# QCD Thermodynamics from Effective Field Theories

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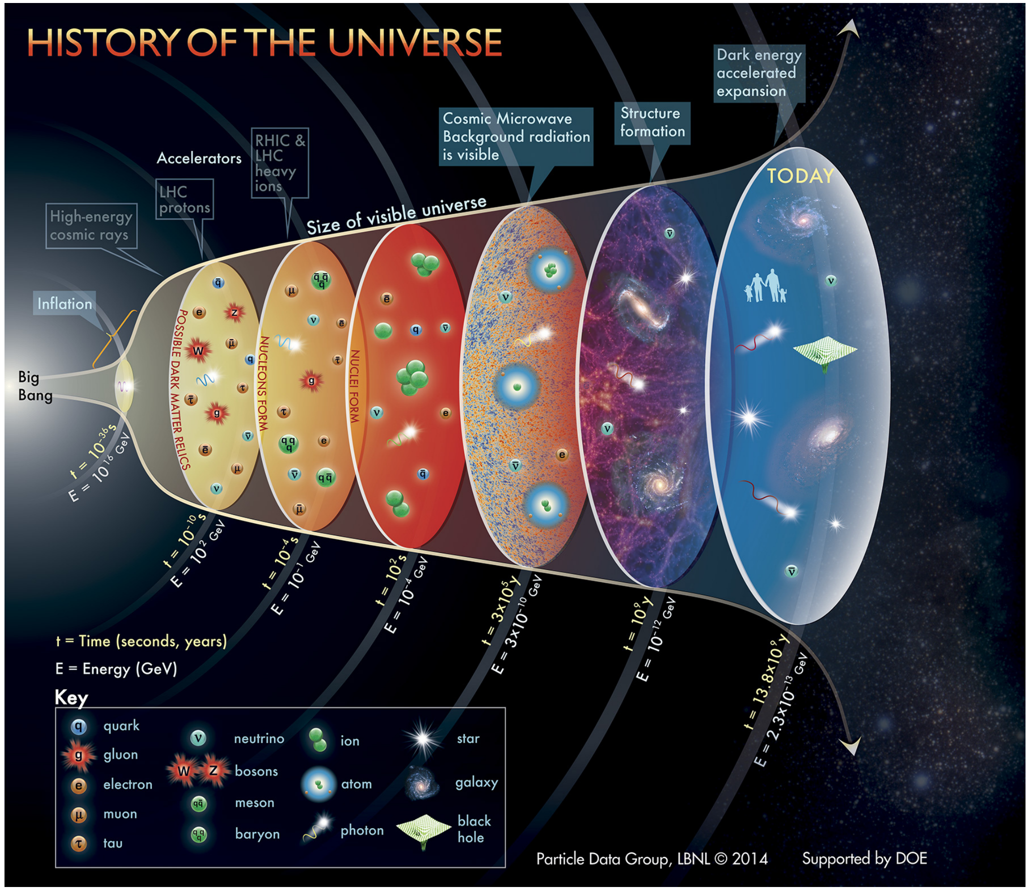
**Selected Issues:**

- interplay between chiral symmetry breaking and confinement
- phases of hot/dense QCD
- fate of hadrons near the phase transition

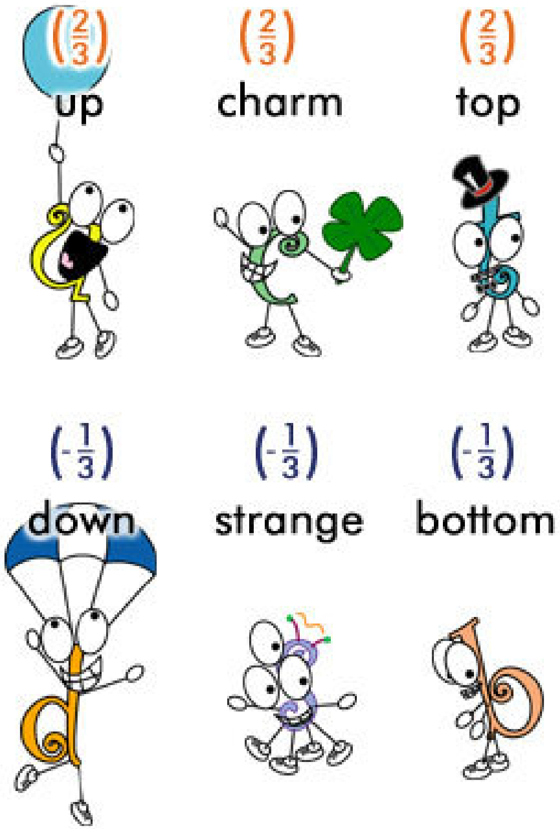
# What is the origin of matter?




- after EW phase transition: Standard Model (EM & Weak plus Strong int.)
- QCD phase transition is the last transition in the evolution of the Universe!

up, down quarks ($\sim 5$ MeV): SM ingredients
Where does the nucleon mass ($\sim 1$ GeV) come from?
10 % from EW/Higgs, and 90 % from QCD!

- scale and confinement? from elementary particle to composite states?
  $\cdots$ fundamental issues in non-abelian gauge theories!




**Baryons qqq and Antibaryons $\bar{q}\bar{q}\bar{q}$**
Baryons are fermionic hadrons.
There are a few of the many types of baryons.

| Symbol | Name | Quark content | Electric charge | GeV/c$^2$ | Spin |
|---|---|---|---|---|---|
| **p** | proton | **uud** | 1 | 0.938 | 1/2 |
| $\bar{\mathbf{p}}$ | antiproton | $\bar{\mathbf{u}}\bar{\mathbf{u}}\bar{\mathbf{d}}$ | –1 | 0.938 | 1/2 |
| **n** | neutron | **udd** | 0 | 0.940 | 1/2 |
| **Λ** | lambda | **uds** | 0 | 1.116 | 1/2 |
| $\Omega^-$ | omega | **sss** | –1 | 1.672 | 3/2 |

$\sim 150$

**Mesons $q\bar{q}$**
Mesons are bosonic hadrons
There are a few of the many types of mesons.

| Symbol | Name | Quark content | Electric charge | GeV/c$^2$ | Spin |
|---|---|---|---|---|---|
| $\pi^+$ | pion | $\mathbf{u}\bar{\mathbf{d}}$ | +1 | 0.140 | 0 |
| $K^-$ | kaon | $\mathbf{s}\bar{\mathbf{u}}$ | –1 | 0.494 | 0 |
| $\rho^+$ | rho | $\mathbf{u}\bar{\mathbf{d}}$ | +1 | 0.770 | 1 |
| $B^0$ | B-zero | $\mathbf{d}\bar{\mathbf{b}}$ | 0 | 5.279 | 0 |
| $\eta_c$ | eta-c | $\mathbf{c}\bar{\mathbf{c}}$ | 0 | 2.980 | 0 |

$\sim 200$

- effective field theories:
  - "particles" contained: either fundamental or composite
  - based on fundamental/emergent symmetries
  - broad spectrum of applications to QCD(-like), SM, BSM

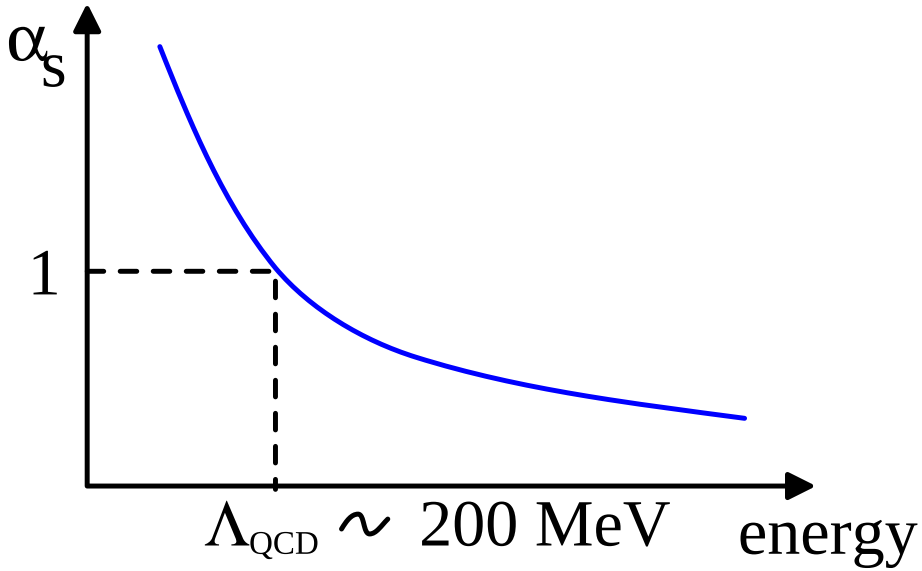
# Quantum Chromodynamics (QCD) underlies Hadron Physics

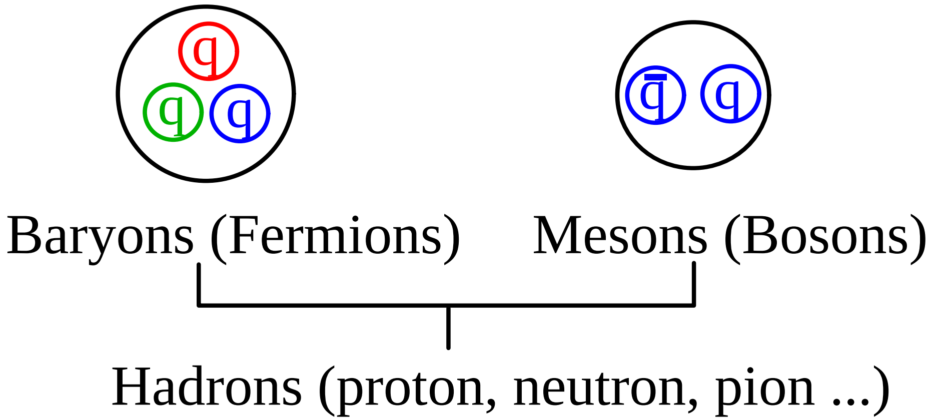
- theory of strong interaction among quarks ($q$) and gluons ($F_{\mu\nu}$)

$$SU_c(3) \;:\; \mathcal{L} = \bar{q}_f^a \left(i\gamma_\mu D^\mu_{ab} - \delta_{ab} m_f\right) q_f^b - \frac{1}{4} F^\alpha_{\mu\nu} F^{\mu\nu}_\alpha$$

colors $= (r\,,g\,,b)$ and flavors $= (u, d, s, c, b, t)$

**asymptotic freedom**
**$\Rightarrow$ color confinement**




- hadrons are "colorless": $qqq\,, \bar{q}q$ $\sim$ SU(3)$_c$ singlet
- exotic hadrons: tetraquarks ($\bar{q}\bar{q}qq$), pentaquarks ($\bar{q}qqqq$), dibaryons ($qqqqqq$), glueballs

## QCD ground state in low energy/temperature/density

- color confinement: only $SU(3)_c$ singlet observed
- dynamical breaking of (approximate) $SU(2)_f$ chiral symmetry: pions as NG bosons, no degenerate parity partners

$$L = -\frac{1}{4} F_{\mu\nu}F^{\mu\nu} + \bar{q}(i\not{D} - m)q$$

color confinement

instantons

trace anomaly

$U(1)_A$ anomaly

dynamical chiral symmetry breaking

pion as NG boson

nuclear force

**changing external parameter(s): different ground state!**

∗ how are color-singlet states formed? exotic hadrons?

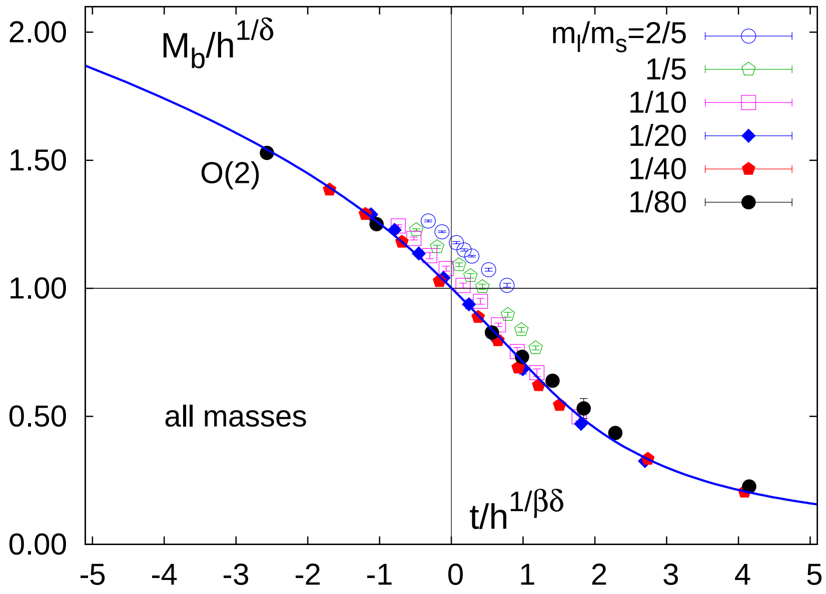
∗ dynamical origin of hadronic interactions and masses?

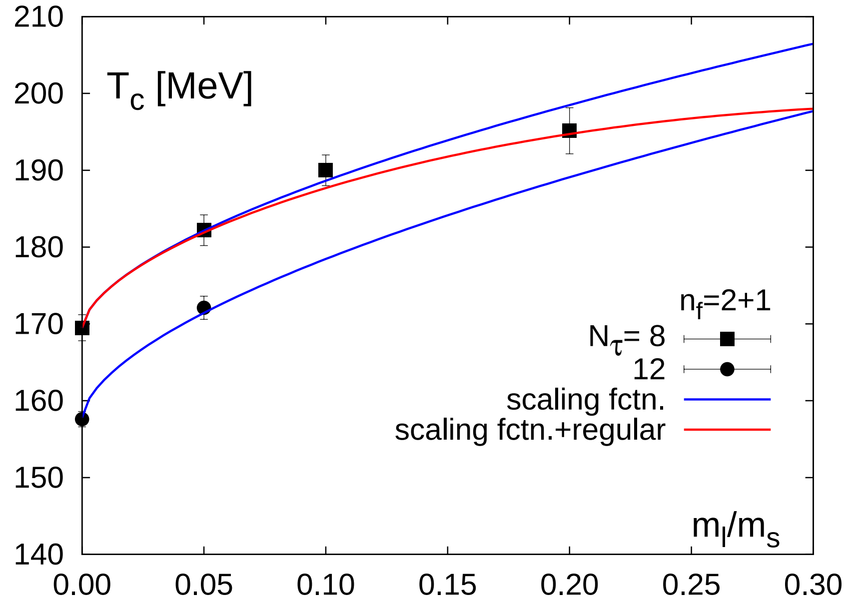
∗ composition of QCD matter at finite temperature/density?

# I. Confinement vs. Dynamical Chiral symmetry Breaking

# Chiral crossover at finite temperature

- chiral restoration in $N_f = 2$ QCD (massless quarks): O(4) universality class [Pisarski and Wilczek ('84)]




- confirmed by Lattice QCD simulations:
  also true in QCD with physical $m_\pi$! [Karsch ('11), Bazavov et al., ('12)]
- chiral crossover in QCD governed by O(4)
  - important guide in building models
  - critical behaviors independent of models! — universality

# QCD thermodynamics at $\mu_q \simeq 0$ from lattice simulations

- deconfinement and SU(2) chiral restoration set in at $T_{\rm pc} \sim 155$ MeV.

  $\hookrightarrow$ conserved charges $\qquad\qquad$ $\hookrightarrow$ melting chiral condensate

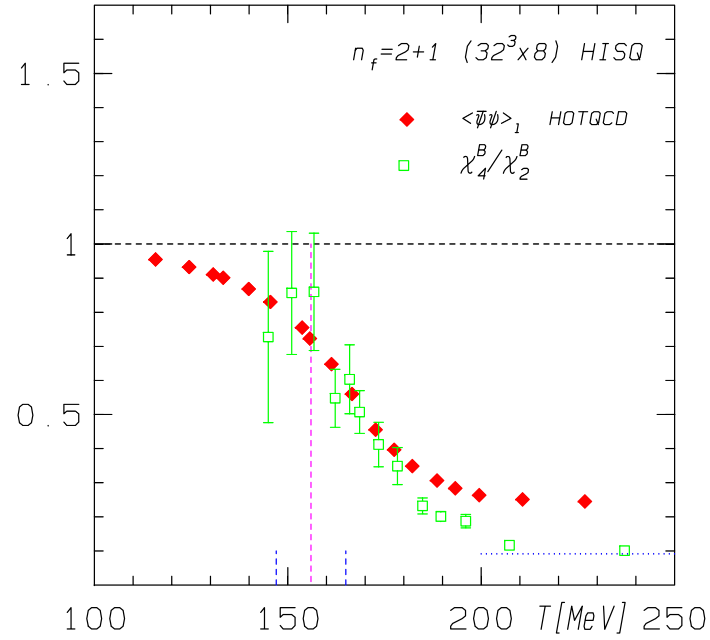
- fluct. of conserved charges with:

$$\mu = B\mu_B + S\mu_S + Q\mu_Q$$

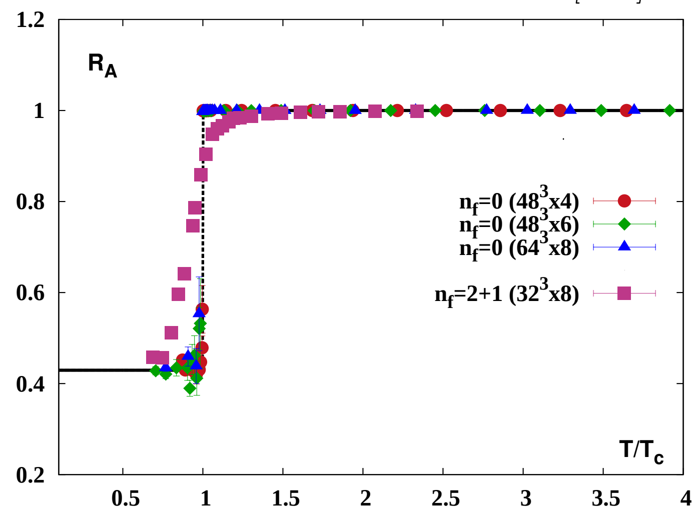
e.g. kurtosis of baryon number fluctuations

$$\kappa = \chi_4^B/\chi_2^B = B^2 \to \begin{cases} 1\,, & T \ll T_c \\ 1/9\,, & T \gg T_c \end{cases} ,$$

$$\chi_n^B = \partial^n(P/T^4)/\partial(\mu_B/T)^n$$




- Polyakov-loop sus. and their ratios:
  a clear remnant of $Z(3)$
  in $R_A = \chi_{\rm absolute}/\chi_{\rm real}$! [Lo et al. ('13)]
  cf. pure YM: exact $Z(3)$, deconf p.t.



## Confinement vs. dynamical chiral symmetry breaking (D$\chi$SB)

**to which extent does D$\chi$SB contain information on confinement?**

- Banks-Casher relation: low-lying Dirac eigenmodes generate $\langle \bar{q}q \rangle$.

$$\langle \bar{q}q \rangle = - \lim_{m_q \to \infty} \lim_{V \to \infty} \pi \rho(0) \,, \quad \rho(\lambda) = \frac{1}{V} \sum_n \langle \delta(\lambda - \lambda_n) \rangle$$

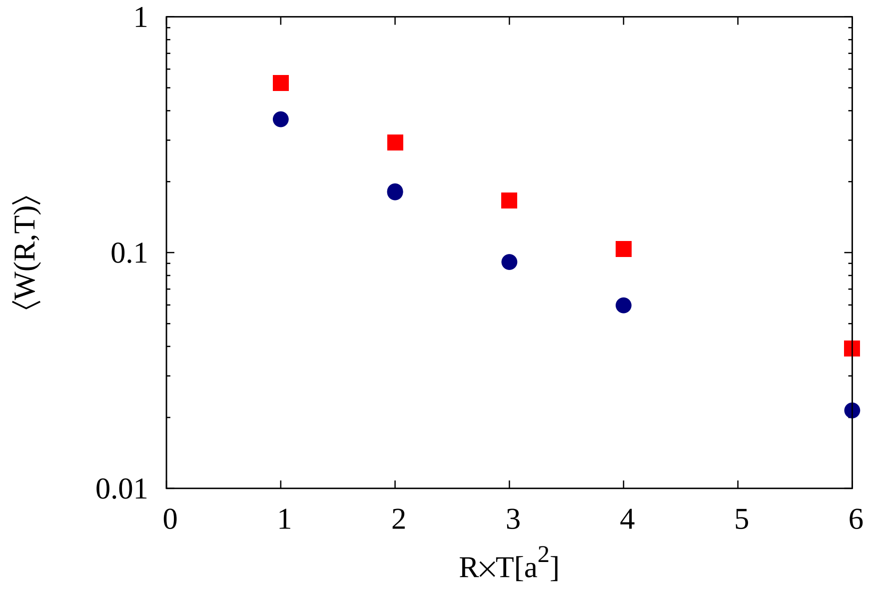
**removal of low-lying Dirac modes $\Rightarrow$ NO D$\chi$SB**
**Q. does confinement disappear simultaneously?**

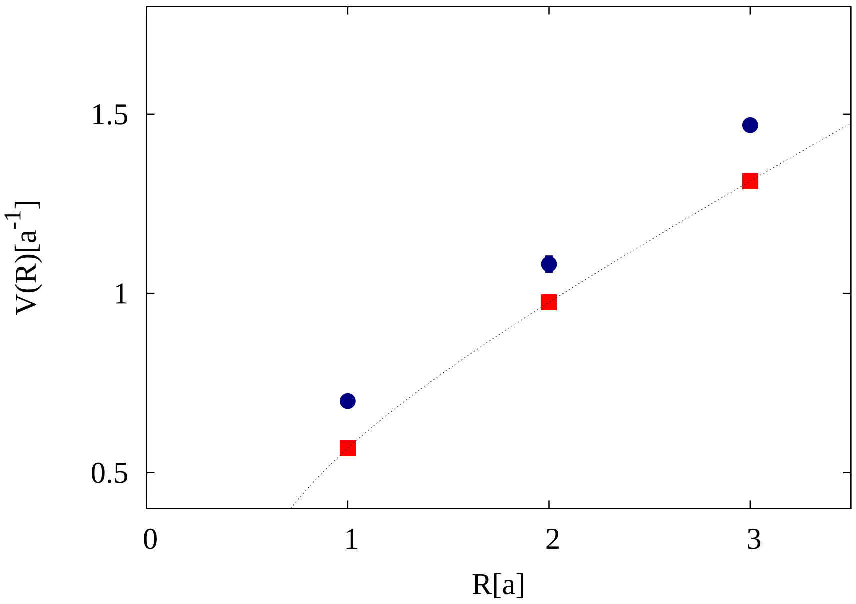
- linking Polyakov loop to spectral function of *lattice* Dirac operator
  [Gattringer ('06); Bruckmann, Gattringer, Hagen ('07); Synatschke, Wipf, Langfeld ('08)]
  manifestly gauge invariant formalism
  [Gongyo, Iritani, Suganuma ('12); Doi, Iritani, Suganuma ('13,14)]

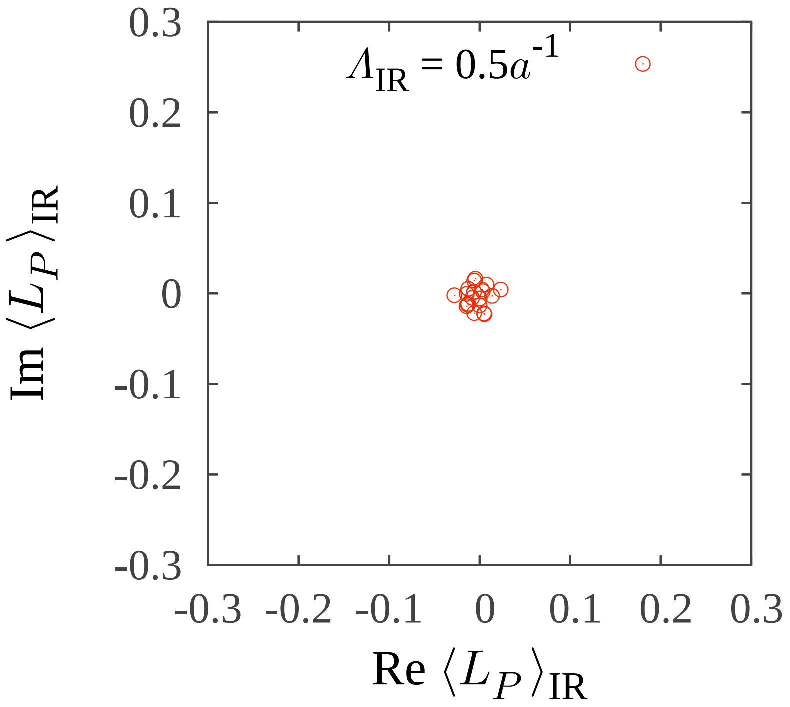
**NO particular Dirac-modes that crucially affect confinement!**

$\Downarrow$

**disappearance of D$\chi$SB DOES NOT mean deconfinement?**




■ full vs. ● w/o low-lying modes

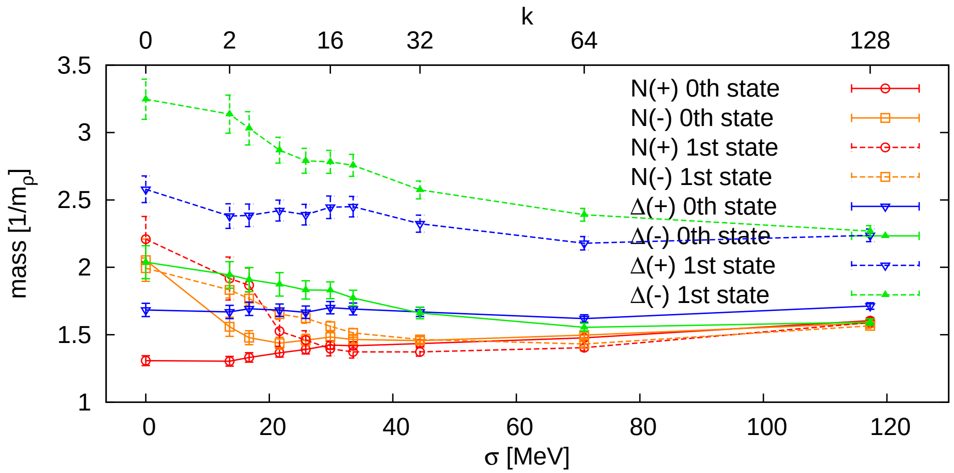
- Wilson loop, quark potential: $\langle W \rangle \propto e^{-\sigma RT}$: slope parameter = string tension ⇒ unchanged!

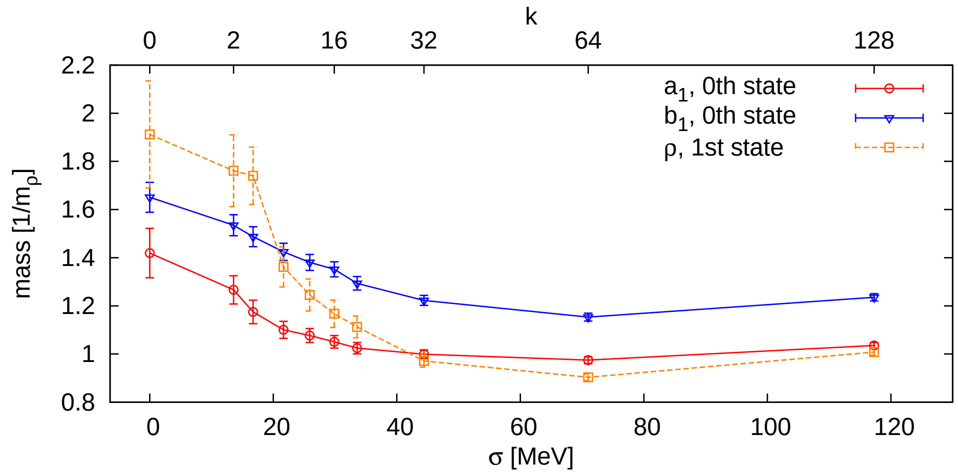
  [Gongyo, Iritani, Suganuma ('12)]

- Polyakov loop susceptibilities unmodified

  [Doi, Redlich, CS, Suganuma ('15)]

# Fate of hadron masses toward chiral symmetry restoration

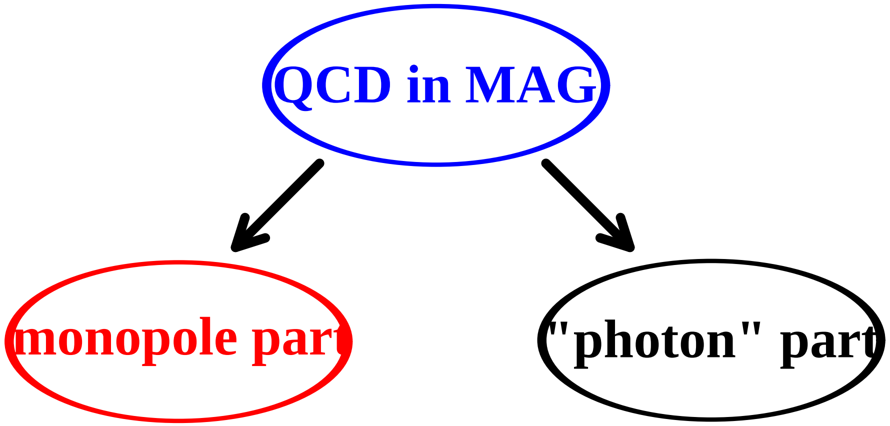


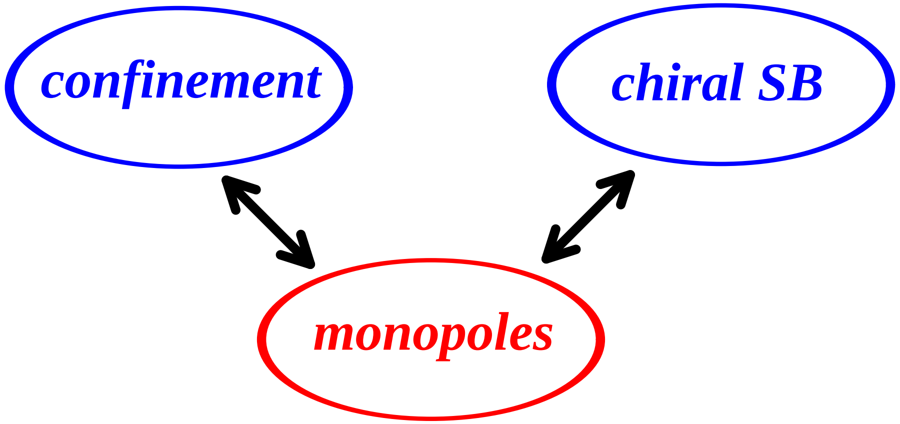




- hadron masses vs. truncation level on lattice [Glozman, Lang, Schrock ('12)]
  - removal of lowest Dirac-eigenmodes $\Leftrightarrow$ NO $\langle \bar{q}q \rangle$
    $\Rightarrow$ parity partners degenerate *and* stay quite massive!
  - *no* universal scaling ($m_{\rm meson} \sim 2m$, $m_{\rm baryon} \sim 3m$) found
    $\Rightarrow$ the system remains confined!
- LQCD at $\mu = 0$: $m_{N^-} \stackrel{T \to T_{\rm ch}}{\longrightarrow} m_{N^+} \sim m_{N^+}^{(\rm vac)}$ [Aarts et al., ('15) ]

at high density: $T_{\rm ch} \sim T_{\rm dec}$ or $T_{\rm ch} \neq T_{\rm dec}$?

## Topology and confinement ['t Hooft ('75); Mandelstam ('76)]




QCD with Maximal Abelian Gauge (MAG) ['t Hooft ('81); Ezawa, Iwasaki ('82)]
confinement and D$\chi$SB induced by monopole condensation

**How monopoles and Dirac zero modes are related?**

- instantons responsible for D$\chi$SB, but not for conf. [Diakonov, Petrov ('86)]
  zero modes *localized*, difficult to yield confinement?
- SU(2) instantons and dyons: simultaneous transition [Larsen, Shuryak ('14,'15)]
  conf. and D$\chi$SB induced by dyon-antidyon interactions
- is it so exotic if the two transitions happen separately?
  NO! cf. adjoint QCD, $T_{\rm ch} \sim 8T_{\rm dec}$

# II. Nature of Hadronic Matter at High Density

## How to suppress unphysical d.o.f. at $T = 0$? [Benic, Mishustin, CS ('15)]

- IR cutoff $b$: NJL, SD eq., AdS/QCD [Ebert, Feldmann, Reinhardt ('96); $\cdots$]
  $1/b \sim$ typical size of a hadron $\Rightarrow$ modified FD distribution functions

$$n_q = \theta(\vec{p}^2 - b^2) f_q\,, \quad n_{N_\pm} = \theta(\alpha^2 b^2 - \vec{p}^2) f_{N_\pm}\,, \quad f_X = 1/1 + e^{\beta(E_X \mp \mu_X)}\,.$$

- asymptotic behavior at high density:
  *non-int. quark gas, restored chiral symmetry*
  $\Rightarrow$ density dep. IR cutoff, upper limit $\alpha_{\max} = 2^{-1/3} \sim 0.8$

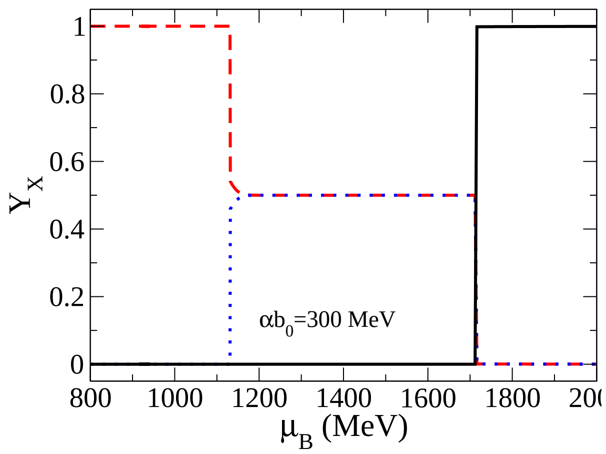
- model setup: $\Omega = \sum_{X=N_\pm,q} \Omega_X + V_\sigma + V_\omega + V_\chi + V_b$

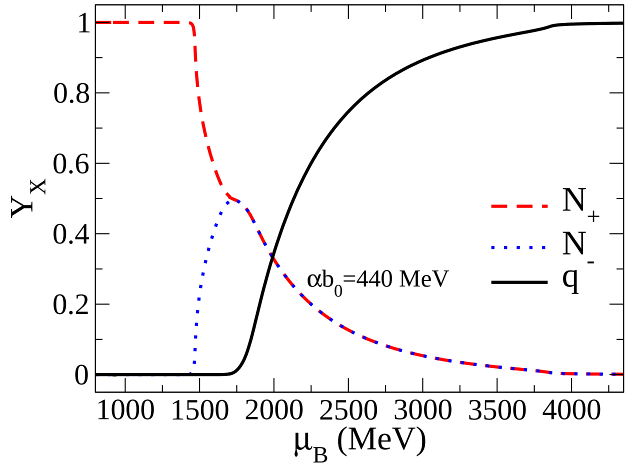
$$V_b = -\frac{\kappa_b^2}{2} b^2 + \frac{\lambda_b}{4} b^4\,.$$

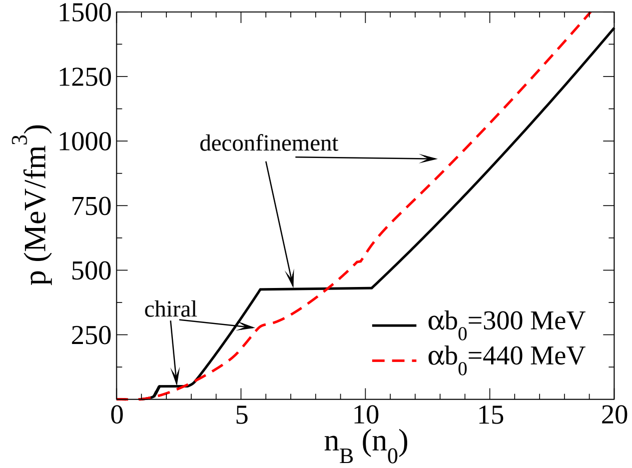
  parameter fixing: $(\kappa_b, \lambda_b) \leftarrow (\epsilon_{\text{vac}}, T_{\text{ch}})$ from lattice QCD

- deconfinement criteria

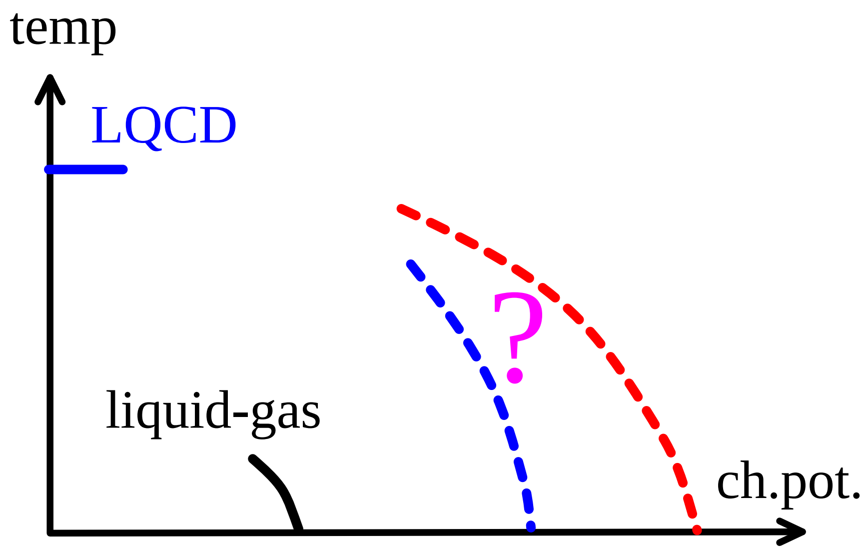
$$Y_{N_+} + Y_{N_-} = Y_q\,; \quad Y_{N_\pm} = \frac{\rho_{N_\pm}}{\rho_B}\,, \quad Y_q = \frac{1}{3}\frac{\rho_q}{\rho_B}\,.$$



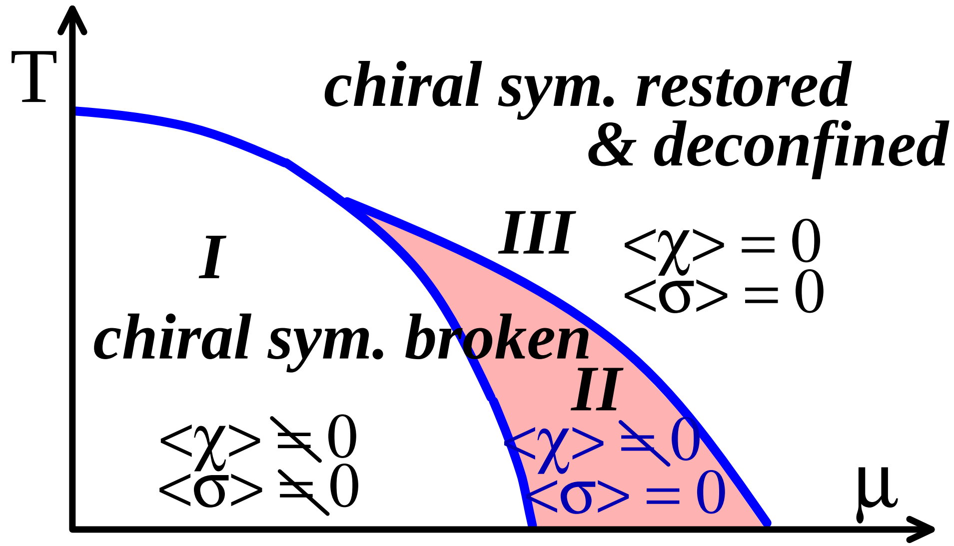



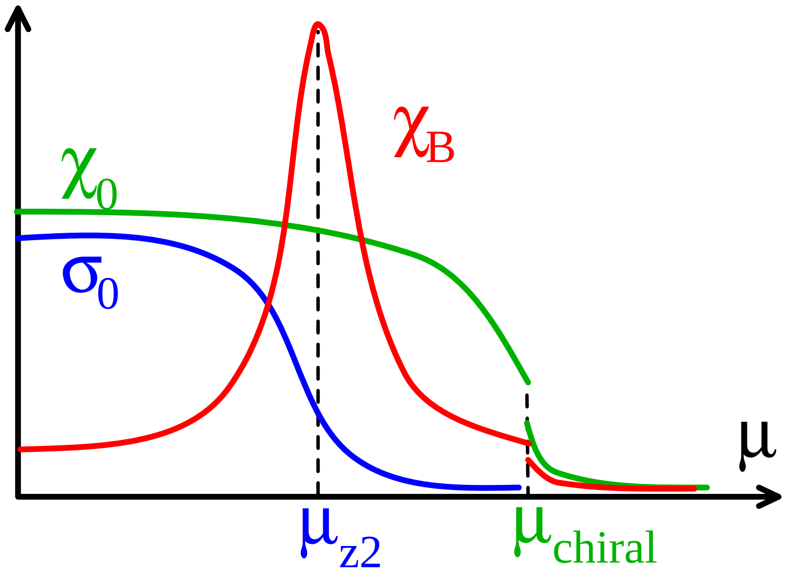

- quark-meson-nucleon hybrid model at $T = 0$ and large $\mu_B$
  $\Rightarrow$ $\rho_{\rm ch}$ separated from $\rho_{\rm dec}$ $\alpha$-dep.: $\Delta\rho_{\rm cr}/\rho_0 \sim$ 4-12
- decreasing $\Delta\rho_{\rm cr}(T)$
  due to bosonic thermal fluctuations
- holographic QCD approach:
  - large $N_c$ limit
  - less parameters



# Role of a tetraquark: more exotic phases

[Harada-CS-Takemoto (09)]




- symmetry breaking: $SU(N_f)_L \times SU(N_f)_R \to SU(N_f)_V \times Z_{N_f} \to SU(N_f)_V$
  2-quark state $\sigma \sim \bar{q}q$ and 4-quark state $\chi \sim (\bar{q}q)^2 + \bar{q}\bar{q}\text{-}qq$
- 3 phases: baryon number flutc. $\chi_B$ becomes max. when $\langle\sigma\rangle \to 0$ , whereas ch.sym. remains broken by $\langle\chi\rangle \neq 0$
- multiple critical points, the same universality class as the "ordinary" CP
  $\Leftrightarrow$ different universality from anomaly induced CP [Hatsuda et al. (06-07)]
  $\because U(1)_B$ is broken in CFL phase.

- hadron mass spectra

| phase I: $\sigma_0 \neq 0\,, \chi_0 \neq 0$ | phase II: $\sigma_0 = 0\,, \chi_0 \neq 0$ |
|---|---|
| $SU(2)_V$ | $SU(2)_V \times (Z_2)_A$ |
| $m_S \neq 0\,, m_P = 0$<br>$m_V \neq m_A$ | $m_S \neq m_P \neq 0\,, m_{P'} = 0$<br>$m_V \neq m_A$ |
| $m_{N^+} \neq m_{N^-}$ | $m_{N^+} = m_{N^-} \neq 0$ |

| phase I: $\sigma_0 \neq 0\,, \chi_0 \neq 0$ | phase II: $\sigma_0 = 0\,, \chi_0 \neq 0$ |
|---|---|
| $SU(3)_V$ | $SU(3)_V \times (Z_3)_A$ |
| $m_S \neq 0\,, m_P = 0$<br>$m_V \neq m_A$ | $m_S = m_P \neq 0\,, m_{P'} = 0$<br>$m_V \neq m_A$ |
| $m_{N^+} \neq m_{N^-}$ | $m_{N^+} = m_{N^-} \neq 0$ |

- $N_f = 2 + 1$:
  (non-strange) $m_S \neq m_P \quad m_{N^+} \simeq m_{N^-}$
  (strange) $m_S \simeq m_P \quad m_{N^+} \simeq m_{N^-}$
  $\Rightarrow$ early onset of $\chi$SR for the strange mesons and non-/strange baryons!

## Toward QCD p.t.: more and more hadrons activated!

- role of higher-lying hadrons
  - V/A spectra: importance of heavier states towards $T_{\rm ch}$ [Hohler, Rapp ('14)]
  - role of higher KK modes in open moose model [Son, Stephanov ('03)]
    correct high-energy behavior for current correlator
  - nuclear matter saturation in Walecka model
    density-dep. parameters: many-body effects integrated out

## How to handle them?

- holographic QCD models: $1/N_c$ corrections?
- microscopic approach: lattice QCD, DS/FRG
  lattice ch. condensates at $\mu = 0$: $T$-dep. int. of charmed mesons
  $\Rightarrow D_s$ more sensitive to $O(4)$ than kaons [CS ('14); CS, Redlich ('14)]
- 4d effective theories: higher resonances, careful treatment of broad resonances [Broniowski et al. ('15); Friman et al. ('15)]

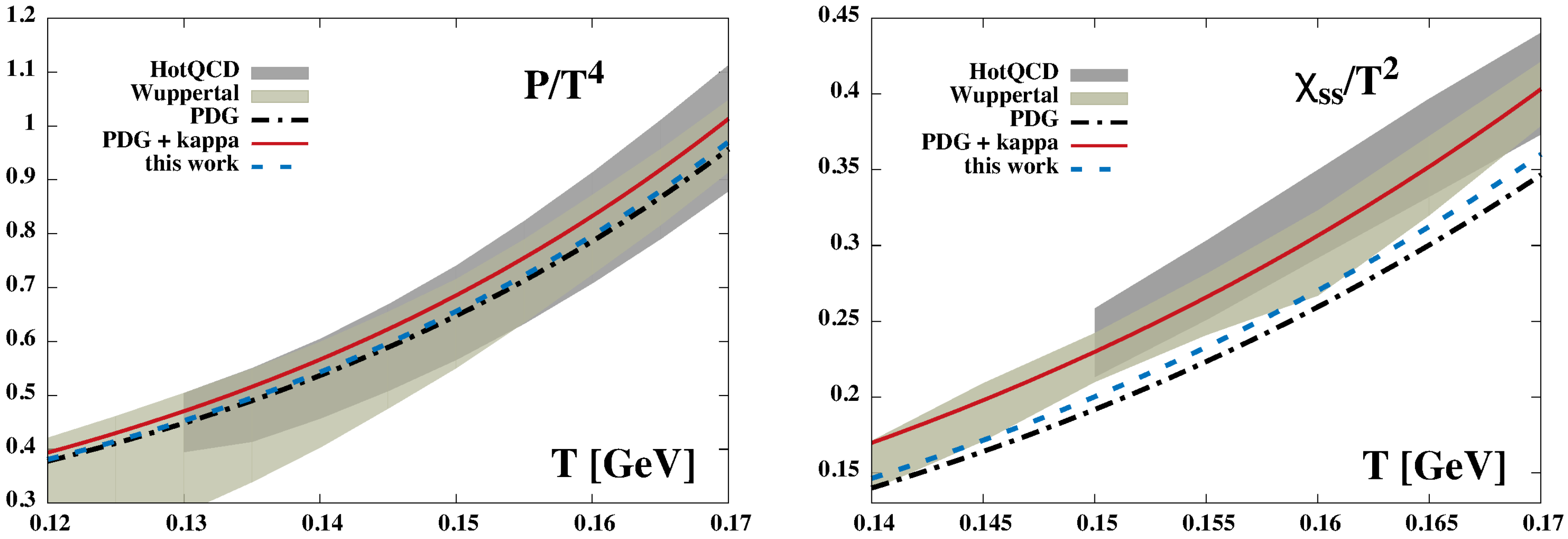
## Low-lying scalar states: $\sigma$ and $\kappa$ mesons ... broad!

- how to deal with broad resonances? [Broniowski et al. ('15); Friman et al. ('15)]
  - S-wave $K\pi$ scattering $\sim \kappa(800), K^*(1430)$ resonances
  - break down of Breit-Wigner spectral function
  - S matrix approach: empirical $K\pi$ phase shift as $input$

    $\Rightarrow$ consequences of the broad width for thermodynamics!

- fluctuations in LQCD: missing strange state? [Bazavov et al. ('14)]

  putting $\kappa$ into HRG compatible with LGT!?... CAUTION! [Pok et at. ('15)]



## Summary

- **Dynamical chiral symmetry breaking vs. confinement**
  - various fluctuations: remnant of underlying symmetry
  - why $T_{\rm ch} \simeq T_{\rm dec}$ at $\mu = 0$?: PNJL/QM models, instanton-dyon picture
  - not clear at high density: Dirac eigenmode expansion, model w/ IR cutoff, cf. adjoint QCD: $T_{\rm dec} \neq T_{\rm ch}$
- **Nature of hadrons near QCD phase transition**
  - exotic phases at high density, new CPs, $\chi_B$
  - role of higher-lying hadrons
  - careful treatment of broad resonances required
  - the problem of missing strange states unresolved
- **Our goal: from hadrons to quarks and gluons**
  *multifaceted studies of gauge dynamics* guided by symmetry, topology, ideal limits AND available data from LQCD simulations and HIC