Fluctuations of conserved charges in lattice QCD

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LQCD: chiral transition happens at $T_c \approx 154$ MeV

What is deconfinement in QCD ? At what temperature does it happen

What is the nature of the deconfined matter ?

⇒ Fluctuations of conserved charges

Chemical freezout and fluctuations of conserved charges

QCD thermodynamics at non-zero chemical potential

Taylor expansion :

$$\frac{p(T,\mu_B,\mu_Q,\mu_S)}{T^4} = \sum_{i,j,k} \frac{1}{i!j!k!} \chi_{ijk}^{BQS} \cdot \left(\frac{\mu_B}{T}\right)^i \cdot \left(\frac{\mu_Q}{T}\right)^j \cdot \left(\frac{\mu_S}{T}\right)^k$$ hadronic

$$\frac{p(T,\mu_u,\mu_d,\mu_s)}{T^4} = \sum_{i,j,k} \frac{1}{i!j!k!} \chi_{ijk}^{uds} \cdot \left(\frac{\mu_u}{T}\right)^i \cdot \left(\frac{\mu_d}{T}\right)^j \cdot \left(\frac{\mu_s}{T}\right)^k$$ quark

$$\chi_{ijk}^{abc} = T^{i+j+k} \frac{\partial^i}{\partial \mu_a^i} \frac{\partial^j}{\partial \mu_b^j} \frac{\partial^k}{\partial \mu_c^k} \frac{1}{VT^3} \ln Z(T,V,\mu_a,\mu_b,\mu_c)|_{\mu_a=\mu_b=\mu_c=0}$$

Taylor expansion coefficients give the fluctuations and correlations of conserved charges, e.g.

$$\chi_2^X = \chi_X = \frac{1}{VT^3}(\langle X^2 \rangle - \langle X \rangle^2) \qquad \chi_{11}^{XY} = \frac{1}{VT^3}(\langle XY \rangle - \langle X \rangle \langle Y \rangle)$$

⬇

information about carriers of the conserved charges (hadrons or quarks)

⬇

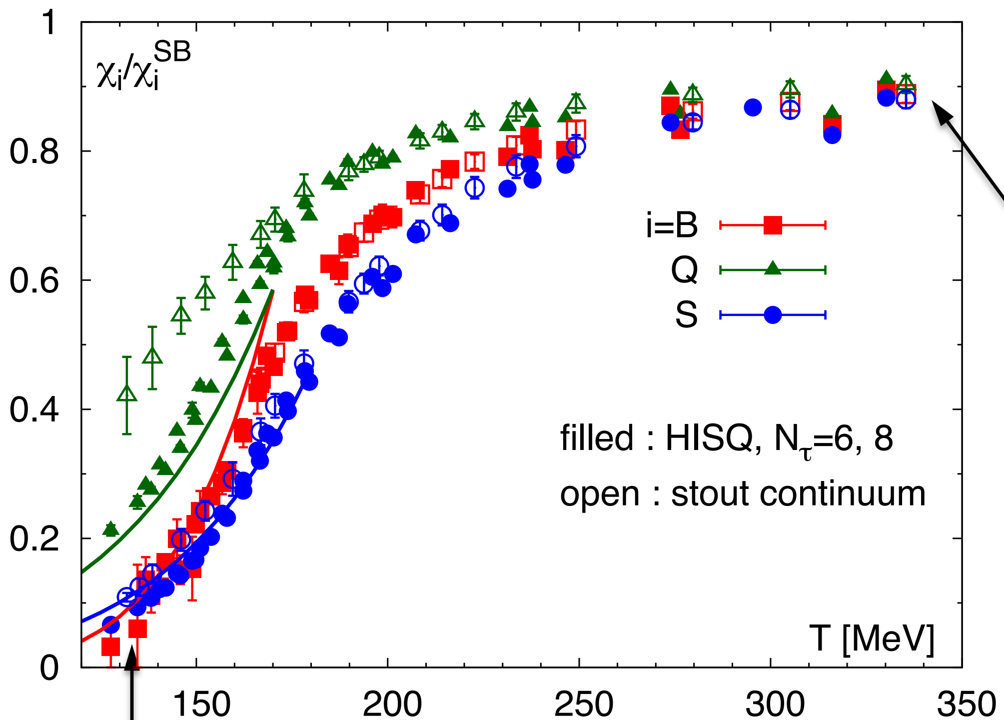
probes of deconfinement

Deconfinement : fluctuations of conserved charges

$$\chi_B = \frac{1}{VT^3}\left(\langle B^2\rangle - \langle B\rangle^2\right)$$ baryon number

$$\chi_Q = \frac{1}{VT^3}\left(\langle Q^2\rangle - \langle Q\rangle^2\right)$$ electric charge

$$\chi_S = \frac{1}{VT^3}\left(\langle S^2\rangle - \langle S\rangle^2\right)$$ strangeness



conserved charges are carried by massive hadrons

Ideal gas of massless quarks :

$$\chi_B^{SB} = \frac{1}{3} \qquad \chi_Q^{SB} = \frac{2}{3}$$

$$\chi_S^{SB} = 1$$

conserved charges carried by light quarks

HotQCD: PRD86 (2012) 034509

BW: JHEP 1201 (2012) 138,

Deconfinement of strangeness

Partial pressure of strange hadrons in uncorrelated hadron gas:

$$P_S = \frac{p(T) - p_{S=0}(T)}{T^4} = M(T)\cosh\left(\frac{\mu_S}{T}\right) +$$

$$B_{S=1}(T)\cosh\left(\frac{\mu_B - \mu_S}{T}\right) + B_{S=2}(T)\cosh\left(\frac{\mu_B - 2\mu_S}{T}\right) + B_{S=3}(T)\cosh\left(\frac{\mu_B - 3\mu_S}{T}\right)$$

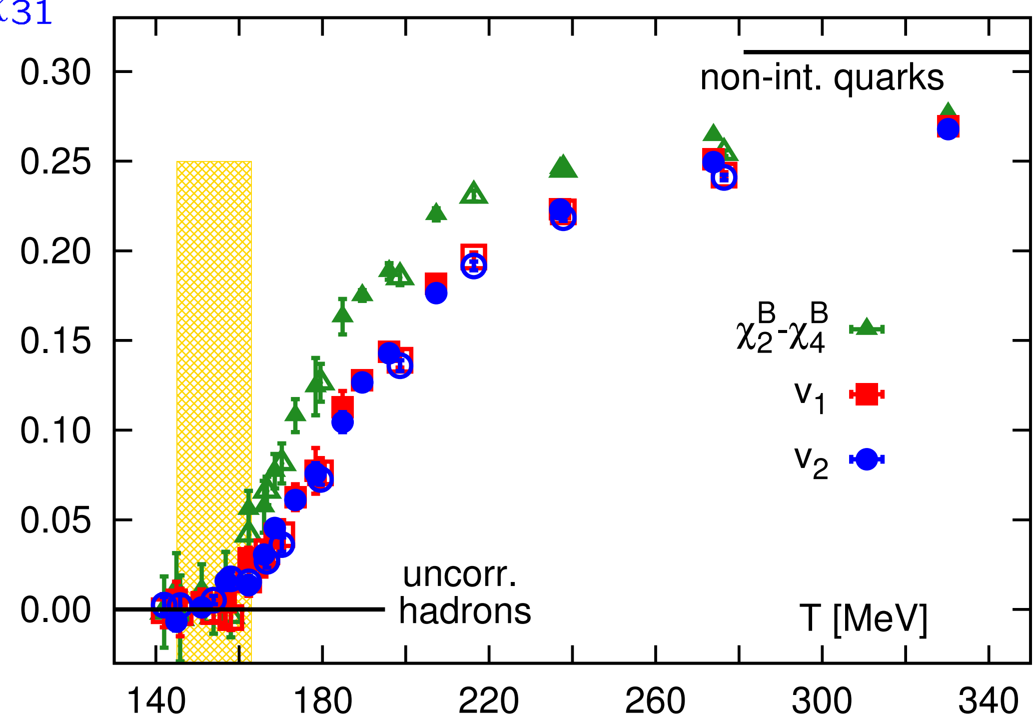
Strange hadrons are heavy
⇒ treat them as Boltzmann gas

$$v_1 = \chi_{31}^{BS} - \chi_{11}^{BS}$$

$$v_2 = \frac{1}{3}\left(\chi_4^S - \chi_2^S\right) - 2\chi_{13}^{BS} - 4\chi_{22}^{BS} - 2\chi_{31}^{BS}$$

should vanish !

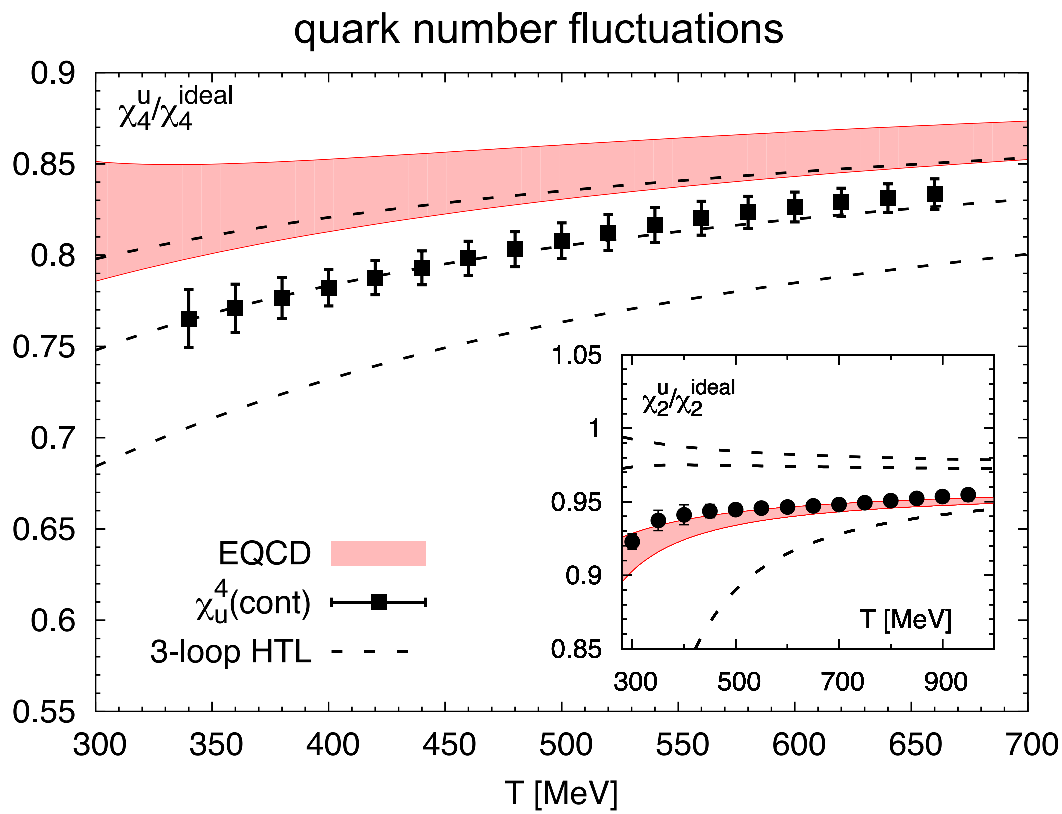
- v_1 and v_2 do vanish within errors at low T
- v_1 and v_2 rapidly increase above the transition region, eventually reaching non-interacting quark gas values

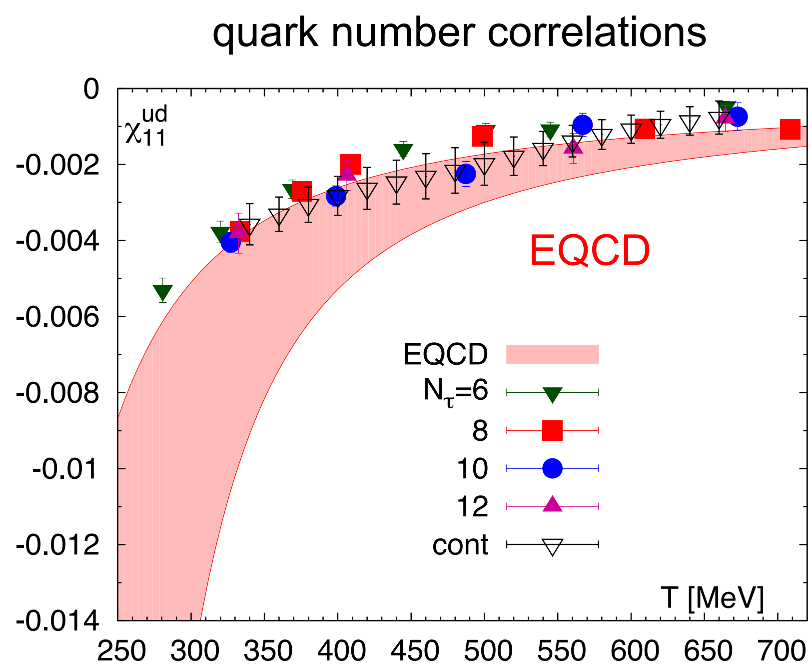


Bazavov et al, PRL 111 (2013) 082301

Quark number fluctuations at high T

At high temperatures quark number fluctuations can be described by weak coupling approach due to asymptotic freedom of QCD





- Lattice results converge as the continuum limit is approached
- Good agreement between lattice and the weak coupling approach for 2nd and 4th order quark number fluctuations as well as for correlations

Bazavov et al, PRD88 (2013) 094021, Ding et at, PRD92 (2015) 074043

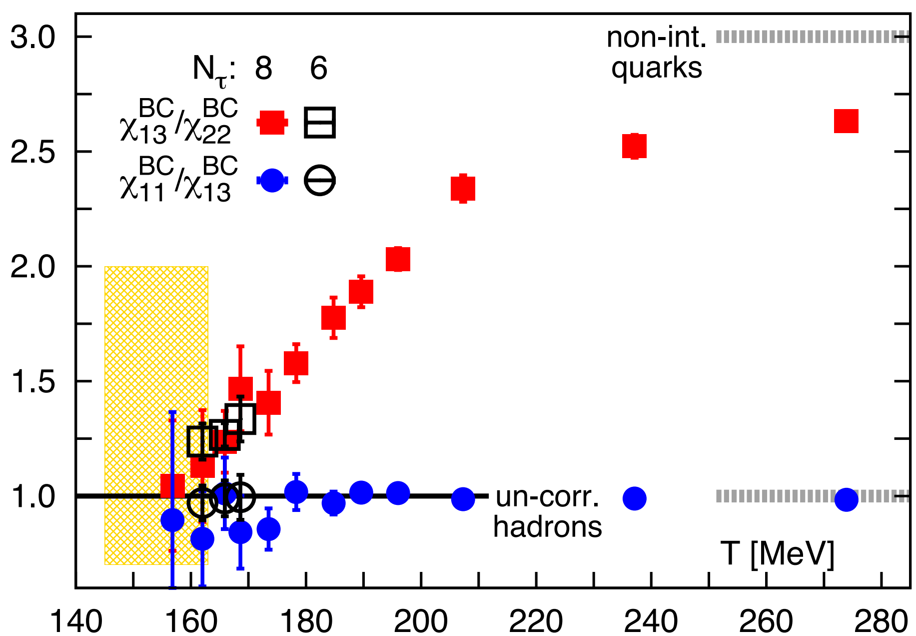
What about charm hadrons ?

$$\chi_{nml}^{XYC} = T^{m+n+l} \frac{\partial^{n+m+l} p(T, \mu_X, \mu_Y, \mu_C)/T^4}{\partial \mu_X^n \partial \mu_Y^m \partial \mu_C^l}$$

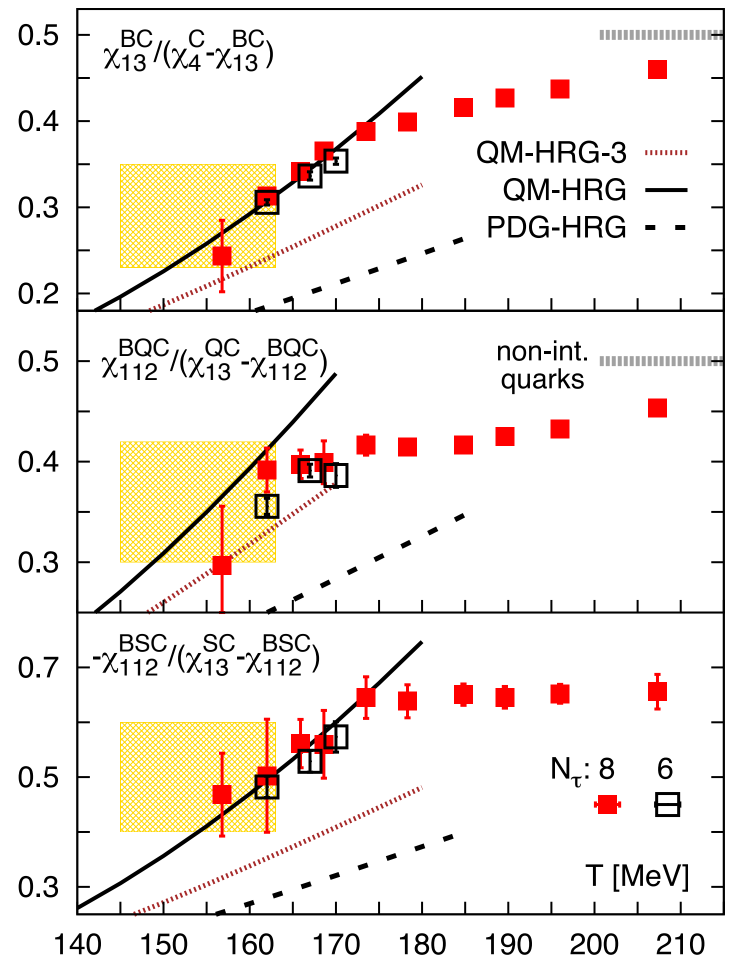
Bazavov et al, Phys.Lett. B737 (2014) 210

$m_c \gg T$ ⇨ only |*C*|=*1* sector contributes

In the hadronic phase all *BC*-correlations are the same !



Charm baryon to meson pressure



Hadronic description breaks down just above T_c
⇒ open charn deconfines above T_c

The charm baryon spectrum is not well known (only few states in PDG), HRG works only if the “missing” states are included

Quasi-particle model for charm degrees of freedom

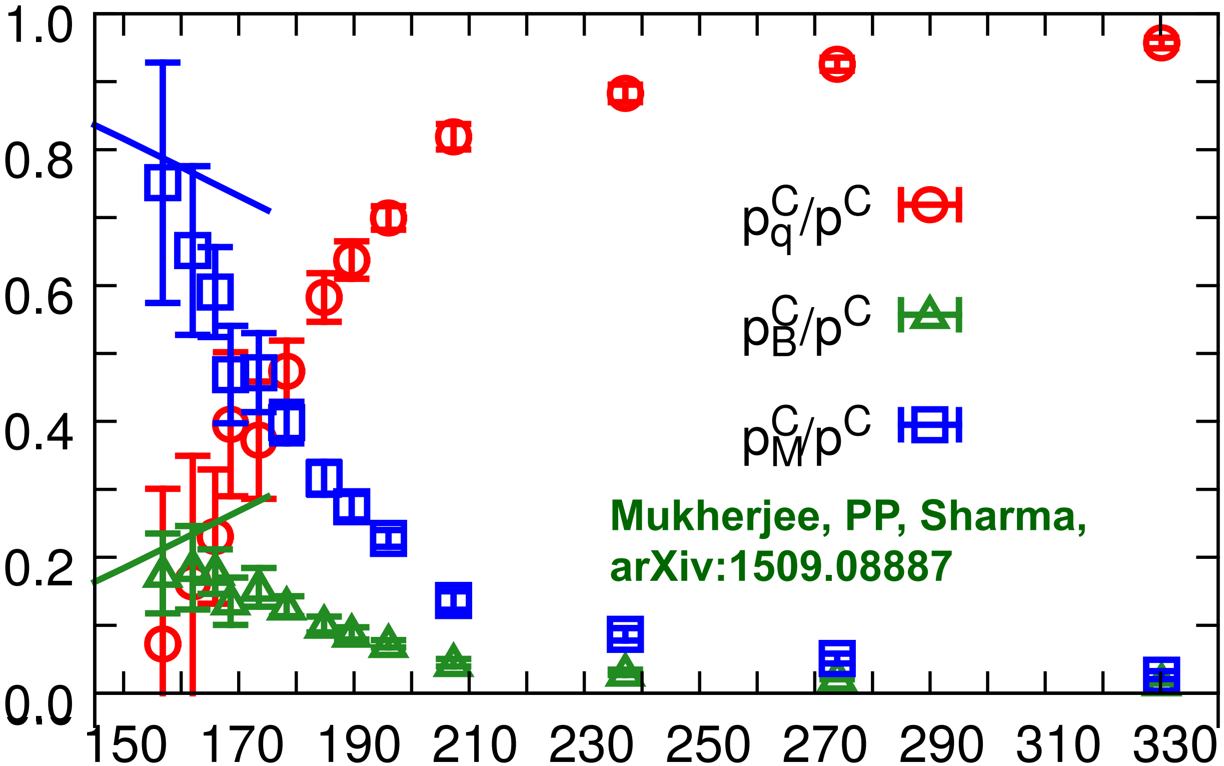
Charm dof are good quasi-particles at all T because $M_c >> T$ and Boltzmann approximation holds

$$p^C(T,\mu_B,\mu_c) = p_q^C(T)\cosh(\hat\mu_C + \hat\mu_B/3) + p_B^C(T)\cosh(\hat\mu_C + \hat\mu_B) + p_M^C(T)\cosh(\hat\mu_C)$$

$$\hat\mu_X = \mu_X/T$$

$$\chi_2^C,\ \chi_{13}^{BC},\ \chi_{22}^{BC} \Rightarrow p_q^C(T), p_M^C(T), p_B^C(T)$$

Partial meson and baryon pressures described by HRG at T_c and dominate the charm pressure then drop gradually, charm quark only dominant dof at $T>200$ MeV



Partial pressures drop because hadronic excitations become broad at high temperatures (bound state peaks merge with the continuum)

See
Jakovac, PRD88 ('13), 065012
Biro, Jakovac, PRD('14)065012

Vice versa for quarks

The freeze-out temperature in heavy ion collisions

Freeze-out temperature as function of baryon potential :

Bazavov et al, arXiV:1509.05786

$$T_f(\mu_B) = T_{f,0}\left(1 - \kappa_2^f \bar{\mu}_B^2 - \kappa_4^f \bar{\mu}_B^4\right) \qquad \bar{\mu}_B \equiv \mu_B / T_{f,0}$$

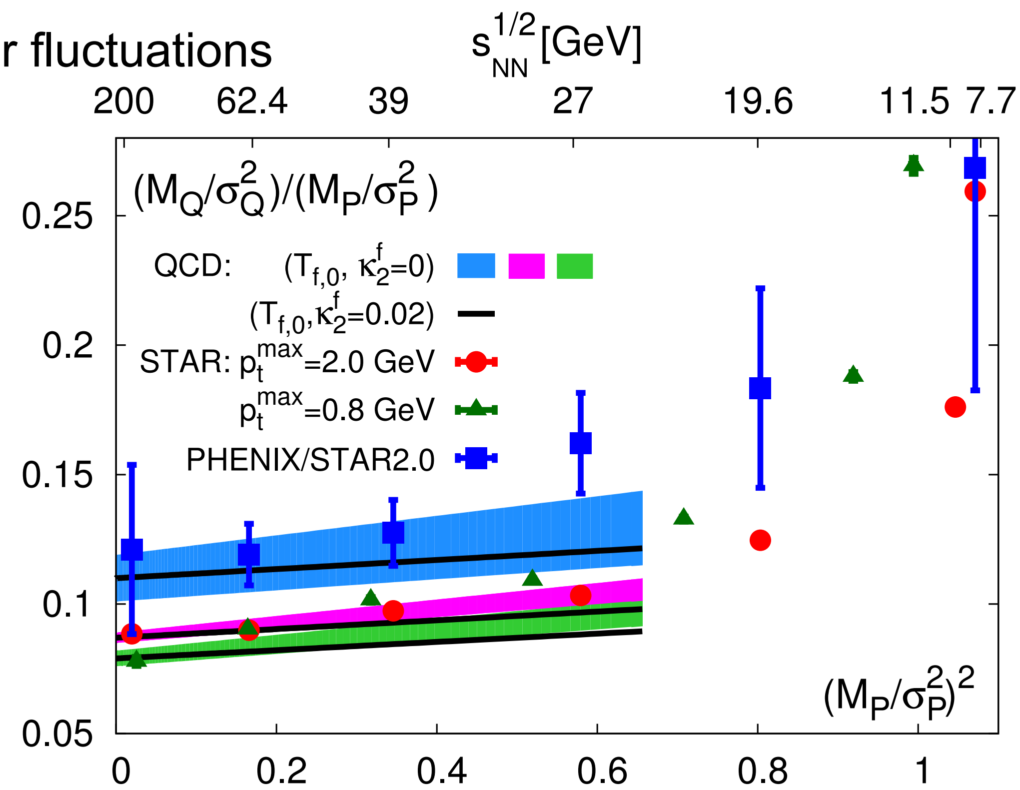
$T_{f,0}, \kappa_2^f$ $\Rightarrow$ charge and baryon number fluctuations

$$R_{12}^X(T,\mu) \equiv \frac{M_X}{\sigma_X^2} = \frac{\chi_1^X(T,\mu)}{\chi_2^X(T,\mu)}$$

can be measured experimentally
for baryon number and electric charge

$$R_{12}^{QB} \equiv R_{12}^Q / R_{12}^B = r\sigma_B^2/\sigma_Q^2$$

can be calculated on the lattice



LHS: Exp.

RHS: LQCD

$$R_{12}^{QB} = R_{12}^{QB,0} + \left(R_{12}^{QB,2} - \kappa_2^f T_{f,0} \frac{\mathrm{d}R_{12}^{QB,0}}{\mathrm{d}T}\Big|_{T_{f,0}}\right)\hat{\mu}_B^2$$

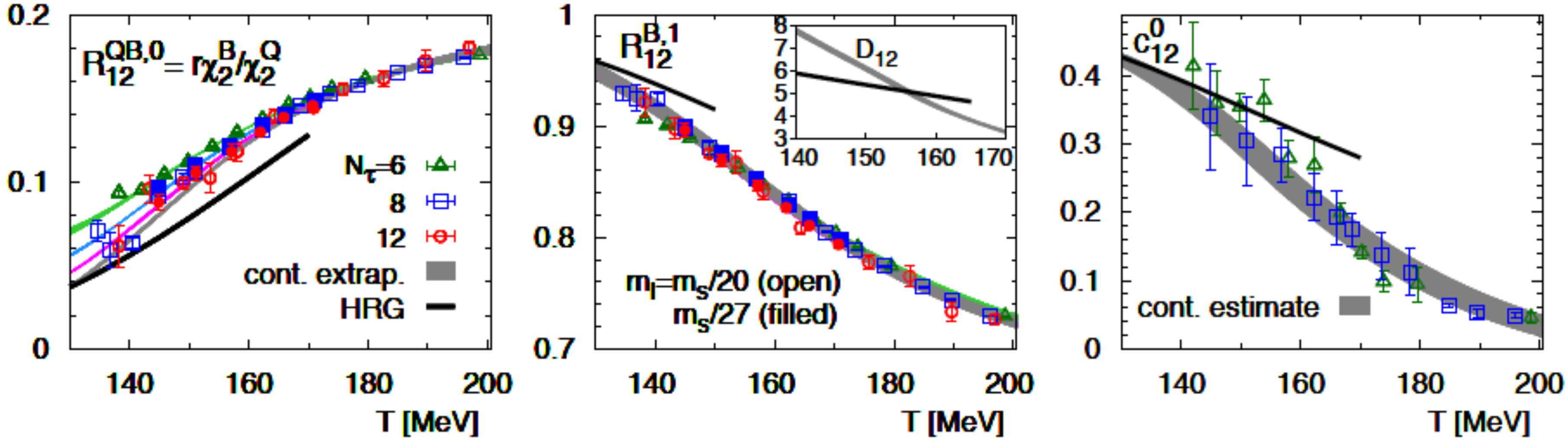
$T_c = (145 - 155)\mathrm{MeV}$

$\kappa_f <\simeq 0$

Summary

- Hadron resonance gas (HRG) can describe various thermodynamic quantities at low temperatures
- Deconfinement transition can be studied in terms of fluctuations and correlations of conserved charges, it manifest itself as a abrupt breakdown of hadronic description that occurs around the chiral transition temperature
- Charm hadrons can exist above T_c and are dominant dof for $T<180$ MeV
- For $T > (300\text{-}400)$ MeV weak coupling expansion works well for certain quantities (e.g. quark number susceptibilities), more work is needed to establish the connection between the lattice and the weak coupling results
- Comparison of lattice and HRG results for certain charm correlations hints for existence of yet undiscovered excited baryons
- The freeze-out curve as function of baryon density can be determined from the the charge and baryon number fluctuations; the curvature of the freeze-out line is very small or even negative

Back-up slides



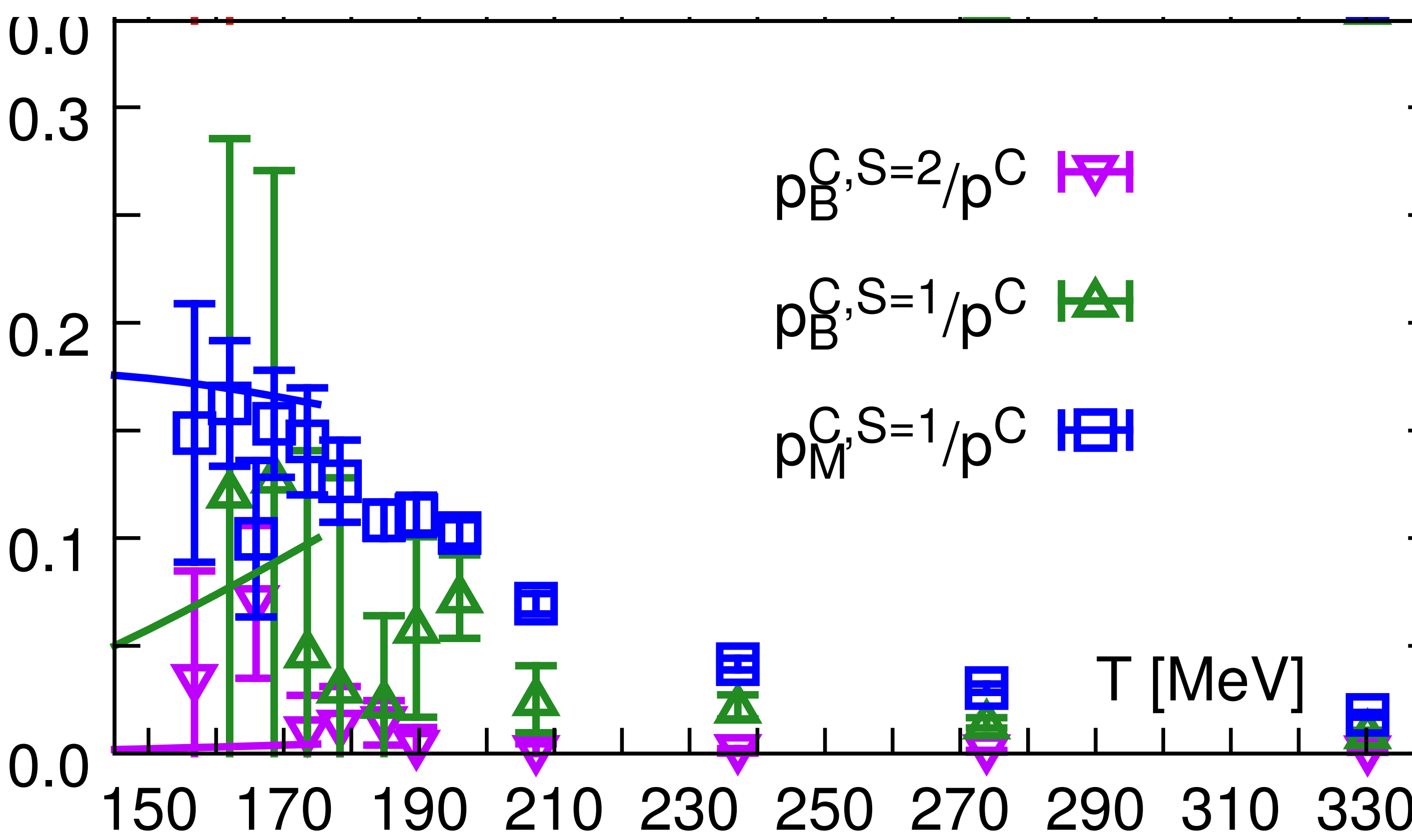
	STAR0.8	STAR2.0	PHENIX/STAR2.0
a_{12}	0.079(3)	0.087(2)	0.110(9)
c_{12}	0.858(101)	0.329(74)	0.559(352)
$T_{f,0}$ [MeV]	145(2)	147(2)	155(4)
$c_{12}^0(T_{f,0})$	0.343(31)	0.326(32)	0.265(52)
$D_{12}(T_{f,0})$	7.04(44)	6.62(36)	5.27(78)
κ_2^f	-0.073(16)	-0.001(12)	-0.056(67)

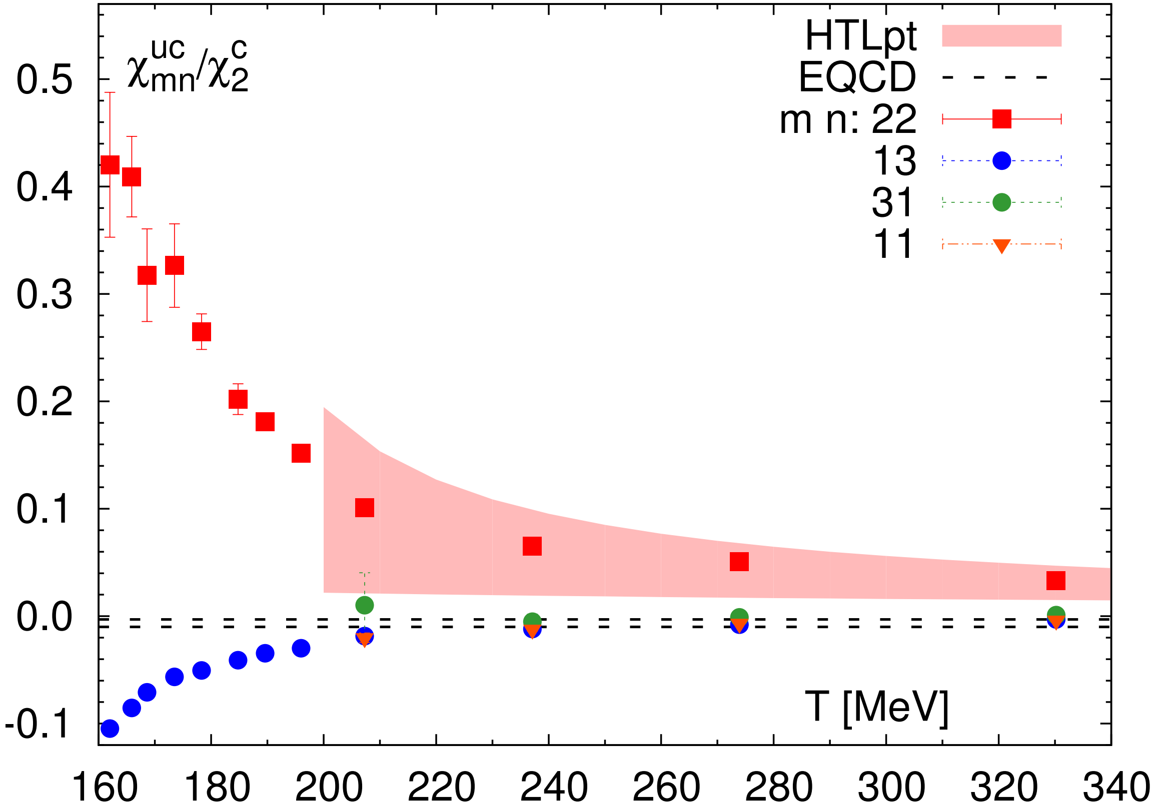
No quarks carrying both strangeness and charm

⇒ non-zero pressures in this sector implies bound charm strange bound states

Charm-strange meson and baryon pressures are consisten with HRG at T_c

$$p^{C,S}(T,\mu_B,\mu_S,\mu_C) = p_M^{C,S=1}(T)\cosh\left(\hat{\mu}_S + \hat{\mu}_C\right) + \sum_{j=1}^{2} p_B^{C,S=j}(T)\cosh\left(\mu_B - j\mu_S + \mu_C\right).$$





Quark mass effects cancel in the ratio

High T (*T>250* MeV) : $\chi^{uc}_{22} \gg \chi^{uc}_{13} \sim \chi^{uc}_{31} \sim \chi^{uc}_{11}$

Low T: correlations are large (bound states ?)