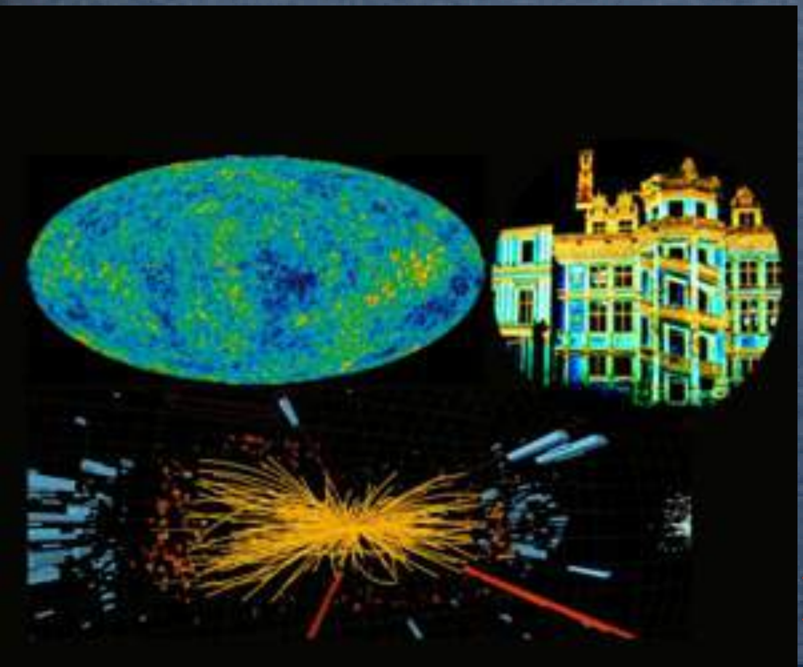
UNIVERSITY OF
Southampton
School of Physics
and Astronomy

elusives
neutrinos, dark matter & dark energy physics

Theoretical Models for the Neutrino Mass and Mixing Pattern



Steve King
2nd June 2016

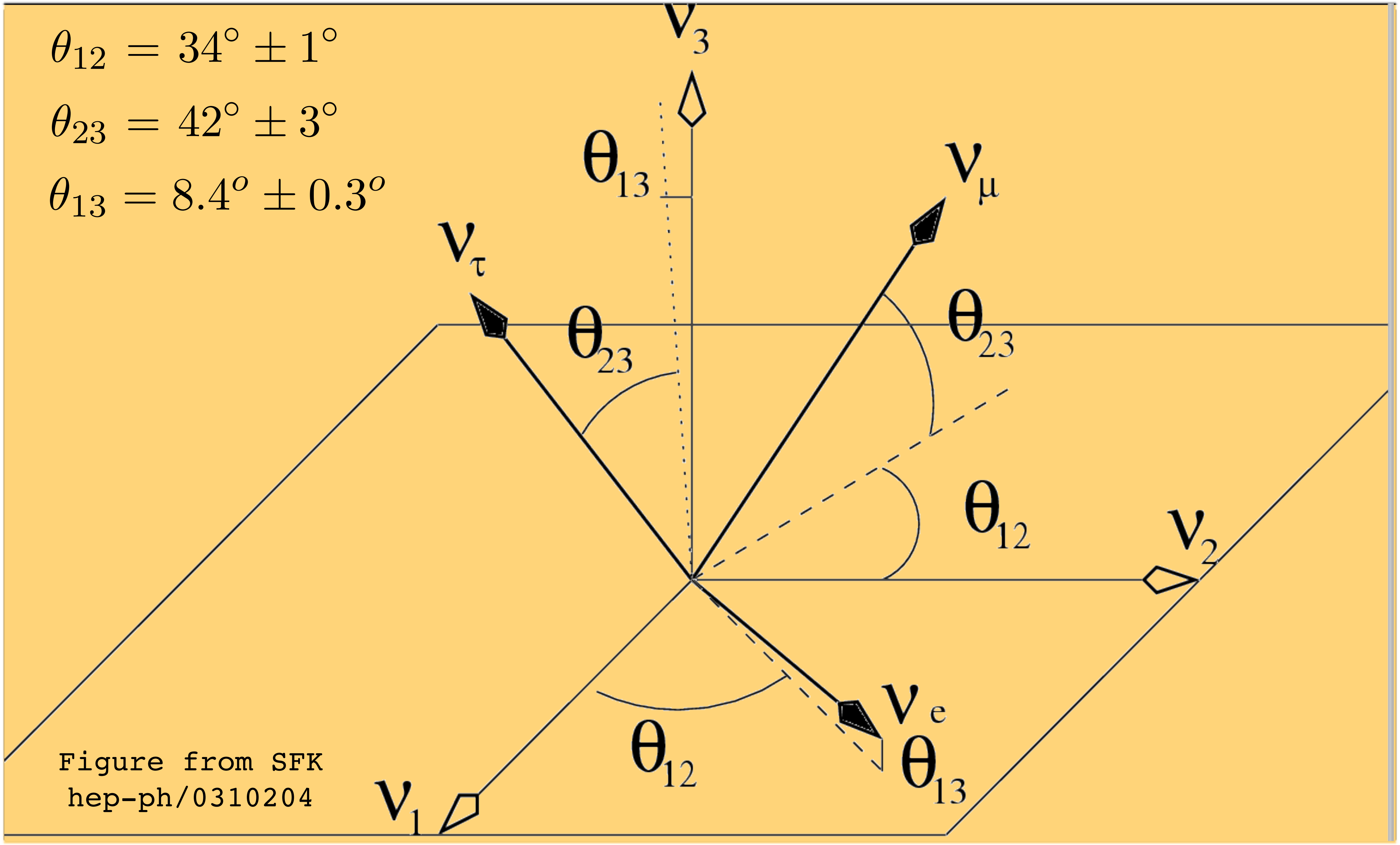
28th Rencontres de Blois
Particle Physics and Cosmology

Neutrino Mass & Mixing

Three things we have learned from the neutrino experiments

1. Neutrinos have tiny masses << m_e
2. Neutrinos mix a lot (unlike the quarks)
3. First (and so far only) new physics Beyond the Standard Model

Lepton Mixing Angles



$$\theta_{12} = 34^\circ \pm 1^\circ$$

$$\theta_{23} = 42^\circ \pm 3^\circ$$

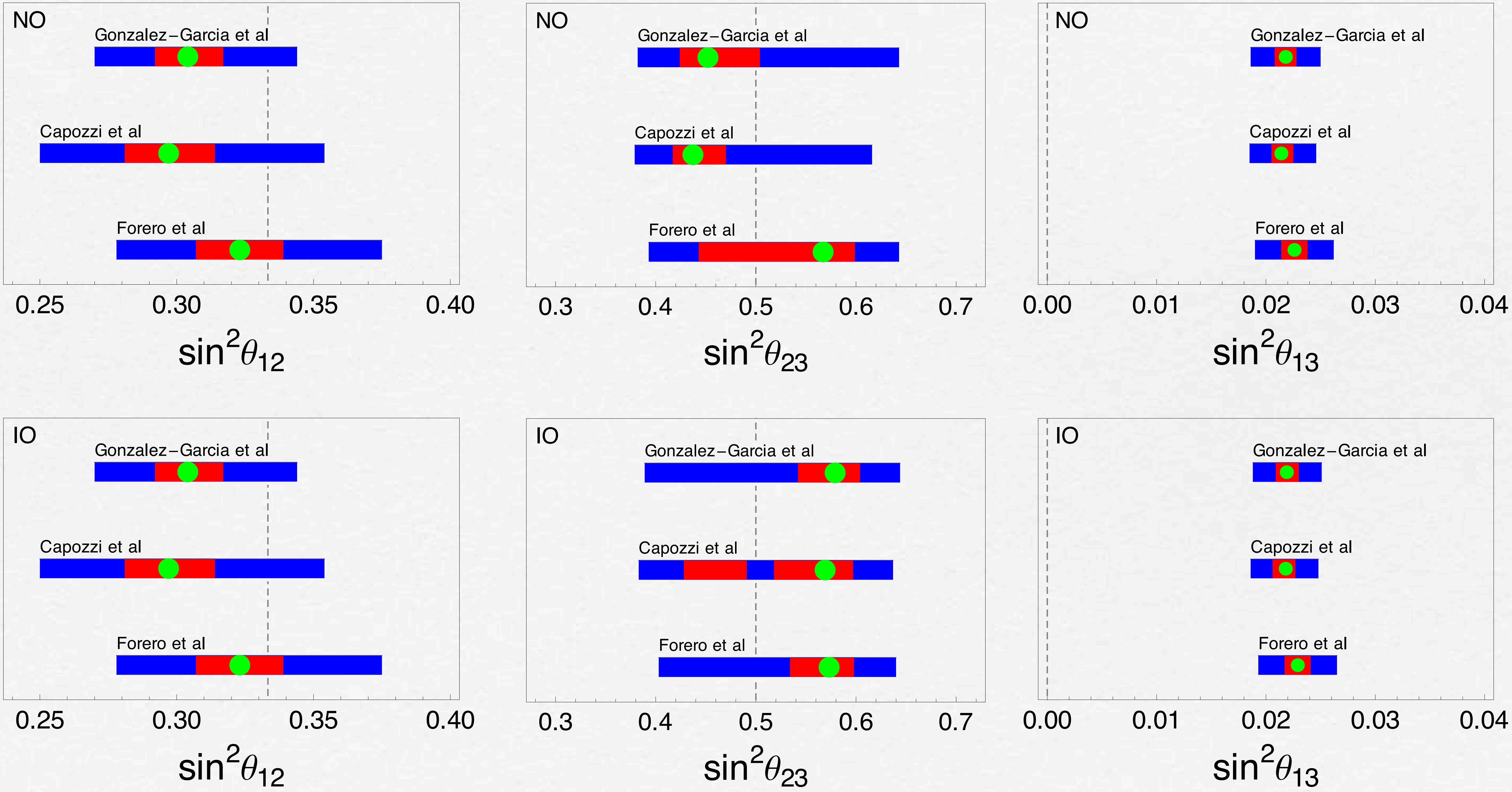
$$\theta_{13} = 8.4^o \pm 0.3^o$$

Figure from SFK
hep-ph/0310204

Gonzalez-Garcia et al = Gonzalez-Garcia, Maltoni, Salvado, Schwetz
Fogli et al = Capozzi, Fogli, Lisi, Marrone, Montanino, Palazzo
Forero et al = Forero, Tortola, Valle

Lepton Mixing

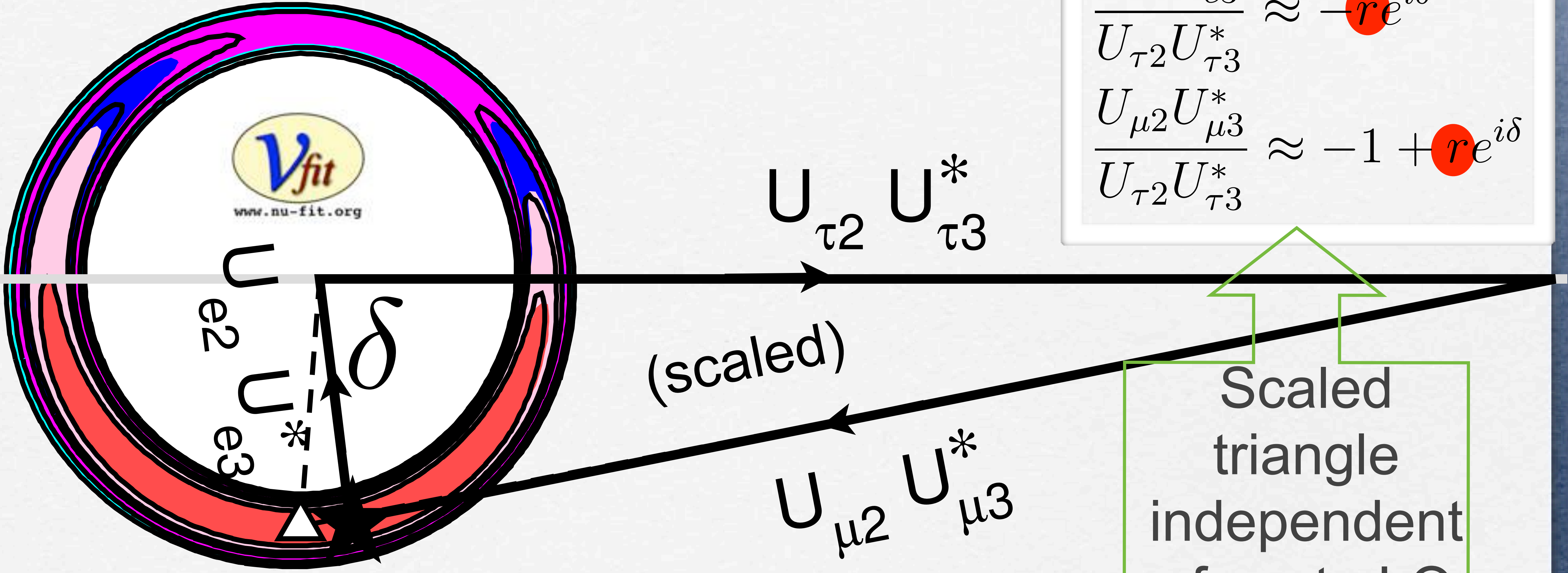
Plots from updated review by SFK and Luhn



CP phase $\delta \sim -90^\circ ?$

$$\frac{U_{e2}U^*_{e3}}{U_{\tau2}U^*_{\tau3}} \approx -re^{i\delta}$$

$$\frac{U_{\mu2}U^*_{\mu3}}{U_{\tau2}U^*_{\tau3}} \approx -1+re^{i\delta}$$



Scaled triangle independent of s,a to LO

arXiv:0710.0530

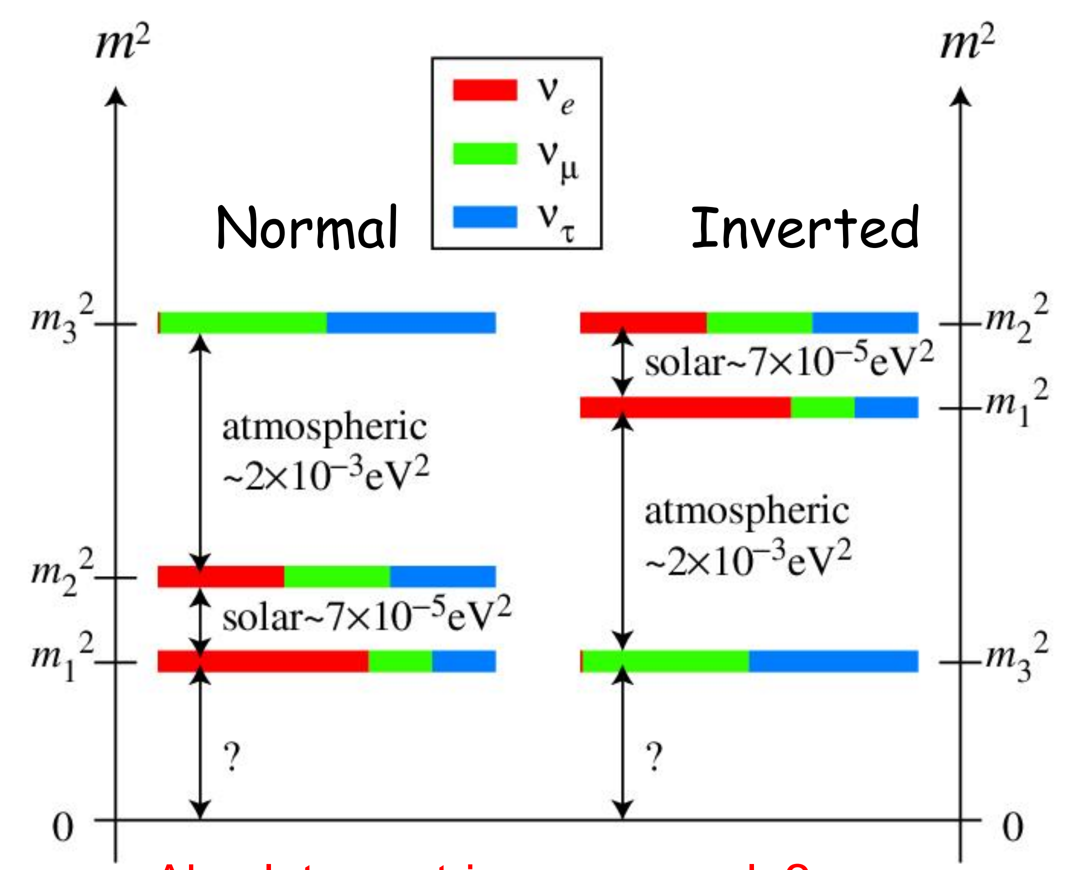
$$\sin\theta_{12} = \frac{1}{\sqrt{3}}(1+s)\,, \qquad \sin\theta_{23} = \frac{1}{\sqrt{2}}(1+a)\,, \qquad \sin\theta_{13} = \frac{r}{\sqrt{2}}$$

s = solar

a = atmospheric

r = reactor

What is the Origin of Small Neutrino Masses?



Absolute neutrino mass scale?

Majorana

$$m^\nu \overline{\nu}_L \nu_L^c$$

Violates L

or

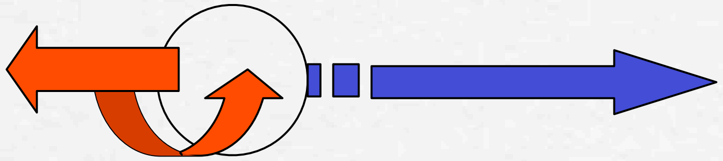
Dirac

$$m^\nu \overline{\nu}_L \nu_R$$

Conserves L

The Electron Mass

e_L^-

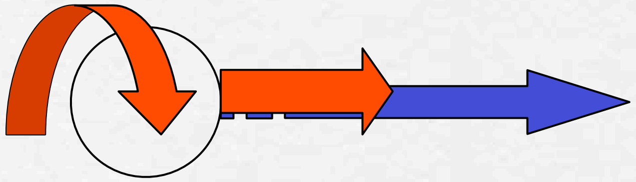


Left-handed electron

Dirac

$$m_e \overline{e_L^-} e_R^-$$

e_R^-



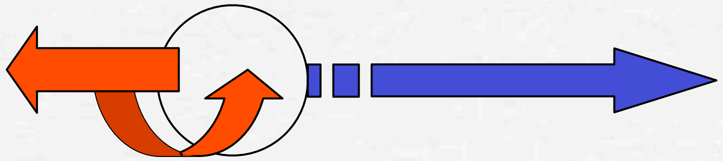
Right-handed electron

Neutrino Mass

Left-handed neutrino

Right-handed neutrino

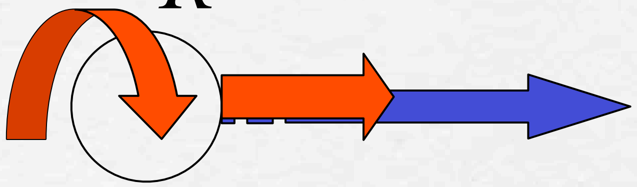
ν_L



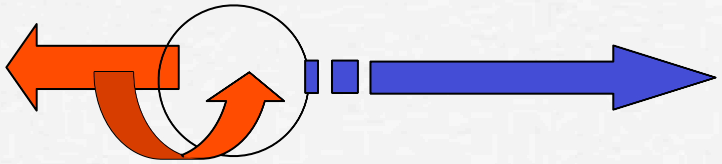
Dirac

$$m^{\nu}_{LR}\overline{\nu}_L\nu_R$$

ν_R



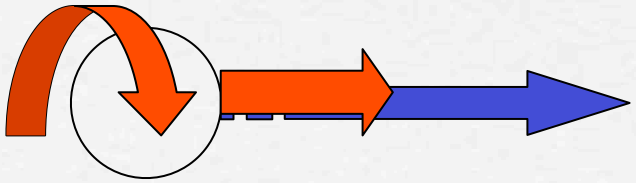
ν_L



Majorana

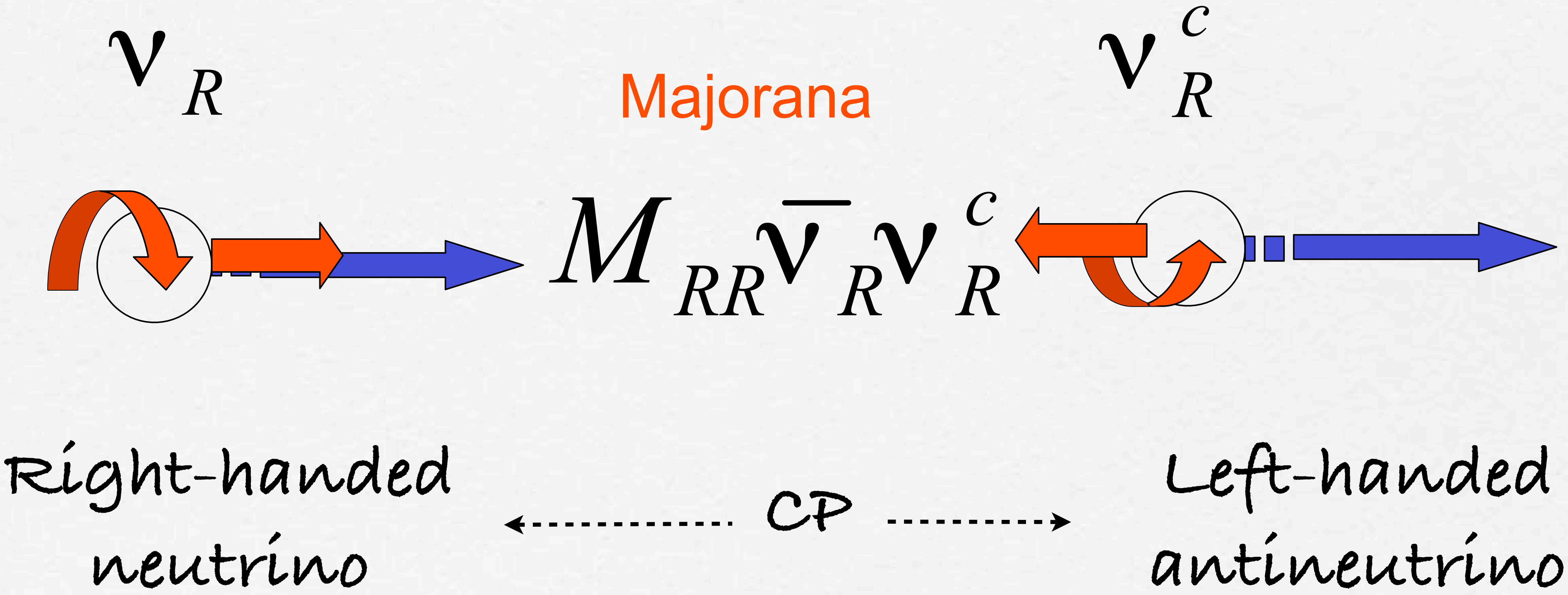
$$m^{\nu}_{LL}\overline{\nu}_L\nu_L^c$$

ν_L^c



Left-handed neutrino ← CP → Right-handed antineutrino

Right-handed neutrino mass



ν_R

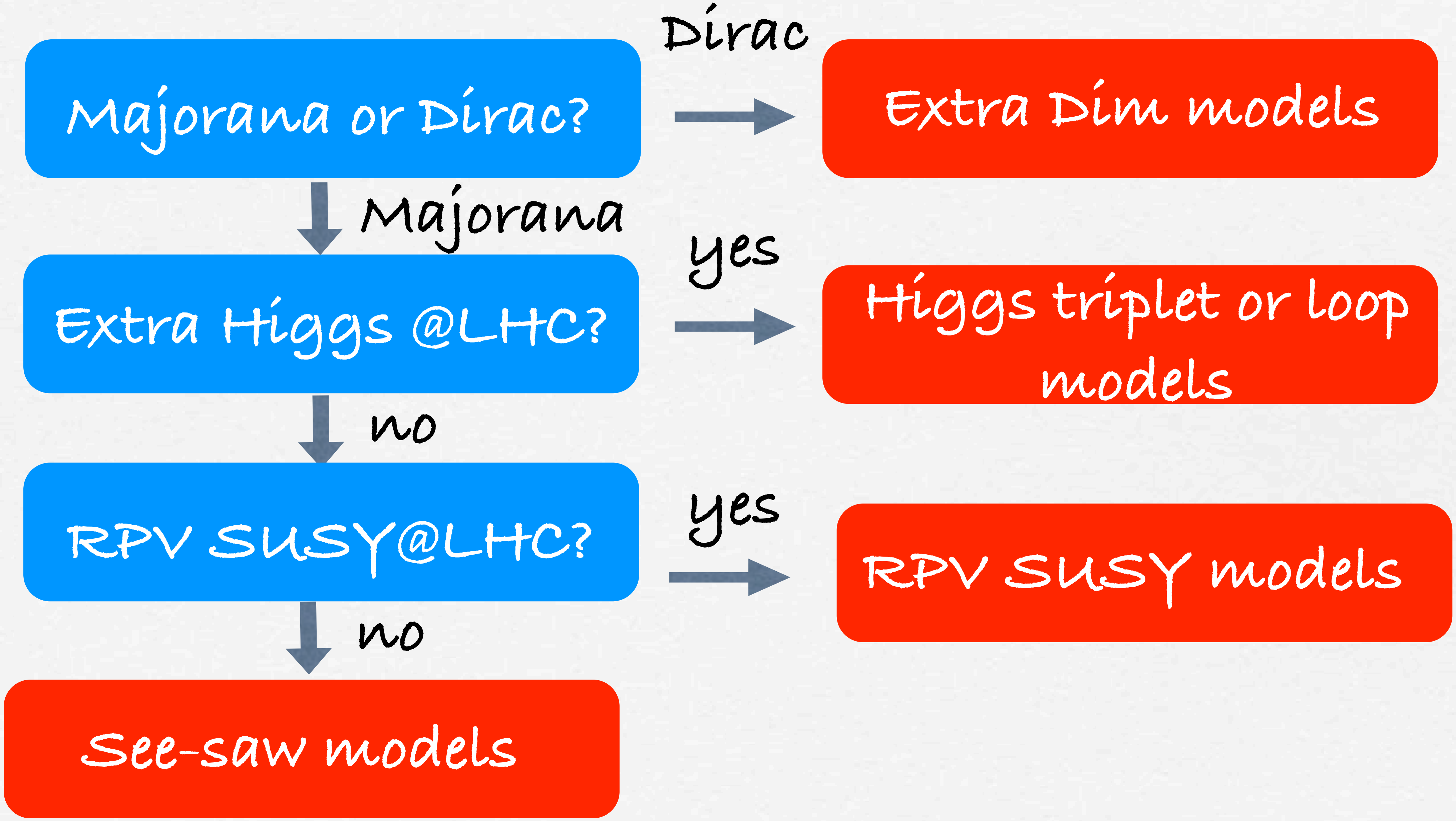
Majorana

ν_R^c

$$M_{RR}\overline{\nu}_R\nu_R^c$$

Right-handed neutrino ←------ CP ------→ Left-handed antineutrino

Roadmap of neutrino mass models



Majorana or Dirac?

Dirac

Extra Dim models

Majorana

Extra Higgs @LHC?

yes

Higgs triplet or loop models

no

RPV SUSY@LHC?

yes

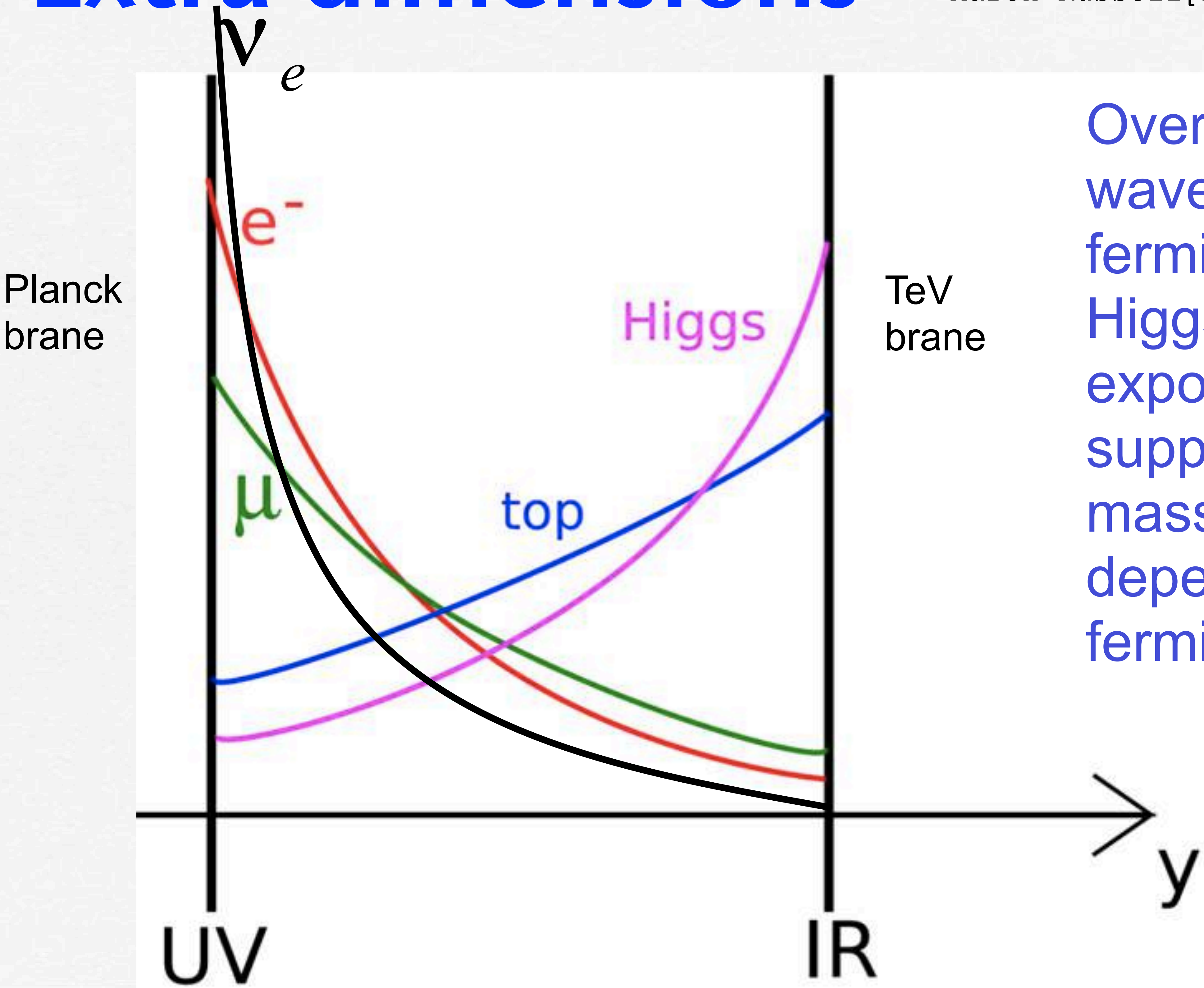
RPV SUSY models

no

See-saw models

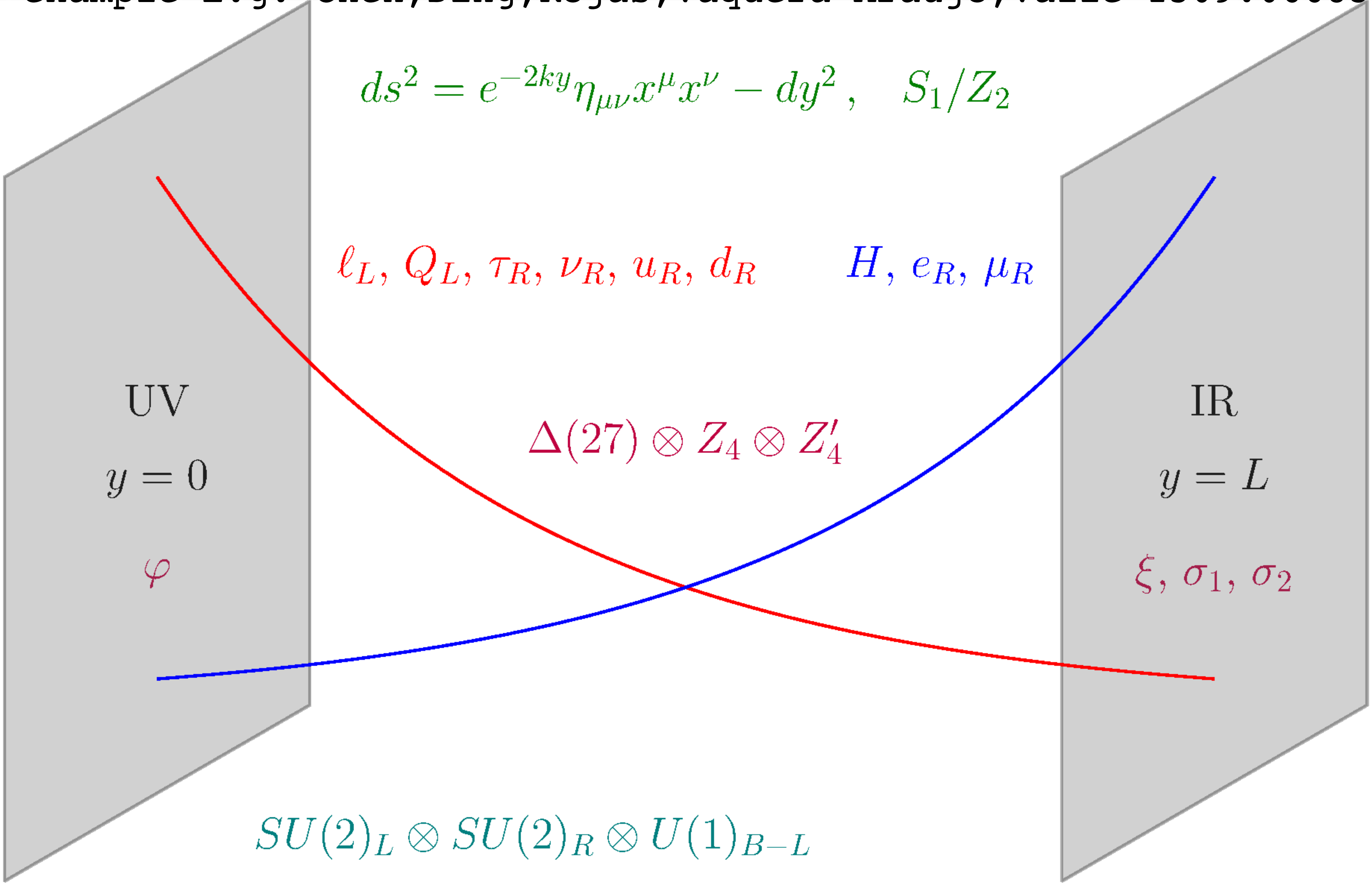
Extra dimensions

Arkani-Hamed,Dimopoulos,Dvali,
March-Russell[hep-ph/9811448]...



Overlap wavefunction of fermions with Higgs gives exponentially suppressed Dirac masses, depending on the fermion profiles

Recent example E.g. Chen,Ding,Rojas,Vaquera-Araujo,Valle 1509.06683



$$ds^2 = e^{-2ky}\eta_{\mu\nu}x^\mu x^\nu - dy^2\,, \quad S_1/Z_2$$

$\ell_L,\, Q_L,\, \tau_R,\, \nu_R,\, u_R,\, d_R$

$H,\, e_R,\, \mu_R$

UV

$y=0$

φ

$\Delta(27)\otimes Z_4\otimes Z_4'$

IR

$y=L$

$\xi,\, \sigma_1,\, \sigma_2$

$SU(2)_L\otimes SU(2)_R\otimes U(1)_{B-L}$

$SU(2)_L\otimes U(1)_Y$

$SU(2)_D\otimes U(1)_{B-L}$

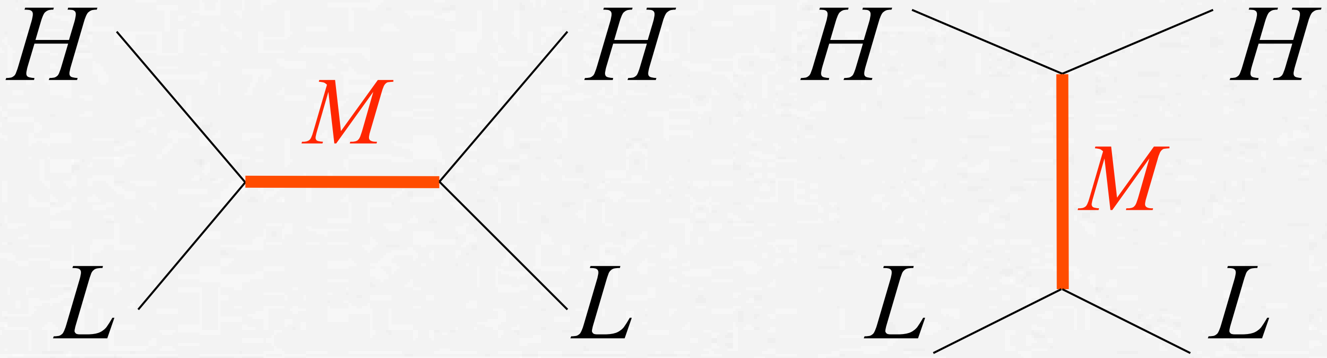
Majorana Neutrino Mass

Renormalisable $\Delta L = 2$ operator $\lambda_\nu LL\Delta$ where Δ is light Higgs triplet with VEV < 8GeV from ρ parameter

Non-renormalisable $\Delta L = 2$ operator $\frac{\lambda_\nu}{M} LLHH = \frac{\lambda_\nu}{M} \left\langle H^0 \right\rangle^2 \overline{\nu}_{eL} \nu_{eL}^c$ Weinberg

This is nice because it gives naturally small Majorana neutrino masses $m_{LL} \sim \langle H^0 \rangle^2/M$ where M is some high energy scale

The high mass scale can be associated with some heavy particle of mass M being exchanged (can be singlet or triplet)



e.g. see-saw with RH neutrinos...introduces a new RH mass spectrum

The RH neutrino Spectrum

M_{GUT}

Figure from
SFK 1510.02091

$\uparrow M_R$

Classic See-Saw (Leptogenesis)
Minkowski...

TeV

Low Scale See-Saw (LHC)

GeV

Asaka-Shaposhnikov
Nu-MSM

BAU (resonant lepto)
Pilaftsis et al

MeV

keV

WDM Merle review...

eV

LSND, Reactor Anomaly,...

meV

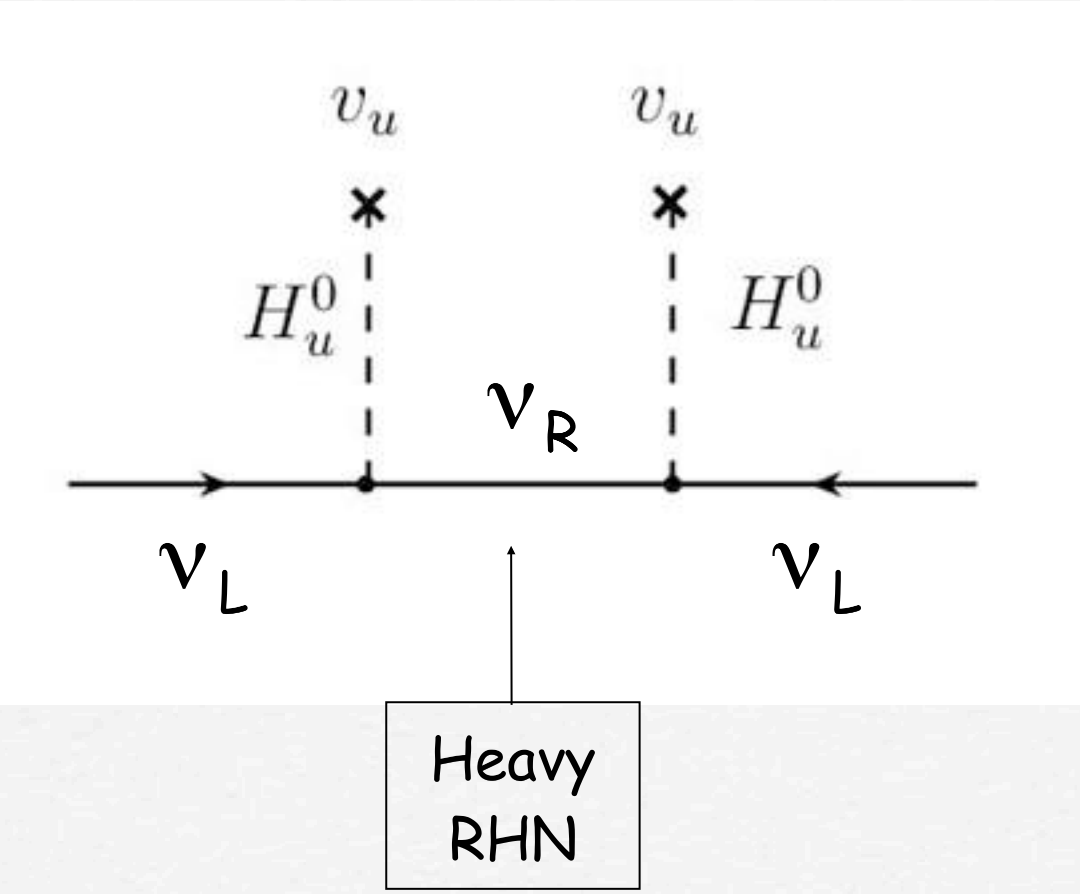
Extra radiation (Planck)

low energy nu
mass matrix

$$m^\nu = -m_D \frac{1}{M_R} m_D^T$$

Type I see-saw mechanism

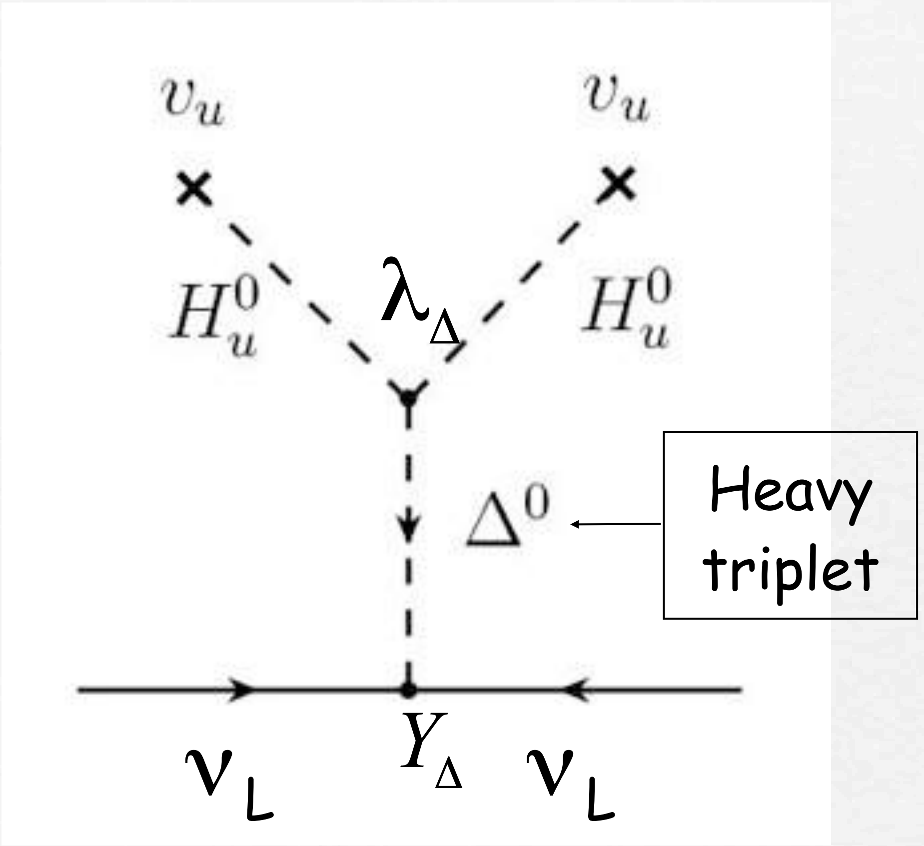
P. Minkowski (1977); Glashow; Yanagida;
Gell-Mann, Ramond, Slansky;
Mohapatra, Senjanovic; Schechter and Valle



$$m_{LL}^{I} \approx -m_{LR} M_{RR}^{-1} m_{LR}^{T}$$

Type II see-saw mechanism (SUSY)

Lazarides, Magg, Mohapatra, Senjanovic,
Shafi, Wetterich, Schechter and Valle



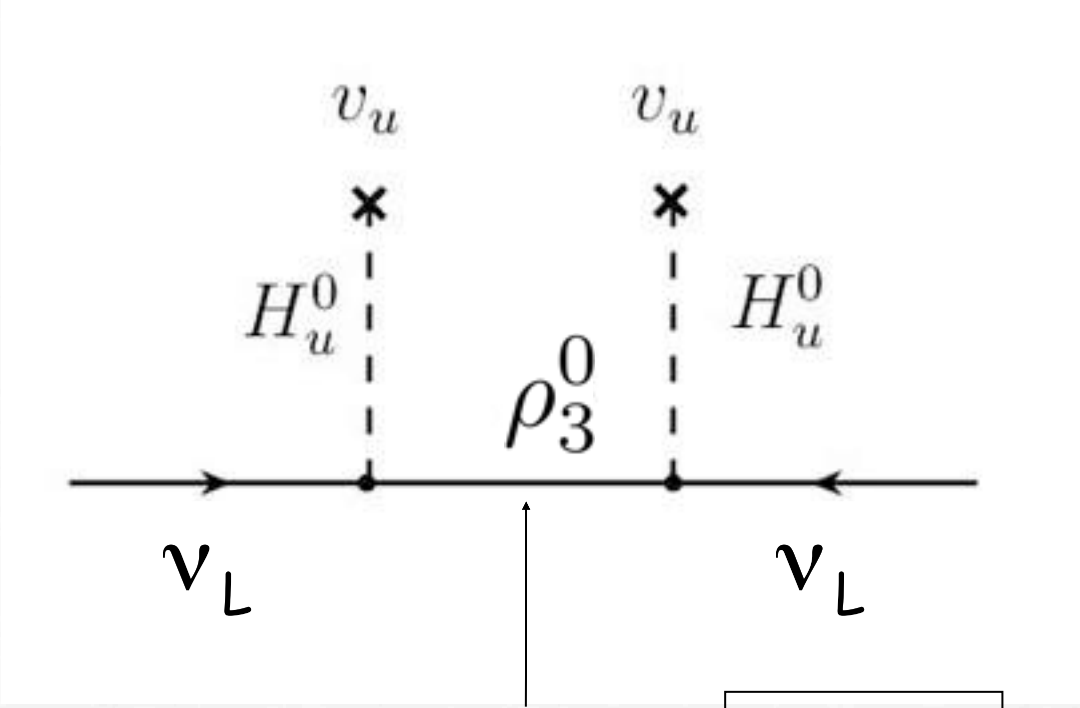
$$m_{LL}^{II} \approx \lambda_\Delta Y_\Delta \frac{v_u^2}{M_\Delta}$$

Type III see-saw mechanism

Foot, Lew, He, Joshi; Ma…

Supersymmetric adjoint SU(5)

Perez et al; Cooper, SFK, Luhn,…



SU(2)$_L$ fermion triplet

$$\boxed{M_\rho \rho\rho}$$

$$\boxed{m_{LL}^{III} \approx -m_{LR} M_\rho^{-1} m_{LR}^T}$$

Type III

See-saw mechanisms with extra singlets S

Inverse see-saw

Wyler, Wolferstein; Mohapatra, Valle

$$\begin{pmatrix} 0 & M_D & 0 \\ M_D^T & 0 & M \\ 0 & M^T & \mu \end{pmatrix} \qquad M_\nu = M_D M^{T^{-1}} \mu M^{-1} M_D^T$$

M ≈ TeV➔LHC

Linear see-saw

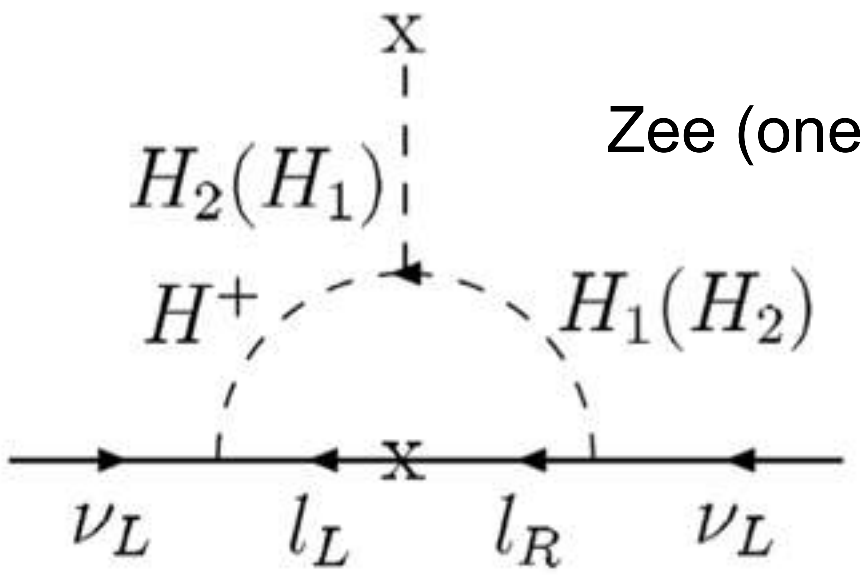
Malinsky, Romao, Valle

$$\begin{pmatrix} 0 & M_D & M_L \\ M_D^T & 0 & M \\ M_L^T & M^T & 0 \end{pmatrix}$$

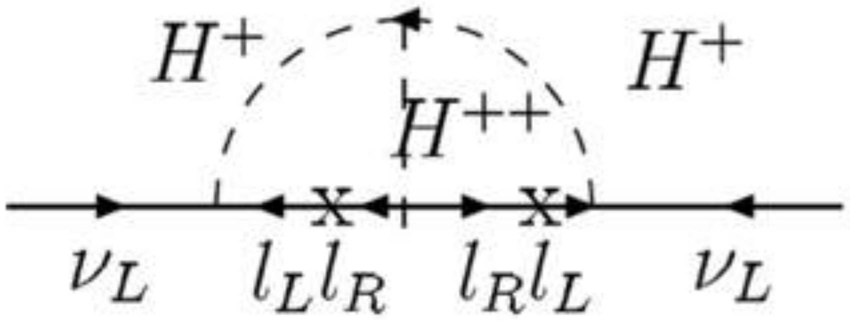
$$M_\nu = M_D (M_L M^{-1})^T + (M_L M^{-1}) M_D{}^T$$

LFV predictions

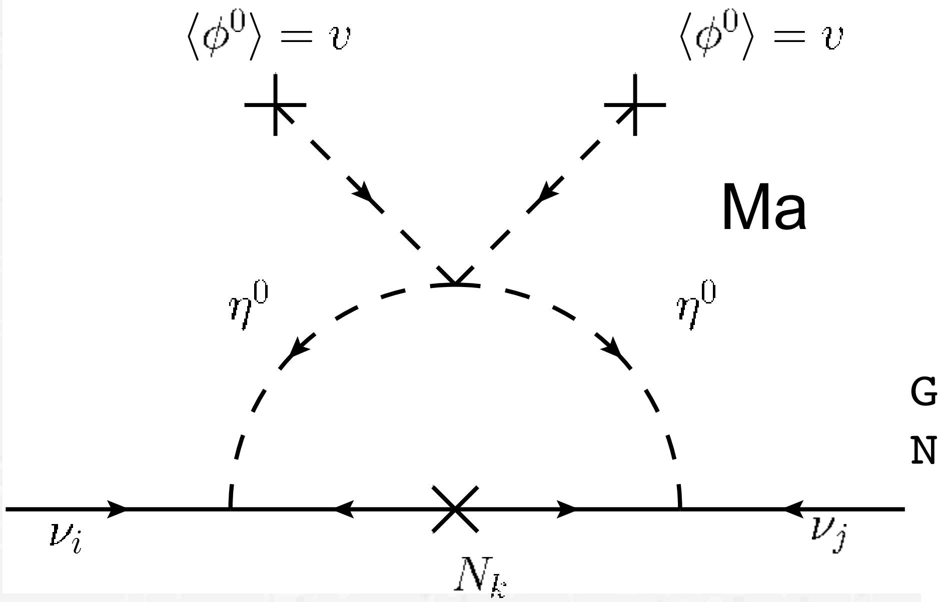
Loop Models of Neutrino Mass



Zee (one loop)

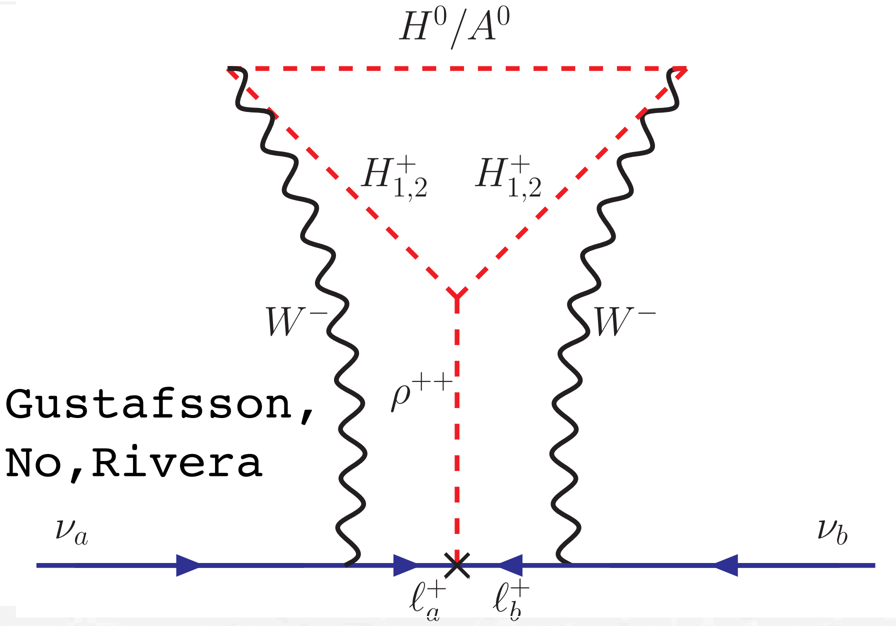


Babu (two loop)



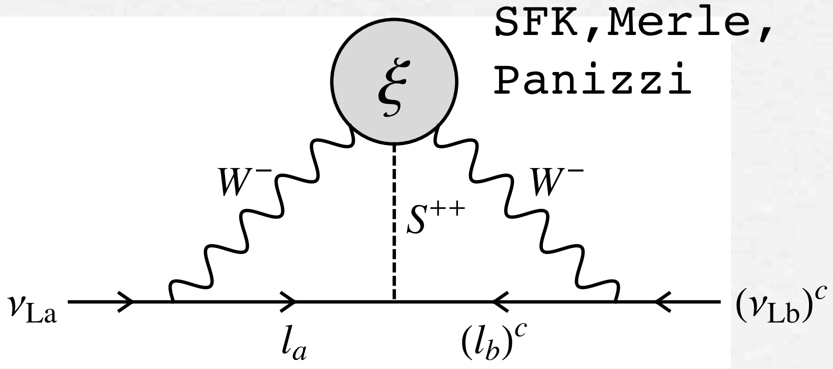
Ma

Scotogenic model



Gustafsson,
No,Rivera

Cocktail model

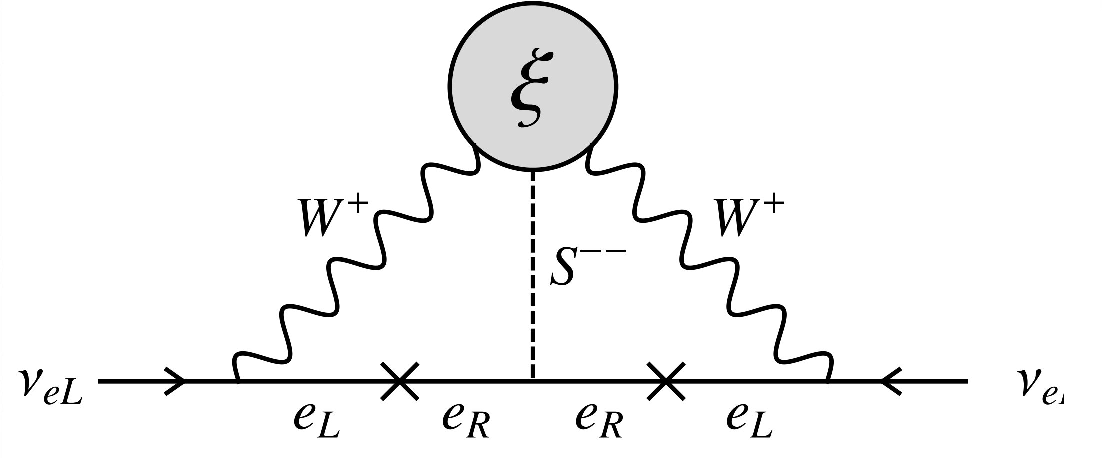


SFK,Merle,
Panizzi

Doubly charged
singlet scalar

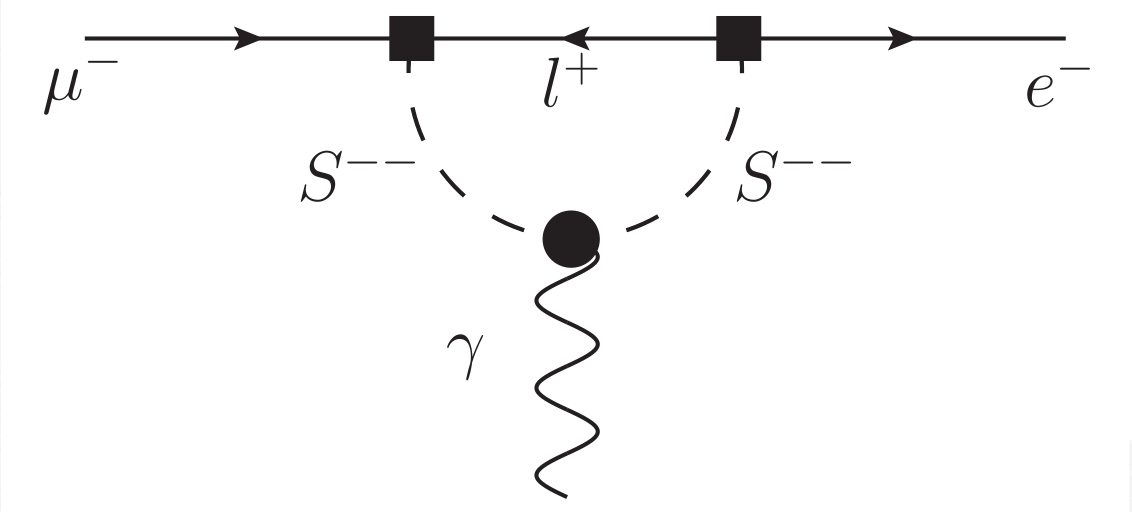
A Loop Model: Complementarity of the Intensity and the Energy Frontiers

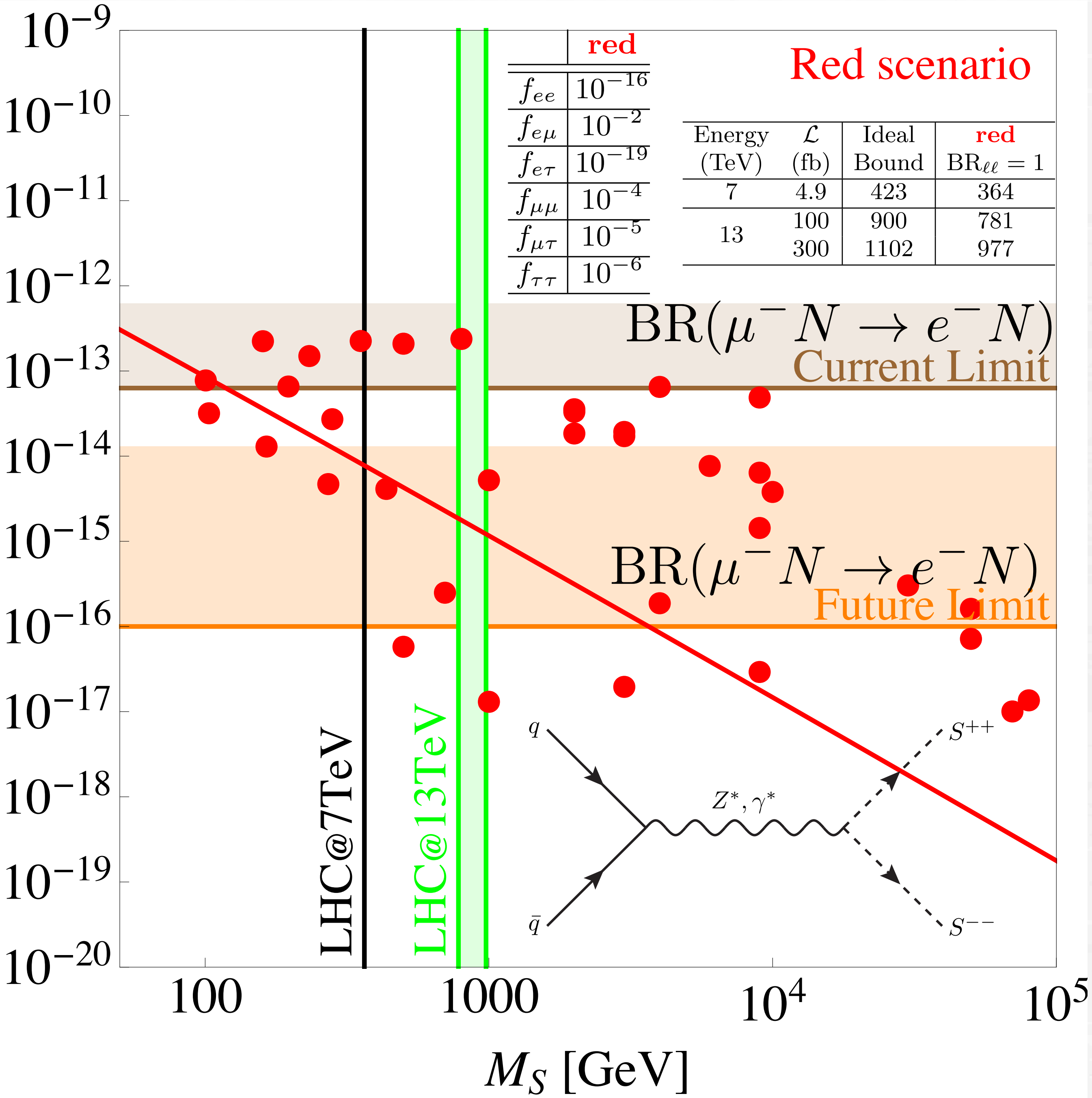
Geib,SFK,Merle,
No,Panizzi,
1512.04391



SFK,Merle,Panizzi,1406.4137

$$(m^{\nu}_{ab}) \sim \begin{pmatrix} m_e^2 f_{ee} & m_e m_\mu f_{e\mu} & m_e m_\tau f_{e\tau} \\ m_e m_\mu f_{e\mu} & m_\mu^2 f_{\mu\mu} & m_\mu m_\tau f_{\mu\tau} \\ m_e m_\tau f_{e\tau} & m_\mu m_\tau f_{\mu\tau} & m_\tau^2 f_{\tau\tau} \end{pmatrix}$$



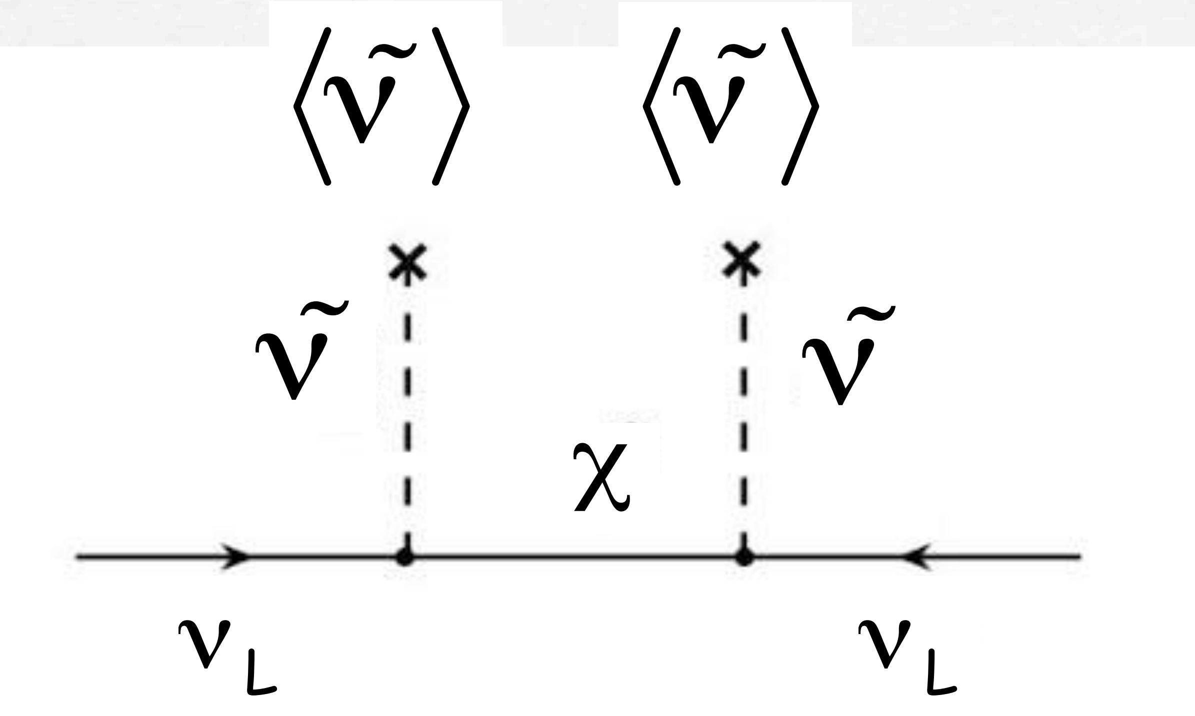


	red
f_{ee}	10^{-16}
$f_{e\mu}$	10^{-2}
$f_{e\tau}$	10^{-19}
$f_{\mu\mu}$	10^{-4}
$f_{\mu\tau}$	10^{-5}
$f_{\tau\tau}$	10^{-6}

Energy (TeV)	$\mathcal{L}$ (fb)	Ideal Bound	**red** $\text{BR}_{\ell\ell} = 1$
7	4.9	423	364
13	100	900	781
	300	1102	977

R-Parity Violating SUSY

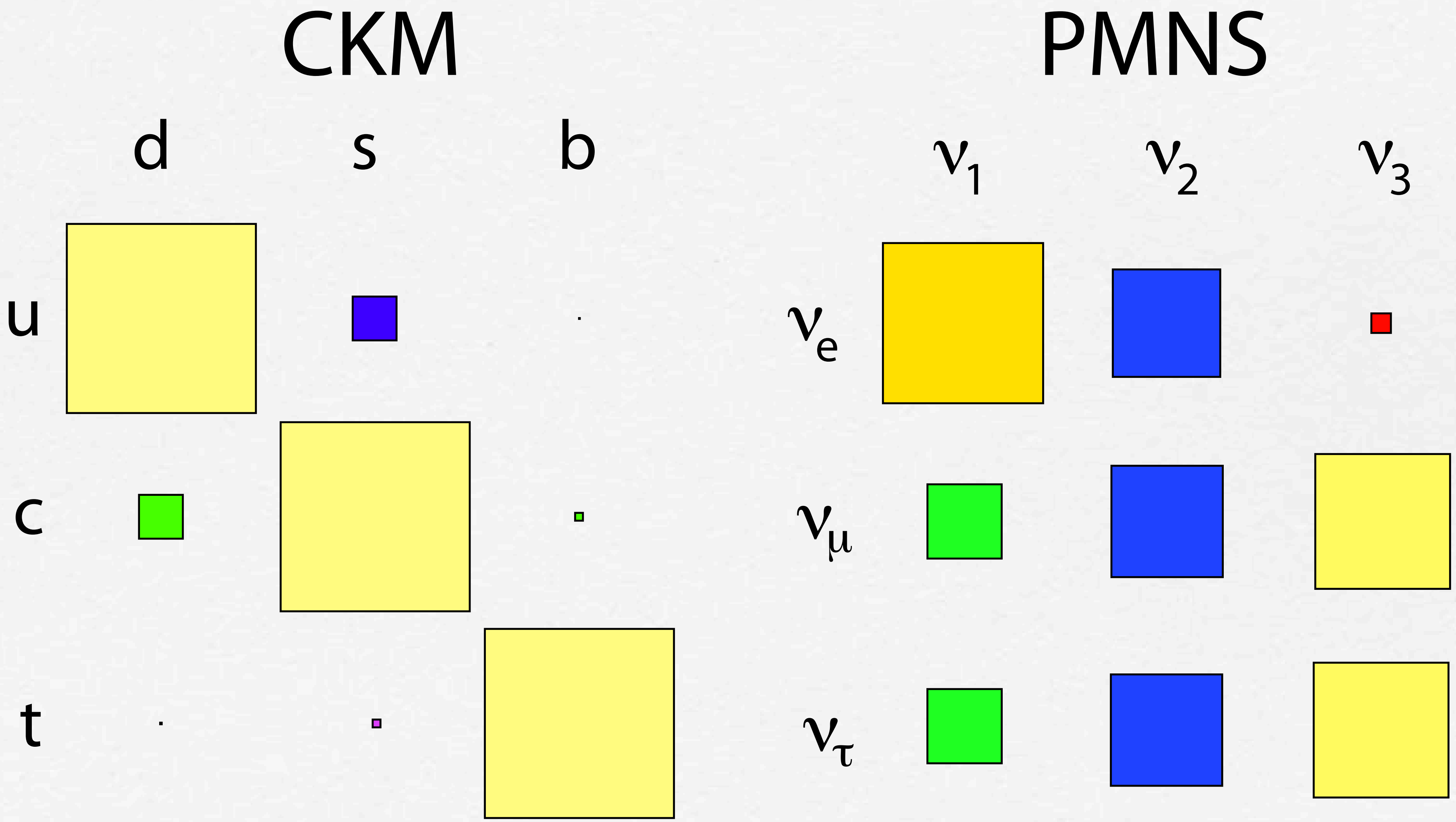
- Majorana masses can be generated via RPV SUSY
- Scalar partners of lepton doublets (slepton doublets) have same quantum numbers as Higgs doublets
- If R-parity is violated then sneutrinos may get (small) VEVs inducing a mixing between neutrinos and neutralinos χ



Hirsch,Diaz,Porod,Romao,Valle [hep-ph/0004115]...

$$m^{\nu}_{LL} \approx \frac{\langle\tilde{\nu}\rangle^2}{M_\chi} \approx \frac{MeV^2}{TeV} \approx eV$$

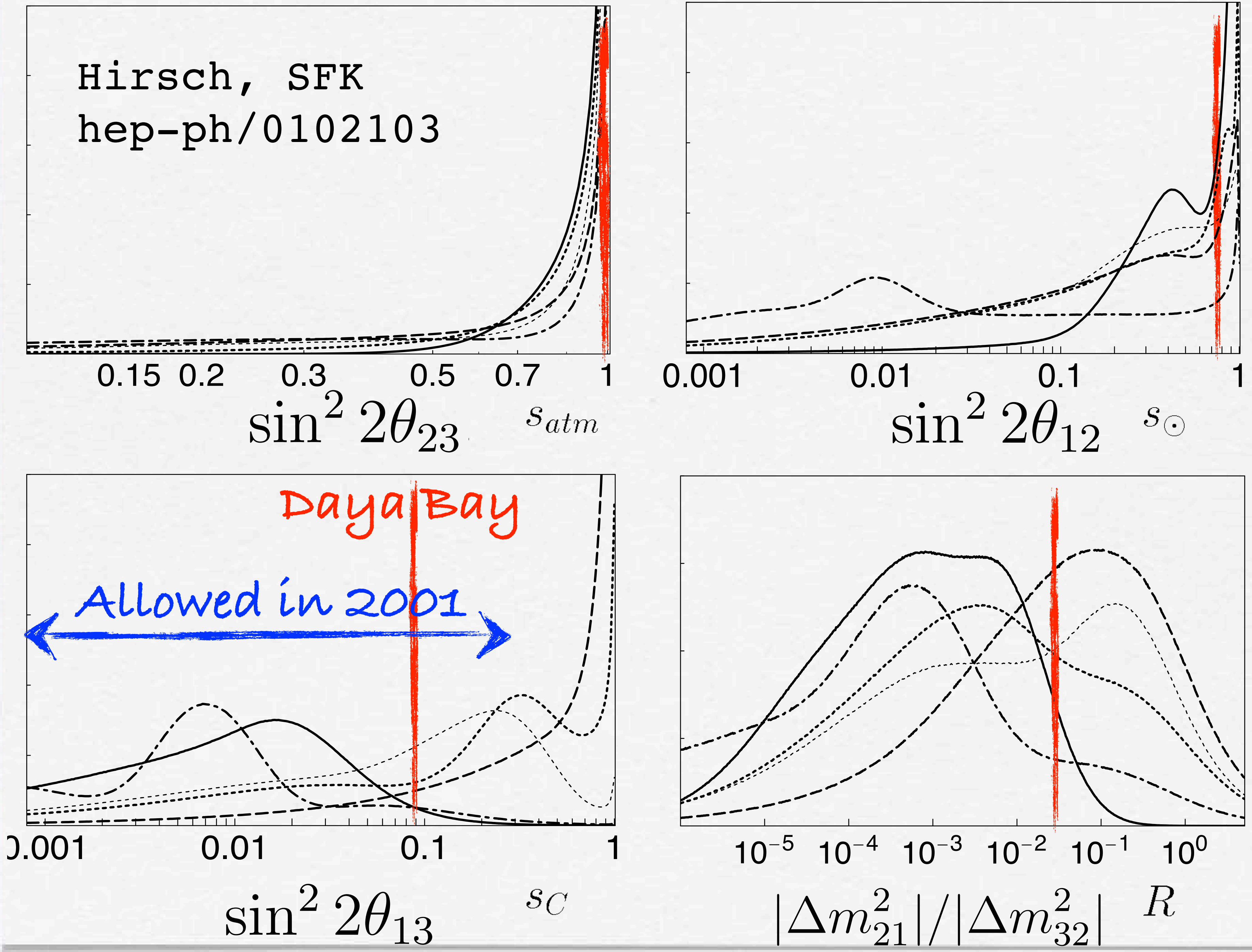
What is the origin of Large Lepton Mixing?



[Stone, 1212.6374]

#1: Anarchy vs U(1) models: inconclusive

Hall,Murayama,Weiner 9911341;
Hirsch,SFK 0102103;
Altarelli,Feruglio,Masina
0210342;...



$$m_{LL}^{anarchy} \sim \begin{pmatrix} - & - & - \\ - & - & - \\ - & - & - \end{pmatrix}$$

dashes

$$m_{LL}^{FC1} \sim \begin{pmatrix} \lambda^4 & \lambda^2 & \lambda^2 \\ \lambda^2 & 1 & 1 \\ \lambda^2 & 1 & 1 \end{pmatrix}$$

full

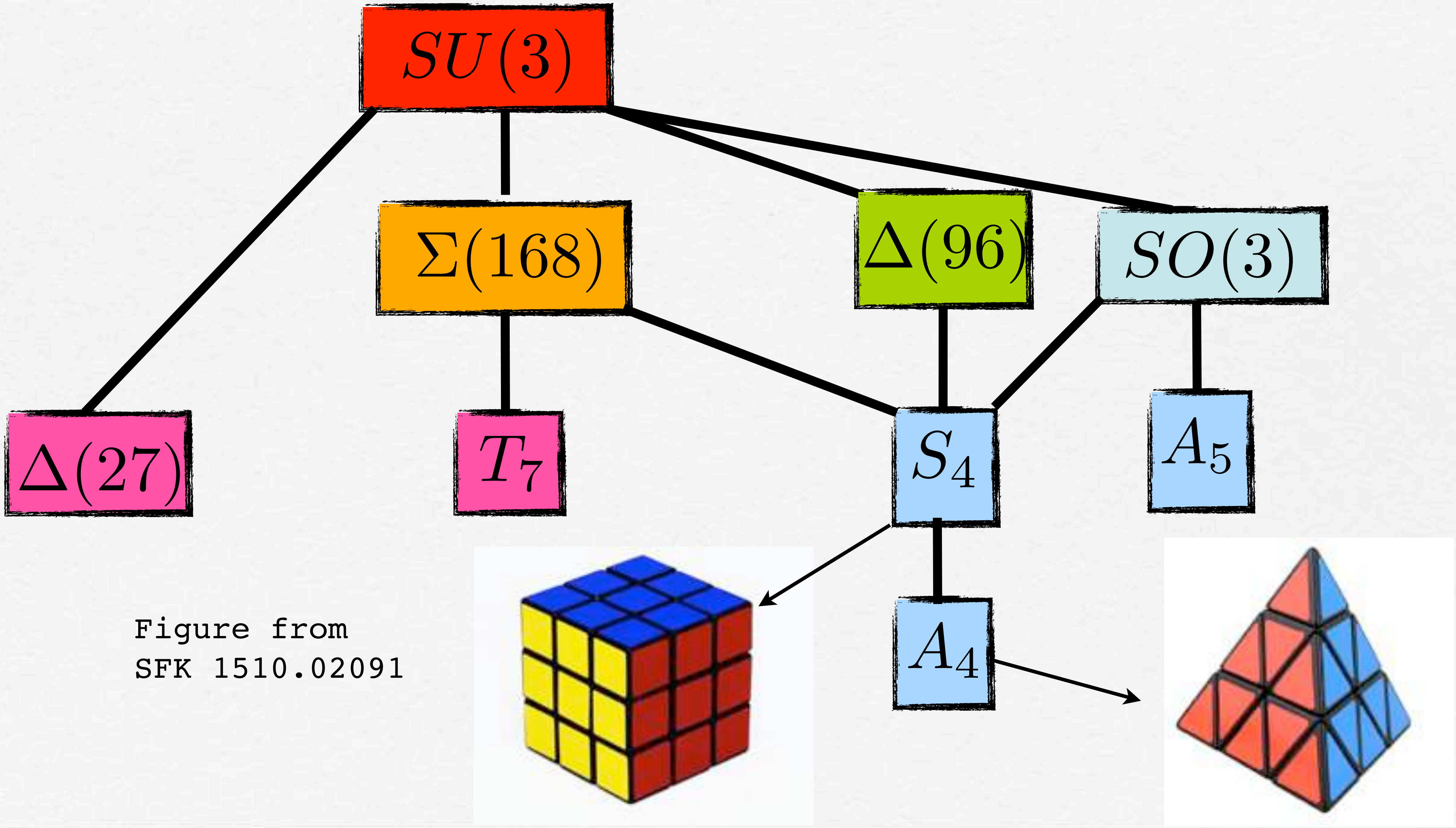
$$m_{LL}^{FC2} \sim \begin{pmatrix} \lambda^7 & \lambda^5 & \lambda^5 \\ \lambda^5 & \lambda^3 & \lambda^3 \\ \lambda^5 & \lambda^3 & \lambda^3 \end{pmatrix}$$

dot-dashes

$$m_{LL}^{FC3} \sim \begin{pmatrix} \lambda & 1 & 1 \\ 1 & \lambda^{-1} & \lambda^{-1} \\ 1 & \lambda^{-1} & \lambda^{-1} \end{pmatrix}$$

thick dots

#2: Flavour Symmetry (non-Abelian)



$SU(3)$

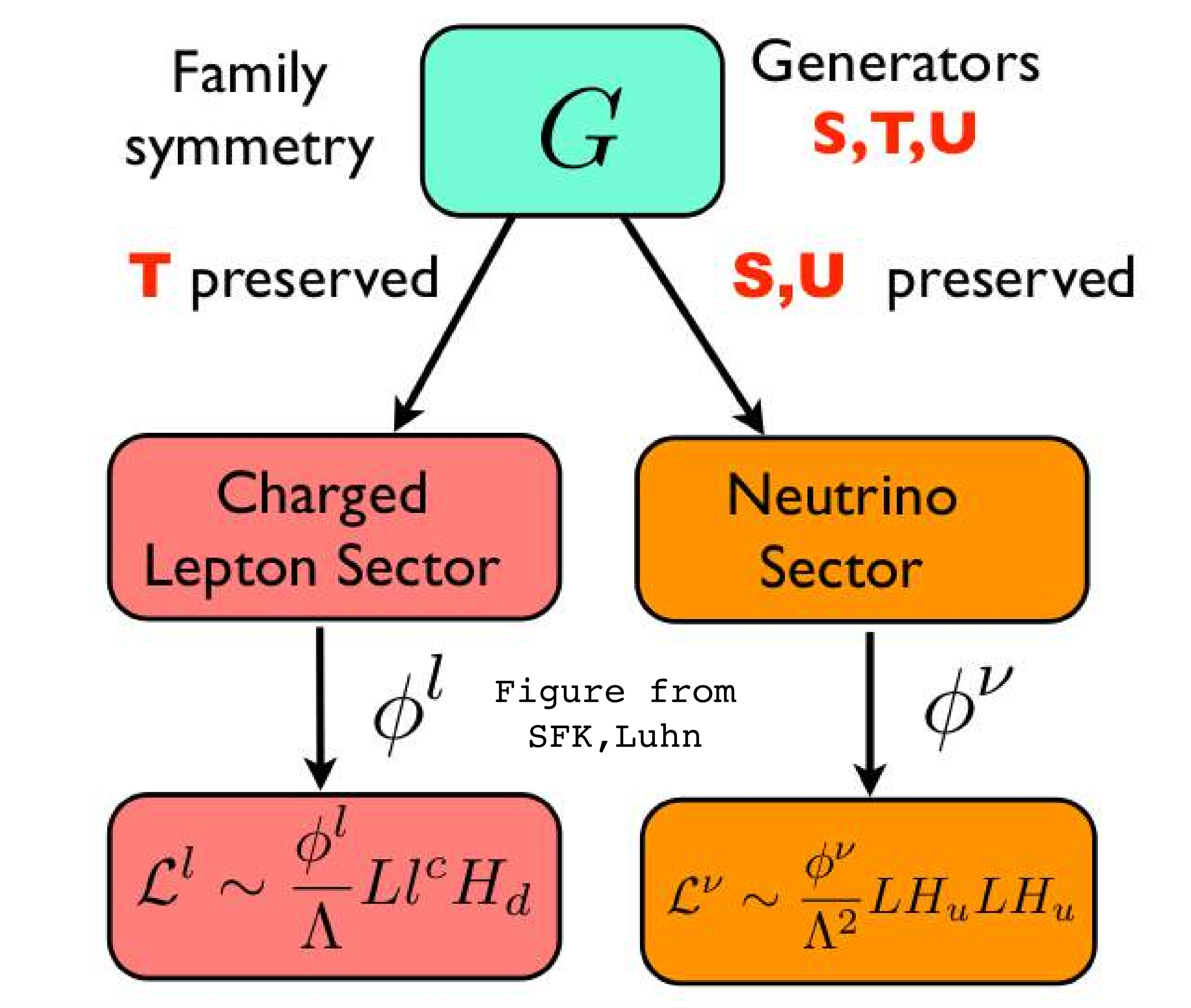
$\Sigma(168)$ $\Delta(96)$ $SO(3)$

$\Delta(27)$ T_7 S_4 A_5

A_4

Figure from
SFK 1510.02091

Direct Models



Family symmetry

G

Generators **S,T,U**

T preserved

S,U preserved

Charged Lepton Sector

Neutrino Sector

ϕ^l

ϕ^ν

$\mathcal{L}^l \sim \frac{\phi^l}{\Lambda} L l^c H_d$

$\mathcal{L}^\nu \sim \frac{\phi^\nu}{\Lambda^2} L H_u L H_u$

`Figure from SFK,Luhn`

$\Delta(6n^2)$

`Luhn et al`

is the only viable symmetry class - predicts zero Dirac CPV

`Holthausen,Lim, Lindner; SFK,Neder,Stuart; Lavoura,Ludl; Fonseca,Grimus`

Semi-Direct Models

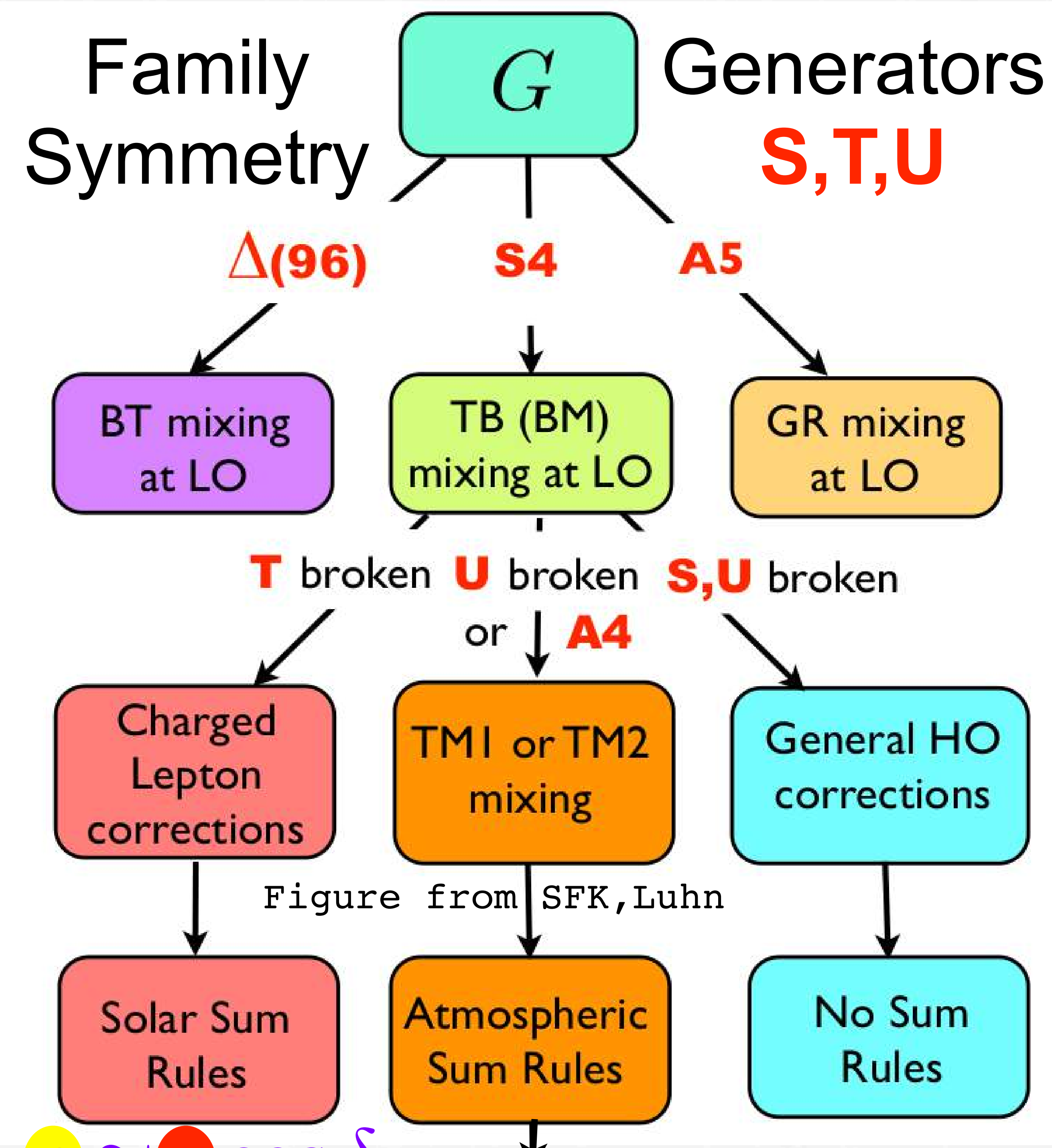


Figure from SFK,Luhn

Klein symmetry and T are partly preserved as subgroups of some family symmetry

$$U_{BM} = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ -\frac{1}{2} & \frac{1}{2} & \frac{1}{\sqrt{2}} \\ \frac{1}{2} & -\frac{1}{2} & \frac{1}{\sqrt{2}} \end{pmatrix} P$$

$$U_{TB} = \begin{pmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix} P$$

TM1 TM2

$a \approx r \cos\delta$

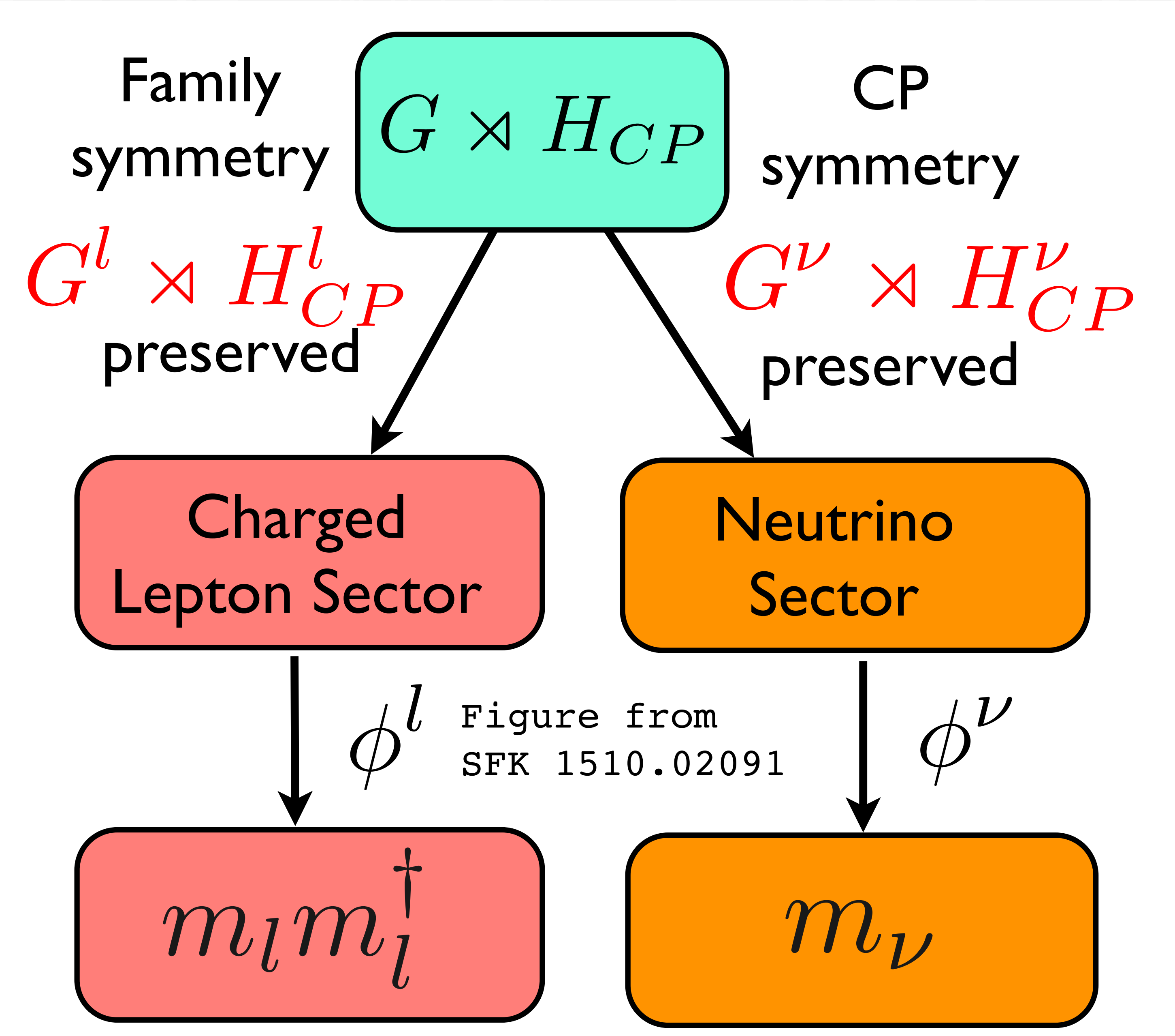
$a \approx -\frac{r}{2} \cos\delta$

SFK,
Antusch,
...

$s \approx r \cos\delta$

CP violation

Feruglio,Hagedorn;
Holthausen,Lindner Schmidt;
Ding,SFK,Luhn,Stuart;
Nishi,Xing; Hagedorn,Meroni,
Molinaro; Ding,SFK,Neder;
Branco, SFK, Varzielas,
Chen,...



Family symmetry

$G \rtimes H_{CP}$

CP symmetry

$G^l \rtimes H^l_{CP}$ preserved

$G^\nu \rtimes H^\nu_{CP}$ preserved

Charged Lepton Sector

Neutrino Sector

ϕ^l

ϕ^ν

$m_l m_l^\dagger$

m_ν

Figure from SFK 1510.02091

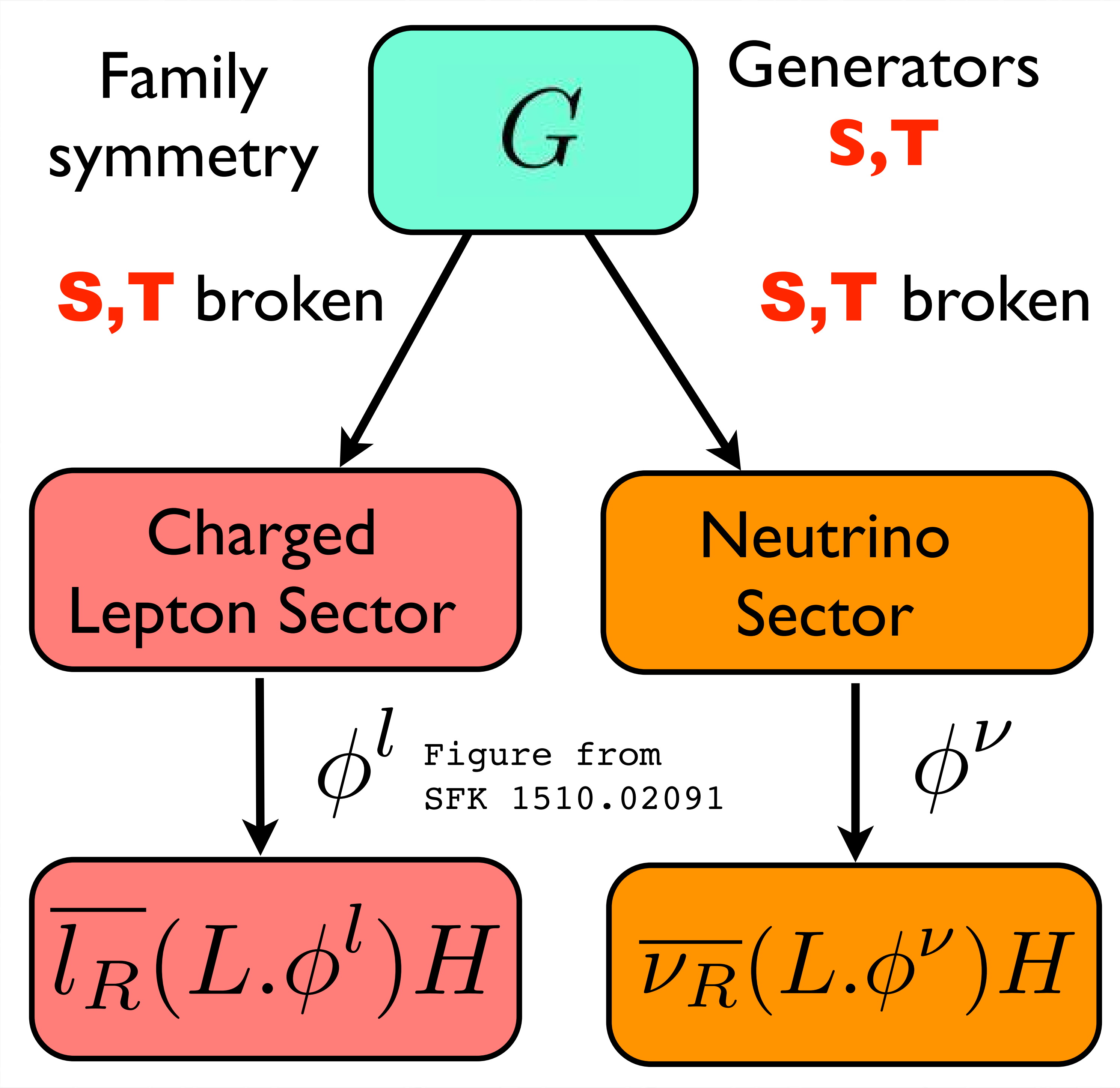
E.g. in semi-direct models where

$$G^\nu \sim Z_2^S$$

predicts maximal Dirac Phase

$$\delta = \pm\pi/2$$

Indirect Models



Family symmetry

G

Generators **S,T**

S,T broken

S,T broken

Charged Lepton Sector

Neutrino Sector

ϕ^l

ϕ^ν

$\overline{l_R}(L.\phi^l)H$

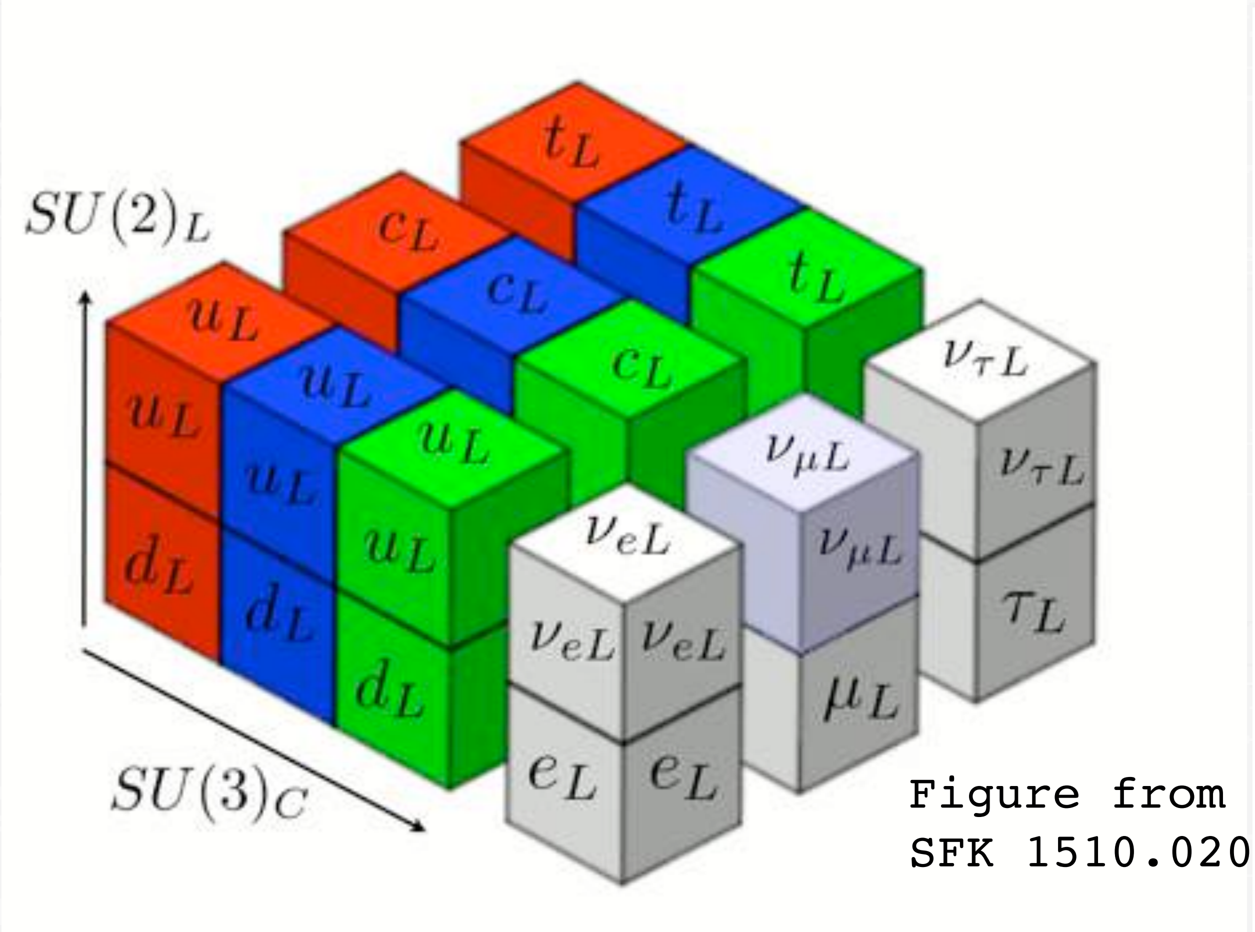
$\overline{\nu_R}(L.\phi^\nu)H$

`Figure from SFK 1510.02091`

`SFK,G.Ross,de Medeiros Varzielas,Antusch,Luhn, Bjorkeroth,...`

Predictions follow from flavon vacuum alignment

What is the minimal natural model of neutrino mass and mixing?



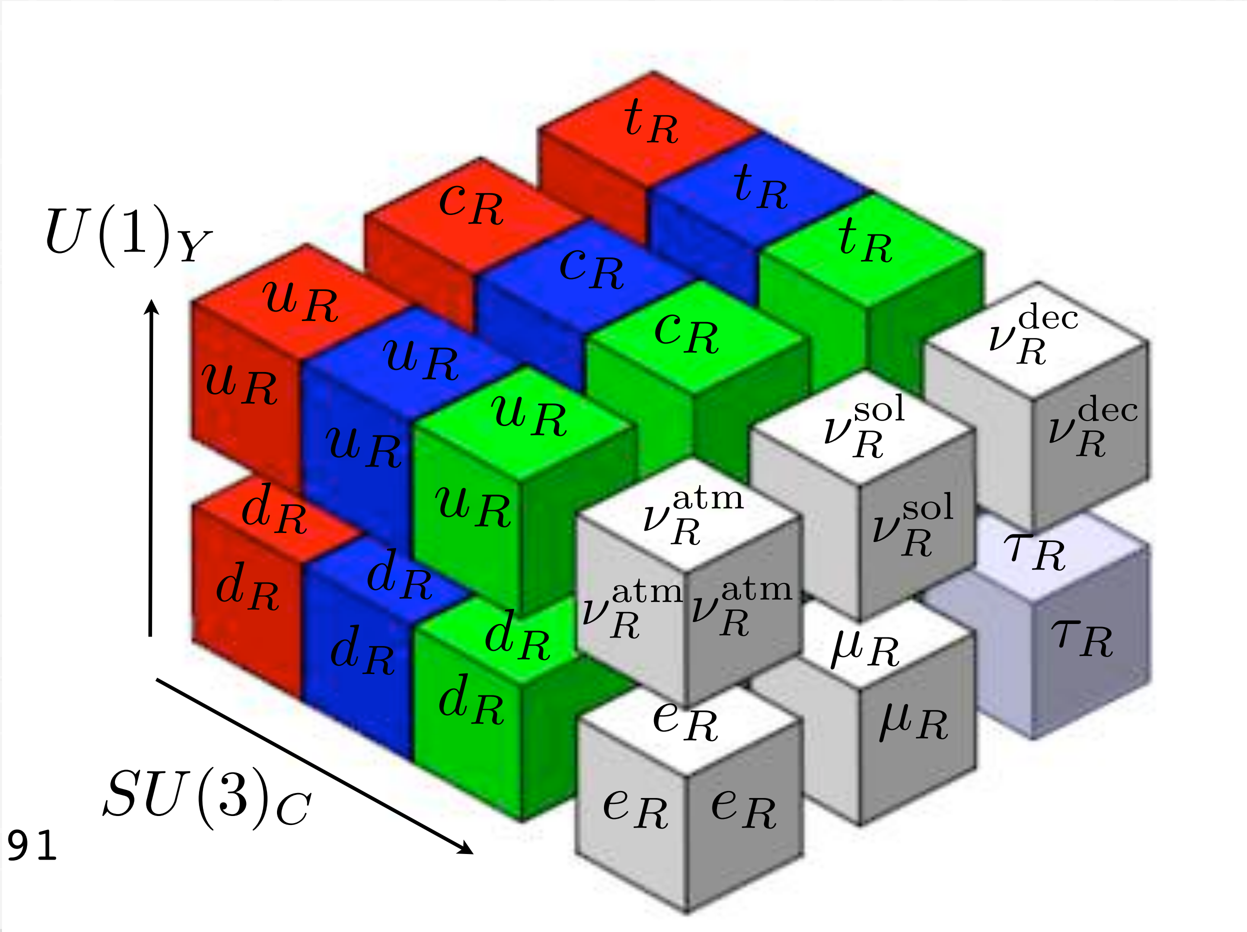


Figure from
SFK 1510.02091

hep-ph/9806440 hep-ph/9912492 hep-ph/0204360

Sequential Dominance

Heavy Majorana

$$M_R = \begin{pmatrix} M_{\rm atm} & 0 & 0 \\ 0 & M_{\rm sol} & 0 \\ 0 & 0 & [M_{\rm dec}] \end{pmatrix}$$

Dirac

$$m^D = \begin{pmatrix} m^D_{e,\rm atm} & m^D_{e,\rm sol} & [m^D_{e,\rm dec}] \\ m^D_{\mu,\rm atm} & m^D_{\mu,\rm sol} & [m^D_{\mu,\rm dec}] \\ m^D_{\tau,\rm atm} & m^D_{\tau,\rm sol} & [m^D_{\tau,\rm dec}] \end{pmatrix} \equiv \begin{pmatrix} m^D_{\rm atm} & m^D_{\rm sol} & [m^D_{\rm dec}] \end{pmatrix}$$

Light Majorana $m^\nu = m^D M_R^{-1} m^{D^T}$ has three contributions, one from each RH neutrino.

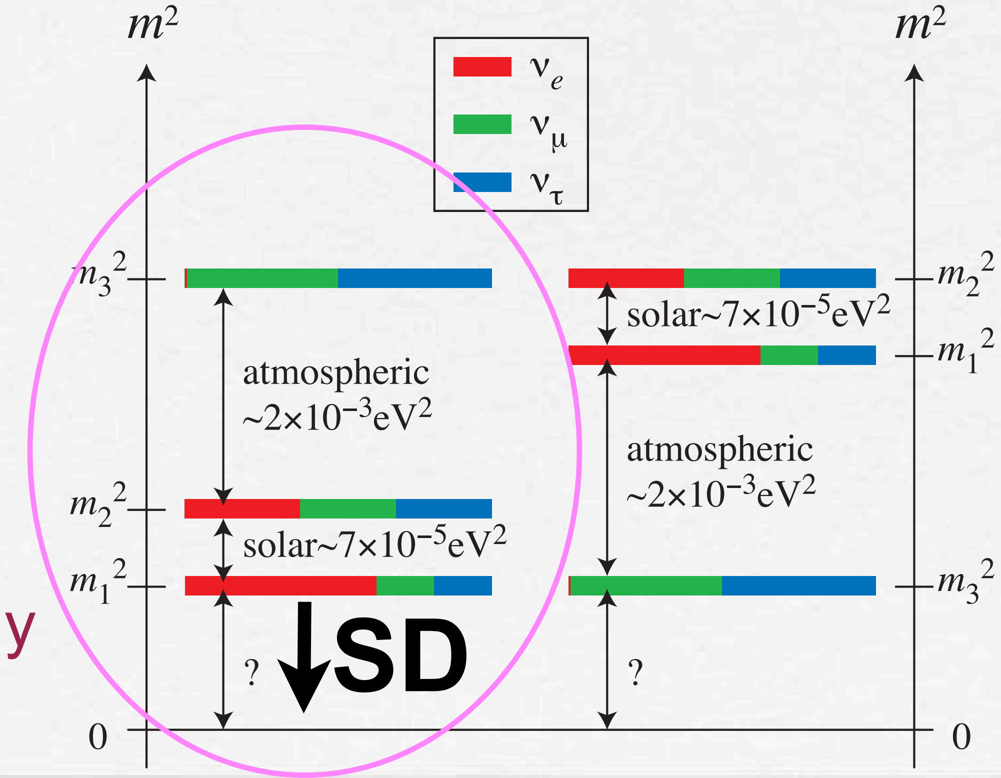
Sequential dominance assumption:

$$\frac{m^D_{\rm atm} {m^D_{\rm atm}}^T}{M_{\rm atm}} \gg \frac{m^D_{\rm sol} {m^D_{\rm sol}}^T}{M_{\rm sol}} \left[\gg \frac{m^D_{\rm dec} {m^D_{\rm dec}}^T}{M_{\rm dec}} \right]$$

$\downarrow$ Responsible for atm nu mass $m_3 \sim 50$ meV

$\downarrow$ Responsible for sol nu mass $m_2 \sim 9$ meV

$\downarrow$ Predicts strong normal hierarchy $m_1 \ll m_2$



1304.6264 1512.07531

Littlest Seesaw Model

2RHN

$$\frac{\phi_{\rm atm}}{\Lambda} L H N^c_{\rm atm} + \frac{\phi_{\rm sol}}{\Lambda} L H N^c_{\rm sol} + M_{\rm atm} N^c_{\rm atm} N^c_{\rm atm} + M_{\rm sol} N^c_{\rm sol} N^c_{\rm sol}$$

Messenger scale

Flavon vevs → $\langle \phi_{\rm atm} \rangle = v_{\rm atm} \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}$ $\langle \phi_{\rm sol} \rangle = v_{\rm sol} \begin{pmatrix} 1 \\ 3 \\ 1 \end{pmatrix}$

Constrained Sequential Dominance "CSD3"

$$m^\nu = \frac{v^2}{\Lambda^2} \left(\frac{\langle \phi_{\rm atm} \rangle \langle \phi_{\rm atm} \rangle^T}{M_{\rm atm}} + \frac{\langle \phi_{\rm sol} \rangle \langle \phi_{\rm sol} \rangle^T}{M_{\rm sol}} \right)$$

After seesaw

Only one phase

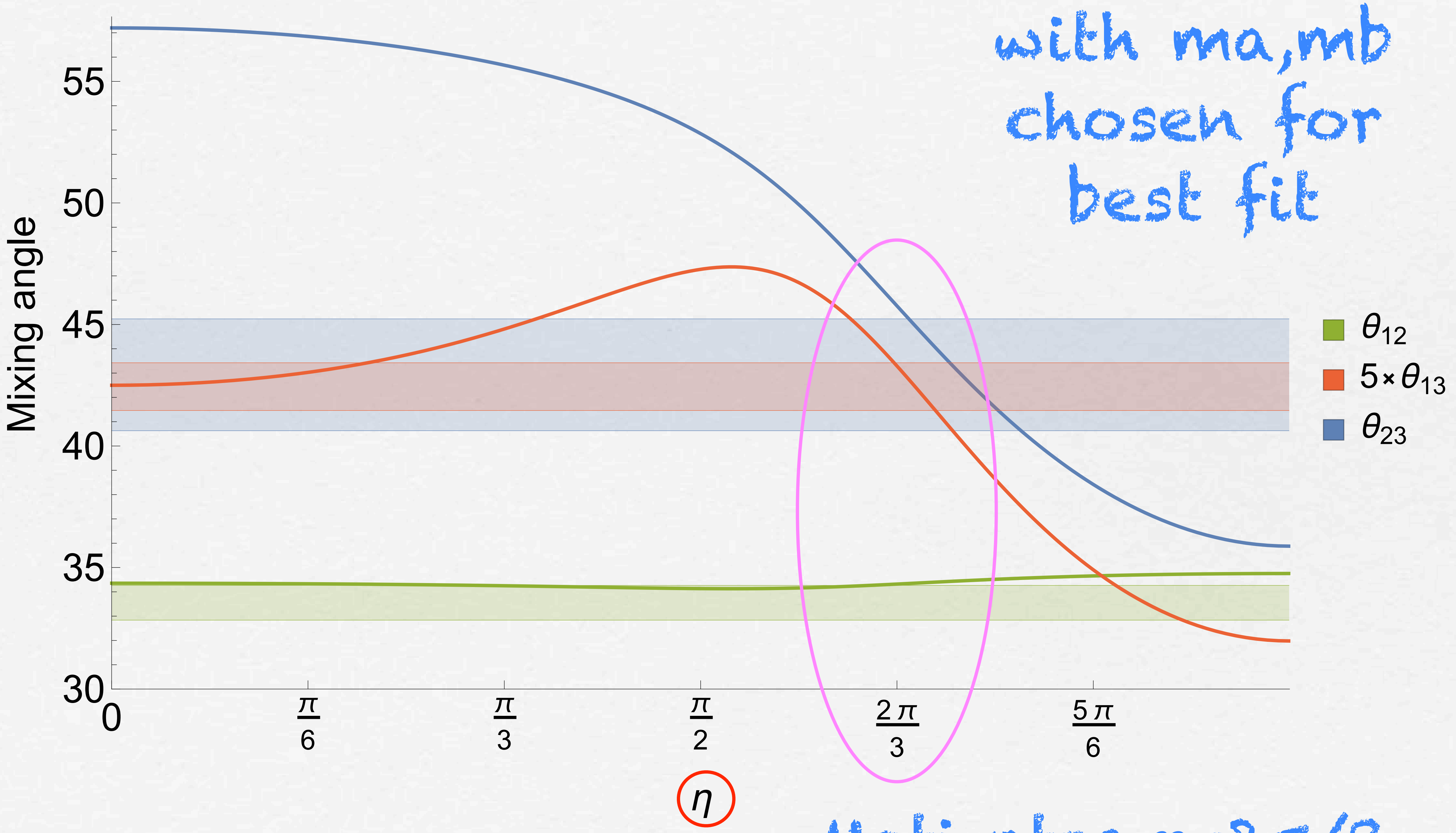
$$m^\nu = m_a \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{pmatrix} + m_b e^{\boxed{i\eta}} \begin{pmatrix} 1 & 3 & 1 \\ 3 & 9 & 3 \\ 1 & 3 & 1 \end{pmatrix}$$

TM1 mixing

Littlest seesaw = smallest number of parameters: m_a, m_b, η

Littlest Seesaw Model

Bjorkeroth et al
1412.6996



with ma,mb
chosen for
best fit

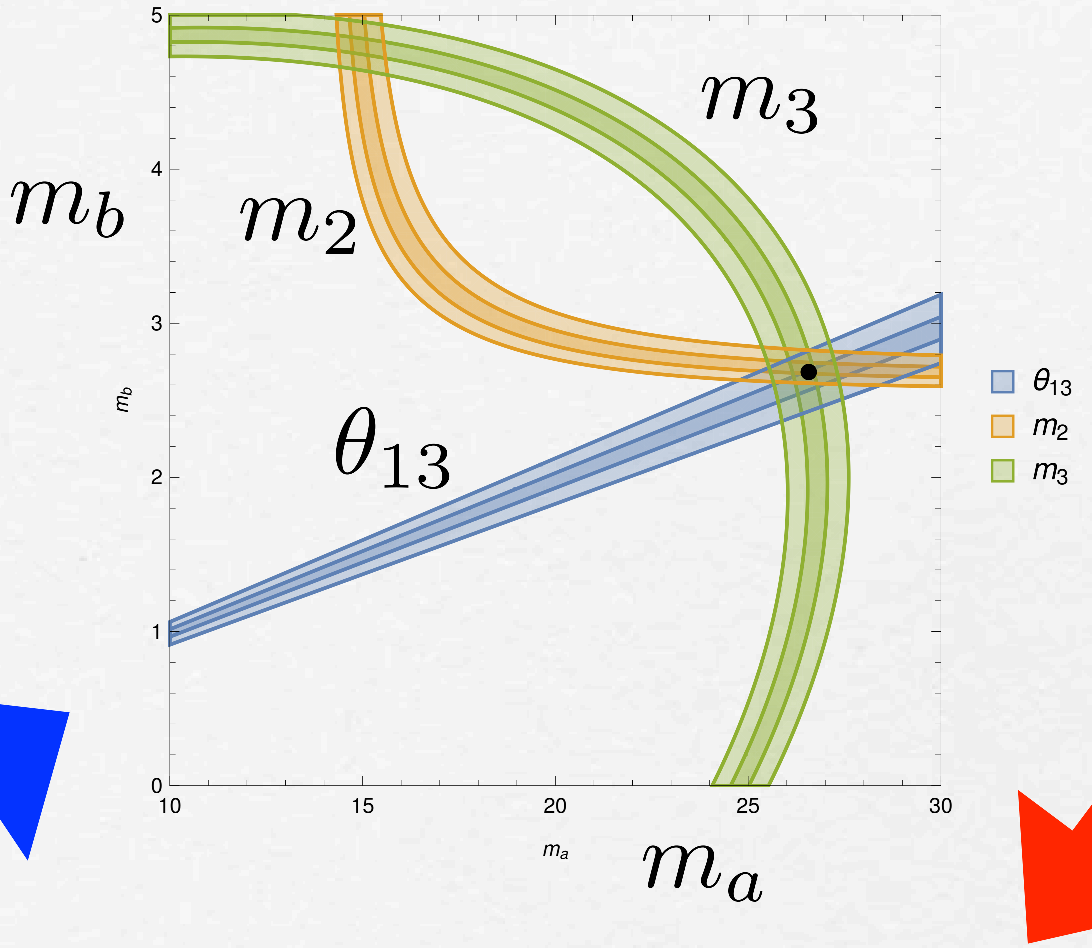
Motivates $\eta=2\pi/3$

Littlest Seesaw Model

Prouse et al
(to appear)

$$m^\nu = m_a \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{pmatrix}$$

$$+ m_b e^{i2\pi/3} \begin{pmatrix} 1 & 3 & 1 \\ 3 & 9 & 3 \\ 1 & 3 & 1 \end{pmatrix}$$



Very predictive: 7 outputs!!

Only two parameters now: m_a, m_b

m_a (meV)	m_b (meV)	η (rad)	θ_{12} (°)	θ_{13} (°)	θ_{23} (°)	$\delta_{\rm CP}$ (°)	m_1 (meV)	m_2 (meV)	m_3 (meV)
26.57	2.684	$\frac{2\pi}{3}$	34.3	8.67	45.8	-86.7	0	8.59	49.8
Value	from	[25]	$33.48^{+0.78}_{-0.75}$	$8.50^{+0.20}_{-0.21}$	$42.3^{+3.0}_{-1.6}$	-54^{+39}_{-70}	0	8.66±0.10	49.57±0.47

Conclusions

- *What is the Origin of Small Neutrino Masses?*

  Roadmap of possibilities e.g. extra dims, loops, seesaw models,...

- *What is the origin of Large Lepton Mixing?*

  Flavour symmetry is really the only way to make progress, but how to implement it? Direct, semi-direct, indirect, CP violation?

- *What is the minimal natural model of neutrino mass and mixing?*

  Littlest Seesaw: 2RHN, CSD3, 2 input parameters, 7 predictions!

Thank YOU