

Pomeron Femtoscopy

from a unitary extension of Bialas-Bzdak model

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p+p @ ISR and @ 7 TeV LHC

Real extension of Bialas-Bzdak

New: focusing ReBB on low t region of $d\sigma/dt$ of pp

Excitation functions

Pomeron Femtoscopy

[arXiv:1204.5617](#)

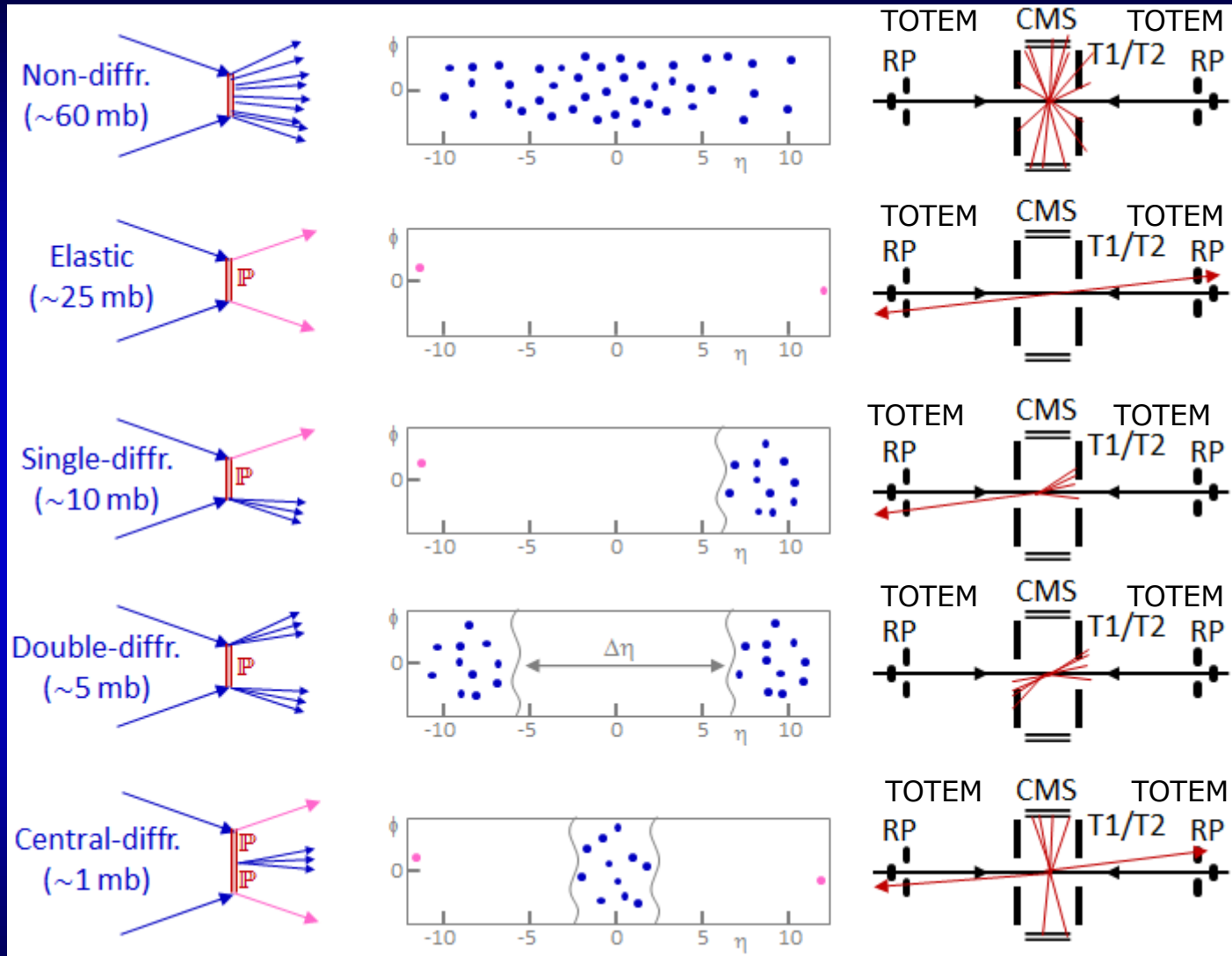
[arXiv:1306.4217](#)

[arXiv:1311.2308](#)

[arxiv:1505.01415](#)

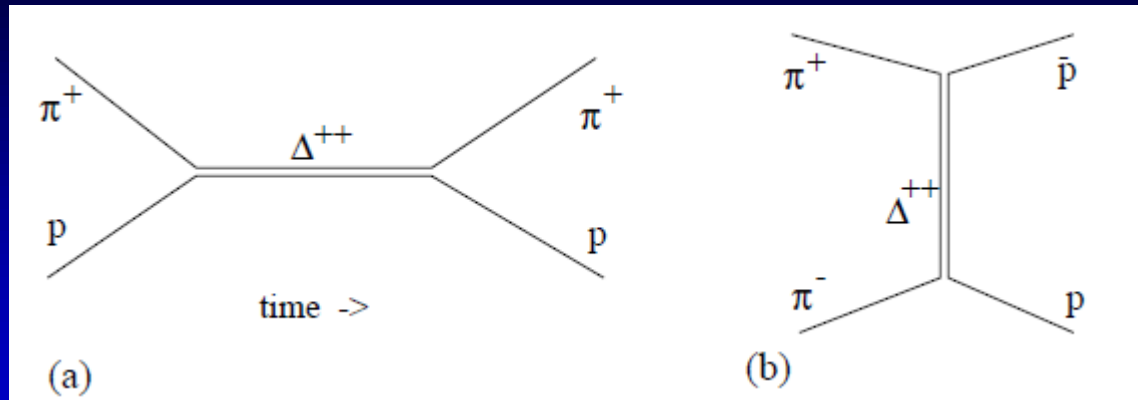
+ manuscript in preparation

Pomeron physics at LHC with TOTEM



Elastic and diffractive scattering: Pomeron (colorless) exchange

Introduction to Pomeron physics



Crossing symmetry:

Resonance production in s-channel: (a)

Interaction through resonance exchange in t-channel: (b)

$$A(s, t) = 16\pi \sum_{l=0}^{\infty} (2l + 1) a_l(s) P_l(\cos \theta),$$

$$a_l \sim 1/(s - m_l^2 + im_l \Gamma_l)$$

R. Engel, [hep-ph/0111396](https://arxiv.org/abs/hep-ph/0111396)

E. Levin, [hep-ph/9808486](https://arxiv.org/abs/hep-ph/9808486)

$$A(s, t) = 16\pi \sum_l (2l + 1) a_l(t) P_l(z_t),$$

$$a_l(t) \sim 1/(t - m_l^2 + im_l \Gamma_l)$$

$$z_t = \cos \theta_t = \frac{2s}{t - s_0} + 1$$

s, t variables for elastic pp:

$$p(p_1) + p(p_2) \rightarrow p(p_3) + p(p_4)$$

$$s = (p_1 + p_2)^2, \quad t = (p_1 - p_3)^2$$

$$s_0 \sim 1\text{GeV}^2$$

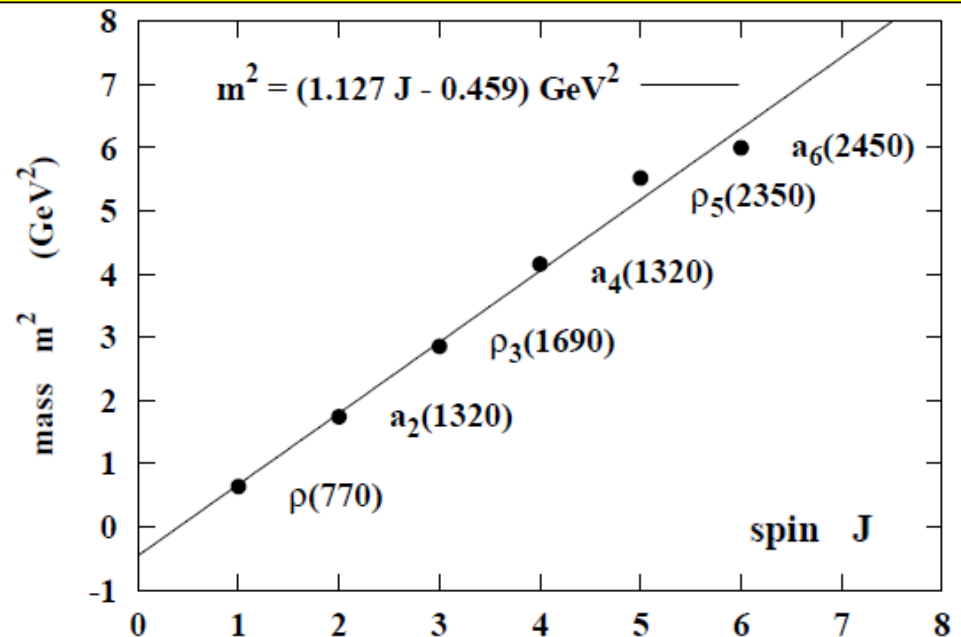
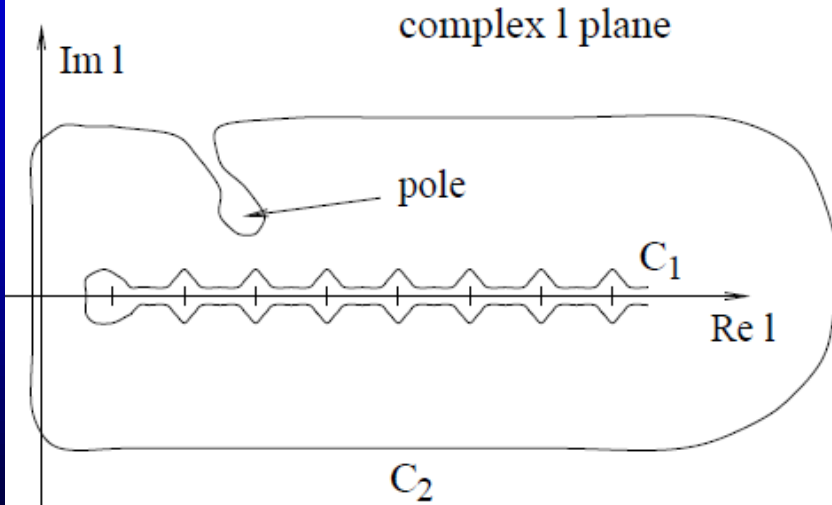
Regge theory

$$A(s, t) = \sum_{\tau=\pm 1} \frac{16\pi}{2i} \int_{C_1} dl (2l + 1) \left(\frac{1 + \tau e^{-i\pi l}}{\sin(\pi l)} \right) a_l(t) P_l(-z_t), \quad \tau = \pm 1$$

$$a_l \sim \frac{1}{t^2 - m_l^2} = \frac{1}{t - m_0^2 - \alpha l} \sim \frac{1}{l - t/a + m_0^2/a} = \frac{1}{l - \alpha(t)}$$

$$m_l^2 = \alpha l + m_0^2$$

Regge trajectories,
pole at $\alpha(t) = l$, $\alpha(t) = \alpha(0) + \alpha'(0) t$



Reggeons

$$A(s, t) = -\frac{1 + \tau e^{-i\pi\alpha(t)}}{\sin(\pi\alpha(t))} \beta(t) P_{\alpha(t)}(-zt)$$

$$P_{\alpha_k(t)} \left(-\frac{2s}{t-s_0} - 1 \right) \xrightarrow{s \rightarrow \infty} \left(\frac{s}{s_0} \right)^{\alpha_k(t)}$$

$$A(s, t) = \sum_k \eta(\alpha_k(t)) \beta_k(t) \left(\frac{s}{s_0} \right)^{\alpha_k(t)}$$

$$\eta(\alpha_k(t)) = -\frac{1 + \tau e^{-i\pi\alpha_k(t)}}{\sin(\pi\alpha_k(t))}$$

Summation is over Regge trajectories,
Reggeons = quasi-particle =
family of resonances with same quantum numbers

T. Regge, Nuovo Cim. 14 (1959) 951

Pomerons and cross-sections

$$\frac{d\sigma_{\text{ela}}}{dt} = \frac{1}{16\pi s^2} |A(s, t)|^2$$

$$\sigma_{\text{tot}} = \frac{1}{s} \Im m (A(s, t = 0))$$

$$\Delta = \alpha(0) - 1$$

$$\sigma_{\text{ela}} = (1 + \rho^2) \frac{g^2}{16\pi} \left(\frac{s}{s_0}\right)^{2\Delta} \exp\{B_{\text{ela}} t\}$$

$$\sigma_{\text{tot}} = g \left(\frac{s}{s_0}\right)^{\Delta}$$

$$B_{\text{ela}} = E$$

From data: $\sigma_{\text{tot}}(s)$ constant or increasing

$$\alpha(0) \gtrsim 1$$

Pomeron (Pomeranchuk-on)

No resonances on the Po

For resonances: Δ

Vacuum quantum number

The Theory of Complex Angular Momenta

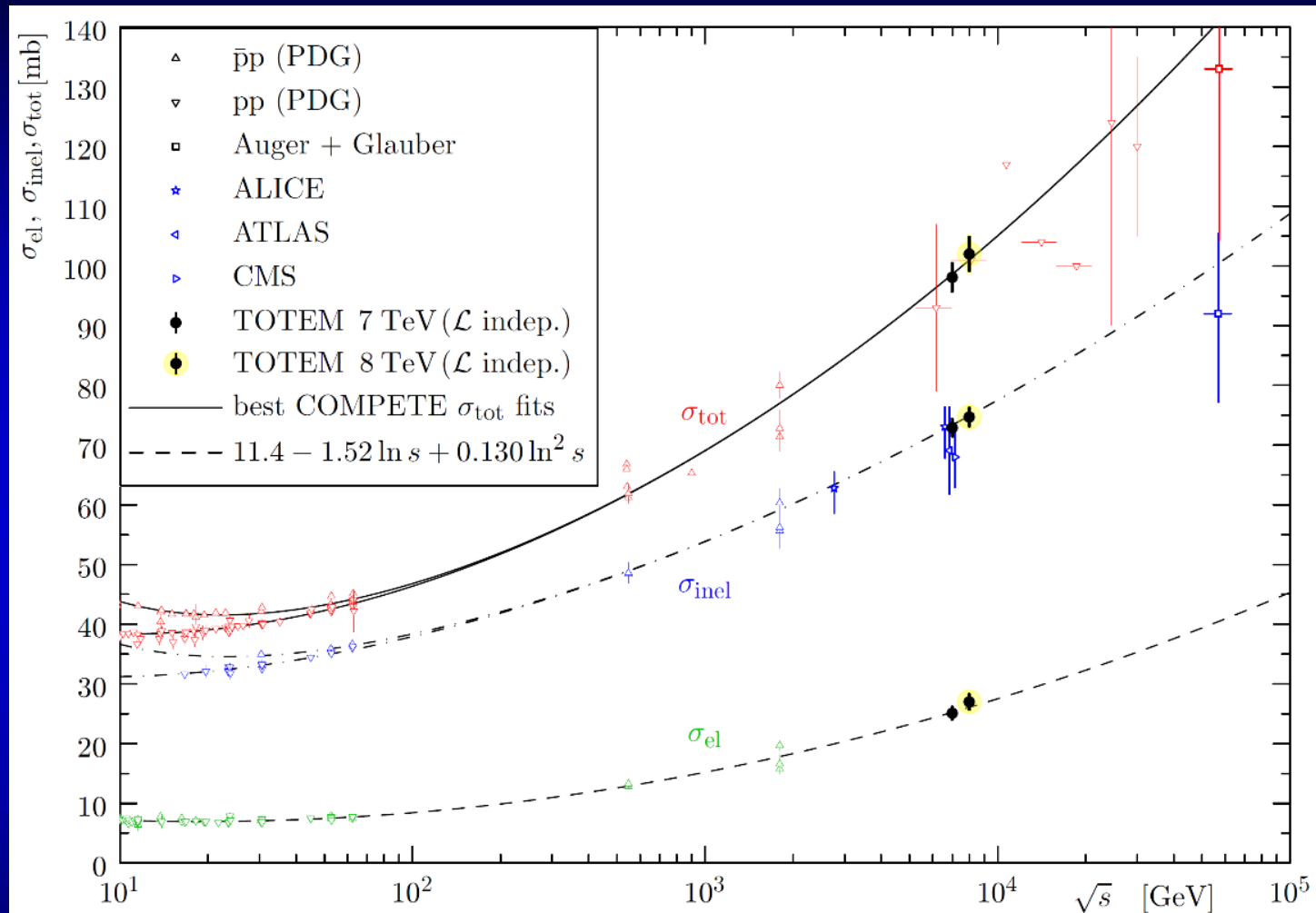
Gribov Lectures on Theoretical Physics

V. N. GRIBOV

CAMBRIDGE MONOGRAPHS
ON MATHEMATICAL PHYSICS

V. N. Gribov and I. Ya. Pomeranchuk, Ph

Pomerons and cross-sections



From LHC data: $\sigma_{\text{tot}}(s)$ increases with $\sim \ln s$

Froissart-Martin bound: $\sigma_{\text{tot}}(s) \leq C \ln^2 s$

Gaussians in b: Pomeron in t!

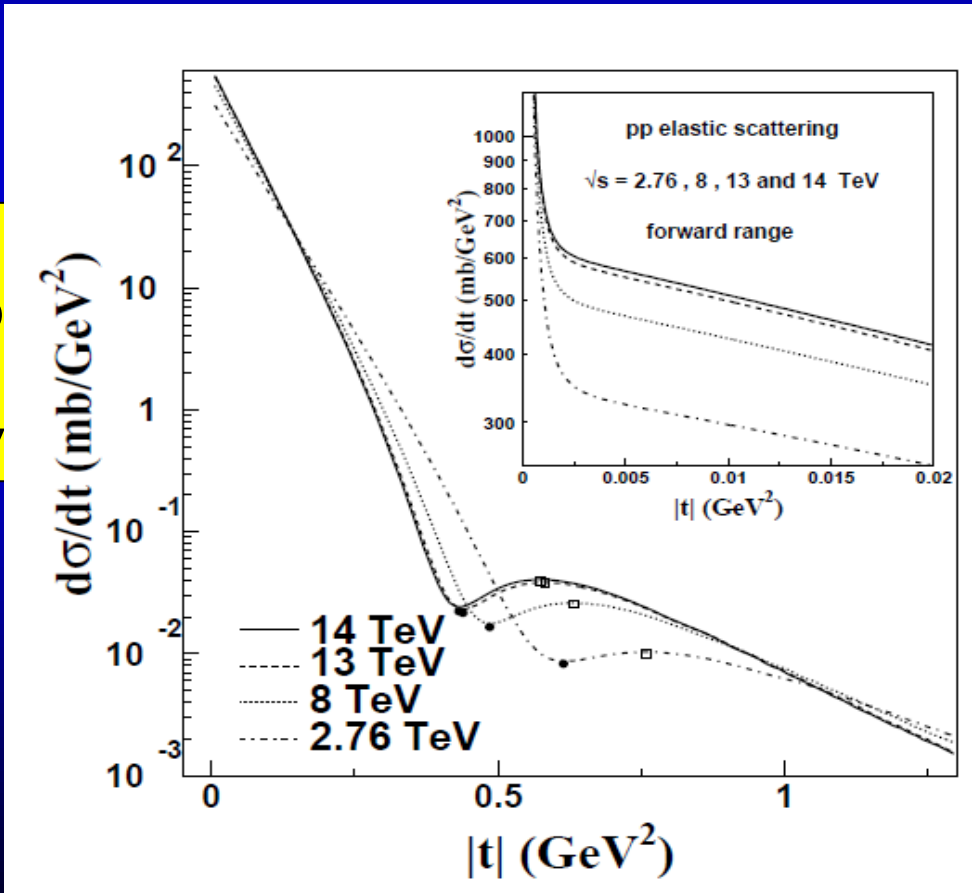
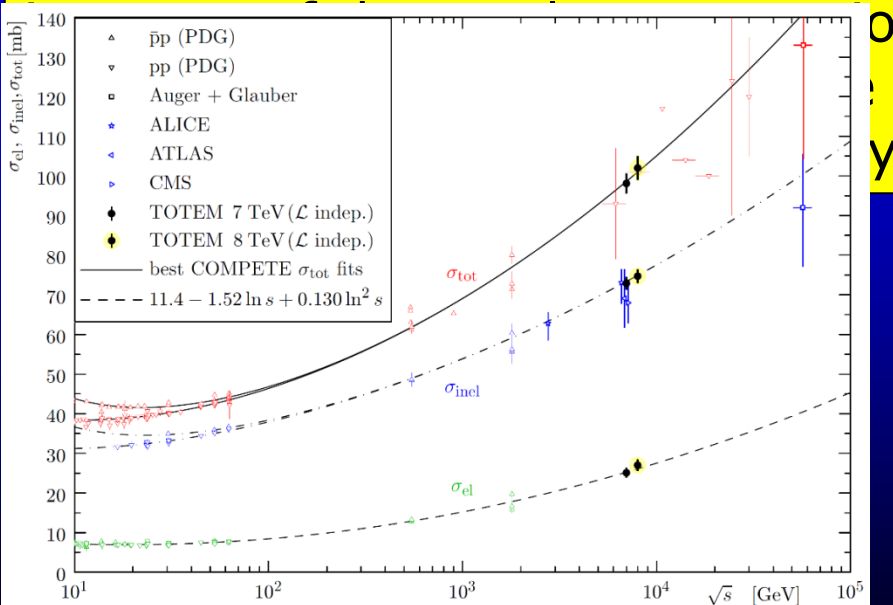
$$g_1(0) g_2(0) (s/s_0)^{\Delta_P} \frac{1}{\pi R^2(s)} e^{-\frac{b_t^2}{R^2(s)}}$$

Gaussian approximation:
elastic amplitude in impact parameter (b) representation

$$\sigma_{tot} = \sigma_{el} + \sigma_{in} = 2\pi R^2$$

$$R^2(s) = R_0^2 + 4\alpha'_P \ln(s/s_0)$$

Gaussian b, Pomeron in t:



S-matrix Unitarity, Optical Theorem

$$SS^\dagger = I,$$

$$S = I + iT$$

$$T - T^\dagger = iTT^\dagger$$

$$2 \operatorname{Im} t_{el}(s, b) = |t_{el}(s, b)|^2 + \sigma(s, b)$$

Black (grey) disc limit (important)

$$\rightarrow \sigma(b) \sim \theta(R-b)$$

Note: diffraction also measures
|Fourier-transform|² images of
sources of elastic scattering

- ideal for femtoscopic studies
- several similarities e.g. non-Gaussian sources etc

Diffraction in quark-diquark models

$$\frac{d\sigma}{dt} = \frac{1}{4\pi} |T(\Delta)|^2.$$

Bialas and Bzdak,
Acta Phys. Polon. B 38 (2007) 159
 $p = (q, d)$ or $p = (q, (q, q))$

$$T(\vec{\Delta}) = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} t_{el}(\vec{b}) e^{i\vec{\Delta} \cdot \vec{b}} d^2b = 2\pi \int_0^{+\infty} t_{el}(b) J_0(\Delta b) b db,$$

$$t_{el}(\vec{b}) = 1 - \sqrt{1 - \sigma(\vec{b})}.$$

$\sigma(b) = b$ dependent prob. of interaction
→ connection to scattering centers

$$\sigma(\vec{b}) = \int_{-\infty}^{+\infty} \dots \int_{-\infty}^{+\infty} d^2s_q d^2s'_q d^2s_d d^2s'_d D(\vec{s}_q, \vec{s}_d) D(\vec{s}'_q, \vec{s}'_d) \sigma(\vec{s}_q, \vec{s}_d; \vec{s}'_q, \vec{s}'_d; \vec{b}),$$

Structure of protons = ?

→ Diffractive pp at ISR (23.5 – 62.5 GeV) and LHC (7 - 8 TeV).

Diffraction a la Bialas and Bzdak

$$D(\vec{s}_q, \vec{s}_d) = \frac{1 + \lambda^2}{\pi R_{qd}^2} e^{-(s_q^2 + s_d^2)/R_{qd}^2} \delta^2(\vec{s}_d + \lambda \vec{s}_q), \quad \lambda = m_q/m_d,$$

$$\sigma(\vec{s}_q, \vec{s}_d; \vec{s}_q', \vec{s}_d'; \vec{b}) = 1 - \prod_{a,b \in \{q,d\}} \left[1 - \sigma_{ab}(\vec{b} + \vec{s}_a' - \vec{s}_b') \right]$$

$$\sigma_{ab}(\vec{s}) = A_{ab} e^{-s^2/R_{ab}^2}, \quad R_{ab}^2 = R_a^2 + R_b^2,$$

The quark-diquark model of Bialas and Bzdak has been analytically integrated in a **Gaussian approximation**, assuming that the real part of forward scattering is negligible.

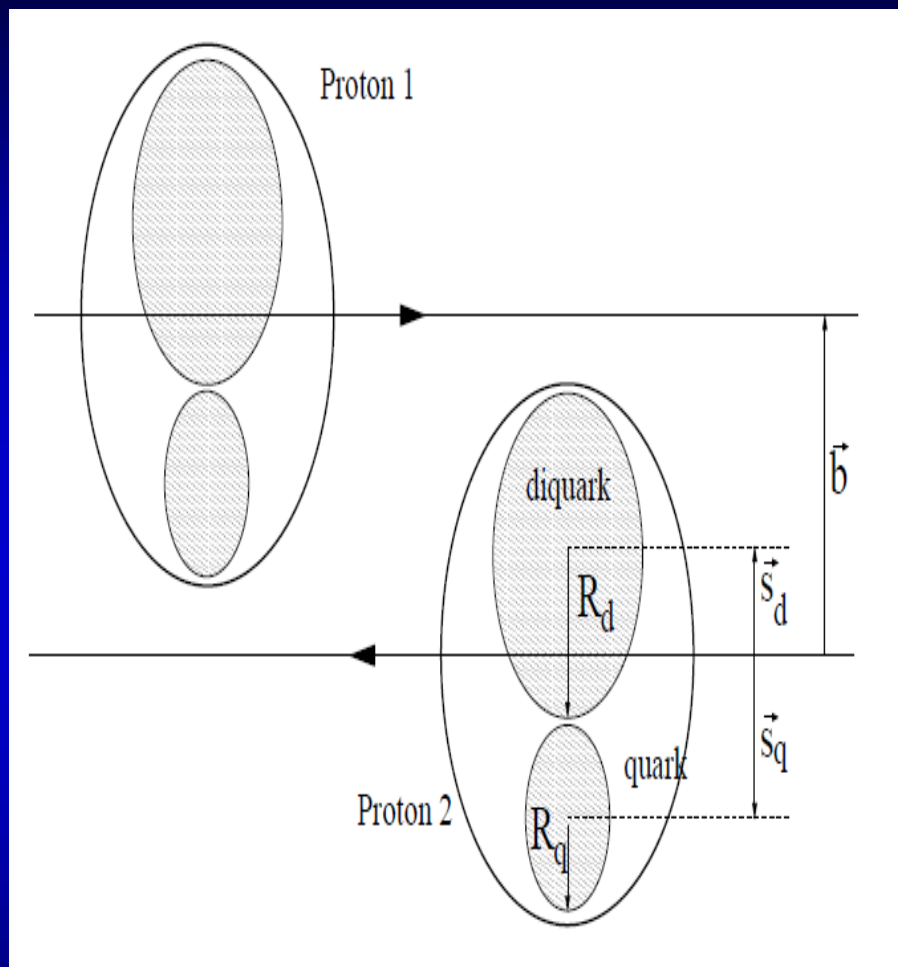
Two different pictures: $p = (q, d)$ or $p = (q, (q, q))$

Note: $p = (q, q, q)$ model fails, quarks are correlated

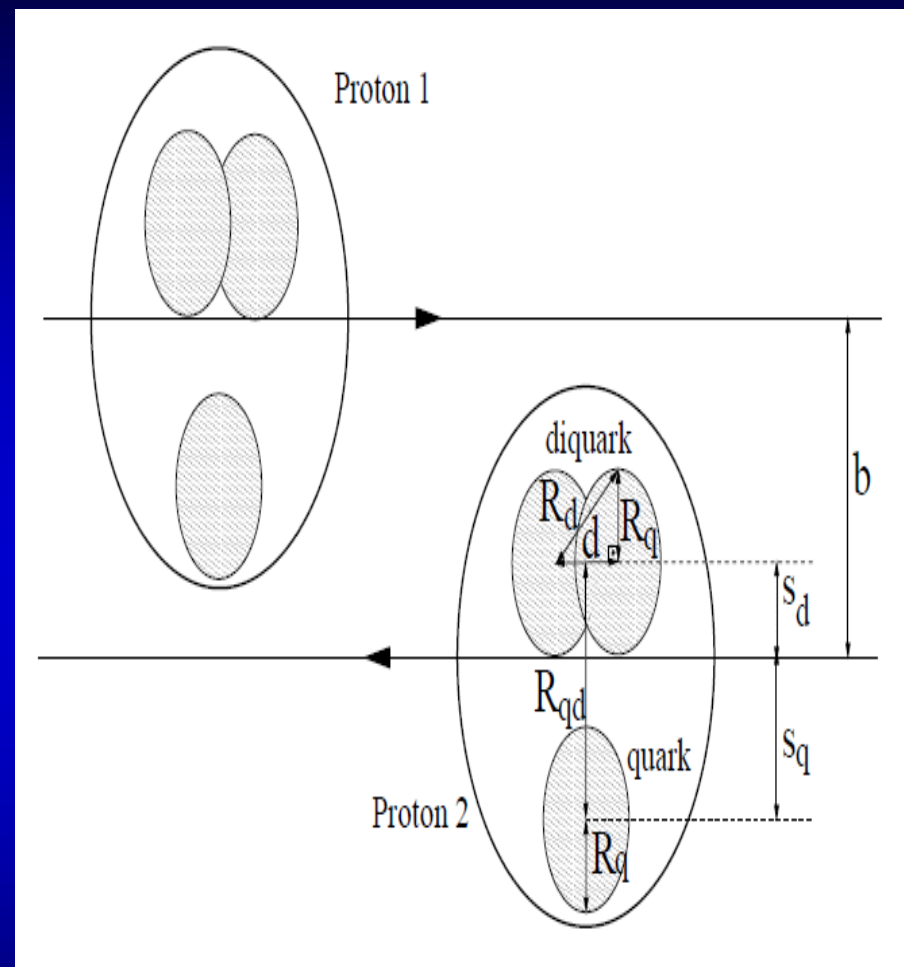
W. Czyz and L. C. Maximon, Annals. Phys. 52 (1969) 59

„Springy” $p = (q, d)$ Pomeron model of Grichine, [arxiv:1404.5768](https://arxiv.org/abs/1404.5768)

Diffractionive pp scattering



$$p = (q, d)$$



$$p = (q, (q, q))$$

Real extended BB model for the dip

$$\frac{d\sigma}{dt} = \frac{1}{4\pi} |T(\Delta)|^2.$$

$$T(\vec{\Delta}) = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} t_{el}(\vec{b}) e^{i\vec{\Delta} \cdot \vec{b}} d^2b = 2\pi \int_0^{+\infty} t_{el}(b) J_0(\Delta b) b db,$$

$$t_{el}(s, b) = i \left(1 - e^{-i \text{Im} \Omega(s, b)} \sqrt{1 - \sigma(s, b)} \right)$$

Bialas-Bzdak obtained
if $\text{Re}(t_{el}) = 0$

$$t_{el}(s, b) = i \left(1 - e^{-\text{Re} \Omega(s, b)} \right) = i \left(1 - \sqrt{1 - \sigma(s, b)} \right)$$

$$\sigma(\vec{b}) = \int_{-\infty}^{+\infty} \dots \int_{-\infty}^{+\infty} d^2s_q d^2s'_q d^2s_d d^2s'_d D(\vec{s}_q, \vec{s}_d) D(\vec{s}'_q, \vec{s}'_d) \sigma(\vec{s}_q, \vec{s}_d; \vec{s}'_q, \vec{s}'_d; \vec{b}),$$

Real extension of an imaginary t_{el}
New parameter $\text{Im} \Omega$ added

ReBB model for the dip (2)

$$\sigma(b) = \int_{-\infty}^{+\infty} \dots \int_{-\infty}^{+\infty} d^2 s_q d^2 s'_q d^2 s_d d^2 s'_d D(\mathbf{s}_q, \mathbf{s}_d) D(\mathbf{s}'_q, \mathbf{s}'_d), \sigma(\mathbf{s}_q, \mathbf{s}_d; \mathbf{s}'_q, \mathbf{s}'_d; \mathbf{b}).$$

$$D(\mathbf{s}_q, \mathbf{s}_d) = \frac{1 + \lambda^2}{R_{qd}^2 \pi} e^{-(s_q^2 + s_d^2)/R_{qd}^2} \delta^2(\mathbf{s}_d + \lambda \mathbf{s}_q), \quad \lambda = \frac{m_q}{m_d},$$

$$\sigma(\mathbf{s}_q, \mathbf{s}_d; \mathbf{s}'_q, \mathbf{s}'_d; \mathbf{b}) = 1 - \prod_{a,b \in \{q,d\}} [1 - \sigma_{ab}(\mathbf{b} + \mathbf{s}'_a - \mathbf{s}_b)]$$

$$\sigma_{ab}(\mathbf{s}) = A_{ab} e^{-s^2/R_{ab}^2}, \quad R_{ab}^2 = R_a^2 + R_b^2, \quad a, b \in \{q, d\}$$

$$\sigma_{qq} : \sigma_{qd} : \sigma_{dd} = 1 : 2 : 4$$

Bialas-Bzdak model is „realized“:
 $p = (q, d)$
 $p = (q, (q, q))$

Gaussians in b
 Pomerons in t?

ReBB model: two choices

$$\text{Im } \Omega(s, b) = -\alpha \cdot \text{Re } \Omega(s, b).$$

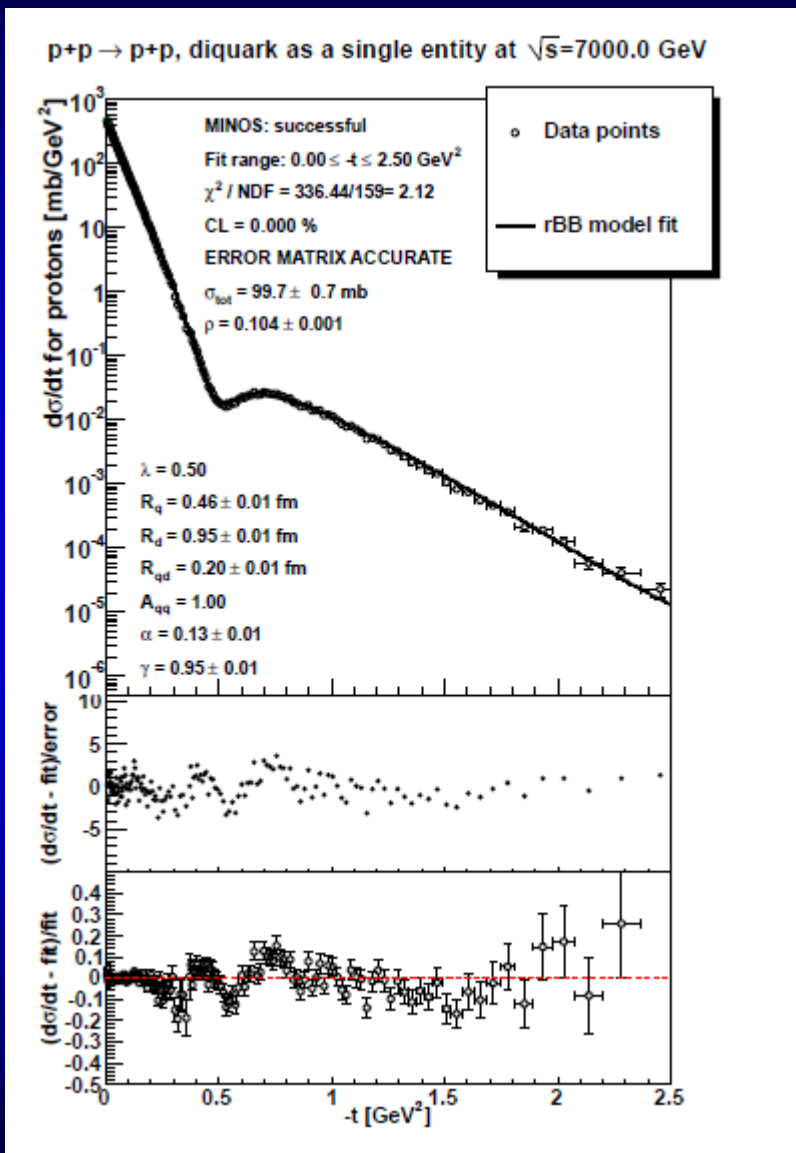
Similar to a constant ρ but not favored by data

$$\text{Im } \Omega(s, b) = -\alpha \cdot \tilde{\sigma}_{inel}(s, b),$$

For small values of α we recover our first attempt, the α BB model

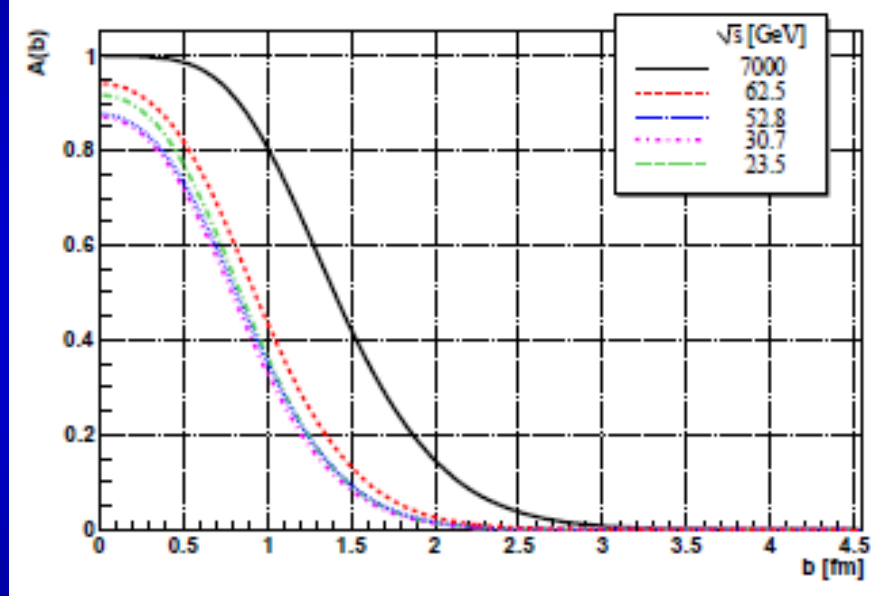
This choice is also favoured by data
T. Cs., F. Nemes, arxiv:1306.4217

ReBB model, combined data sets



Shadow profile function

$$A(s, b) = 1 - |\exp[-\Omega(s, b)]|^2$$

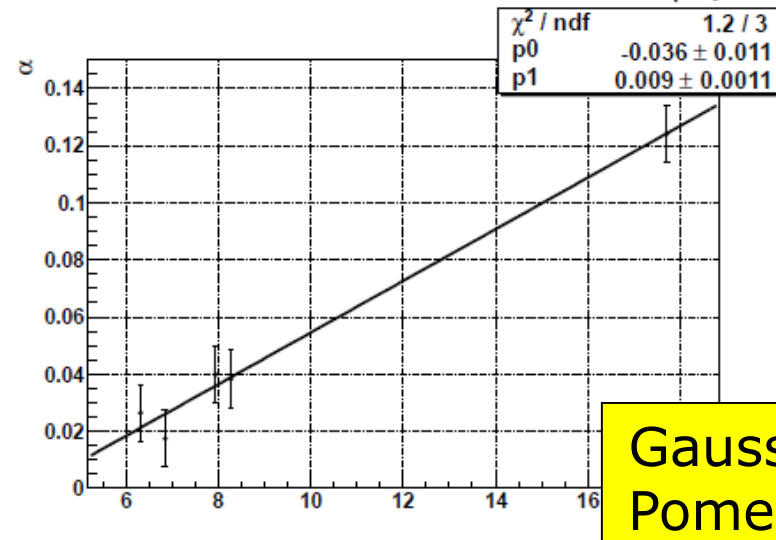
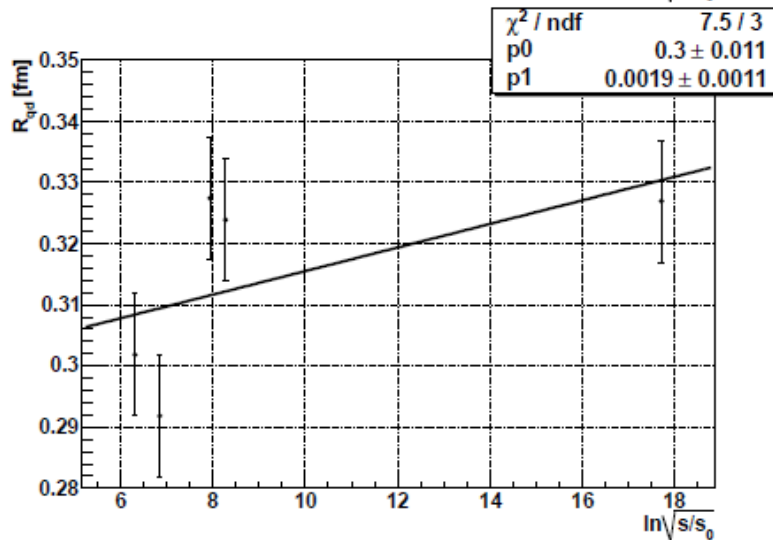
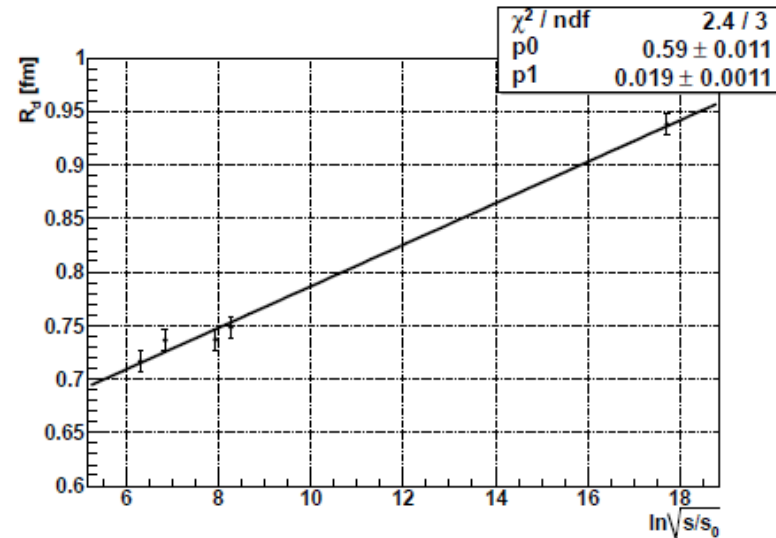
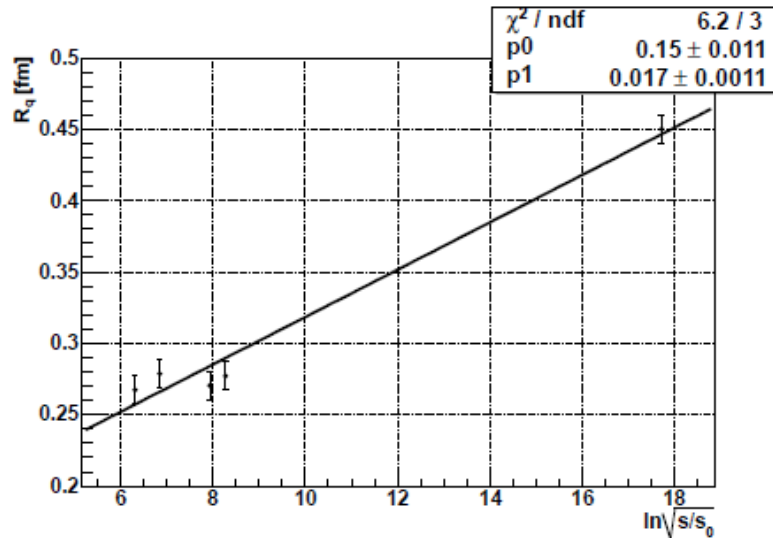


$$\frac{d\sigma}{dt} \rightarrow \gamma \cdot \frac{d\sigma}{dt}$$

$$t_{\text{sep}} = -0.375 \text{ GeV}^2$$

Fit range: $0 \leq -t \leq 2.5$ GeV²,
 not quite OK → check @ 8 TeV

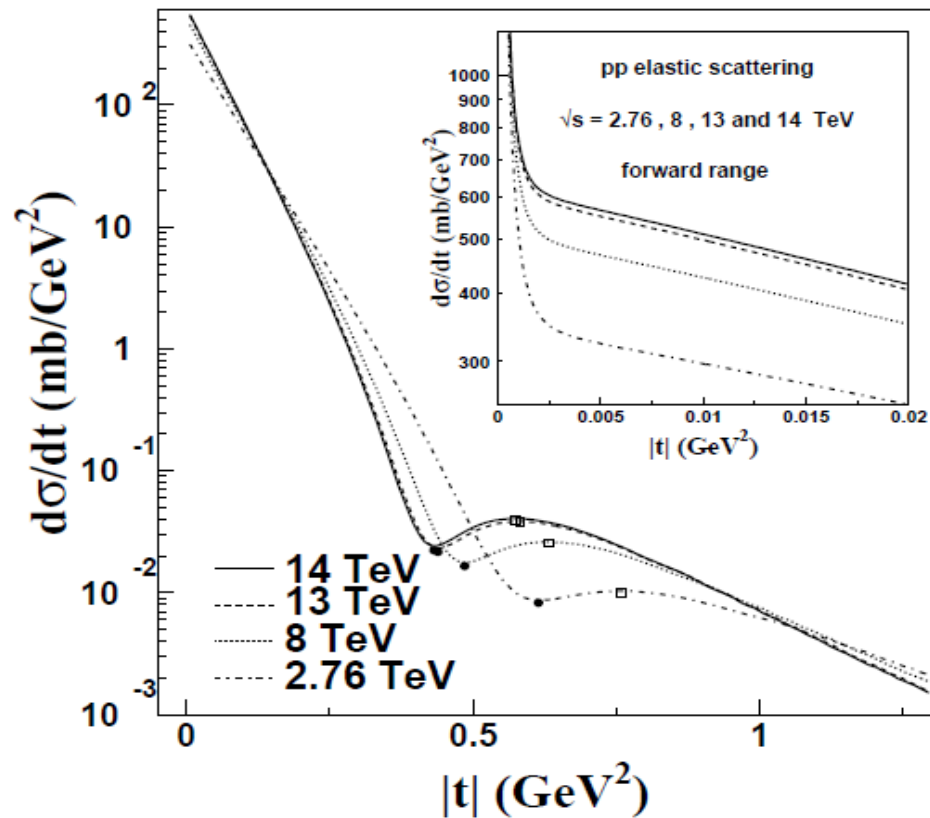
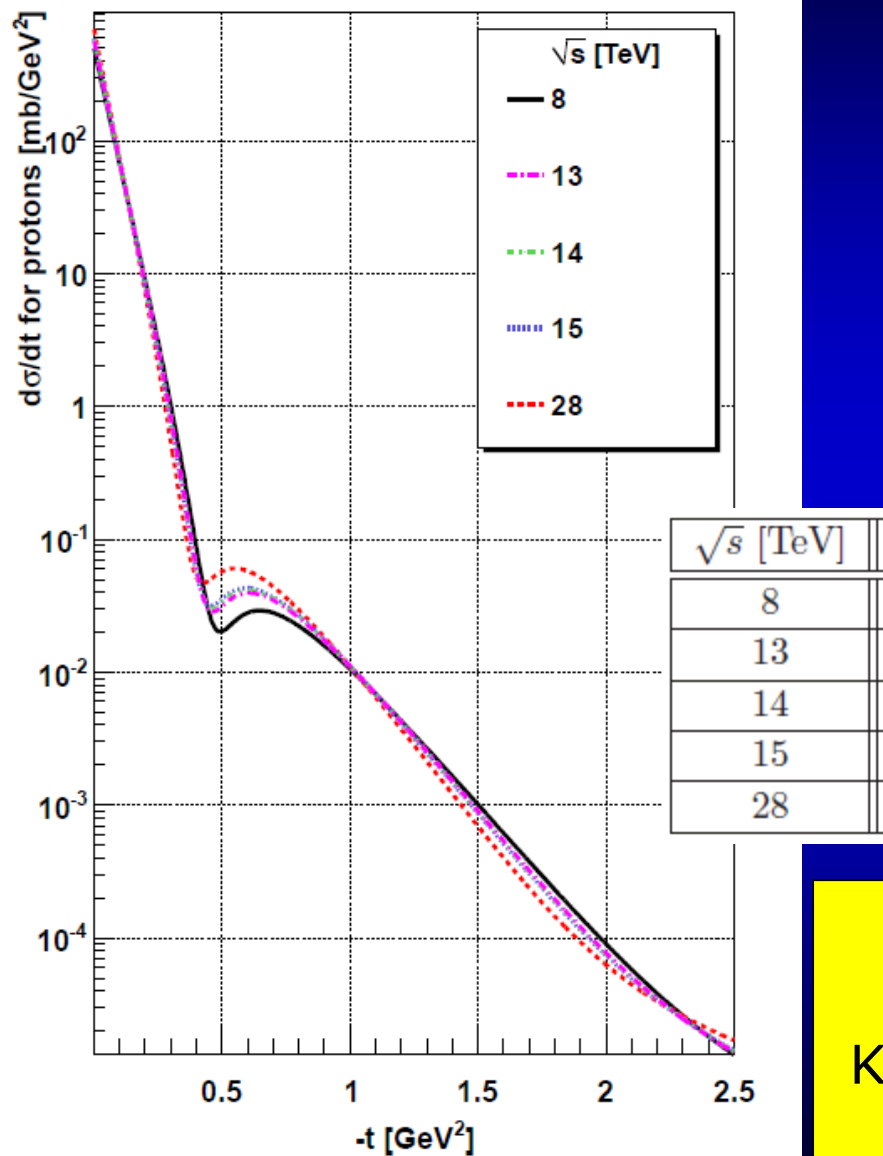
Excitation function: scaling in pp



Gaussians in b
 Pomerons in t!

Geometric scaling: $\{R_q, R_d, R_{qd}, \alpha\} = p_0 + p_1 \ln (s/s_0)$

Excitation function: $d\sigma/dt$



but $t_{\text{dip}} \sigma_{\text{tot}} \sim \text{const}$ (2 %)

Similar to:

K.A. Kohara, T. Kodama, E. Ferreira,
 arXiv:1411.3518

ReBB shadow profile functions

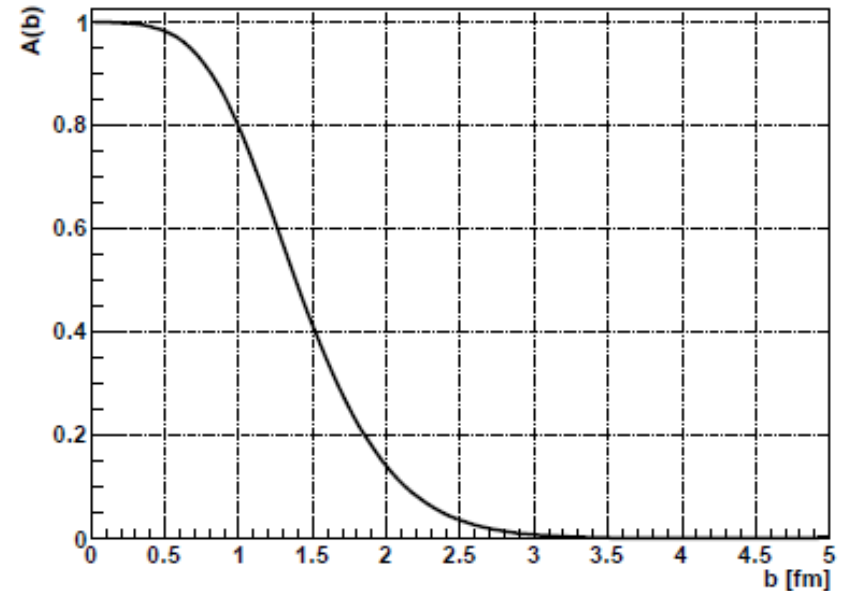
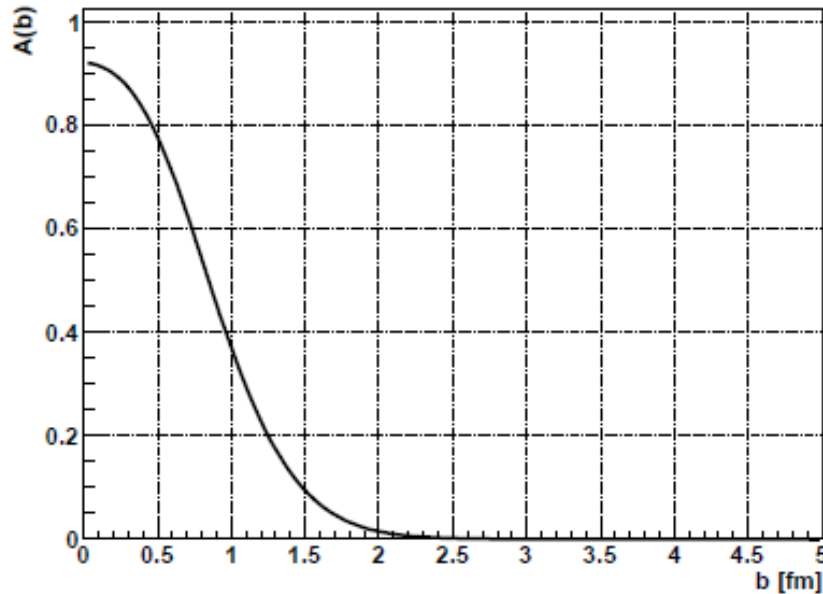
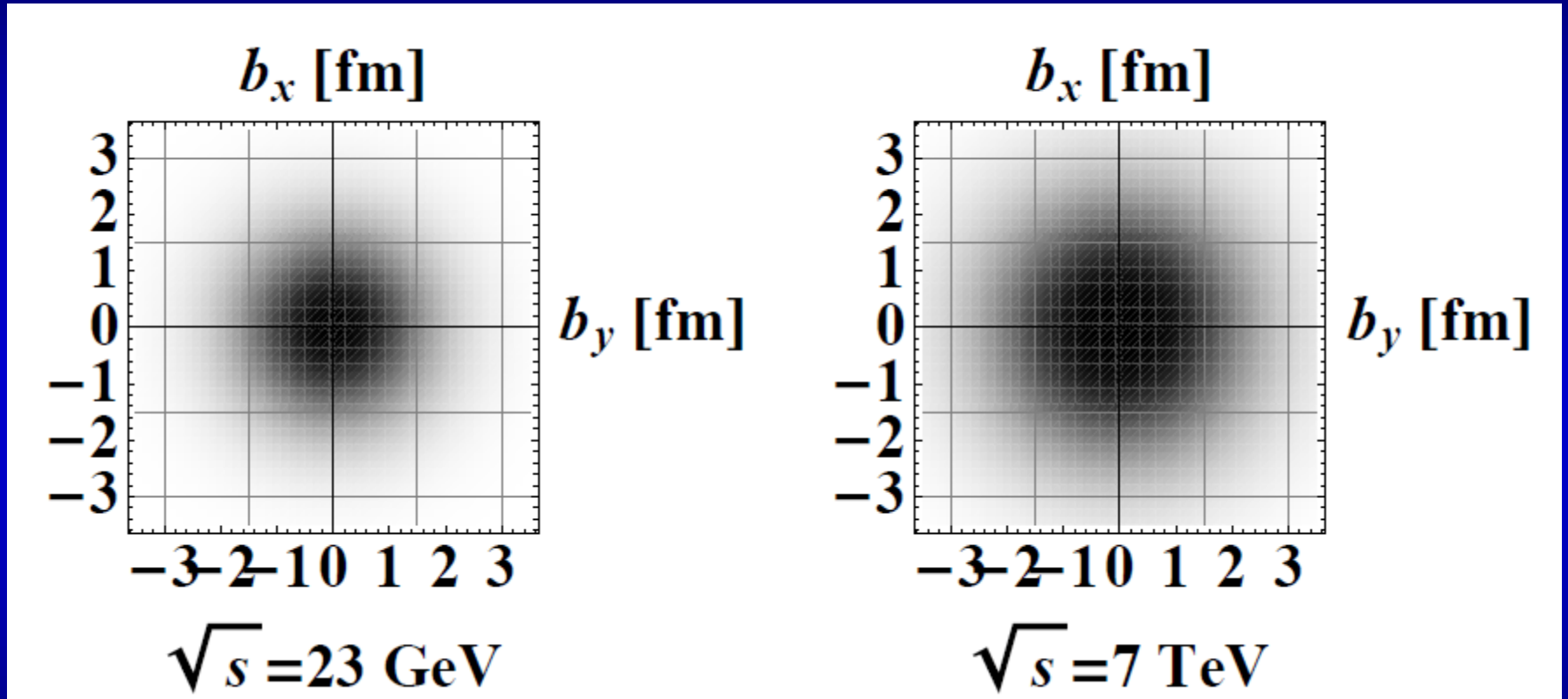


Figure 4: The $A(b) = 1 - |e^{-\Omega(b)}|^2$ shadow profile function. 23.5 GeV (left) and 7 TeV (right).

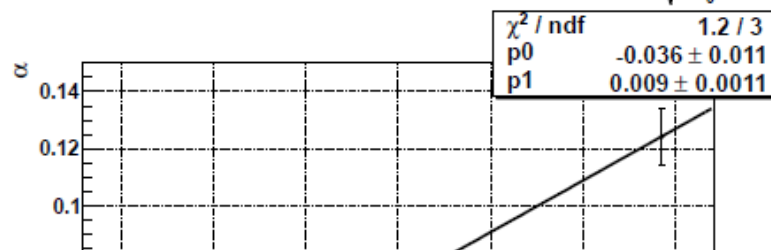
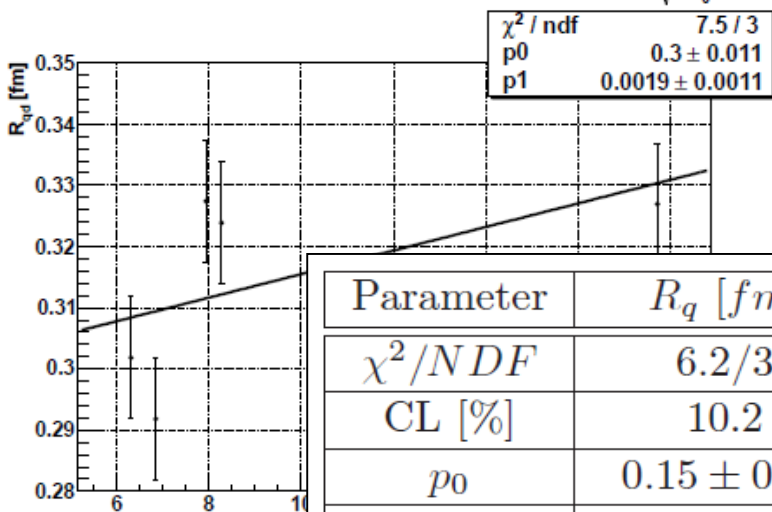
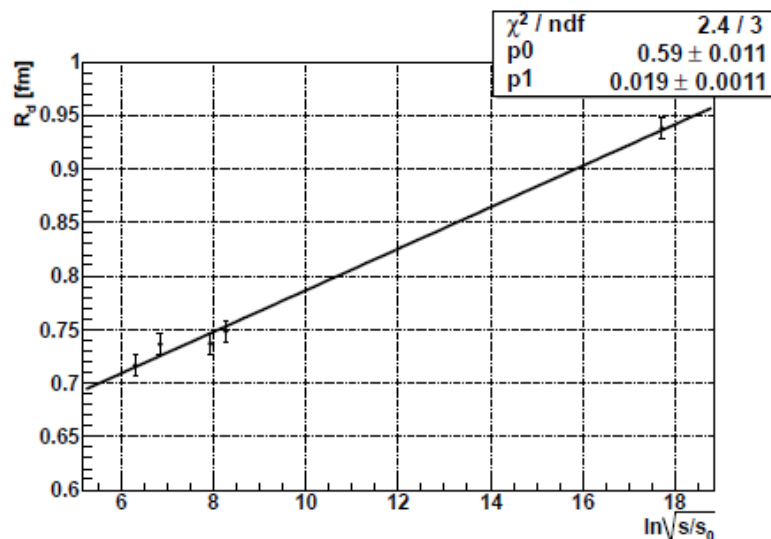
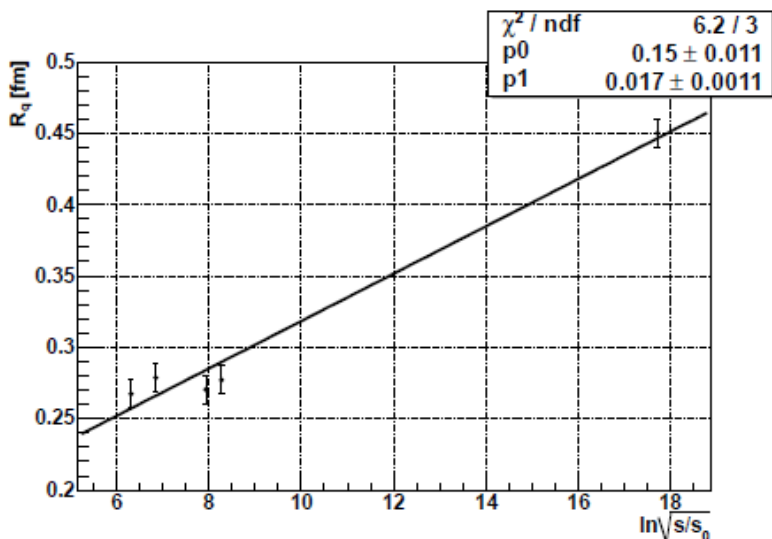
Indication of saturation at 7 TeV: $A(b) \sim 1$ at low b .
 \sim max probability of interaction at low b

Imaging on the sub-femtometer scale at 23 GeV ISR and 7 TeV LHC energy



What about 8 TeV and future LHC energies?

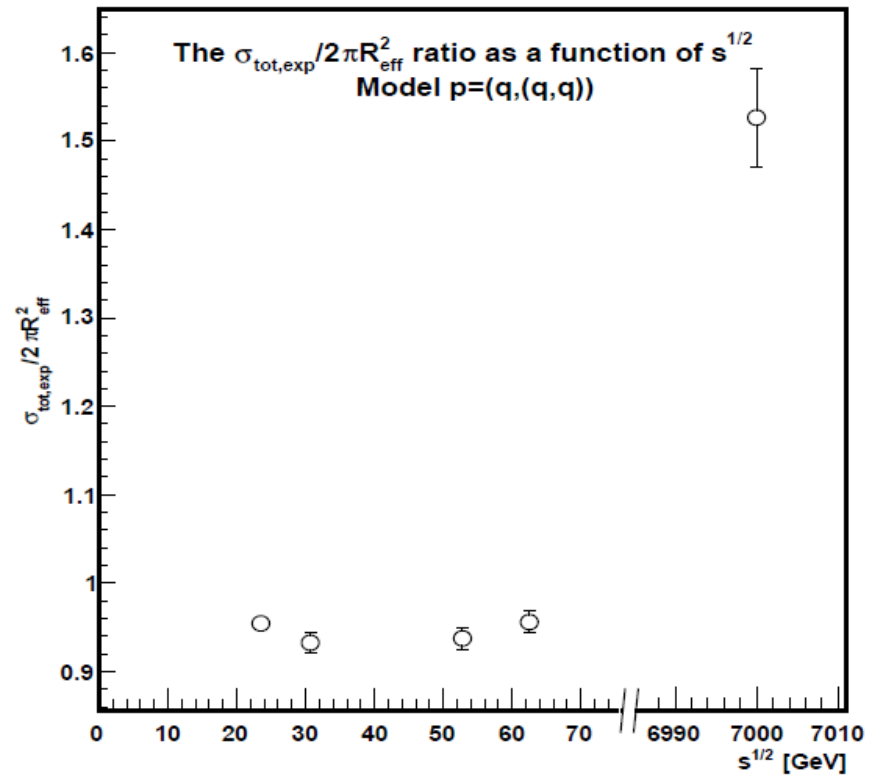
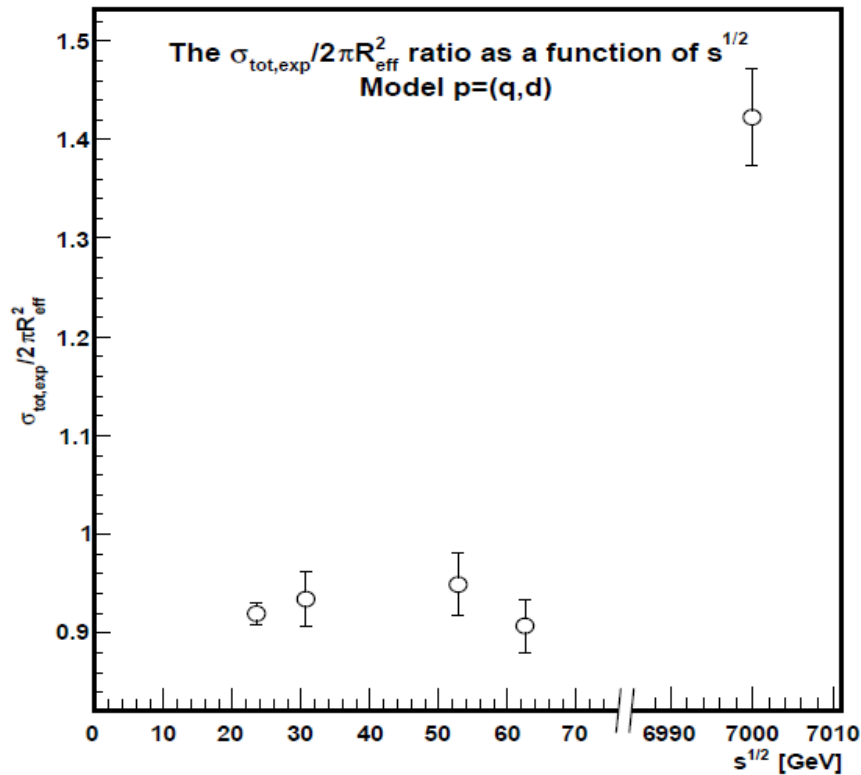
Geometric scaling in pp



Parameter	R_q [fm]	R_d [fm]	R_{qd} [fm]	α
χ^2 / NDF	6.2/3	2.4/3	7.5/3	1.2/3
CL [%]	10.2	49.4	5.8	75.3
p_0	0.15 ± 0.01	0.59 ± 0.01	0.3 ± 0.01	-0.036 ± 0.01
p_1	0.017 ± 0.001	0.019 ± 0.001	0.0019 ± 0.001	0.009 ± 0.001

Geometric scaling: $\{R_q, R_d, R_{qd}, \alpha\} = p_0 + p_1 \ln (s/s_0)$

What have we learned?

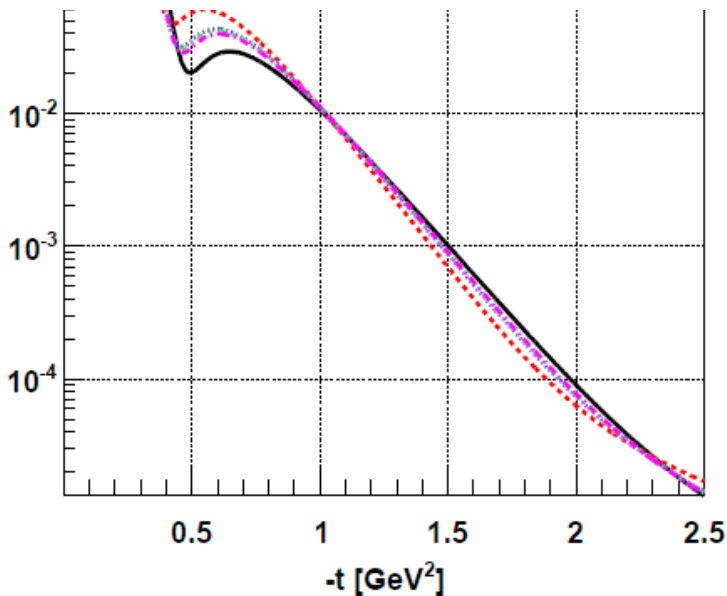
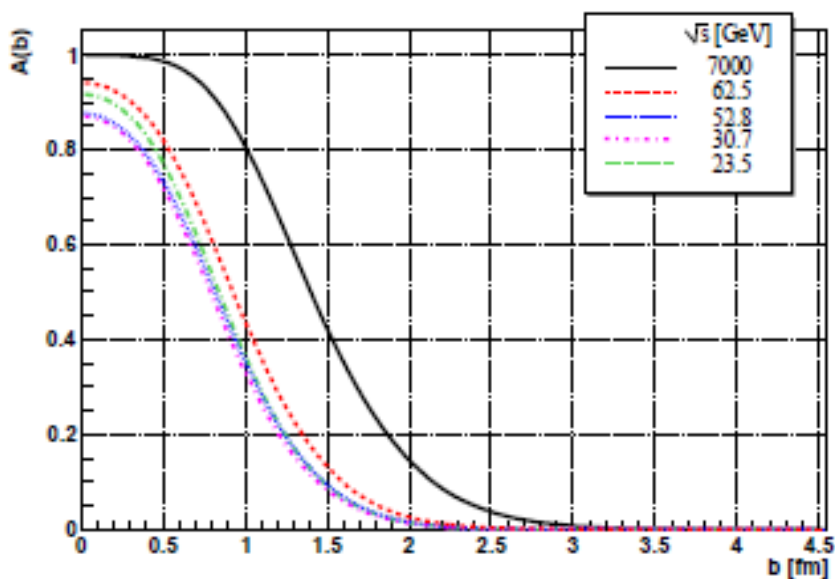


Model independent effective formula:
works \sim well both for BB and α BB models
F. Nemes and T. Cs, [arXiv:1204.5617](https://arxiv.org/abs/1204.5617)
Froissart-Martin bound satisfied (new)!

$$R_{\text{eff}} = \sqrt{R_q^2 + R_d^2 + R_{qd}^2},$$

$$\sigma_{\text{total}} = 2\pi R_{\text{eff}}^2.$$

What have we learned?



BB: Bialas-Bzdak

Gaussian in b :
Pomeron in t

ReBB model:
Pomeron Femtoscopy
on (q,d) level

ReBB model:
Froissart-Martin bound
Automatically satisfied!

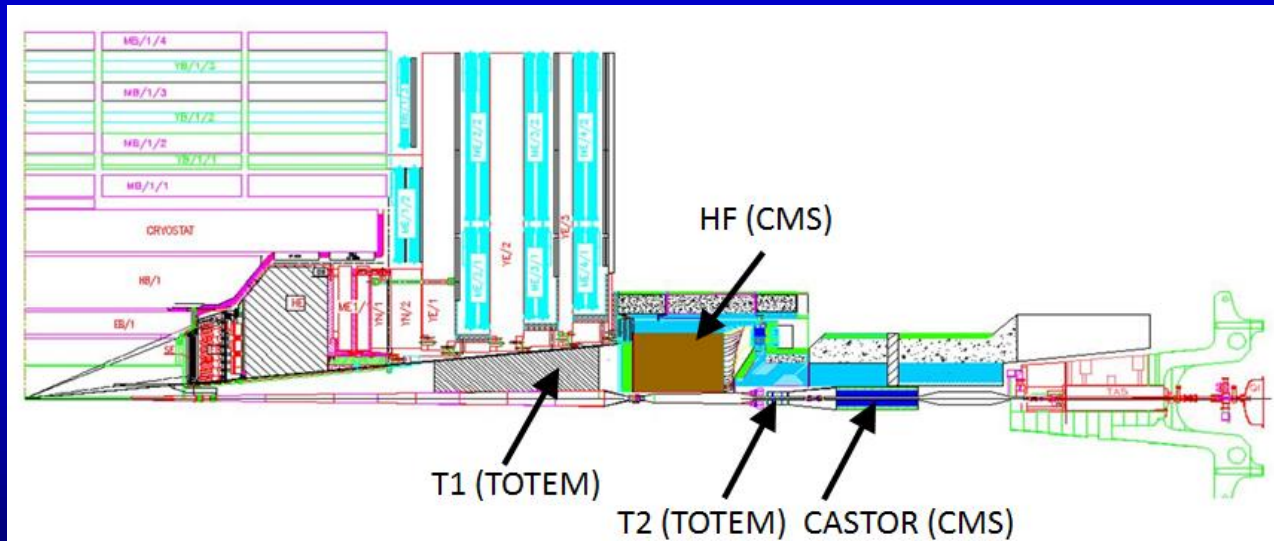
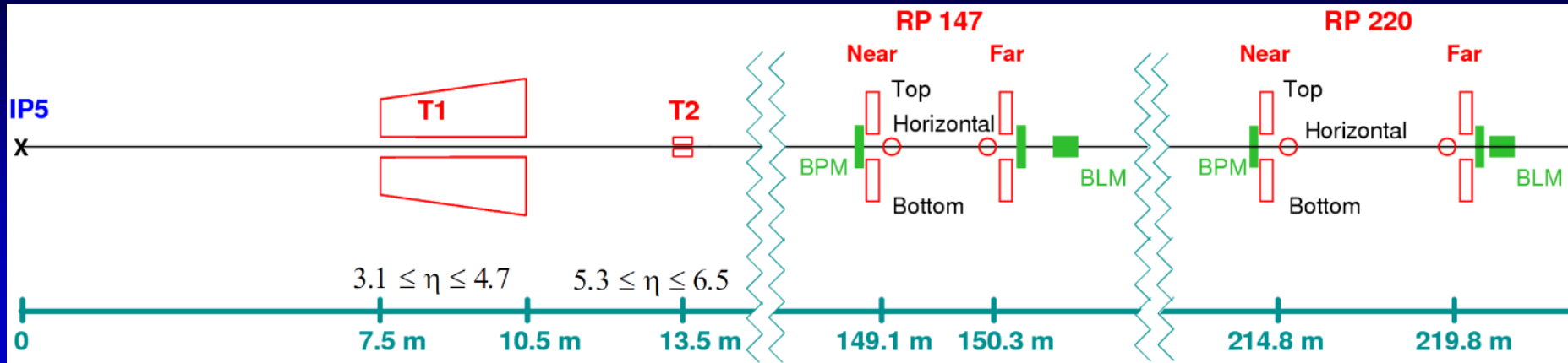
BB \rightarrow AA model

Thank you for your attention!

Questions?

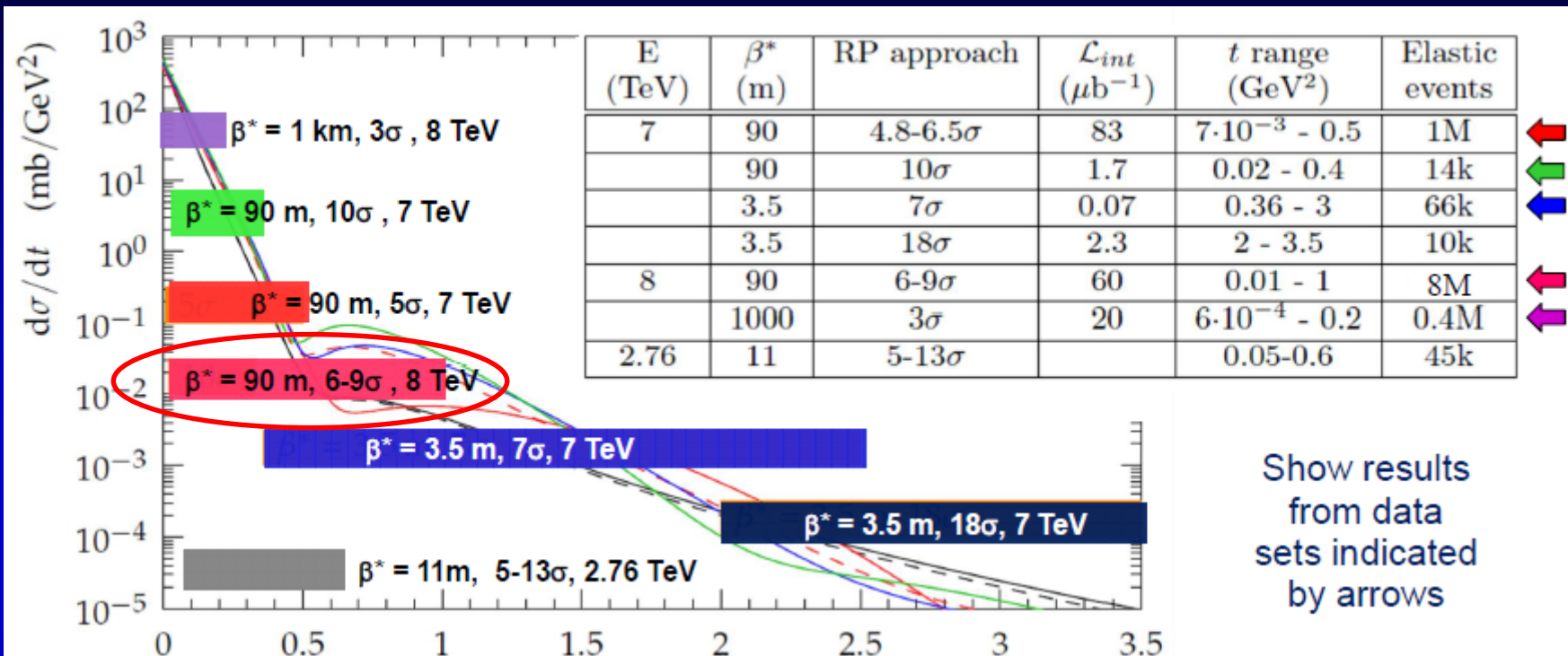
Backup slides - TOTEM

TOTEM – Experimental Setup at IP5



T1, T2: CSC and GEM Inelastic telescopes; RP: Roman Pots
[Details: JINST 3 (2008) S08007]. In this talk: TOTEM Roman Pots 220 m

TOTEM data taking

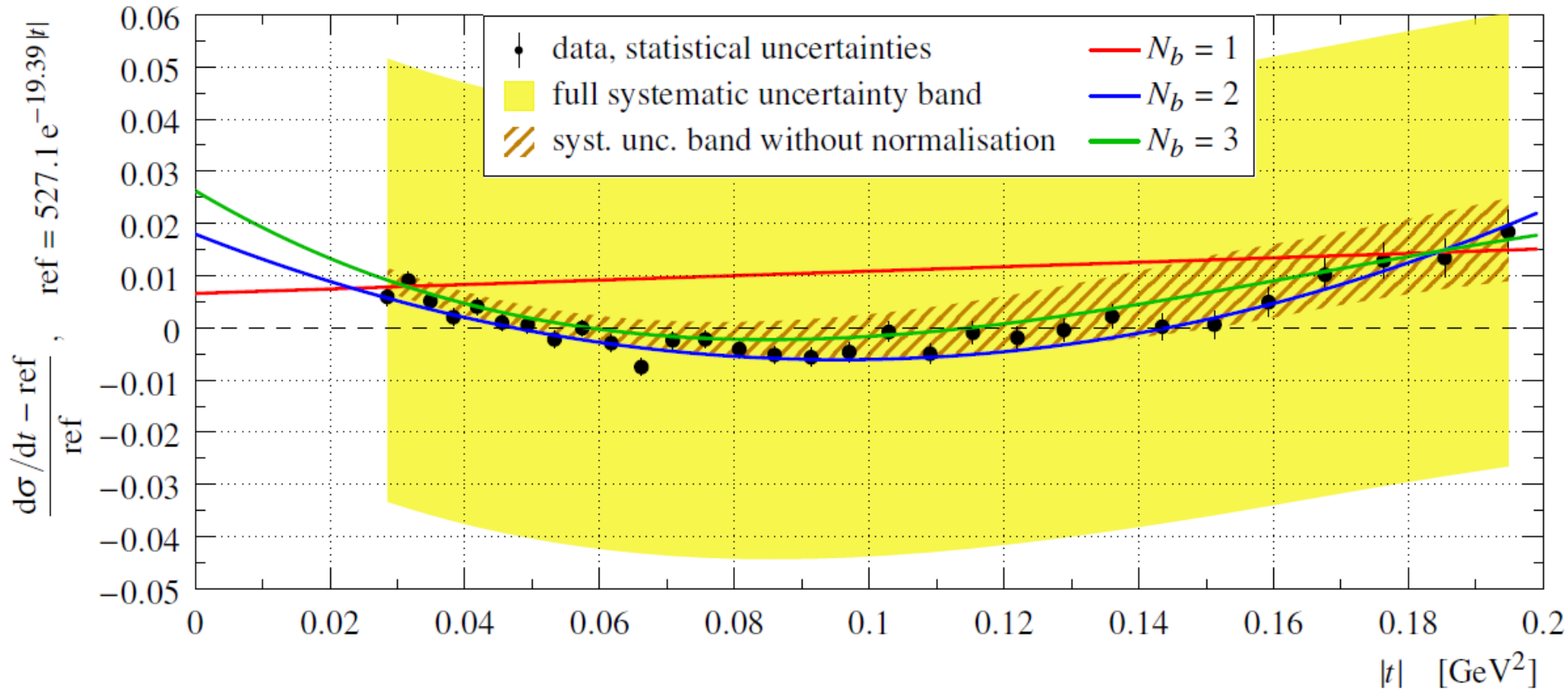


Show results from data sets indicated by arrows

July 2012 data, **special** LHC run , $\beta^* = 90$ m, $\sqrt{s} = 8$ TeV

2 \rightarrow 3 colliding bunch pair, 8×10^{10} p/bunch
 Instantaneous $L \sim 10^{28} \text{ cm}^{-2}\text{s}^{-1}$
 11 h data taking, RP-s at $9.5 \sigma_{\text{beam}}$
 Integrated $L \sim 735 \mu\text{b}^{-1}$
 $7.2 \cdot 10^6$ elastic events

Differential cross-section @ 8 TeV



$$\frac{d\sigma}{dt}(t) = \frac{d\sigma}{dt} \Big|_{t=0} \exp\left(\sum_{i=1}^{N_b} b_i t^i\right),$$

$$\chi^2 = \Delta^T V^{-1} \Delta,$$

$$V = V_{\text{stat}} + V_{\text{syst}}$$

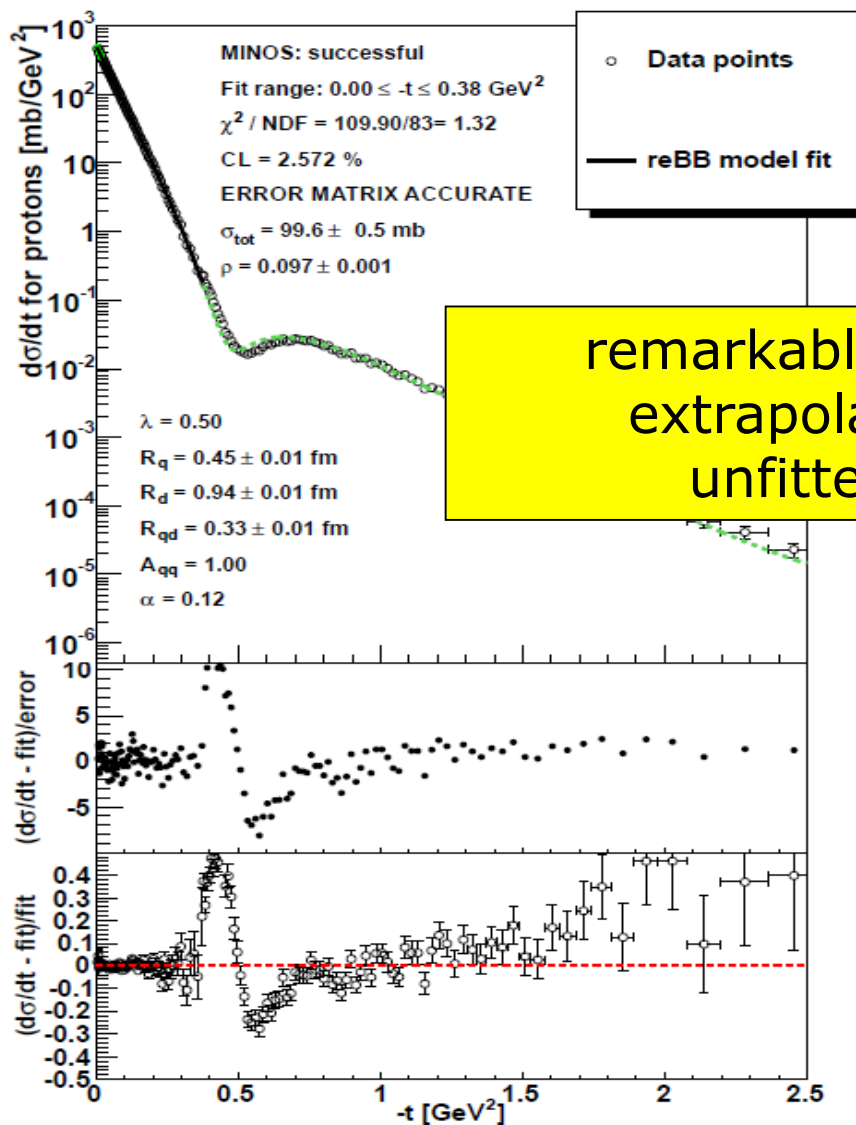
$$\Delta_i = \frac{d\sigma}{dt} \Big|_{\text{bin } i} - \frac{1}{\Delta t_i} \int_{\text{bin } i} f(t) dt,$$

$N_b = 1$ fits excluded. Relative to best exponential, a significant 7.2σ deviation found.

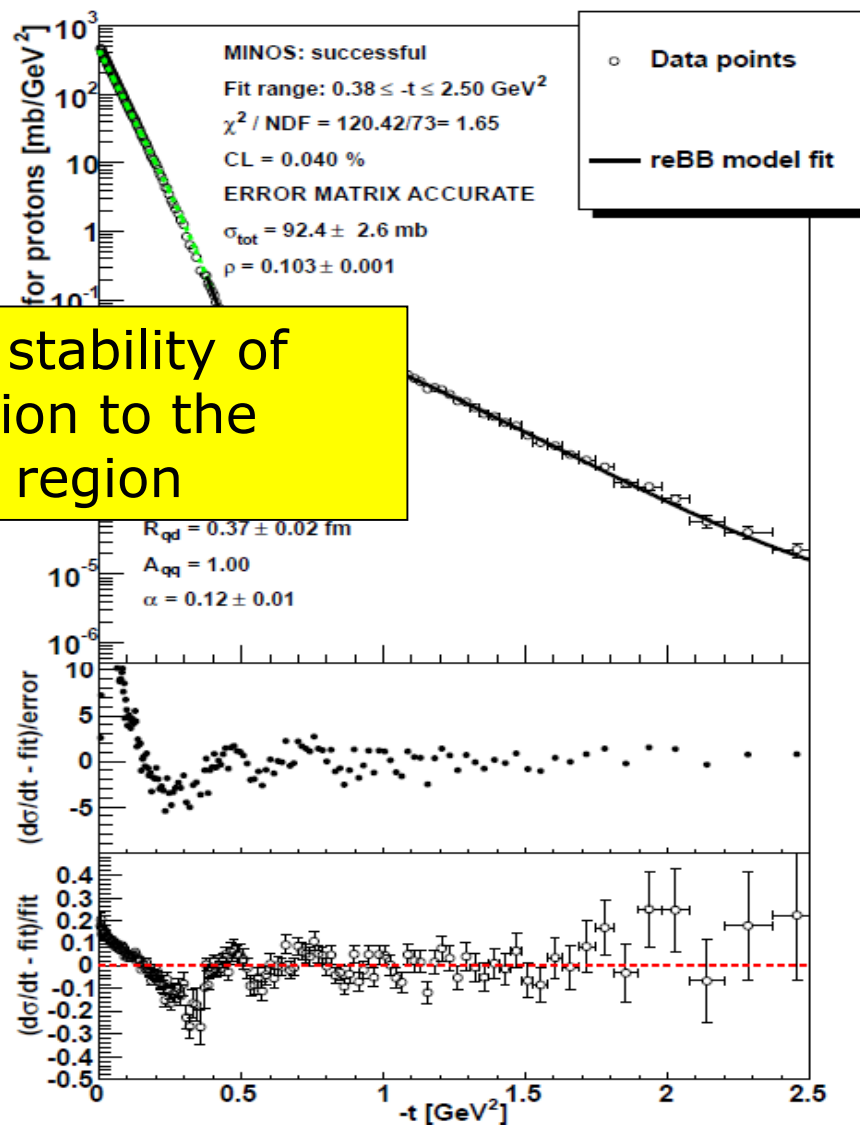
Backup slides – ReBB

ReBB model, fit range studies

p+p → p+p, diquark as a single entity at $\sqrt{s}=7000.0$ GeV



p+p → p+p, diquark as a single entity at $\sqrt{s}=7000.0$ GeV



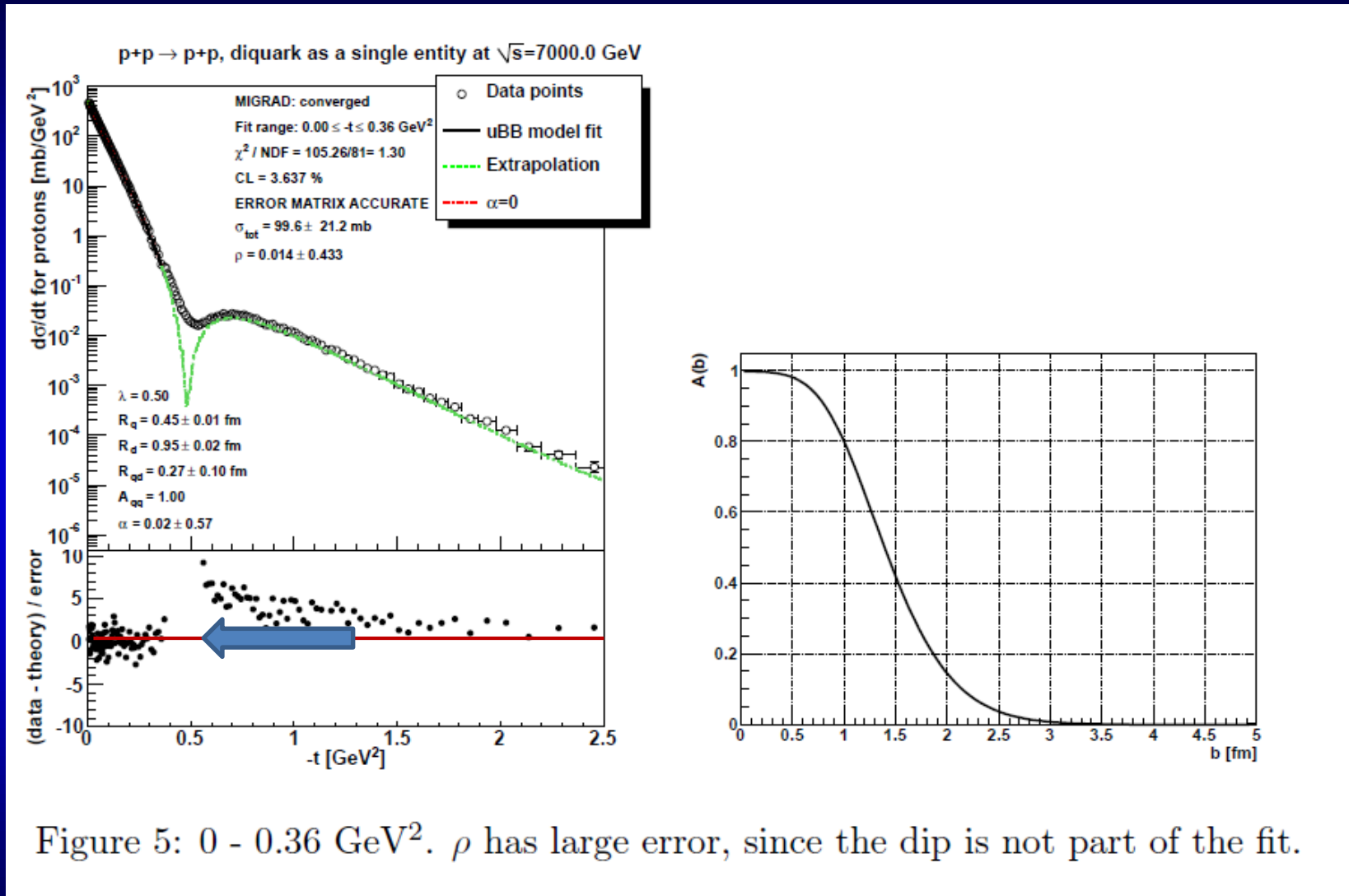
remarkable stability of extrapolation to the unfitted region

fit: $0.36 \leq -t \leq 2.5$ GeV², OK

/arsaw,

fit: $0 \leq -t \leq 2.5$ GeV², ~ OK

Focusing reBB on the low- t region



Saturation is apparent if fit range is limited to $|t| < 0.36$ GeV²

Focusing reBB on even lower -t region

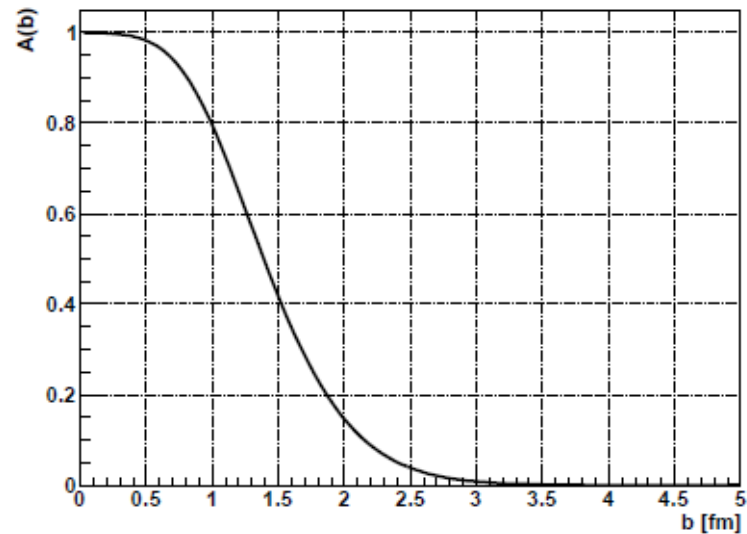
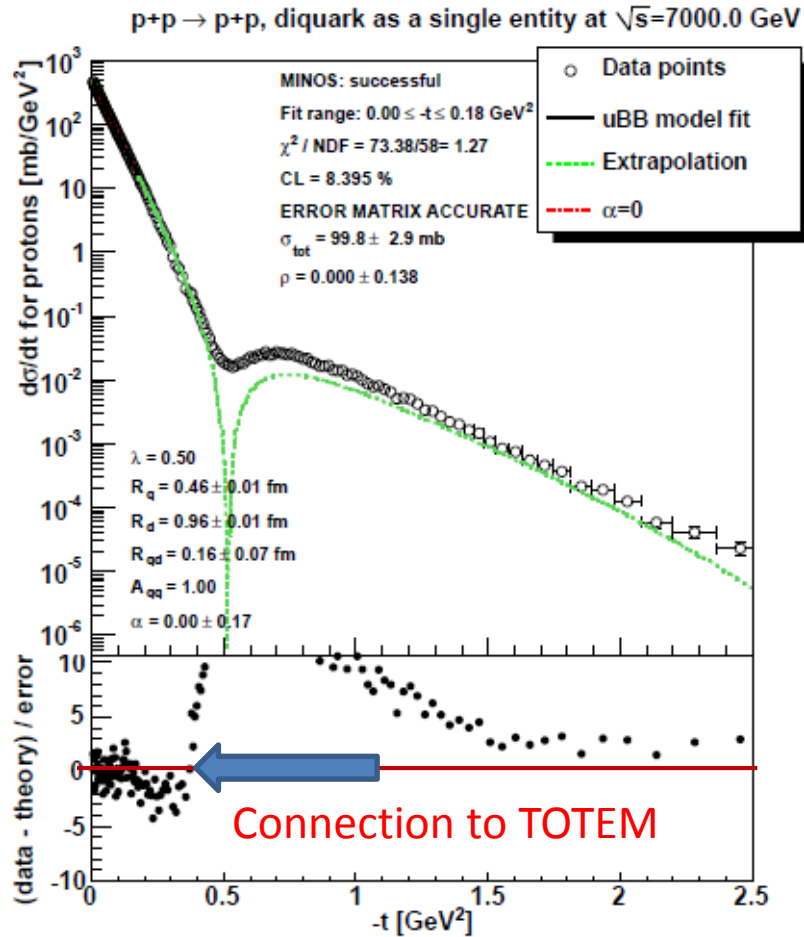


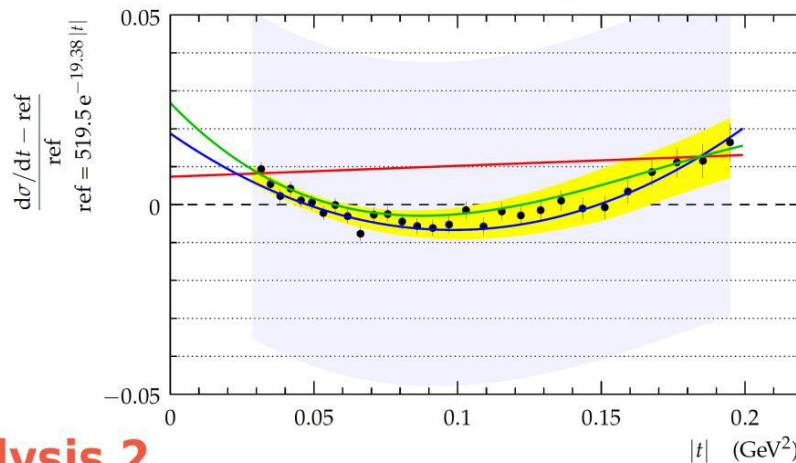
Figure 6: 0 - 0.18 GeV². ρ has large error, since the dip is not part of the fit.

Saturation still apparent, fit range $|t| < 0.18$ GeV²

TOTEM 8 TeV pp data

Analysis 1: fits $A \exp(b_1 t + b_2 t^2 + \dots)$, N_b parameters in exponent

DS4



diagonals combined

Nucl.Phys. B899 (2015) 527-546

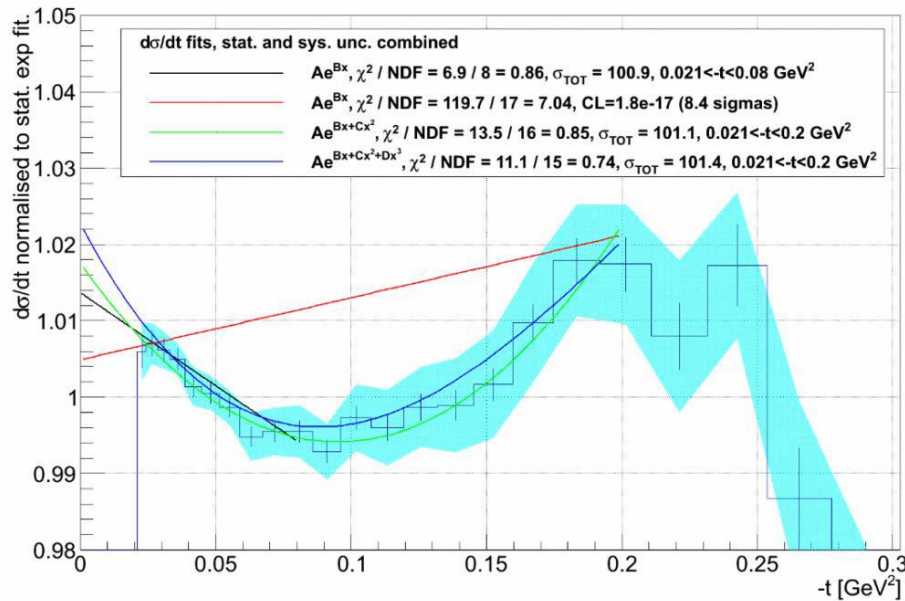
- data (binning)
- | statistical uncertainties
- systematic uncertainty band: analysis+normalisation
- systematic uncertainty band: analysis only

fit parametrisation: $a \exp(\sum_{n=1}^{N_b} b_n t^n)$

fits with statistical and systematic uncertainties:

- $N_b = 1$: $\chi^2/\text{ndf} = 117.5/28 = 4.198 \Rightarrow$ p-value = 6.14×10^{-13} , significance = 7.20σ
- $N_b = 2$: $\chi^2/\text{ndf} = 29.3/27 = 1.085 \Rightarrow$ p-value = 3.47×10^{-1} , significance = 0.94σ
- $N_b = 3$: $\chi^2/\text{ndf} = 25.5/26 = 0.980 \Rightarrow$ p-value = 4.92×10^{-1} , significance = 0.69σ

Analysis 2



dσ/dt fits, stat. and sys. unc. combined

- Ae^{Bx} , $\chi^2/\text{NDF} = 6.9/8 = 0.86$, $\sigma_{\text{TOT}} = 100.9$, $0.021 < -t < 0.08 \text{ GeV}^2$
- Ae^{Bx+Cx^2} , $\chi^2/\text{NDF} = 119.7/17 = 7.04$, $\text{CL} = 1.8e-17$ (8.4 sigmas)
- $Ae^{Bx+Cx^2+Dx^3}$, $\chi^2/\text{NDF} = 13.5/16 = 0.85$, $\sigma_{\text{TOT}} = 101.1$, $0.021 < -t < 0.2 \text{ GeV}^2$
- $Ae^{Bx+Cx^2+Dx^3}$, $\chi^2/\text{NDF} = 11.1/15 = 0.74$, $\sigma_{\text{TOT}} = 101.4$, $0.021 < -t < 0.2 \text{ GeV}^2$

TOTEM pp data at 8 TeV
exponential shape
excluded at $7 + \sigma$

new determination

$$\sigma_{\text{tot}} = (101.4 \pm 2.0) \text{ mb}$$

Theoretical support, from ISR
to LHC energies, unitarity
L. Jenkovszky and A. Lengyel,
arXiv:1410.4106

Non-exponential behaviour in ReBB

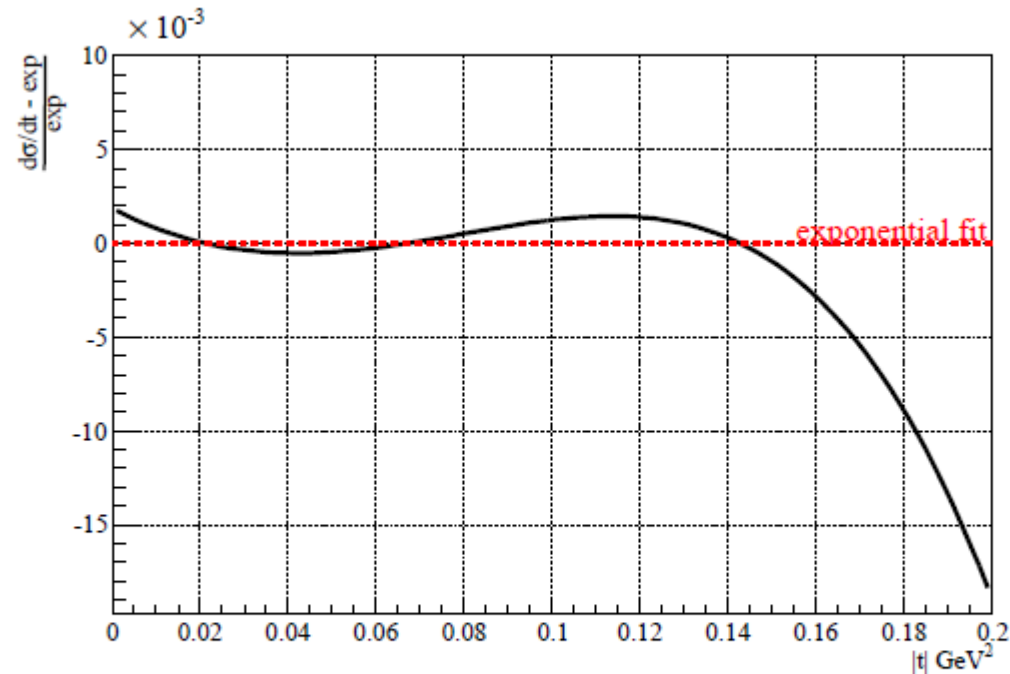
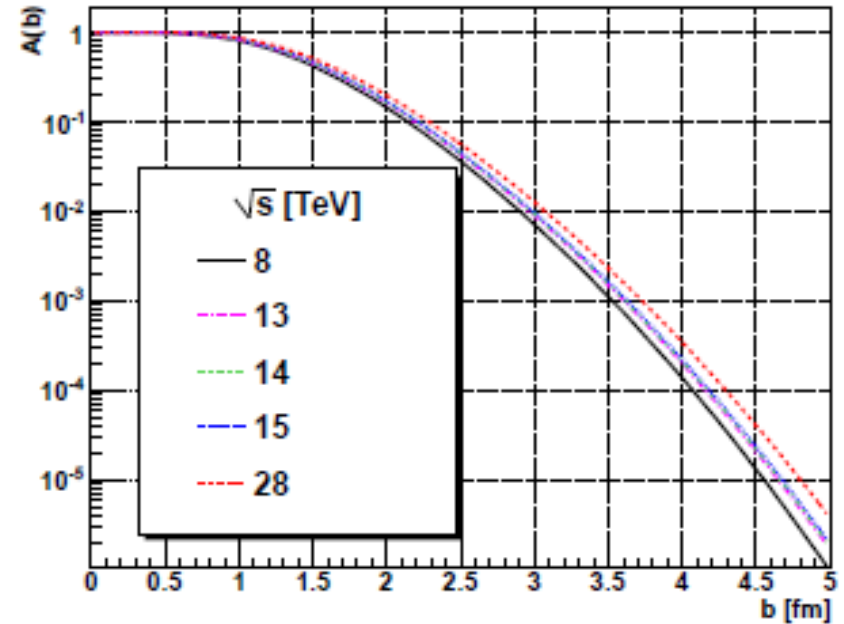
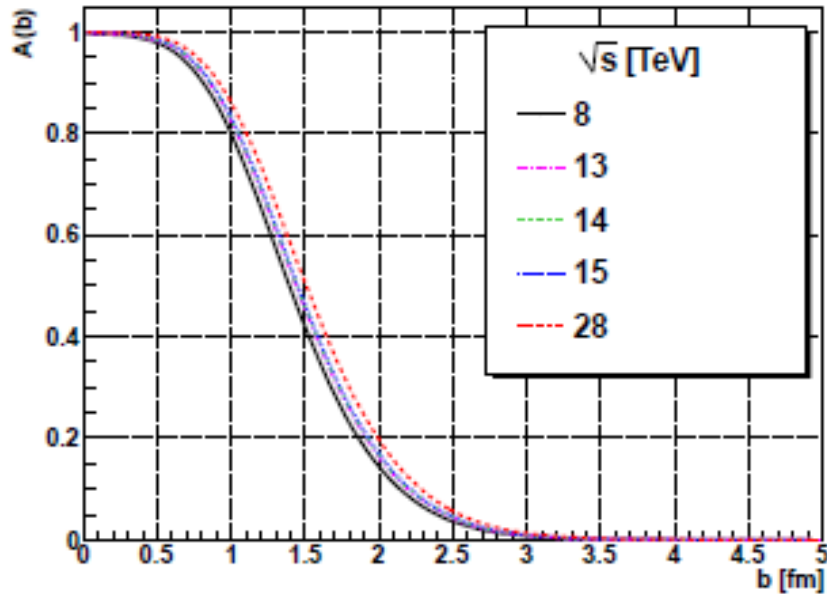


Fig. 5. The ReBB model, fitted in the $0.0 \leq |t| \leq 0.36 \text{ GeV}^2$ range, with respect to the exponential fit of Eq. (33). In the plot only the $0.0 \leq |t| \leq 0.2 \text{ GeV}^2$ range is shown. The curve indicates a significant deviation from the simple exponential at low $|t|$ values.

Similar
non-exponential feature
seen at 7 TeV as in 8 TeV
TOTEM data

Predictions for the shadow profile

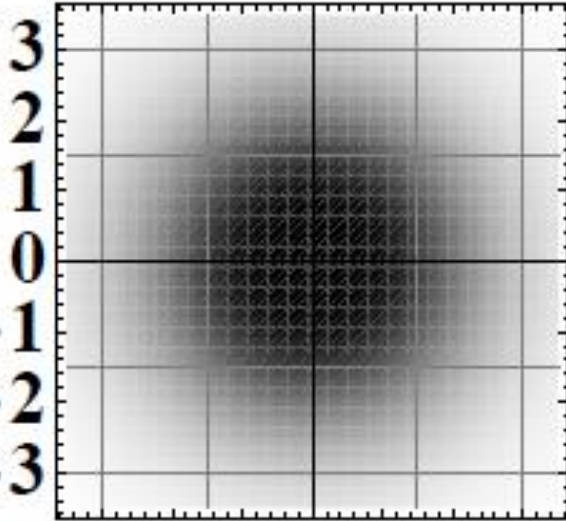


Blacker and Larger,
but not Edgier:
BnEL effect
at LHC energies

Similar to:
K.A. Kohara, T. Kodama,
E. Ferreira,
arXiv:1411.3518
but they also claim
an asymptotic BEL effect

Predictions for the shadow profile

b_x [fm]

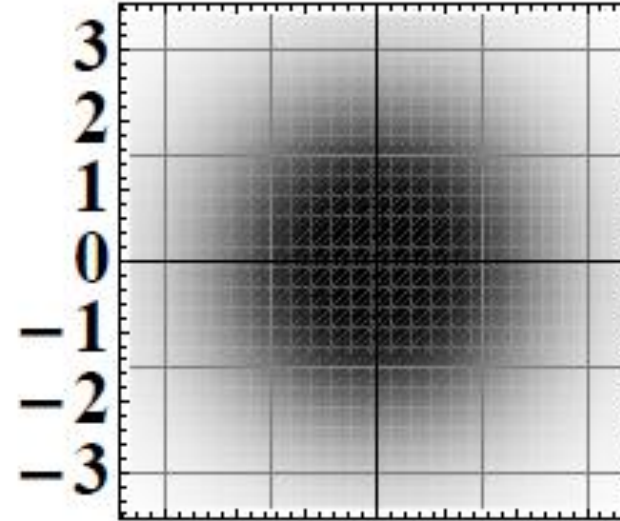


b_y [fm]

-3 -2 -1 0 1 2 3

$\sqrt{s} = 14$ TeV

b_x [fm]



b_y [fm]

-3 -2 -1 0 1 2 3

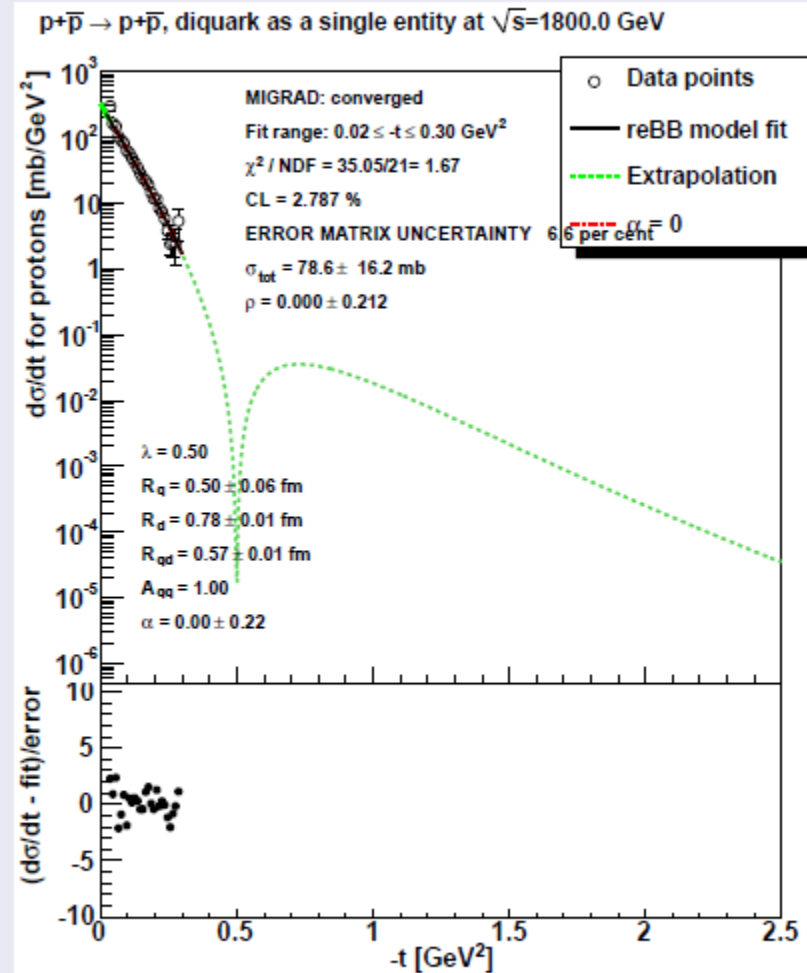
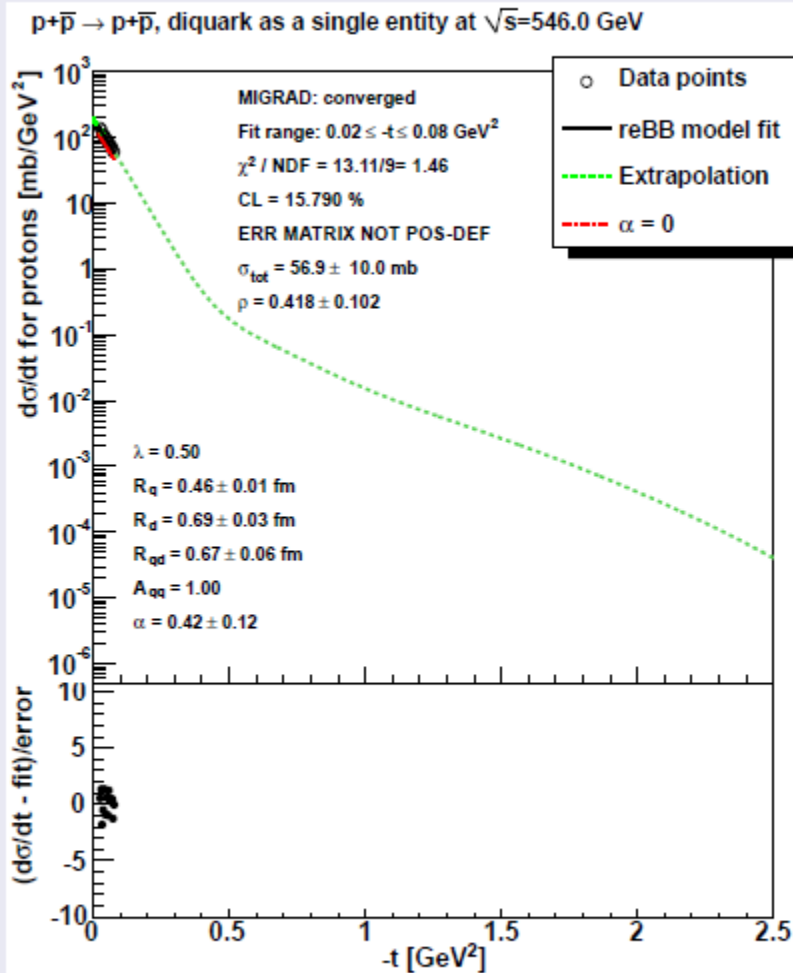
$\sqrt{s} = 28$ TeV

Blacker and Larger,
but not Edgier:
BnEL effect
at LHC energies

Results presented so far:
[arxiv:1505.01415](https://arxiv.org/abs/1505.01415)

New results: $p\bar{p}$ data with ReBB model

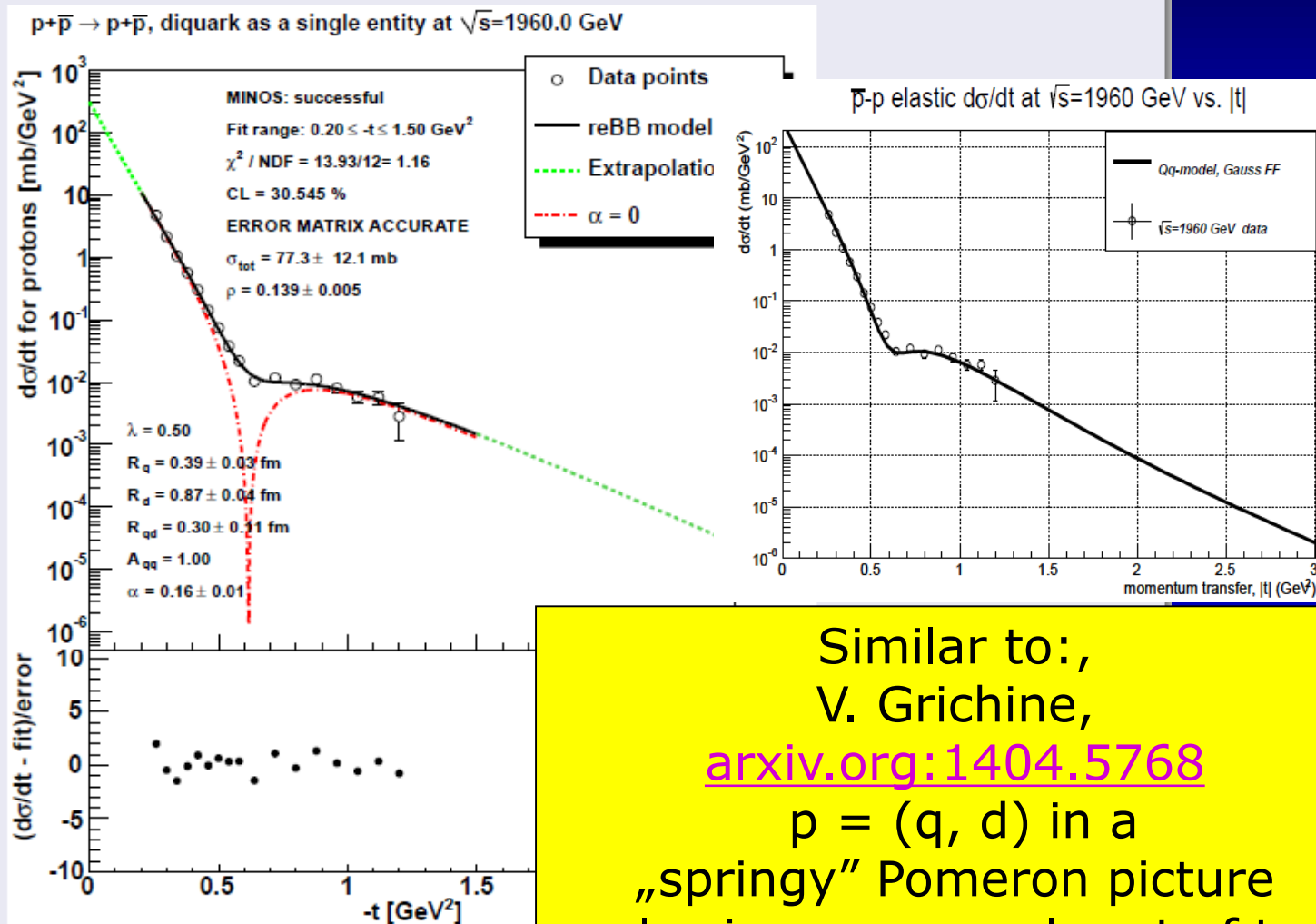
All the usual BB fit parameters are free ($\sqrt{s} = 546 \text{ GeV}, 1.8 \text{ TeV}$)



There are not enough points to pin down the shape.

Tevatron $p\bar{p}$ data with ReBB model

All the usual BB fit parameters are free (1.96 TeV)

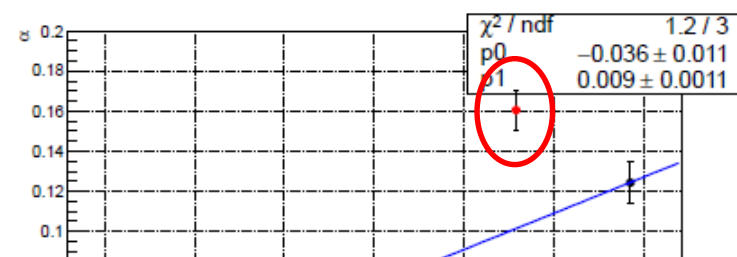
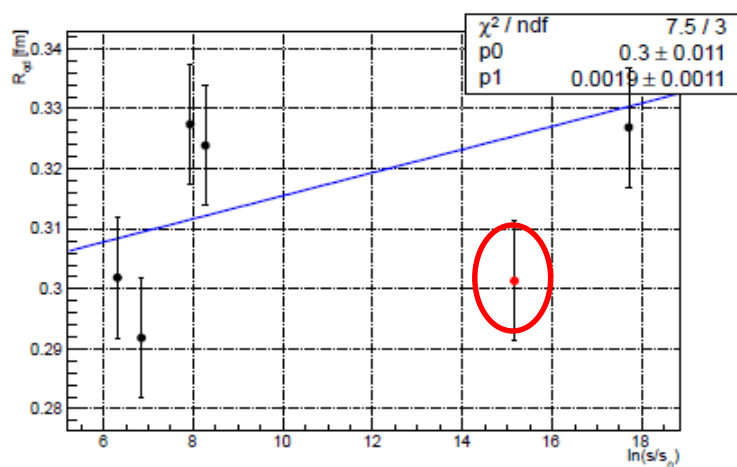
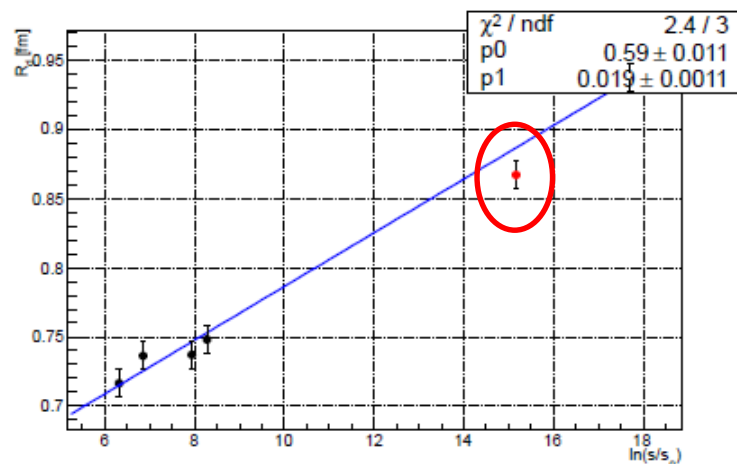
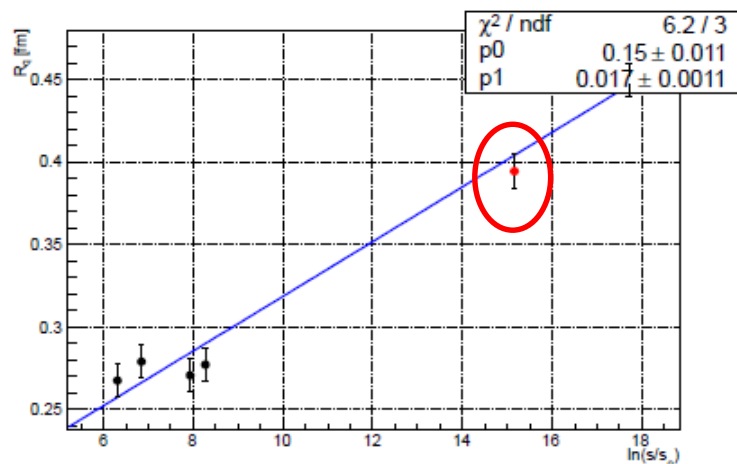


Similar to,
 V. Grichine,
[arxiv.org:1404.5768](https://arxiv.org/abs/1404.5768)
 $p = (q, d)$ in a
 „springy” Pomeron picture
 also increases real part of t_{el}

Ok.

Tevatron $p\bar{p}$ data trends ReBB model

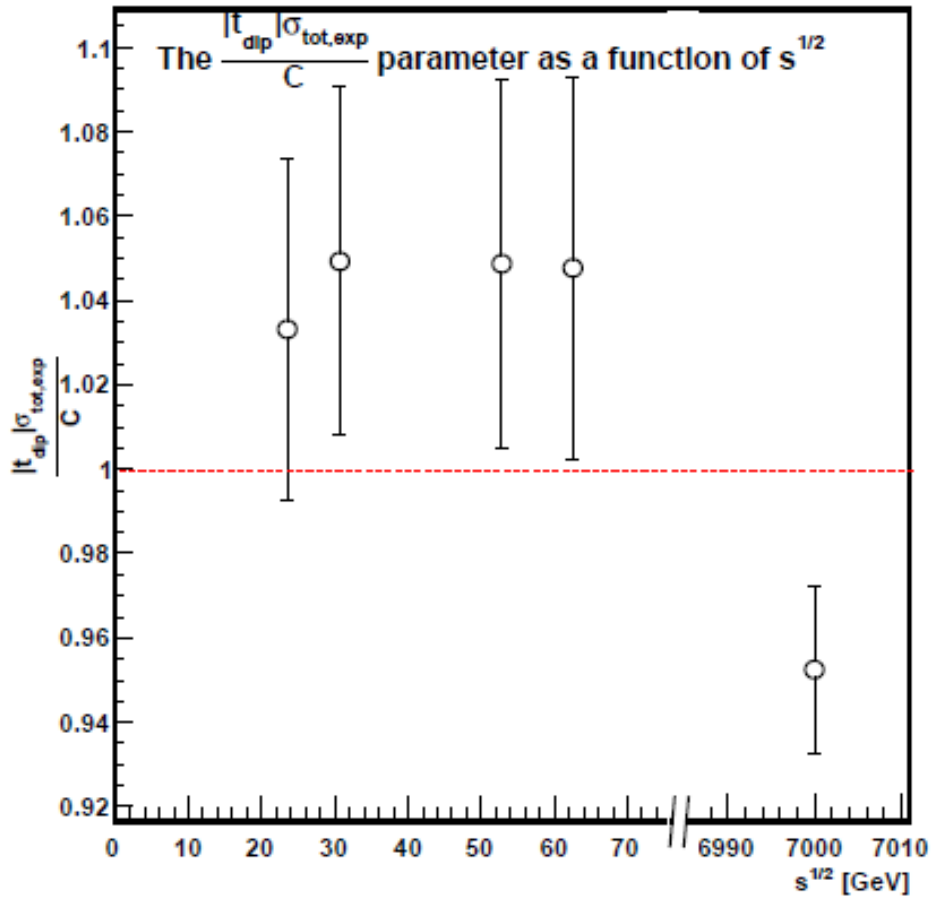
The good fit at $\sqrt{s} = 1.96$ TeV compared with the extrapolations based only on pp fits of our ReBB paper



ReBB model works also for elastic $p\bar{p}$ data but $p\bar{p}$ is more „opaque” than pp .

Backup slides – Discussion

Black disc limit?



Geometric scaling,
but not the black disc limit:

T. Cs. and F. Nemes
arXiv:1306.4217
Int. J. Mod. Phys. A (2014)

$$C(\text{data}) \sim 50 \text{ mb GeV}^2$$

$$\neq$$

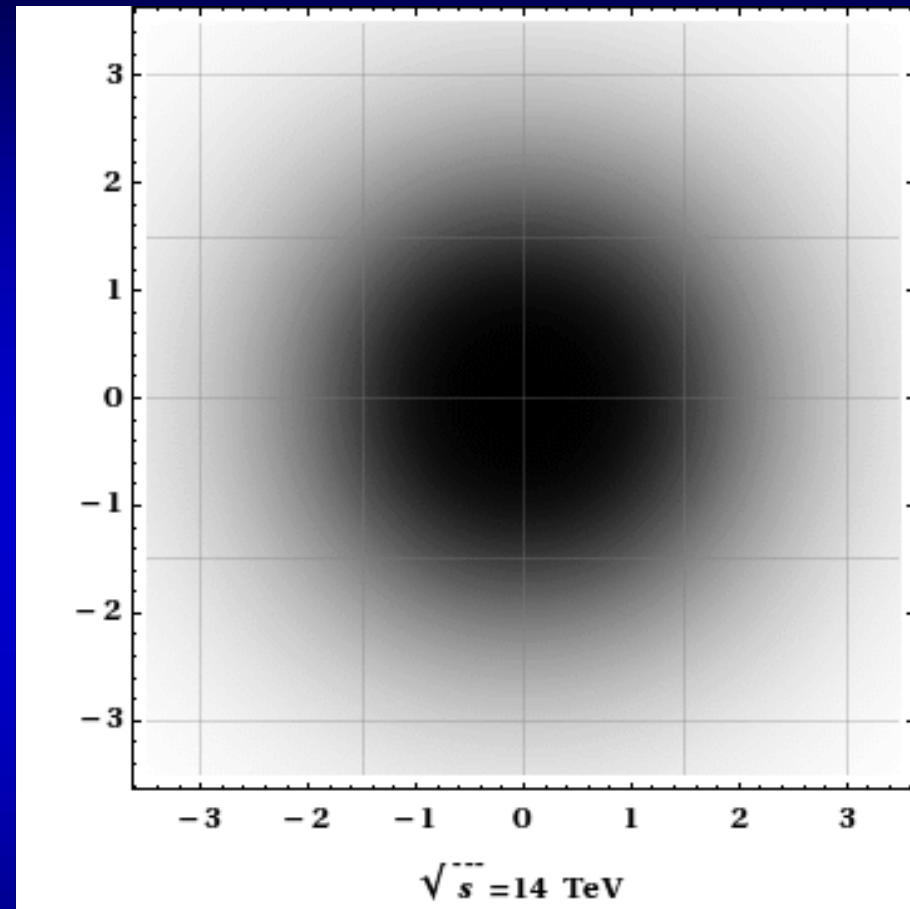
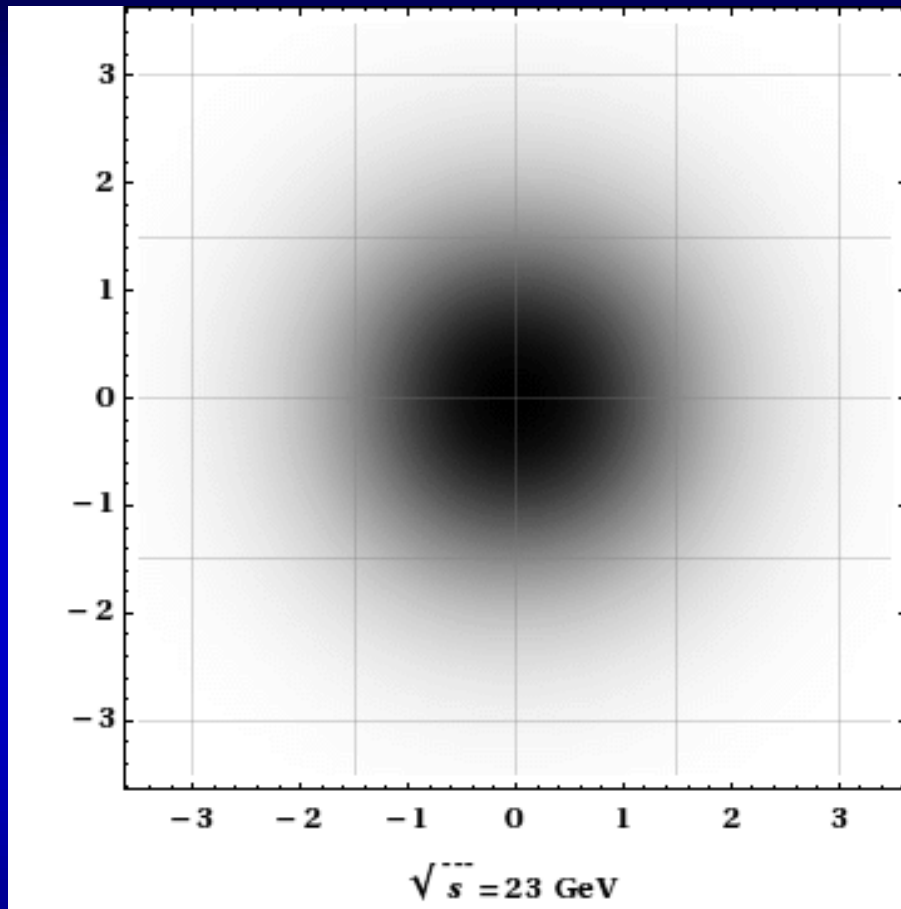
$$C(\text{black}) \sim 36 \text{ mb GeV}^2$$

$$\frac{d\sigma_{black}}{dt} = \pi R^4 \left[\frac{J_1(qR)}{qR} \right]^2$$

$$\sigma_{tot,black} = 2\pi R^2.$$

$$C_{black} = |t_{dip,black}| \cdot \sigma_{tot,black} = 2\pi j_{1,1}^2 (\hbar c)^2 \approx 35.9 \text{ mb GeV}^2$$

Motivation: Is the proton a black disc?

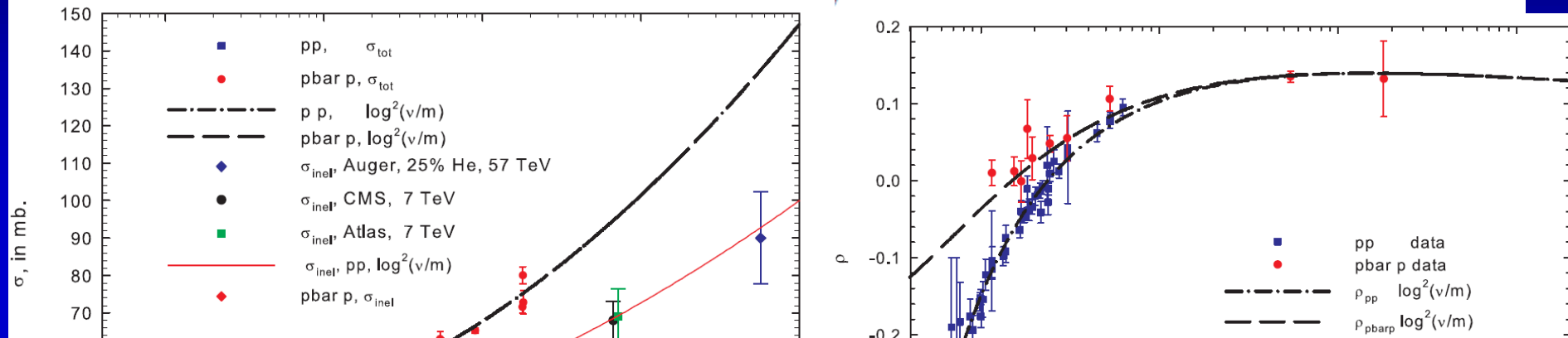


Recent papers by M. Block and F. Halsen address this topic :
Experimental confirmation: the proton is asymptotically a black disc,
arXiv:1109.2041, Phys. Rev. Lett. 107 (2011) 212002

Properties of a black disc

Properties of a black disc: In impact parameter space b , the elastic and total cross sections are given by

$$\sigma_{\text{el}} = 4 \int d^2b |a(b, s)|^2, \quad \sigma_{\text{tot}} = 4 \int d^2b \text{Im} a(b, s). \quad (7)$$



Conclusions: We find that the $\ln^2 s$ Froissart bound for the proton for σ_{tot} [7] and σ_{inel} [9] is saturated and that at infinite s , (1) the experimental ratio $\sigma_{\text{inel}}/\sigma_{\text{tot}} = 0.509 \pm 0.011$, compatible with the black disk ratio of 0.5 and (2) the forward scattering amplitude is purely imaginary. We thus conclude that the proton becomes an expanding black disk at sufficiently ultra-high energies that are probably never accessible to experiment. The theory for these bounds is predicated on the pillar stones of analyticity and unitarity, which have now been experimentally verified up to 57000 GeV. Further, since σ_{tot} has been extrapolated up from the Tevatron, we expect no new large cross section physics between 2000 and 57000 GeV.

Finally, the lowest-lying glueball mass is measured to be $M_{\text{glueball}} = 2.97 \pm 0.03$ GeV. Reproducing these experimental results will be a task of lattice QCD.

arXiv:1109.2041, Phys. Rev. Lett. 107 (2011) 212002

arXiv:1208.4086, Phys.Rev. D86 (2012) 051504

[arXiv:1409.3196](https://arxiv.org/abs/1409.3196)

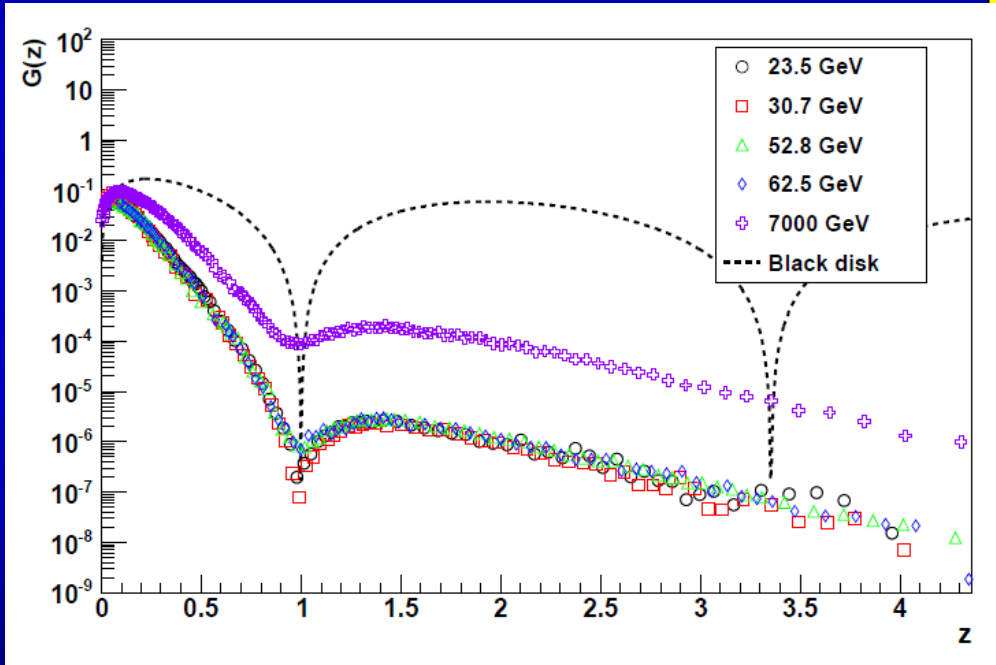
Black Disc (BD) limit?

Scaling but
not in the black disc limit:
T. Cs. and F. Nemes
arXiv:1306.4217
Int. J. Mod. Phys. A (2014)

$$C(\text{data}) \sim 50 \text{ mb GeV}^2$$

$$\neq$$

$$C(\text{black}) \sim 36 \text{ mb GeV}^2$$

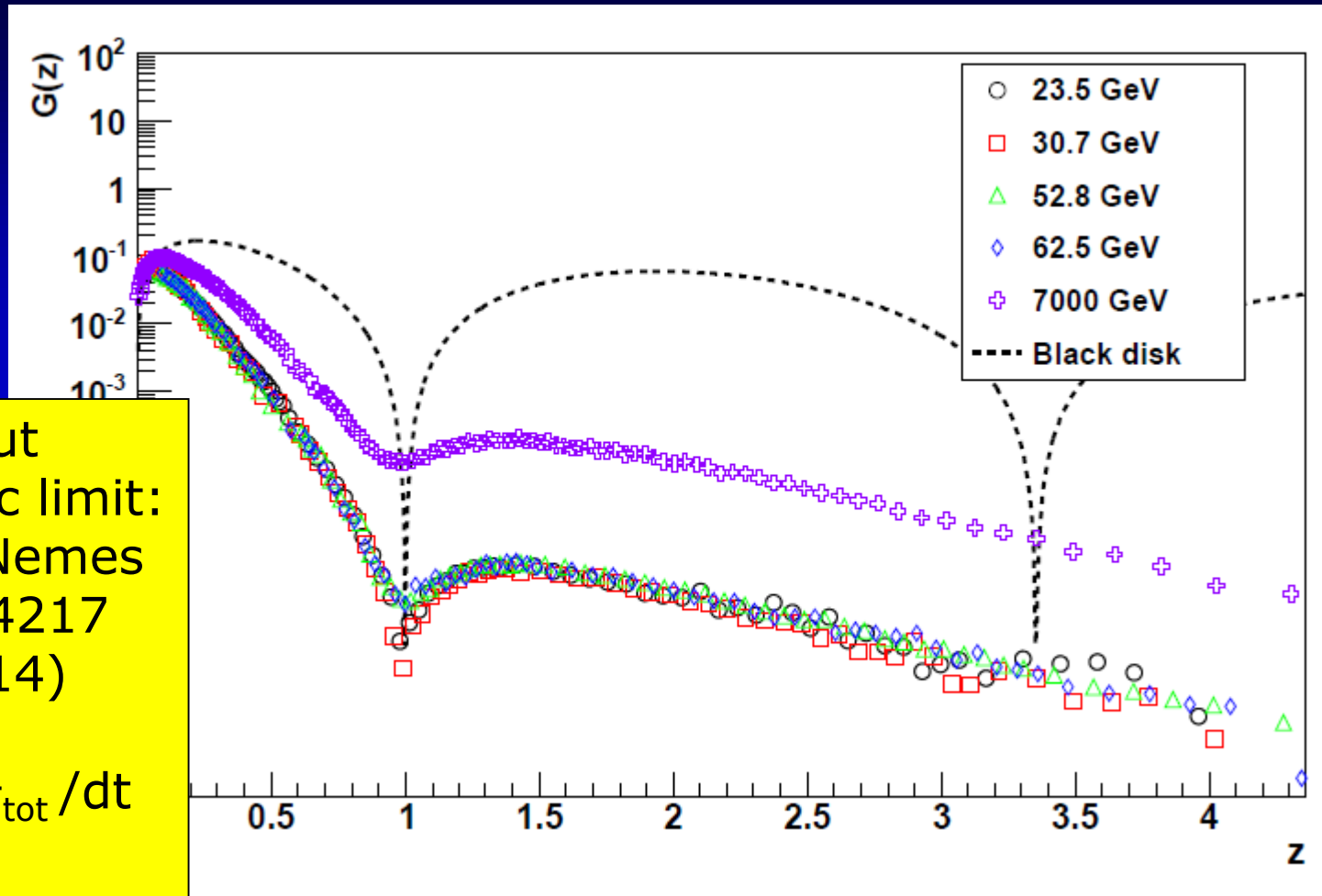


$$\frac{d\sigma_{black}}{dt} = \pi R^4 \left[\frac{J_1(qR)}{qR} \right]^2$$

$$\sigma_{tot,black} = 2\pi R^2.$$

$$C_{black} = |t_{dip,black}| \cdot \sigma_{tot,black} = 2\pi j_{1,1}^2 (\hbar c)^2 \approx 35.9 \text{ mb GeV}^2$$

Geometric scaling, but not BD limit?



Scaling but
not a black disc limit:
T. Cs. and F. Nemes
arXiv:1306.4217
IJMPA (2014)

$$G(z) = t \frac{d\sigma/\sigma_{\text{tot}}}{dt}$$

plotted vs

$$z = t/t_{\text{dip}}$$