

Far-from-equilibrium gauge fields
and
early time dynamics of heavy-ion collisions

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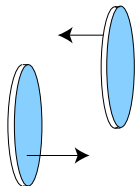
Blois 2016



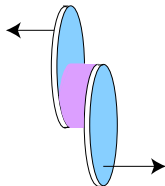
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Motivation

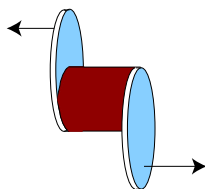
Lorentz contracted nuclei



Pre-thermal plasma



Locally thermalised plasma



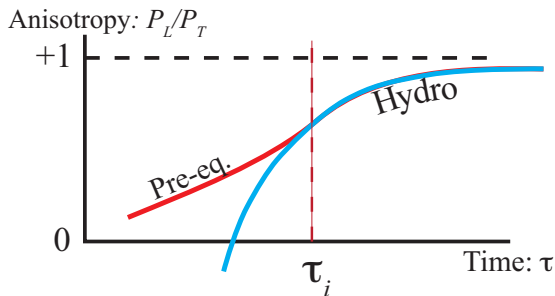
- Quark-Gluon plasma created in heavy-ion collisions flows hydrodynamically
- Hydro as gradient expansion around local thermal equilibrium:

Baier et al. JHEP 0804 (2008) 100

$$\partial_\mu T^{\mu\nu} = 0$$

$$T^{\mu\nu} = T_{\text{Therm. eq.}}^{\mu\nu} - \eta 2\nabla^{\langle\mu} u^{\nu\rangle} + \dots$$

Far-from-equilibrium evolution



- Strong anisotropy $P_L/P_T \ll 1$, sign of large corrections
- Hydro simulations start at *intialization time* τ_i
- At early times *far-from-equilibrium* evolution

Far-from-equilibrium evolution

Limited tool box

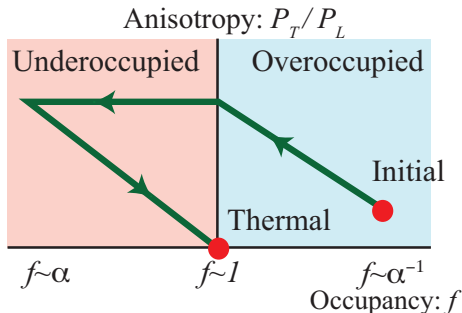
- Lattice techniques suffer from sign problem
- Strong coupling methods (holography) for some special cases

$$\mathcal{N} = 4, N_c \rightarrow \infty, \lambda \rightarrow \infty$$

For generic theories weak coupling methods

- Physics often non-perturbative *strong fields, instabilities, secular divergences ...*
- Effective theory methods
 - Effective kinetic theory
 - Classical field theory
 - Vlasov theory
 - ...

Bottom-up thermalization at weak coupling



- Color Glass Condensate: Initial condition overoccupied

McLerran, Venugopalan PRD49 (1994) 2233-2241 , PRD49 (1994) 3352-3355 ; Gelis et. al Int.J.Mod.Phys. E16 (2007) 2595-2637 , Ann.Rev.Nucl.Part.Sci. 60 (2010) 463-489

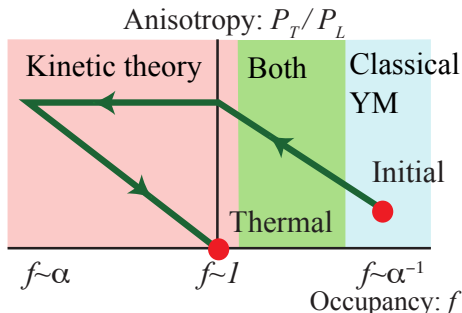
$$f(Q_s) \sim 1/\alpha_s, \quad Q_s \sim 2\text{GeV}$$

- Expansion makes system underoccupied before thermalizing

Baier et al Phys.Lett. B502 (2001) 51-58

$$f(Q_s) \ll 1$$

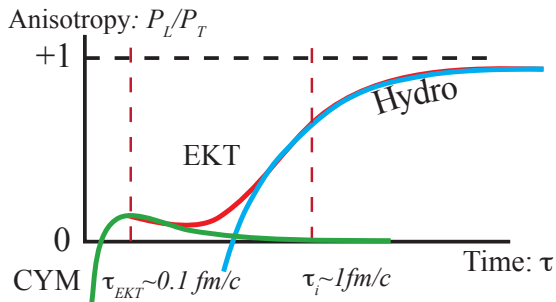
Bottom-up thermalization at weak coupling



- Degrees of freedom:
 - $f \gg 1$: Classical Yang-Mills theory (CYM)
 - $f \ll 1/\alpha_s$: (Semi-)classical particles, Eff. Kinetic Theory (EKT)
- Transmutation of fields to particles: Field-particle duality
Son, Mueller PLB582 (2004) 279-287; Jeon PRC72 (2005) 014907; Mathieu et al EPJ. C74 (2014) 2873 ; AK, Moore PRD89 (2014) 7, 074036

$$1 \ll f \ll 1/\alpha_s$$

Stages of far-from-equilibrium evolution



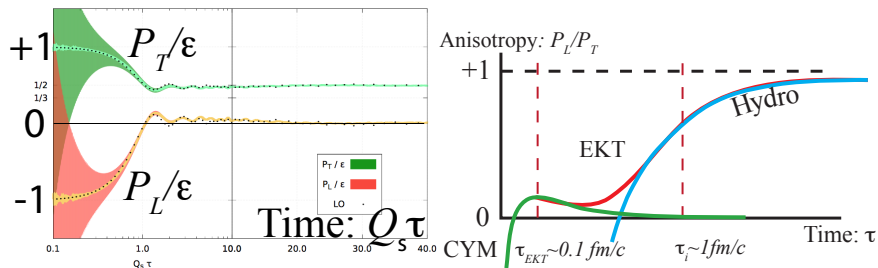
Strategy: Switch from CYM to EKT at τ_{EKT} ,

$$1 \ll f \ll 1/\alpha_s$$

From EKT to hydro at τ_i ,

$$P_L/P_T \sim 1$$

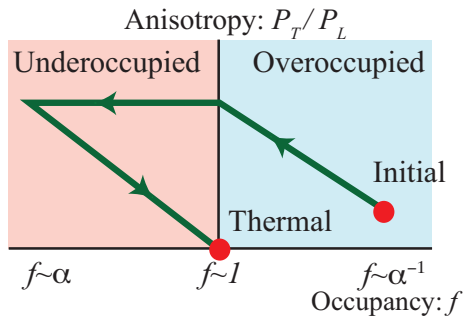
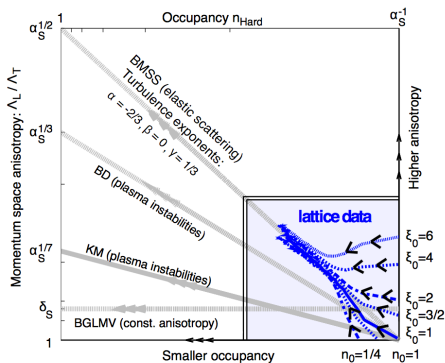
Early times $0 < Q_s \tau \lesssim 1$: classical YM evolution



Epelbaum & Gelis, PRL. 111 (2013) 23230

- Melting of the coherent boost invariant CGC fields
 - After $\tau \sim 1/Q_s$, fields decohere, $P_L > 0$

First stage of the evolution



Berges et al. Phys.Rev. D89 (2014) 7, 074011

- Numerical demonstration of classical/overoccupied part of the diagram
- Classical theory never thermalises or isotropizes
- Before $f \sim 1$, must switch to kinetic theory

$$\frac{df}{dt} = -C_{2\leftrightarrow 2}[f] - C_{1\leftrightarrow 2}[f]$$

The diagram shows two Feynman diagrams. The left diagram, labeled $C_{2\leftrightarrow 2}$, depicts a scattering process where two incoming particles (represented by straight lines with arrows) interact via a gluon exchange (represented by a wavy line) to produce two outgoing particles. The right diagram, labeled $C_{1\leftrightarrow 2}$, shows a process where a single incoming particle (straight line with arrow) interacts with a gluon (wavy line) to produce two outgoing particles (straight lines with arrows). The gluon line is shown as a ladder structure with multiple rungs, representing a series of interactions.

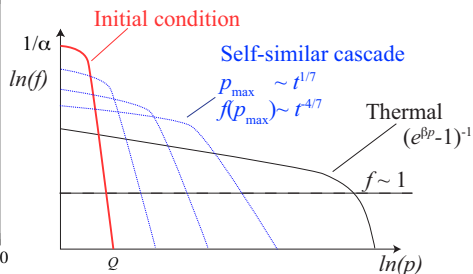
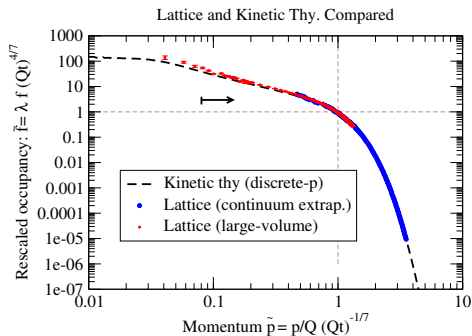
- Soft and collinear divergences lead to nontrivial matrix elements

soft: screening, Hard-loop; collinear: LPM, ladder resum

The diagram shows a complex function represented by a large oval containing a series of vertical rungs (ladder diagrams). This is equated to the real part of a product of two terms, each represented by a horizontal line with vertical rungs and a wavy line at the end, indicating a resummed series of diagrams.

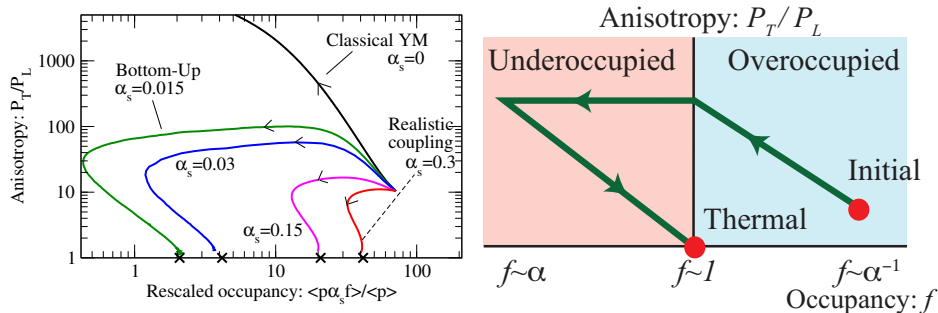
- No free parameters; LO accurate in the $\alpha_s \rightarrow 0$, $\alpha_s f \rightarrow 0$ limit.
- Used for LO transport coefficients in QCD, jet energy loss
Arnold et al. JHEP 0305 (2003) 051; Moore, York PRD79 (2009) 054011; Ghiglieri, Teaney 1502.03730; AK, Wiedemann PLB740 (2015) 172-178; Iancu, Wu JHEP 1510 (2015) 155
- Progress in NLO $\mathcal{O}(\sqrt{\alpha_s})$ Ghiglieri et al. JHEP 1603 (2016) 095

Comparison between CYM and EKT: Isotropic

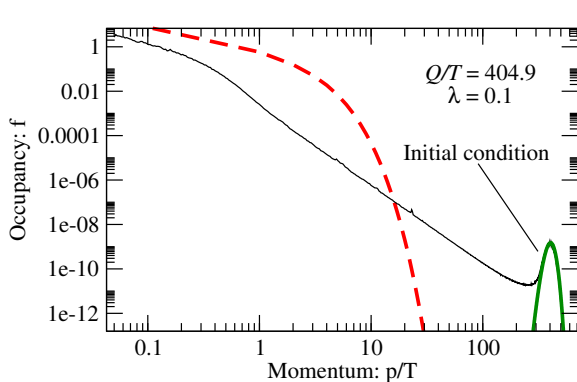


- Quantitative agreement between CYM and EKT for

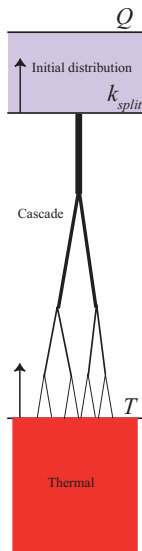
$$1 \ll f \ll 1/\alpha_s$$

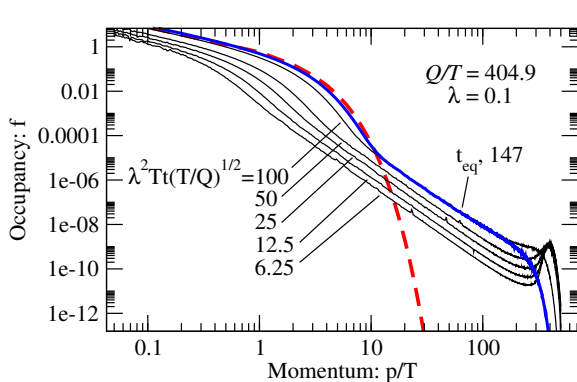


- In the classical limit ($\alpha_s \rightarrow 0, \alpha_s f$ fixed), no thermalization
- At small values of couplings, clear Bottom-Up behaviour
- Features become less defined as α_s grows

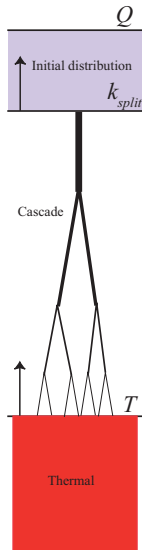


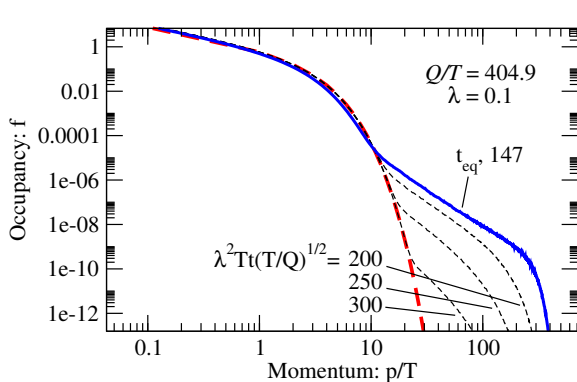
- Soft radiation quick: creation of a thermal bath (“Bottom”)
- Hard particles quenched by interactions with the bath
- Heating the soft bath rises longitudinal pressure



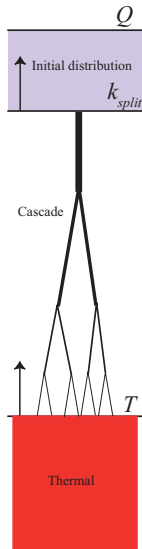


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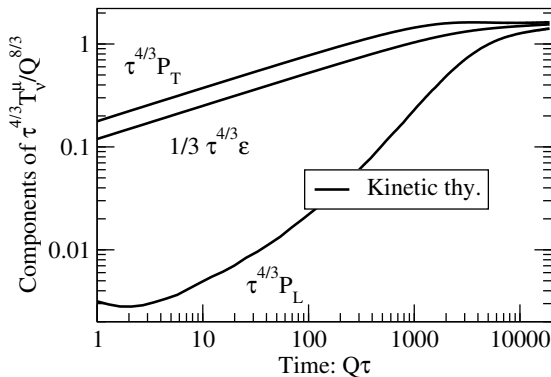




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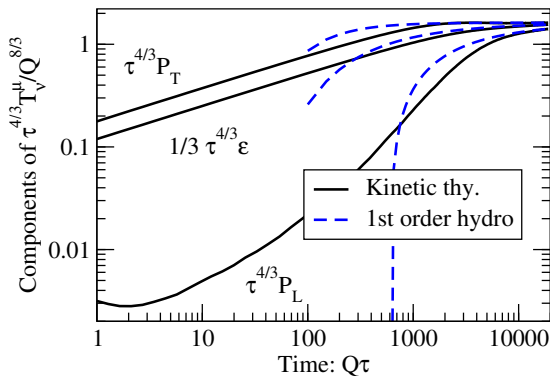


$$\alpha_s = 0.03$$



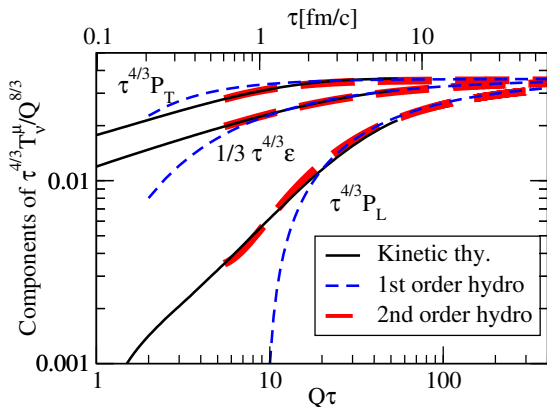
- Kinetic theory converges to hydro smoothly and automatically

$$\alpha_s = 0.03$$



- Kinetic theory converges to hydro smoothly and automatically
- Hydro prediction fixed by perturbative η/s

$$\alpha_s = 0.3$$



- For realistic couplings, hydrodynamics reached around $\lesssim 1\text{fm}/c$.
- Hydro seems to give a good description even when $P_L/P_T \sim 1/5!$
Thermalization \neq hydrodynamization

Other recent directions

Phenomenology: Transverse dynamics, preflow [Keegan et al. 1605.04287](#)

Other theories at weak coupling:

- Several studies in scalar theory
 - Classical [Berges et al. 1508.03073](#)
 - Kinetic theory [Epelbaum et al. JHEP 1509 \(2015\) 117](#)
- Exact solutions to kinetic theory in relaxation time approximation [Denicol et al. PRL 113 \(2014\) 20, 202301](#)

Strong coupling $\mathcal{N} = 4$ SYM [Chesler, Yaffe PRL 106 \(2011\) 021601](#); [van der Schee et al. PRL 111 \(2013\) 22, 222302, arXiv:1507.08195](#)

Approaching from the hydro side:

- What limits the convergence of hydrodynamics [Heller, Spalinski PRL 115, 072501 \(2015\)](#)
- Modifying hydro to work further from equilibrium, aHydro [Phys.Rev. C92 \(2015\) no.4, 044912](#)

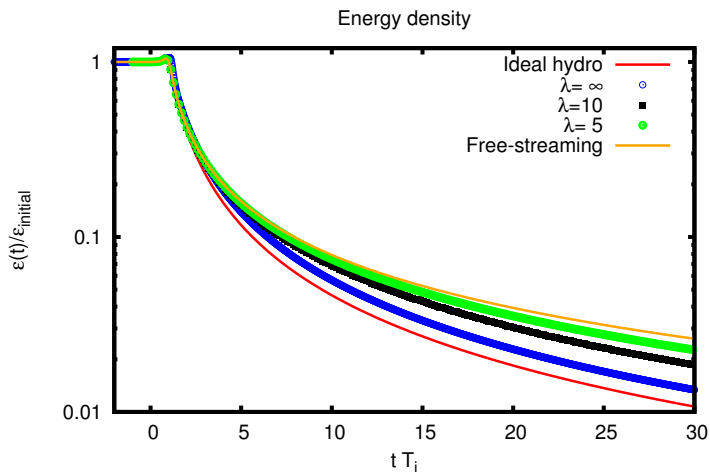
Summary

- Development of generic leading order perturbative far-from-equilibrium methodology in QCD
- Combination of classical Yang-Mills simulations and effective kinetic theory allows to follow the time evolution from highly occupied initial condition to thermal equilibrium.
- Applications beyond heavy-ion collisions
reheating, parametric resonance, relics, phase transitions

Backup slides

Weakly or strongly coupled thermalization?

Apples to apples comparison of weak and strong coupling

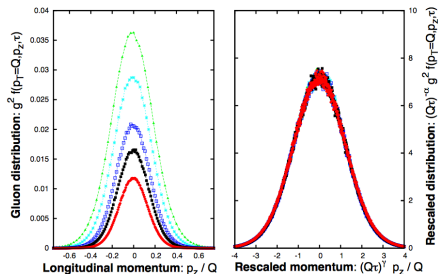


Comparison between CYM and EKT: Expanding

In non-pert classical regime $1 \ll f \ll 1/\alpha_s$

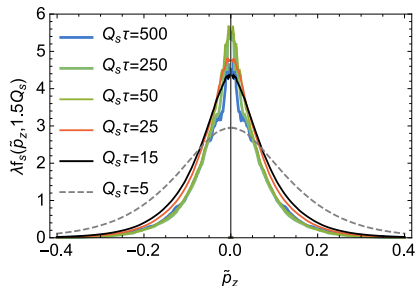
$$f(p_z, p_\perp, \tau) = (Q_s \tau)^{-2/3} f_S((Q_s \tau)^{1/3} p_z, p_\perp),$$

CYM $\alpha_s f \ll 1$ limit but $f \gg 1$



Berges et al. Phys.Rev. D89 (2014) 7, 074011

EKT $f \gg 1$ limit but $f \ll 1/\alpha_s$



AK, Zhu 1506.06647