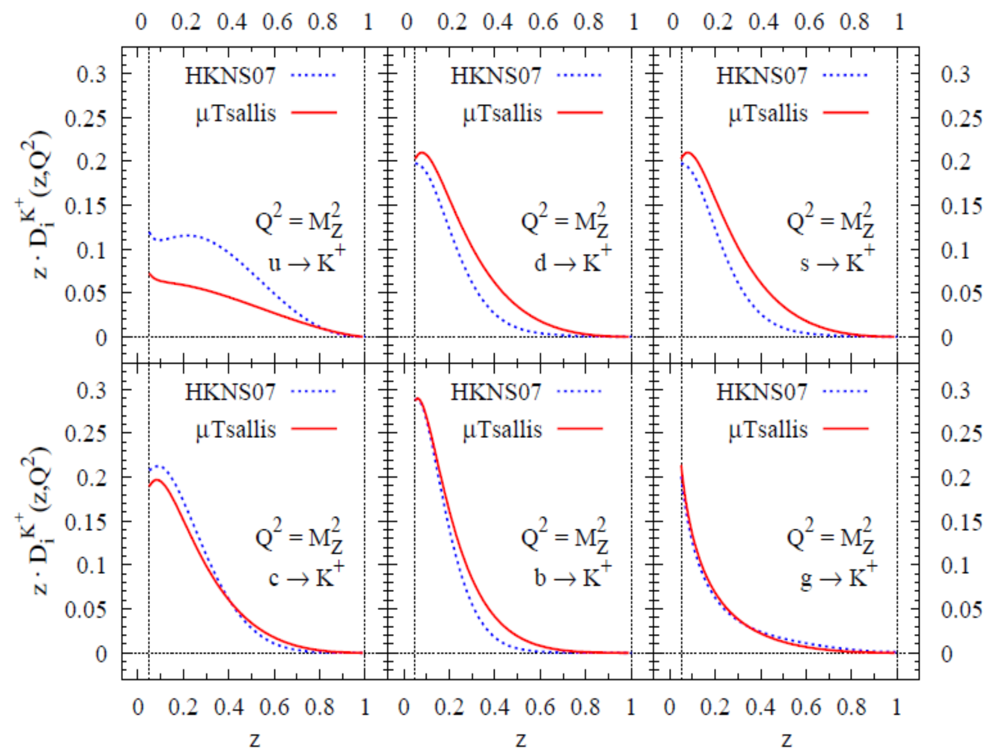
- Fit ansatz for polinomial and Tsallis-pareto like cases

$$D_i^h(z,Q_0^2) = N_i^h z^{\alpha_i^h} (1-z)^{\beta_i^h}$$

$$D_i^h(x) = N_i^h \cdot (1-x) \cdot \left[1 - \hat{q}_i^h \hat{T}_i^h \ln(1-x)\right]^{-1/\hat{q}_i^h}$$

- Comparison of channels in pion and kaon production

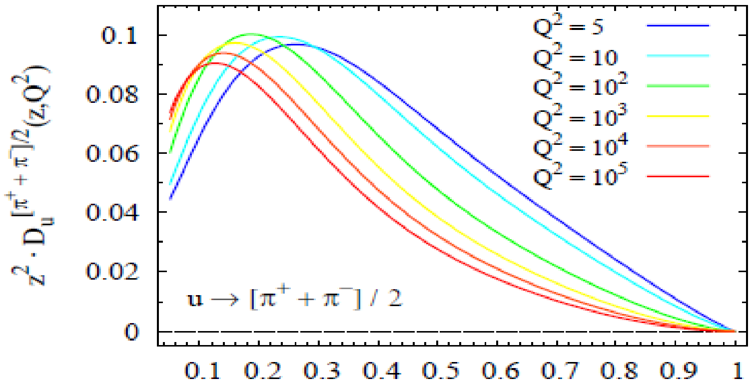


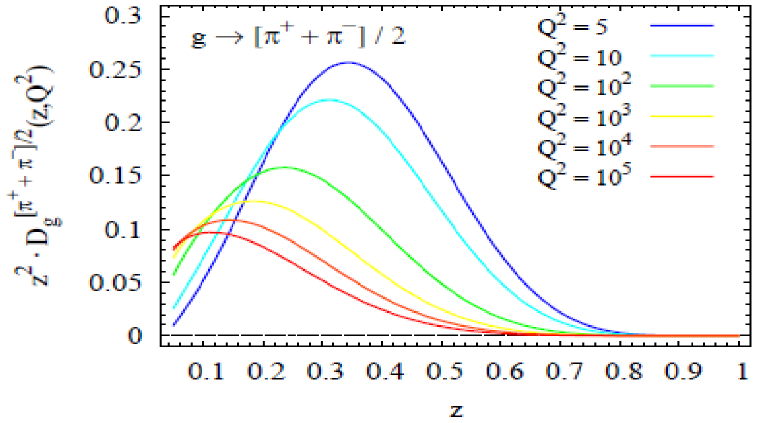


Fragmentation Functions: DGLAP evolution

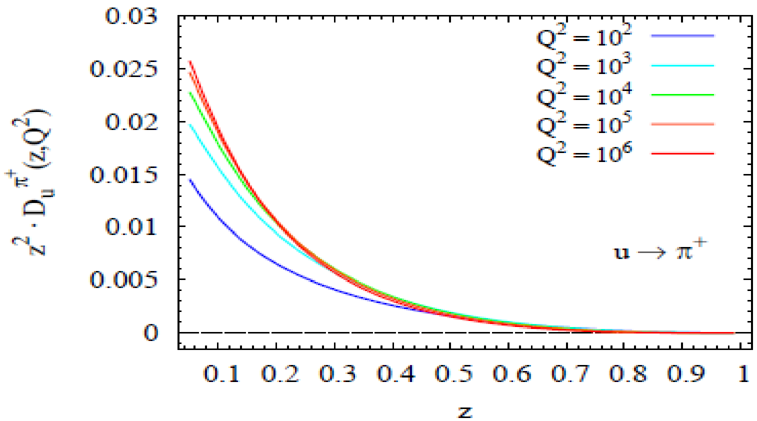
- DGLAP evolution for q & g channels in pion production

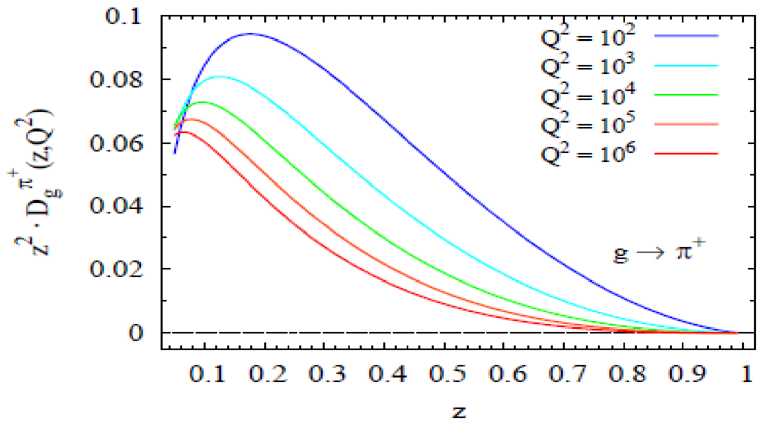
$$D_i^h(z,Q_0^2) = N_i^h z^{\alpha_i^h}(1-z)^{\beta_i^h}$$





$$D_i^h(x) = N_i^h \cdot (1-x) \cdot \left[1 - \hat{q}_i^h \hat{T}_i^h \ln(1-x)\right]^{-1/\hat{q}_i^h}$$

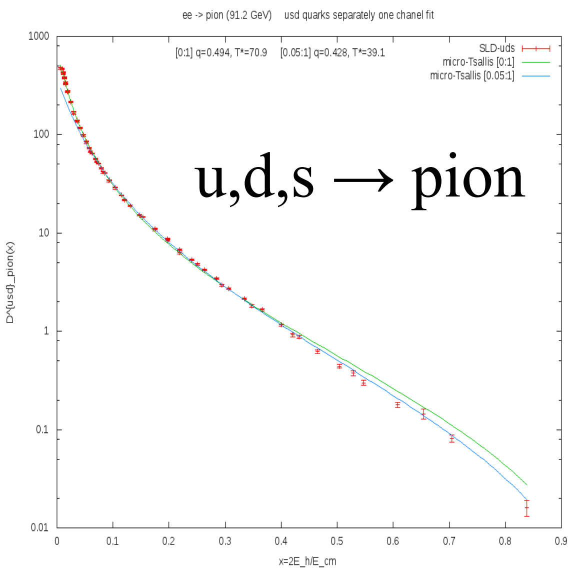


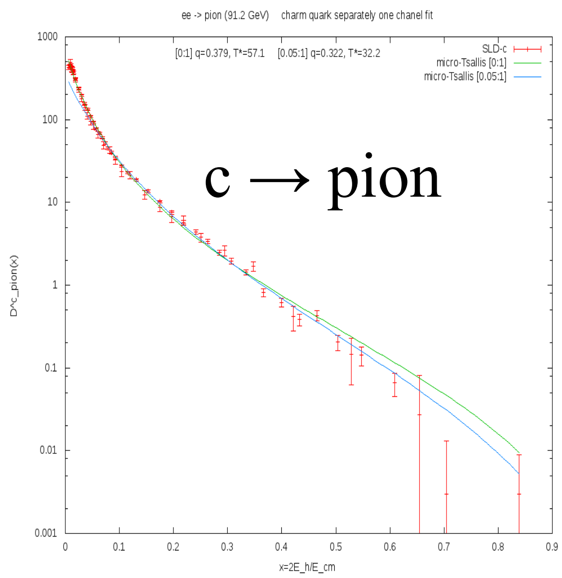


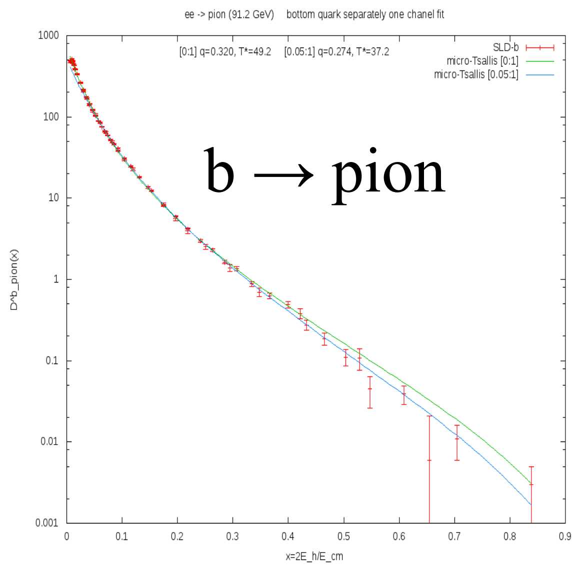
Fragmentation Functions: Comparison to SLD data

- Comparison of channels SLD Collaboration:

$$D_i^h(x) = N_i^h \cdot (1-x) \cdot \left[1 - \hat{q}_i^h \, \hat{T}_i^h \ln(1-x)\right]^{-1/\hat{q}_i^h}$$



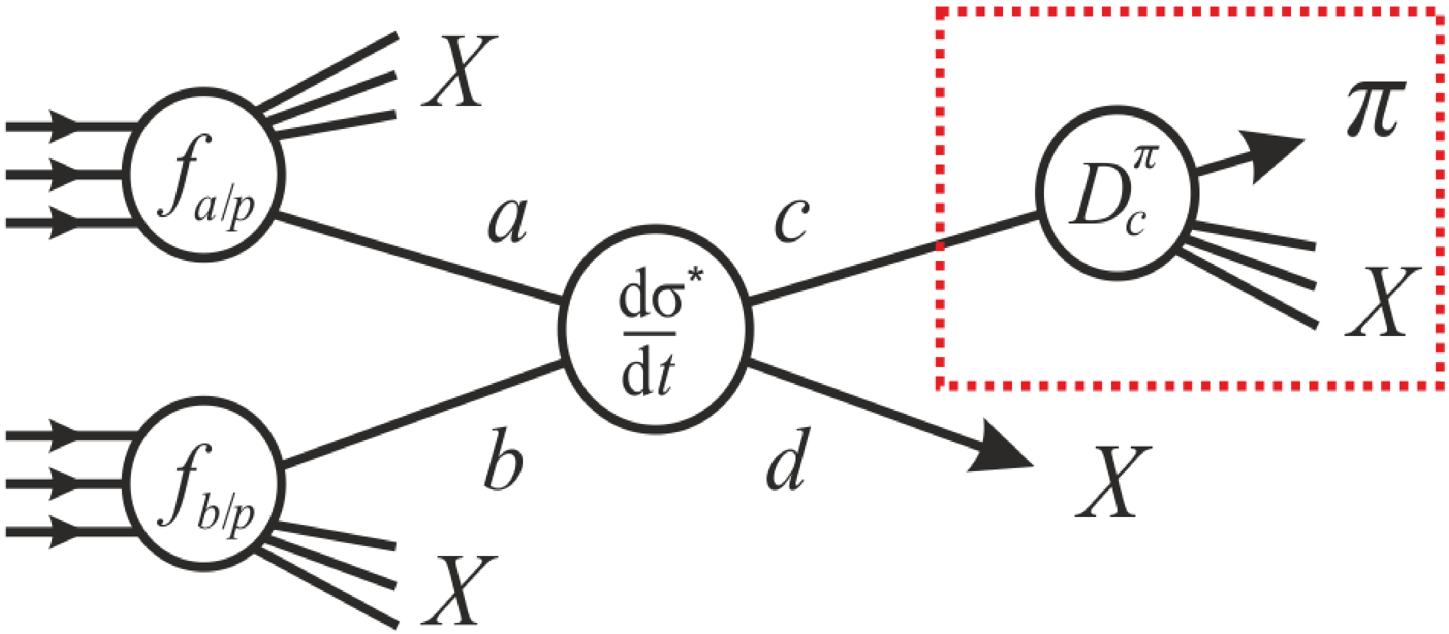




Test of the Tsallis–Pareto like Fragmentation Function Parametriztion in pp Collisions

AKK as a typical FF in a pQCD code (kTpQCDv20)

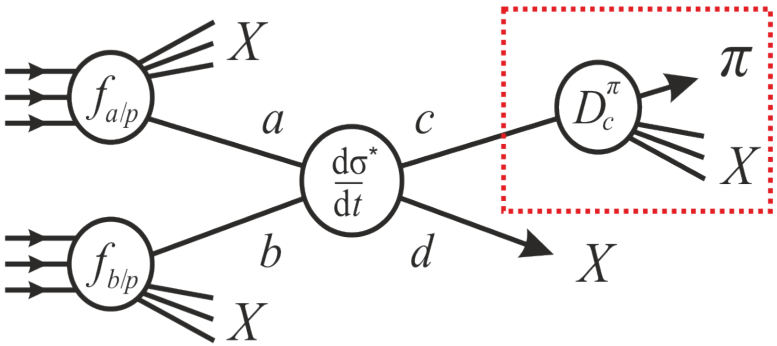
$$D_i^h(z,Q_0^2) = N_i^h z^{\alpha_i^h}(1-z)^{\beta_i^h}\left[1+\gamma_i^h(1-z)^{\delta_i^h}\right],$$



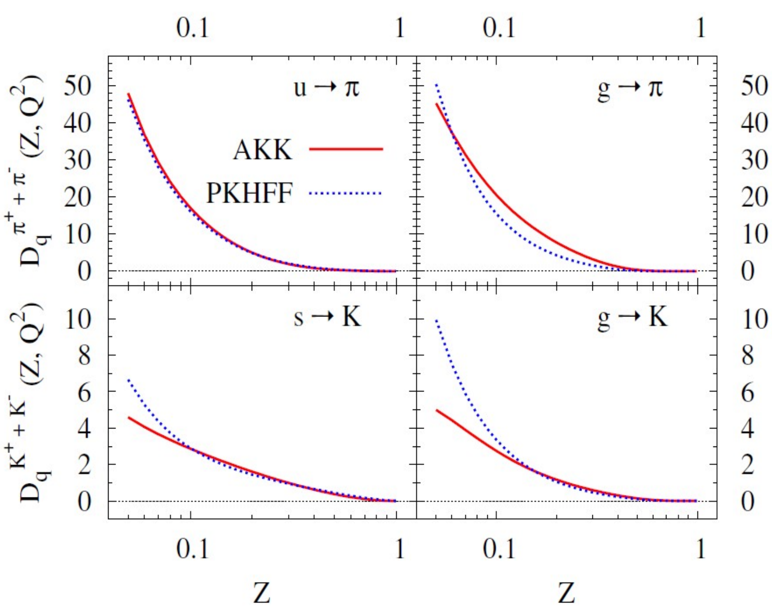
$$E_\pi \frac{d\sigma_\pi^{pp}}{d^3p_\pi} \sim f_{a/p}(x_a,Q^2) \otimes f_{b/p}(x_b,Q^2) \otimes \frac{d\sigma^{ab\to cd}}{d\hat{t}} \otimes \frac{D_c^\pi(z,Q^2)}{\pi z^2},$$

AKK as a typical FF in a pQCD code (kTpQCDv20)

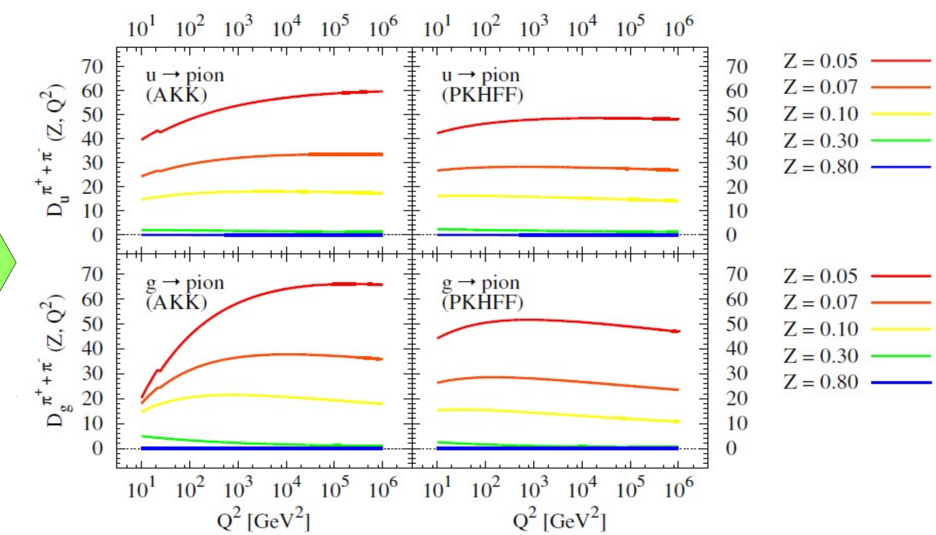
$$D_i^h(z,Q_0^2) = N_i^h z^{\alpha_i^h}(1-z)^{\beta_i^h}\left[1+\gamma_i^h(1-z)^{\delta_i^h}\right],$$



$$D_i^h(z,Q_0^2) = N_i^h z^{\alpha_i^h}(1-z)^{\beta_i^h}$$



DGLAP evolution



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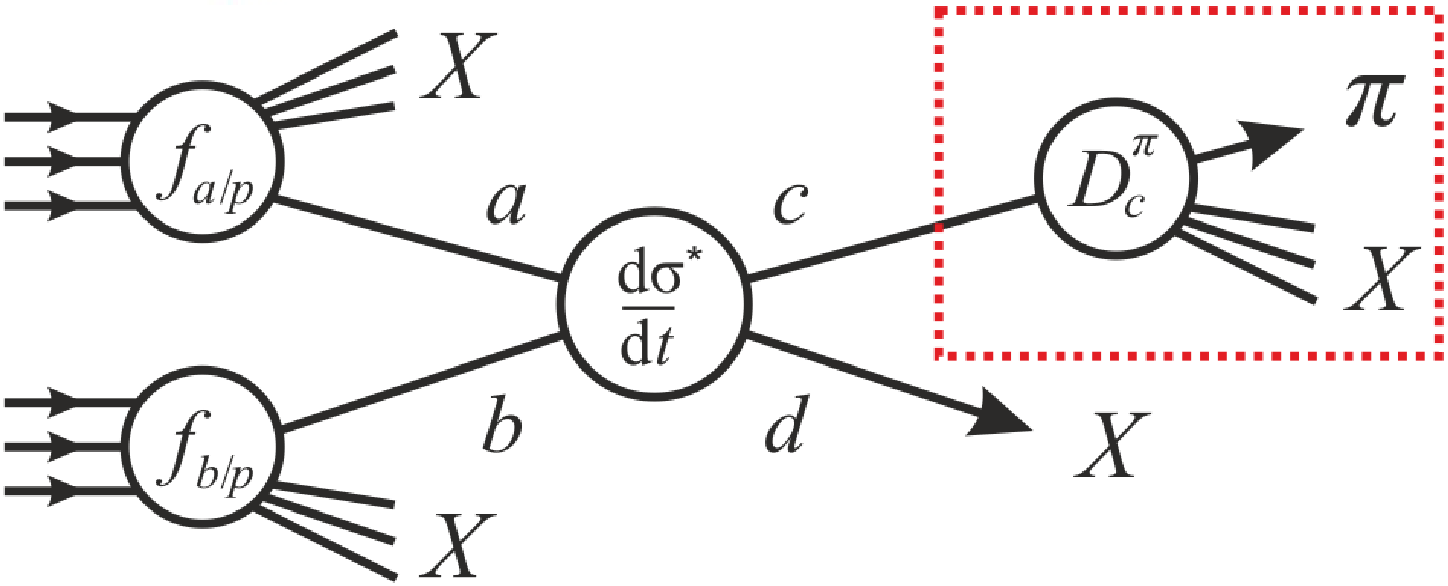
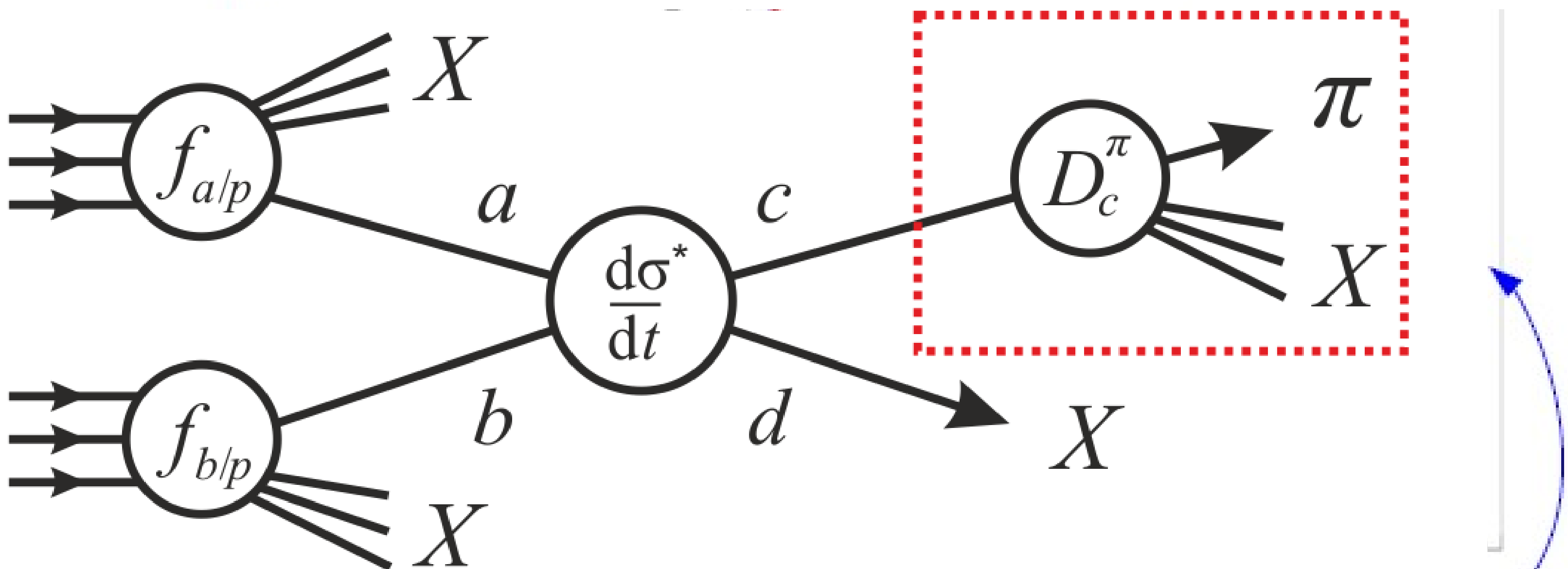
Test of the FF via NLO pQCD code (kTpQCDv20)

$$E_\pi \frac{\mathrm{d}\sigma_\pi^{pp}}{\mathrm{d}^3 p_\pi} \sim f_{a/p}(x_a, Q^2) \otimes f_{b/p}(x_b, Q^2) \otimes \frac{\mathrm{d}\sigma^{ab\to cd}}{\mathrm{d}\hat{t}} \otimes \frac{D_c^\pi(z, Q^2)}{\pi z^2},$$



Test of the FF via NLO pQCD code (kTpQCDv20)

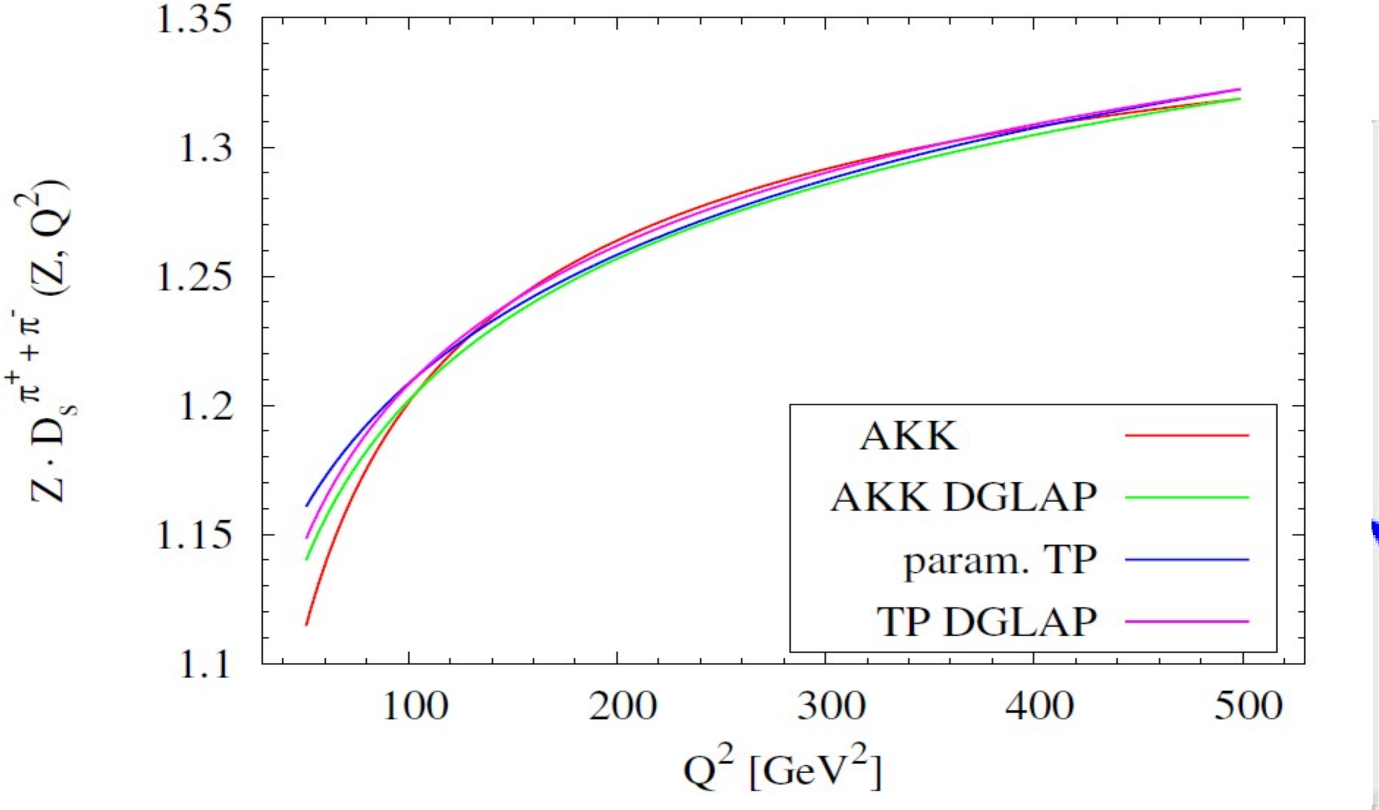
$$E_\pi \frac{\mathrm{d}\sigma_\pi^{pp}}{\mathrm{d}^3 p_\pi} \sim f_{a/p}(x_a, Q^2) \otimes f_{b/p}(x_b, Q^2) \otimes \frac{\mathrm{d}\sigma^{ab\to cd}}{\mathrm{d}\hat{t}} \otimes \frac{D_c^\pi(z, Q^2)}{\pi z^2},$$



$$D_{p_i}^{\pi^+}(z) \sim \left(1 + (q_i - 1)\, z / T_i\right)^{-1/(q_i - 1)}$$

Barnaföldi et. al., *Proceedings of the Workshop Gribov '80* (2010)

Test of the FF via NLO pQCD code (kTpQCDv20)



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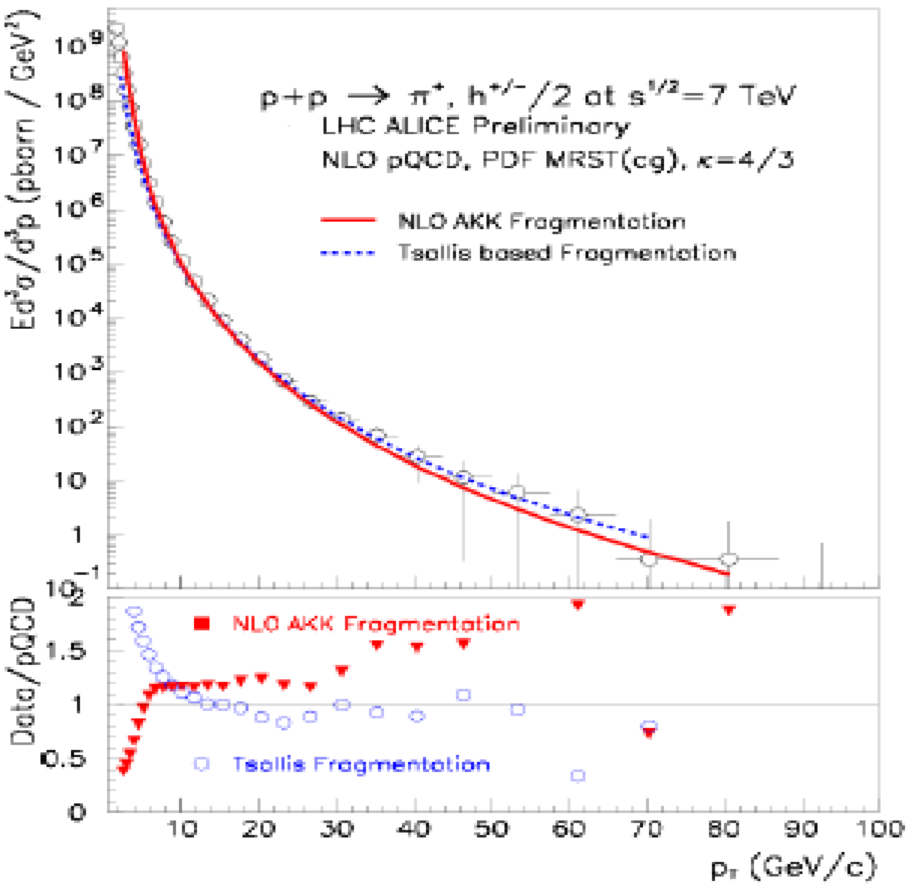

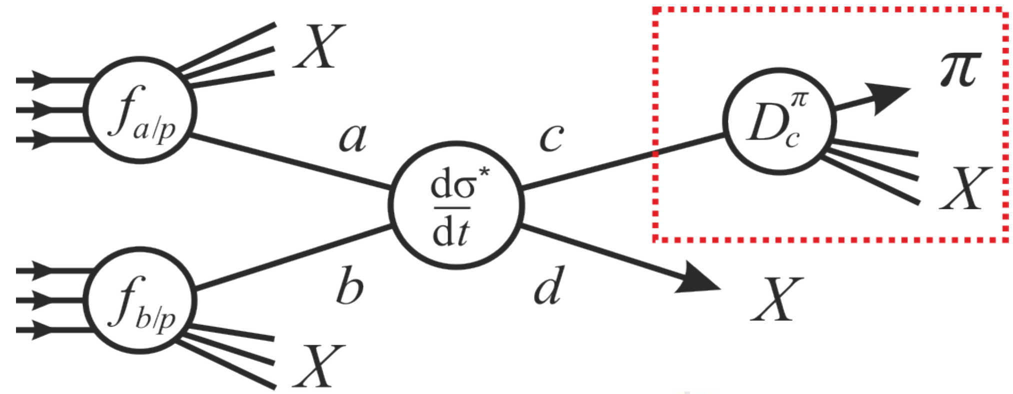

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Barnaföldi et. al., *Proceedings of the Workshop Gribov '80* (2010)

SUMMARY

- Tsalls-Pareto based Fragmentation Functions
  - Test by a standard parametization (HKNS07)
  - Microcanonical Tsallis-Pareto distribution works well
  - Test and comarison of HKNS07 and Tsallis Pareto
  - Comparision to SLD Collaboration data
- FFs based on Tsallis-Pareto parametrization
  - It seems Tsallis-Pareto based FFs are working
  - First results in pp collisions looks promising