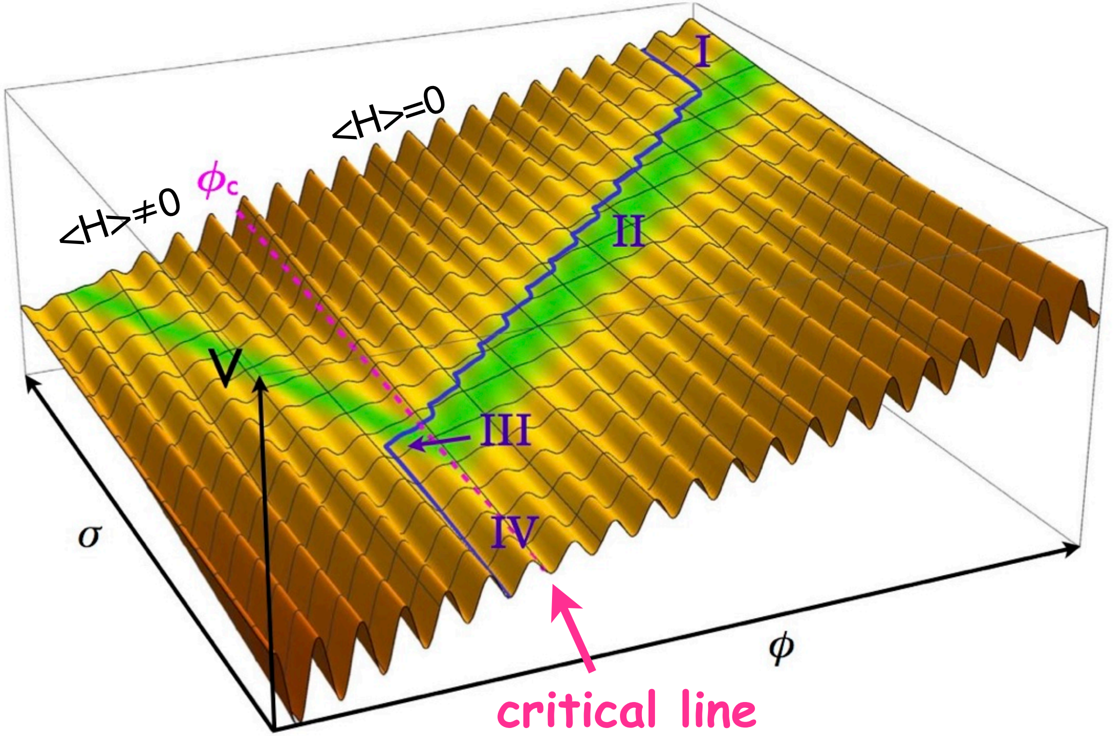
Cosmological Higgs-Axion Interplay for a Naturally Small Electroweak Scale

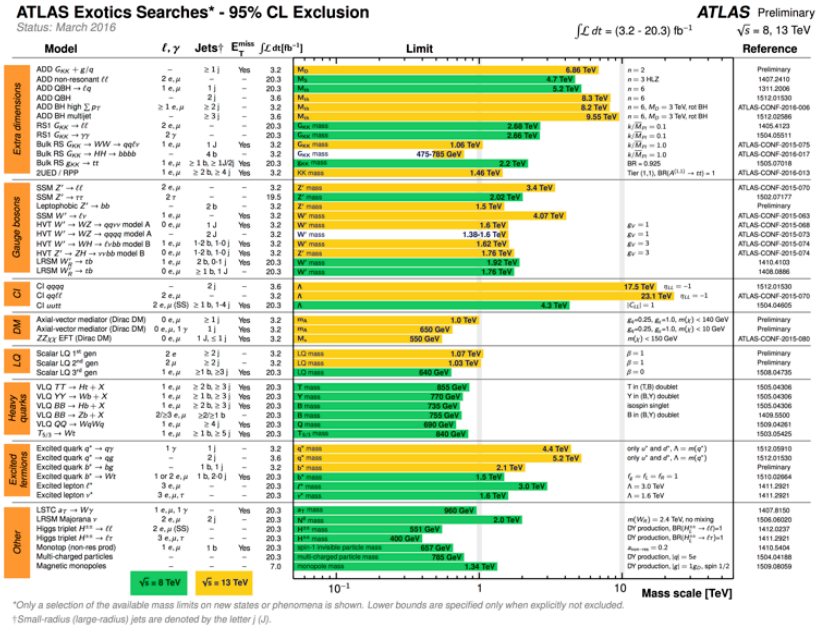
Géraldine SERVANT
DESY & U.Hamburg

Rencontres de Blois, May 31 2016



Cosmological Relaxation of the EW scale:

A newborn paradigm following post-LHC Run I theorists' depression



"It is in moments of **crisis** that new ideas develop," **Gian Giudice**

2015 Theory Highlight

The Relaxion

Graham, Kaplan, Rajendran [1504.07551]

New approach to tackle the Hierarchy problem in particle physics

Purpose of this talk is to discuss:

-the idea

-explicit models

-drawbacks & reasons for improvement

-experimental consequences

J.R. Espinosa, C. Grojean, G. Panico, A. Pomarol, O. Pujolàs, G. Servant, [1506.09217]

The Hierarchy Problem

If Standard Model is an effective field theory below M$_{\rm Planck}$

$$V = m_h^2\, h^2 + \lambda\, h^4$$

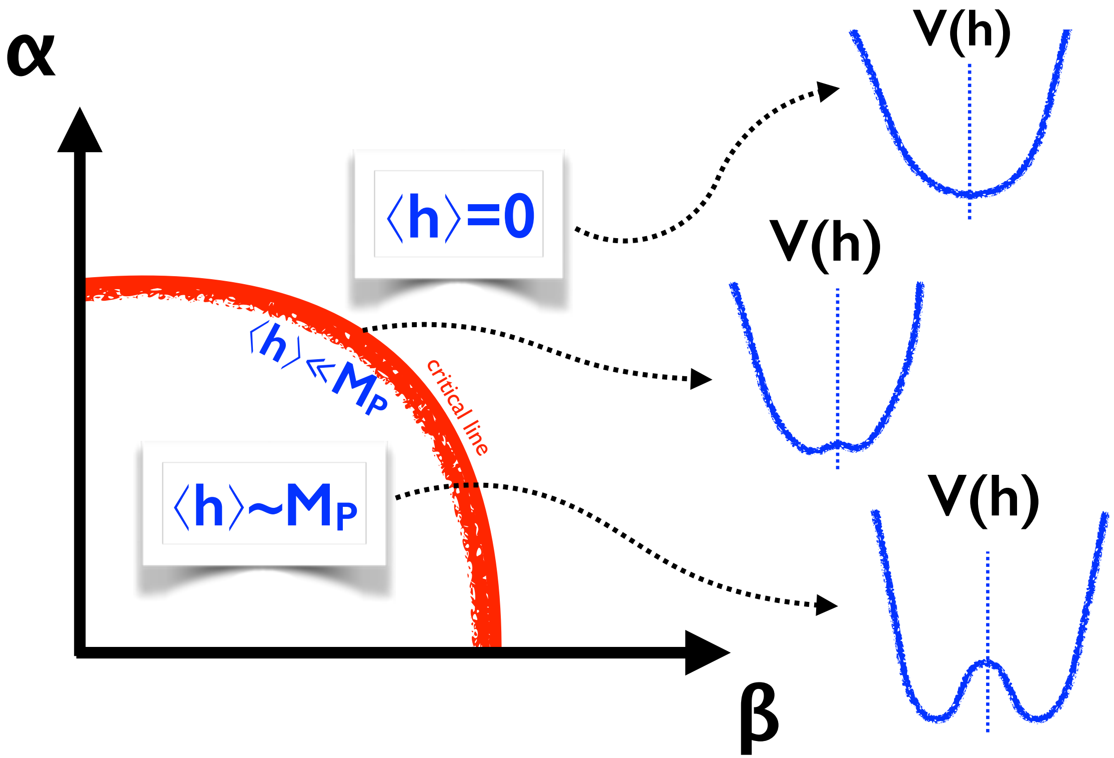
Why $m_h^2 \ll \mathrm{M}_{\mathrm{Planck}}^2$ **?**

The Hierarchy Problem

In high energy completions of the Standard Model where the Higgs potential can be computed in terms of new parameters, α and β:

$$m_h^2 = m_h^2(\alpha,\beta)$$

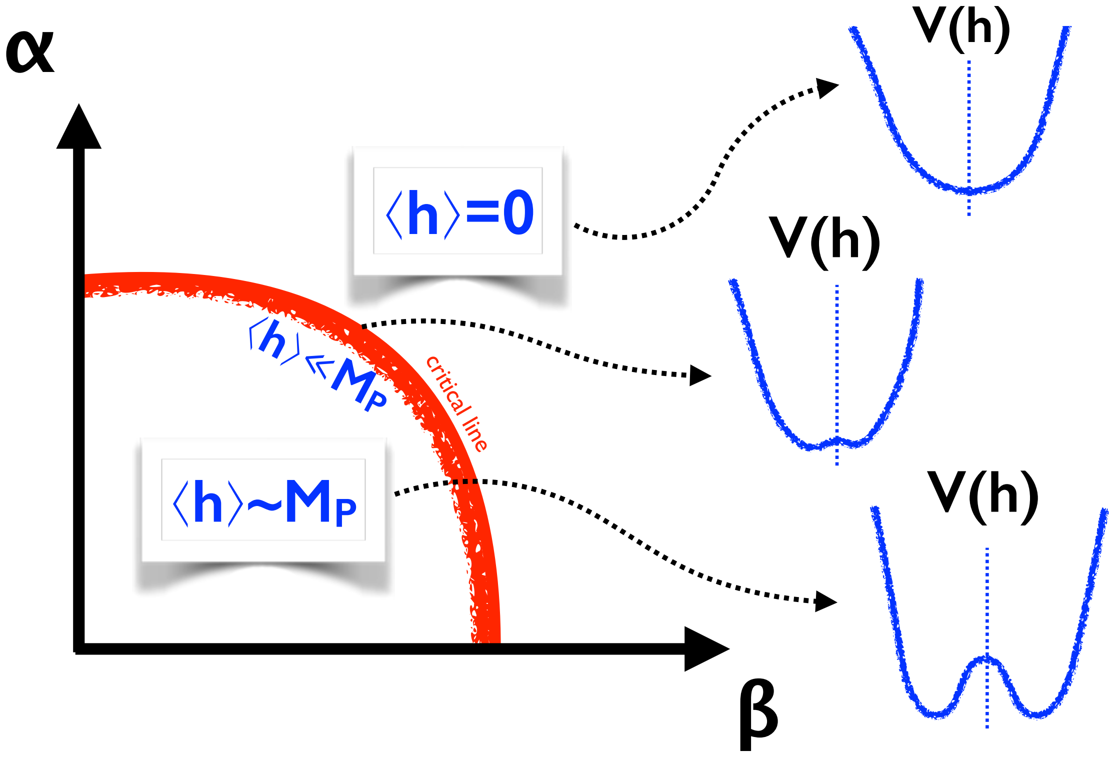
Why does the Higgs vacuum reside so close to the critical line separating the phase with unbroken (<H>=0) from the phase with broken (<H> ≠0) electroweak symmetry?



[Figure Credit:A. Pomarol]

The Hierarchy Problem

$$m_h^2 = m_h^2(\alpha, \beta)$$



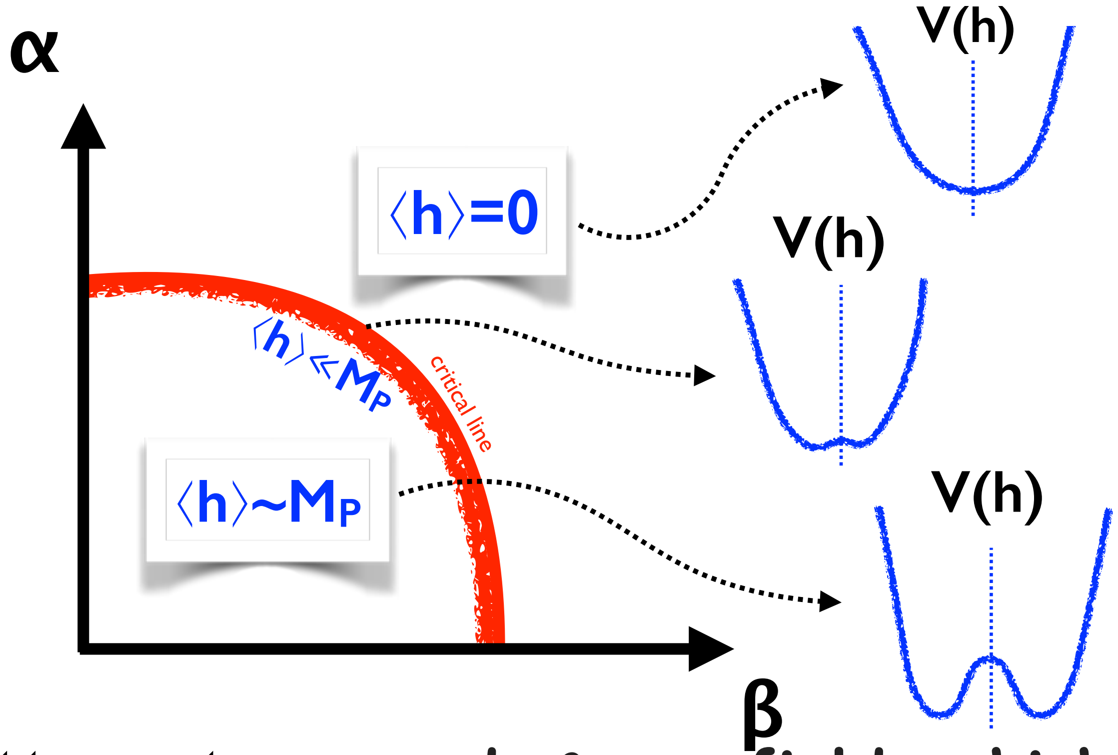
[Figure Credit:A. Pomarol]

Solution 1: Critical line is special line with enhanced symmetry-> Supersymmetry

implications: Susy particles expected at the weak scale

The Hierarchy Problem

$$m_h^2 = m_h^2(\alpha, \beta)$$



New attempt : α and β are fields which have local minima in the broken phase. Cosmological evolution settles them in a minimum close to the critical line.

[Figure Credit: A. Pomarol]

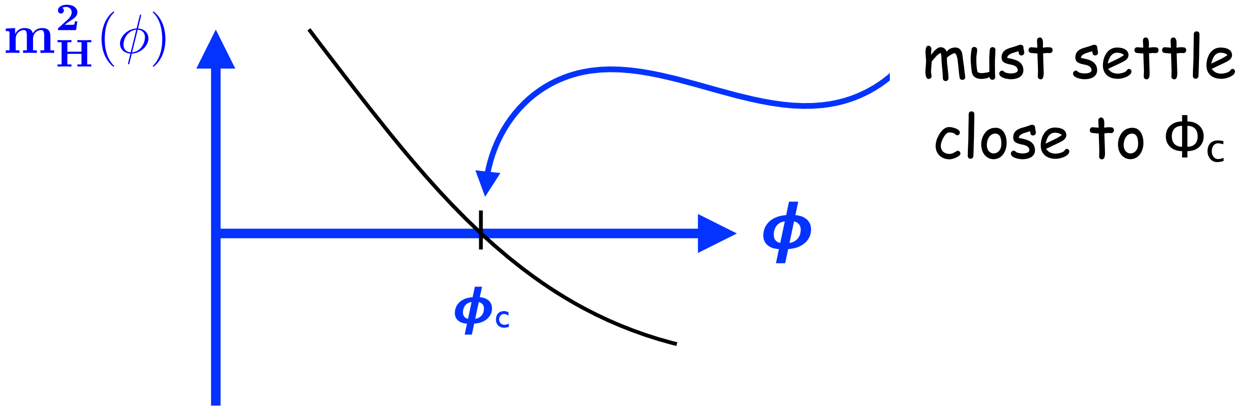
Key idea: Higgs mass parameter is field-dependent

$$m^2|H|^2 \rightarrow m^2(\phi)|H|^2$$

Φ can get a value such that $m^2(\phi) \ll \Lambda^2$

UV cutoff

from a dynamical interplay between H and Φ



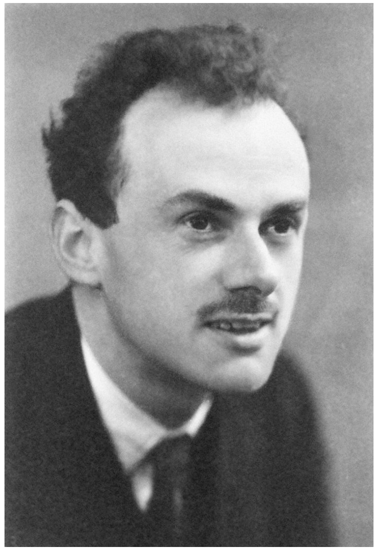
m_H naturally stabilized due to back-reaction of the Higgs field after EW symmetry breaking !

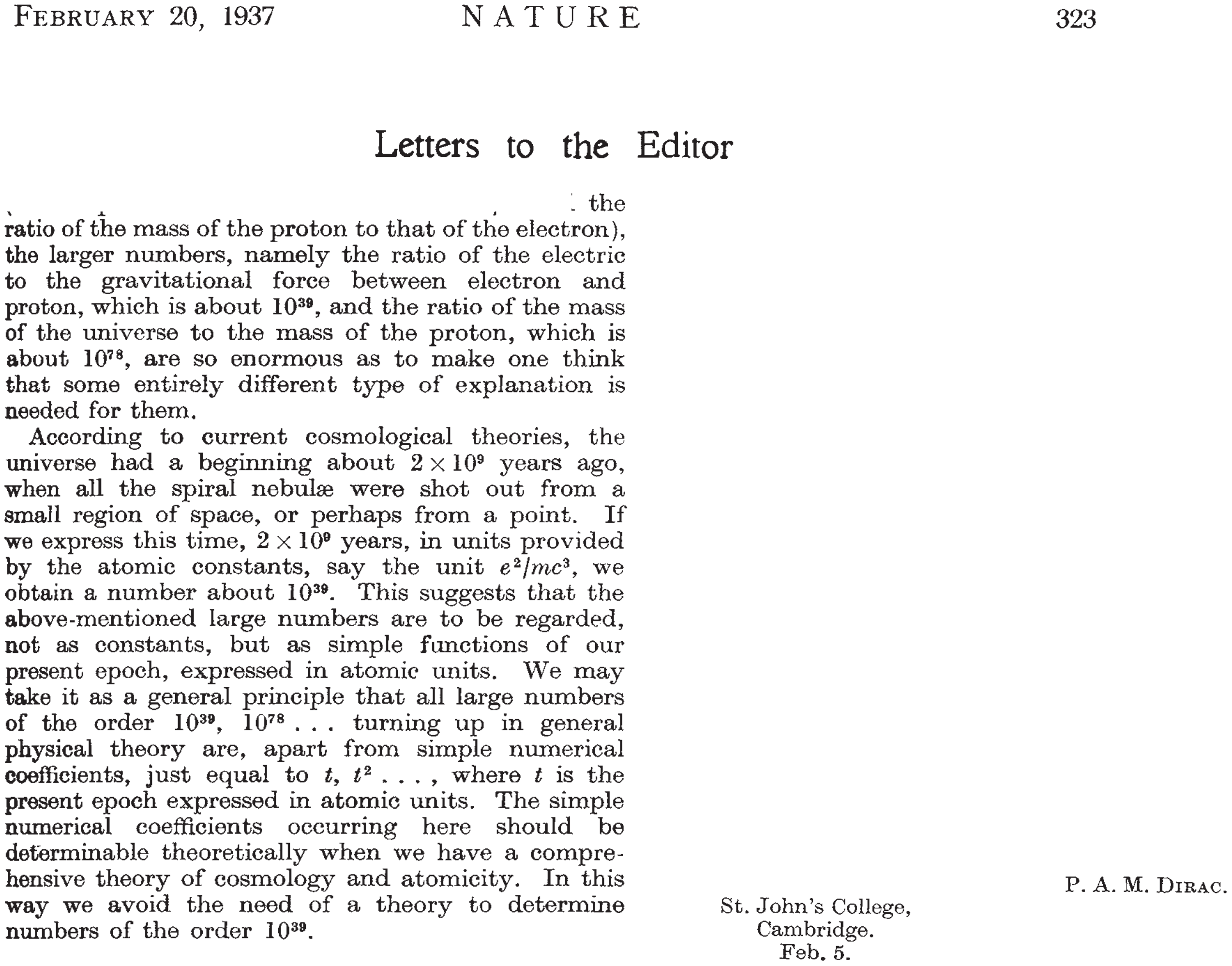
New paradigm:

Hierarchies are induced/created by the time evolution/the age of the Universe

Dramatic implications for strategy to search for new physics explaining the Weak scale

The idea that hierarchies in force scales could have something to do with cosmological evolution goes back to Dirac (hypothetizes a relation between ratio of universe sizes to ratio of force strengths)





FEBRUARY 20, 1937 NATURE 323

Letters to the Editor

the ratio of the mass of the proton to that of the electron), the larger numbers, namely the ratio of the electric to the gravitational force between electron and proton, which is about 10^{39}, and the ratio of the mass of the universe to the mass of the proton, which is about 10^{78}, are so enormous as to make one think that some entirely different type of explanation is needed for them.

According to current cosmological theories, the universe had a beginning about 2×10^9 years ago, when all the spiral nebulæ were shot out from a small region of space, or perhaps from a point. If we express this time, 2×10^9 years, in units provided by the atomic constants, say the unit e^2/mc^3, we obtain a number about 10^{39}. This suggests that the above-mentioned large numbers are to be regarded, not as constants, but as simple functions of our present epoch, expressed in atomic units. We may take it as a general principle that all large numbers of the order 10^{39}, 10^{78} . . . turning up in general physical theory are, apart from simple numerical coefficients, just equal to t, t^2 . . . , where t is the present epoch expressed in atomic units. The simple numerical coefficients occurring here should be determinable theoretically when we have a comprehensive theory of cosmology and atomicity. In this way we avoid the need of a theory to determine numbers of the order 10^{39}.

P. A. M. DIRAC.

St. John's College,
Cambridge.
Feb. 5.

Volume 150B, number 6 PHYSICS LETTERS 24 January 1985

A MECHANISM FOR REDUCING THE VALUE OF THE COSMOLOGICAL CONSTANT

L.F. ABBOTT [1]

Physics Department, Brandeis University, Waltham, MA 02254, USA

Received 30 October 1984

A mechanism is presented for relaxing an initially large, positive cosmological constant to a value near zero. This is done by introducing a scalar field whose vacuum energy compensates for the initial cosmological constant. The compensating sector involves small mass scales but no unnatural fine-tuning of parameters. It is not clear how to incorporate this mechanism into a realistic cosmology.

$$V = \epsilon B/f_B - \Lambda_{\mathrm{ph}}^4 \cos(B/f_B) + V_0,$$

PHYSICAL REVIEW D, VOLUME 70, 063501

Cosmic attractors and gauge hierarchy

Gia Dvali[1] and Alexander Vilenkin[2]
[1]*Center for Cosmology and Particle Physics, Department of Physics, New York University, New York, New York 10003, USA*
[2]*Institute of Cosmology, Department of Physics and Astronomy, Tufts University, Medford, Massachusetts 02155, USA*

(Received 31 July 2003; published 1 September 2004)

We suggest a new cosmological scenario which naturally guarantees the smallness of scalar masses and vacuum expectation values , without invoking supersymmetry or any other (nongravitationally coupled) new physics at low energies. In our framework, the scalar masses undergo discrete jumps due to nucleation of closed branes during (eternal) inflation. The crucial point is that the step size of variation decreases in the direction of decreasing scalar mass. This scenario yields exponentially large domains with a distribution of scalar masses, which is sharply peaked around a hierarchically small value of the mass. This value is the "attractor point" of the cosmological evolution.

Higgs (h) and Axion-like (ϕ) Interplay

3 terms:

$$V(\phi, h) = \boxed{\Lambda^3 g\phi} - \boxed{\frac{1}{2}\Lambda^2\left(1-\frac{g\phi}{\Lambda}\right)h^2} + \boxed{\epsilon\Lambda_c^4\left(\frac{h}{\Lambda_c}\right)^n \cos(\phi/f)}$$

relaxion rolling potential

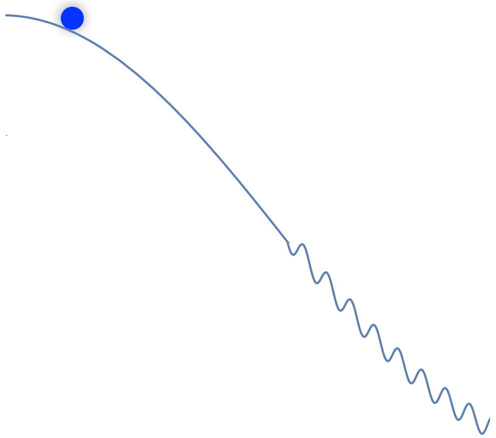
slope for Φ to move forward

relaxion-dependent Higgs mass

Φ scans the Higgs mass

Backreaction sector

barrier stopping Φ when <h> turns on



Higgs (h) and Axion-like (ϕ) interplay

$$V(\phi,h)=\Lambda^3 g\phi-\frac{1}{2}\Lambda^2\left(1-\frac{g\phi}{\Lambda}\right)h^2+\boxed{\epsilon\Lambda_c^4\left(\frac{h}{\Lambda_c}\right)^n\cos(\phi/f)}$$

n=1,2,...

Barrier that stops ϕ when <h> turns on

periodic function for ϕ
as for axion-like states
generated at scale Λ_c

e.g: QCD axion case: n=1, $\Lambda_c \sim \Lambda_{QCD}$

$\epsilon \sim y_u$

Higgs (h) and Axion-like (ϕ) interplay

$$V(\phi,h) = \Lambda^3 g\phi - \frac{1}{2}\Lambda^2\left(1-\frac{g\phi}{\Lambda}\right)h^2 + \epsilon\Lambda_c^4\left(\frac{h}{\Lambda_c}\right)^n \cos(\phi/f)$$

g<<1, breaks the shift symmetry $\phi \rightarrow \phi + c$

ϵ<<1, breaks the shift symmetry

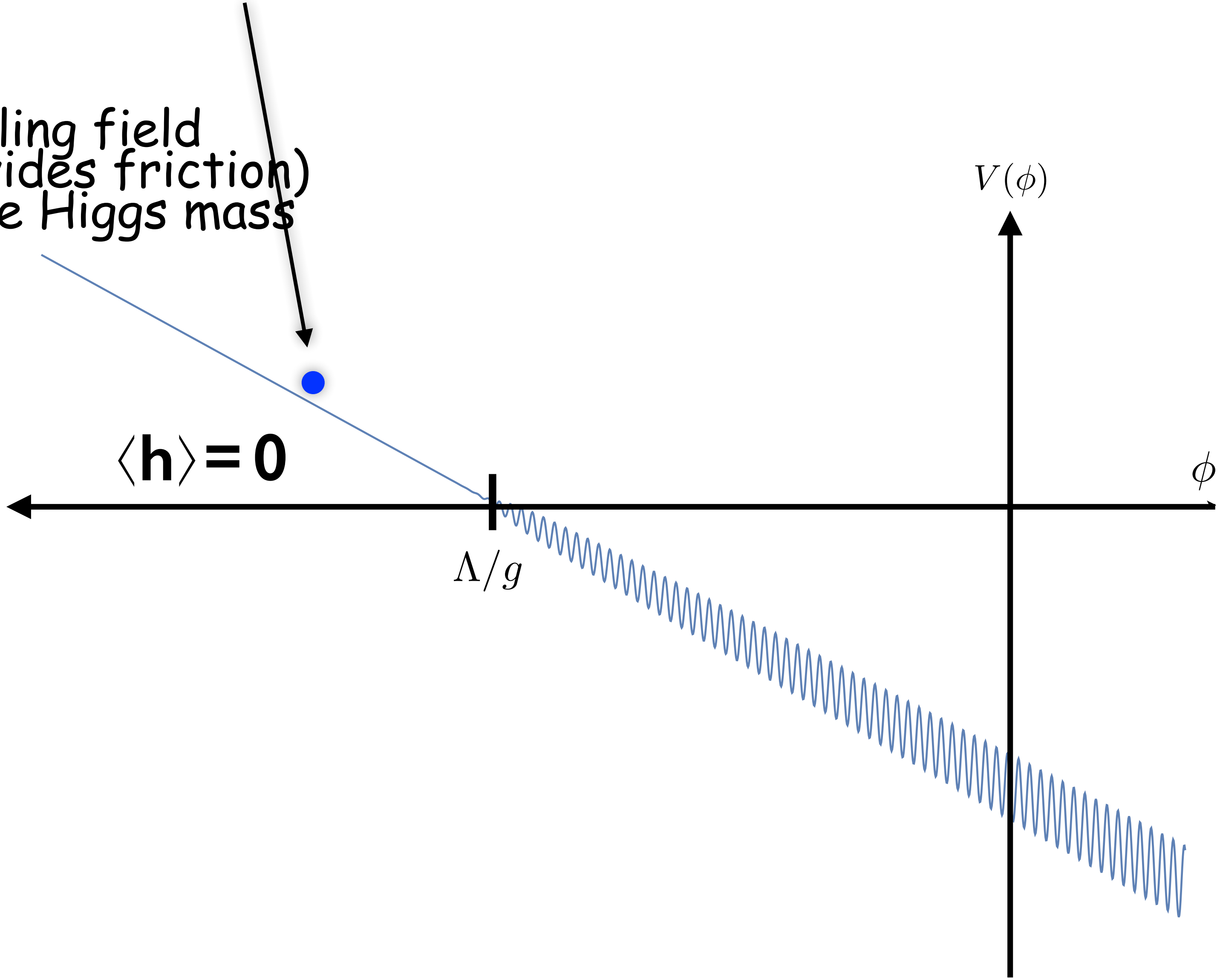
respects $\phi \rightarrow \phi + 2\pi f$

$\phi \rightarrow -\phi$

Potential stable under radiative corrections!

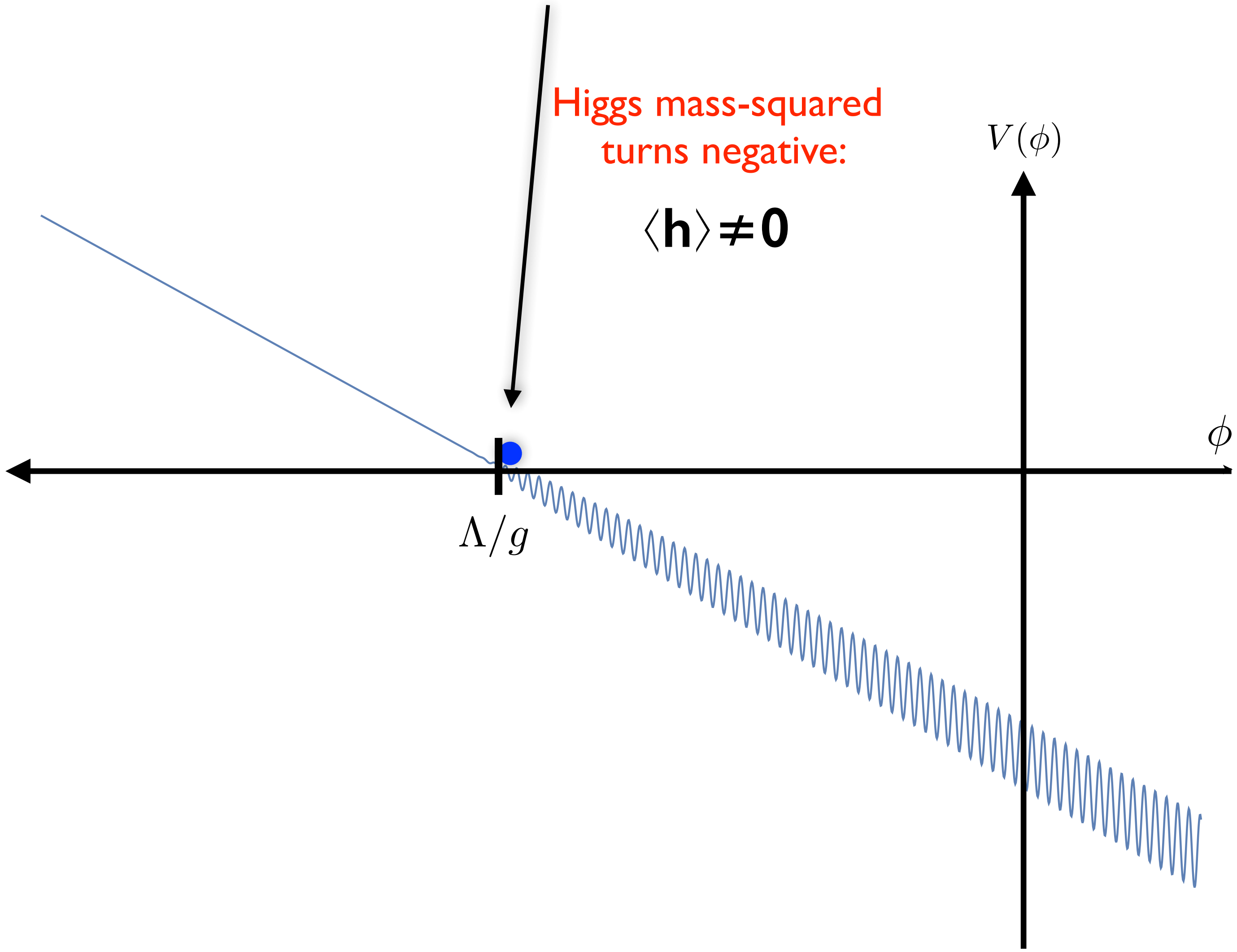
Cosmological evolution

$$V(\phi,h) = \Lambda^3 g\phi - \frac{1}{2}\Lambda^2\left(1-\frac{g\phi}{\Lambda}\right)h^2 + \epsilon\Lambda_c^4\left(\frac{h}{\Lambda_c}\right)^n\cos(\phi/f)$$



Cosmological evolution

$$V(\phi, h) = \Lambda^3 g\phi - \frac{1}{2}\Lambda^2\left(1 - \frac{g\phi}{\Lambda}\right)h^2 + \epsilon\Lambda_c^4\left(\frac{h}{\Lambda_c}\right)^n \cos(\phi/f)$$

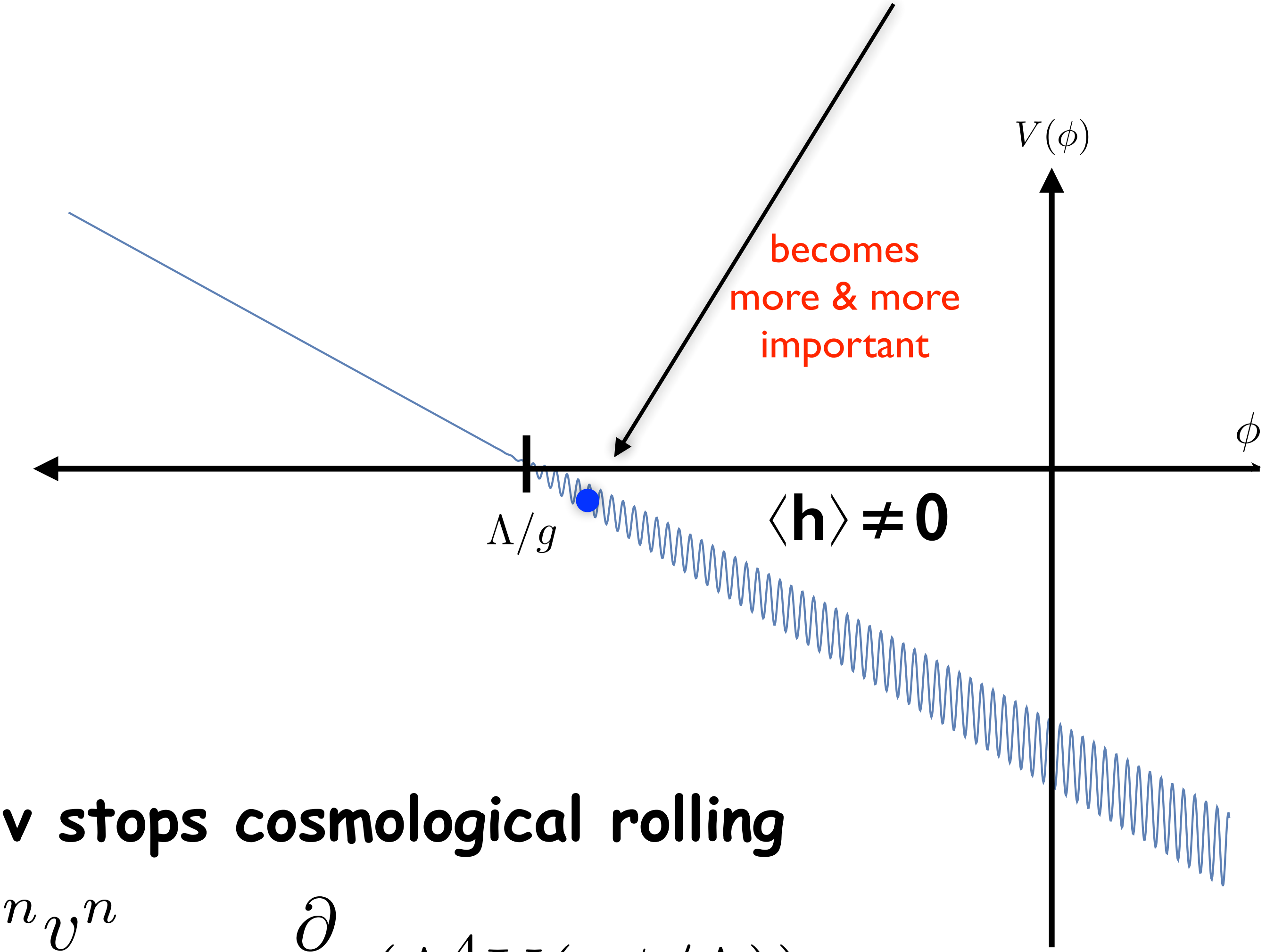


Higgs mass-squared turns negative:

$\langle h\rangle \neq 0$

Cosmological evolution

$$V(\phi,h) = \Lambda^3 g\phi - \frac{1}{2}\Lambda^2\left(1-\frac{g\phi}{\Lambda}\right)h^2 + \epsilon\Lambda_c^4\left(\frac{h}{\Lambda_c}\right)^n \cos(\phi/f)$$

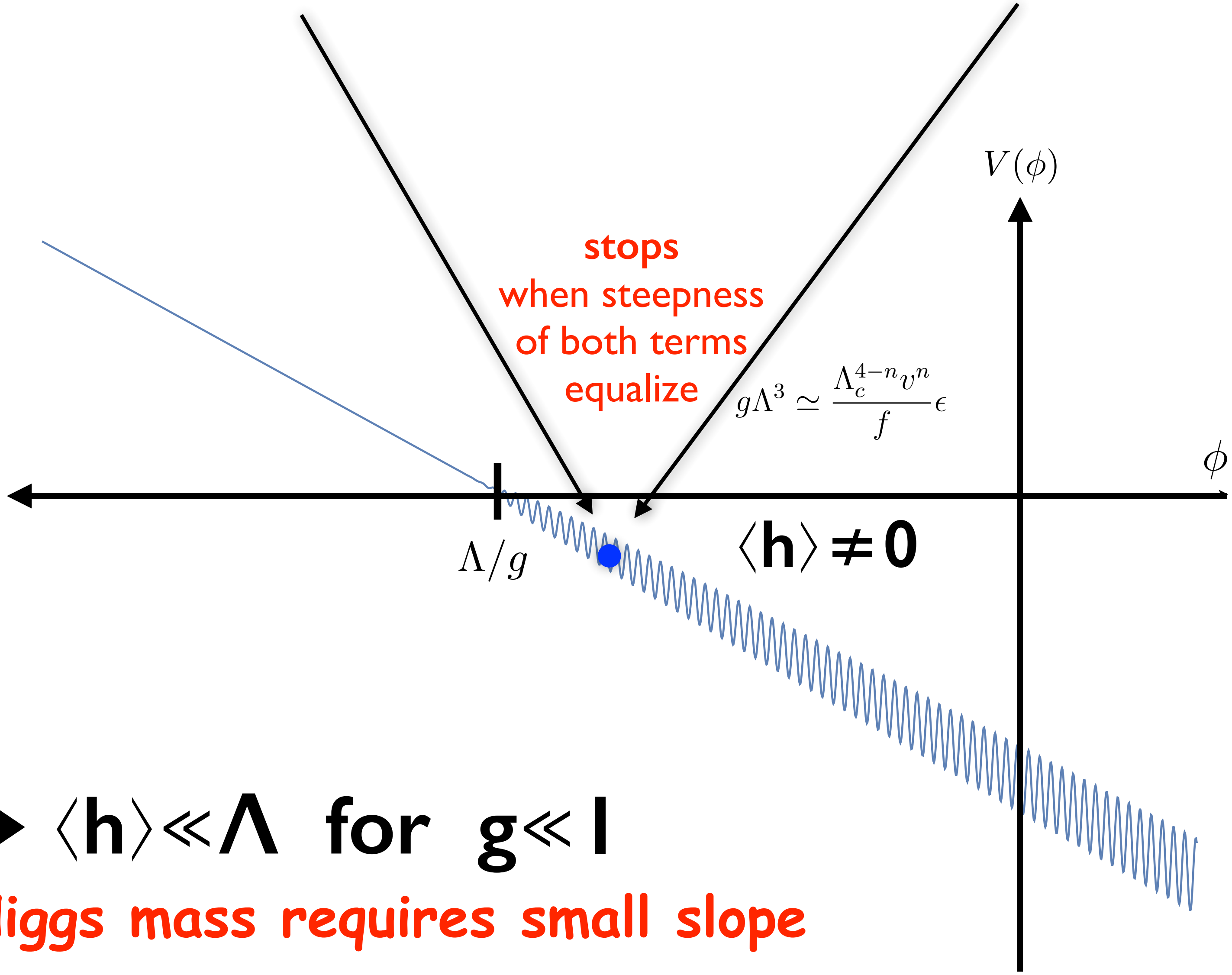


Higgs vev stops cosmological rolling

$$\frac{\Lambda_c^{4-n}v^n}{f}\epsilon \sim \frac{\partial}{\partial\phi}(\Lambda^4 V(g\phi/\Lambda))$$

Cosmological evolution

$$V(\phi,h) = \Lambda^3 g\phi - \frac{1}{2}\Lambda^2\left(1-\frac{g\phi}{\Lambda}\right)h^2 + \epsilon\Lambda_c^4\left(\frac{h}{\Lambda_c}\right)^n \cos(\phi/f)$$



stops
when steepness
of both terms
equalize

$$g\Lambda^3 \simeq \frac{\Lambda_c^{4-n} v^n}{f}\epsilon$$

$\langle h\rangle \neq 0$

➥ $\langle h\rangle \ll \Lambda$ for $g \ll 1$

small Higgs mass requires small slope

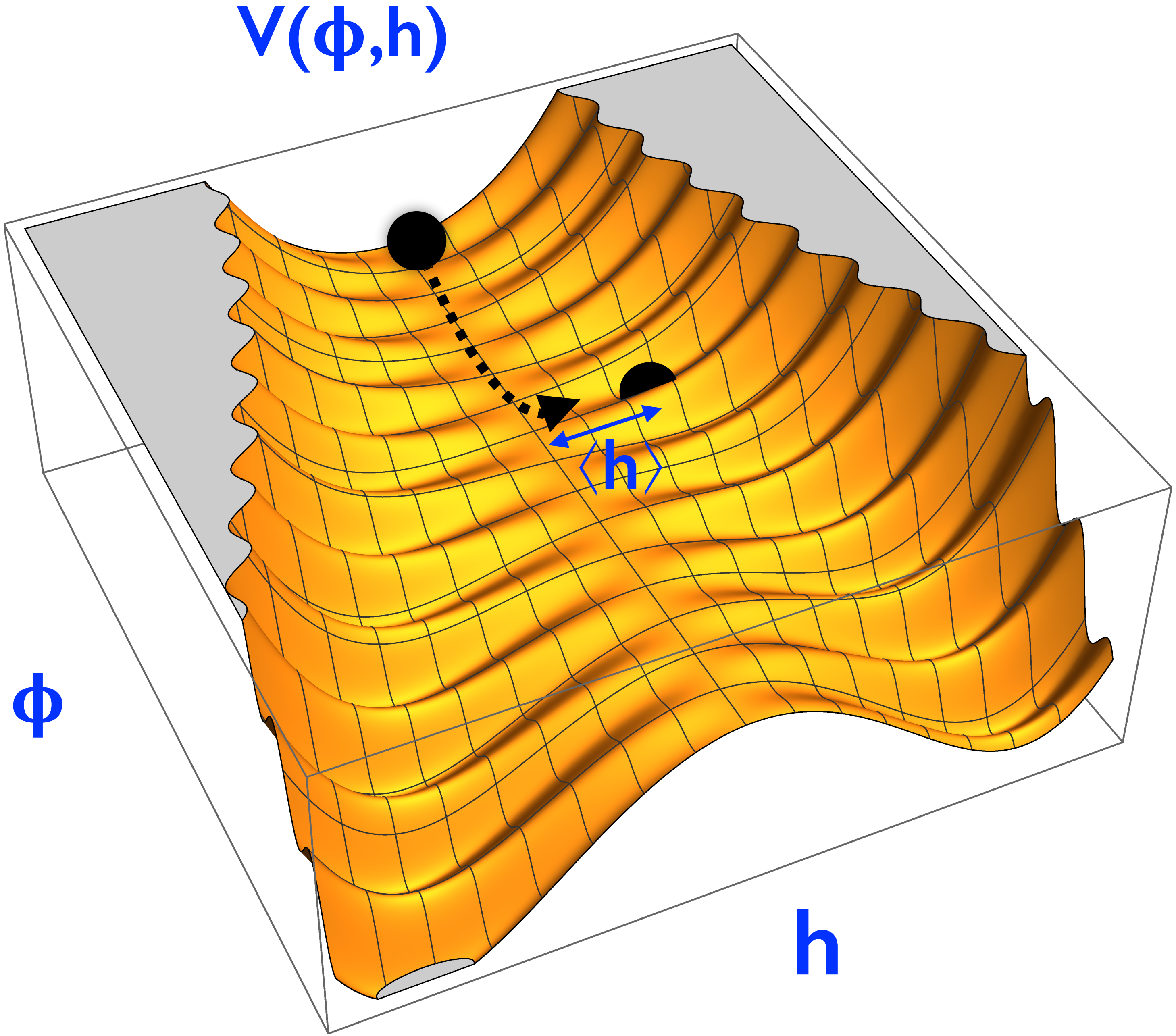
Cosmological evolution

$$V(\phi, h) = \Lambda^3 g\phi - \frac{1}{2}\Lambda^2 \left(1 - \frac{g\phi}{\Lambda}\right) h^2 + \epsilon \Lambda_c^4 \left(\frac{h}{\Lambda_c}\right)^n \cos(\phi/f)$$

Large field excursions for ϕ needed

$$\phi \sim \Lambda/g \gg \Lambda$$

No dependence on initial conditions, provided that this takes place during inflation


V(ϕ,h)
⟨h⟩
ϕ
h

Conditions:

Slow rolling:

$$H_I > \frac{\Lambda^2}{M_P}$$

ensures that the energy density stores in Φ does not affect inflation

from friction due to inflation

Needed to avoid overshooting the EW range vacua

$$N_{efolds} \gtrsim \frac{H_I^2}{g^2\Lambda^2}$$

for complete scanning of the Higgs mass

Classical rolling

classical displacement over one Hubble time > quantum fluctuation

$$\frac{1}{H_I}\frac{d\phi}{dt} = \frac{1}{H_I^2}\frac{dV}{d\phi} = \frac{g\Lambda^3}{H_I^2} > H_I$$

-> $H_I \lesssim g^{1/3}\Lambda$

putting all these constraints together tells you what is the maximal cutoff Λ

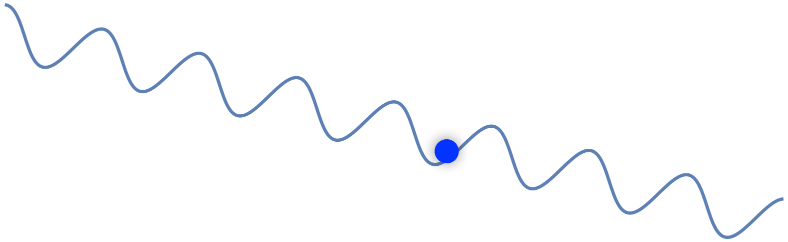
n=2: $\Lambda \lesssim (v^4 M_P^3)^{1/7} \simeq 2 \times 10^9\,\mathrm{GeV}$

Origin of $\epsilon\Lambda_c^4 \left(\frac{h}{\Lambda_c}\right)^n \cos(\phi/f)$

For n=1: From QCD condensate $\Lambda_c = \Lambda_{QCD}$

$$\frac{\phi}{f}\tilde{G}^{\mu\nu}G_{\mu\nu} \longrightarrow m_u(h)\langle q\bar{q}\rangle \cos(\phi/f)$$

but leads to $\theta_{QCD} \sim 1$ due to the tilt!



Problem solved if the tilt disappears at the end of inflation but one gets

Λ≲30 TeV

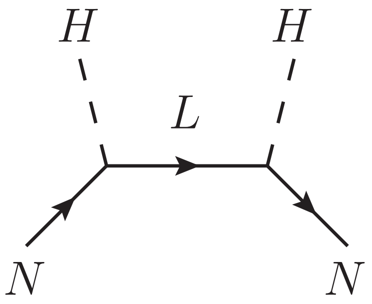
Origin of $\epsilon\Lambda_c^4\left(\frac{h}{\Lambda_c}\right)^n\cos(\phi/f)$

For n=2: $\epsilon\Lambda_c^2|H|^2\cos(\phi/f)$ **gauge invariant, no need to rely on QCD**

Similarly to QCD, the anomalous interaction term $\frac{\phi}{f}G'_{\mu\nu}\tilde{G}'^{\mu\nu}$

can be rotated away by a chiral rotation for N , and replaced by the term

$m_N e^{i\phi/f}\bar{N}N + h.c \to \Lambda^3 m_N\cos(\phi/f)$ where $\langle\bar{N}N\rangle \sim \Lambda^3$



$m_N \sim y^2|H|^2/m_L$

but $\epsilon\Lambda_c^4\cos(\phi/f)$ is generated by closing H in loop

Origin of $\epsilon\Lambda_c^4\left(\frac{h}{\Lambda_c}\right)^n\cos(\phi/f)$

For n=2: $\epsilon\Lambda_c^2|H|^2\cos(\phi/f)$ **gauge invariant,**

no need to rely on QCD

but $\epsilon\Lambda_c^4\cos(\phi/f)$ is generated by closing H in loop

and will stop □ before the Higgs vev develops

for the Higgs VEV to be responsible for stopping the rolling of phi, we need

$$\Lambda_c \lesssim v$$

coincidence problem!! similar to the mu pb in the MSSM

Important drawback: weak scale is put by hand.

Solution: make the envelop of the oscillatory potential field-dependent

[1506.09217]

Cosmological Higgs-Axion INterplay (CHAIN)

J.R. Espinosa, C. Grojean, G. Panico, A. Pomarol, O. Pujolàs, G. Servant, [1506.09217]

Two-scanners potential:

$$\boxed{V(\phi,\sigma,H) = \Lambda^4\left(\frac{g\phi}{\Lambda}+\frac{g_\sigma\sigma}{\Lambda}\right) + m^2(\phi)|H|^2 + A(\phi,\sigma,H)\cos(\phi/f)}$$

$$A(\phi,\sigma,H) \equiv \epsilon\Lambda^4\left(\beta + c_\phi\frac{g\phi}{\Lambda} - c_\sigma\underbrace{\frac{g_\sigma\,\sigma}{\Lambda}} + \frac{|H|^2}{\Lambda^2}\right)$$

generated at loop level

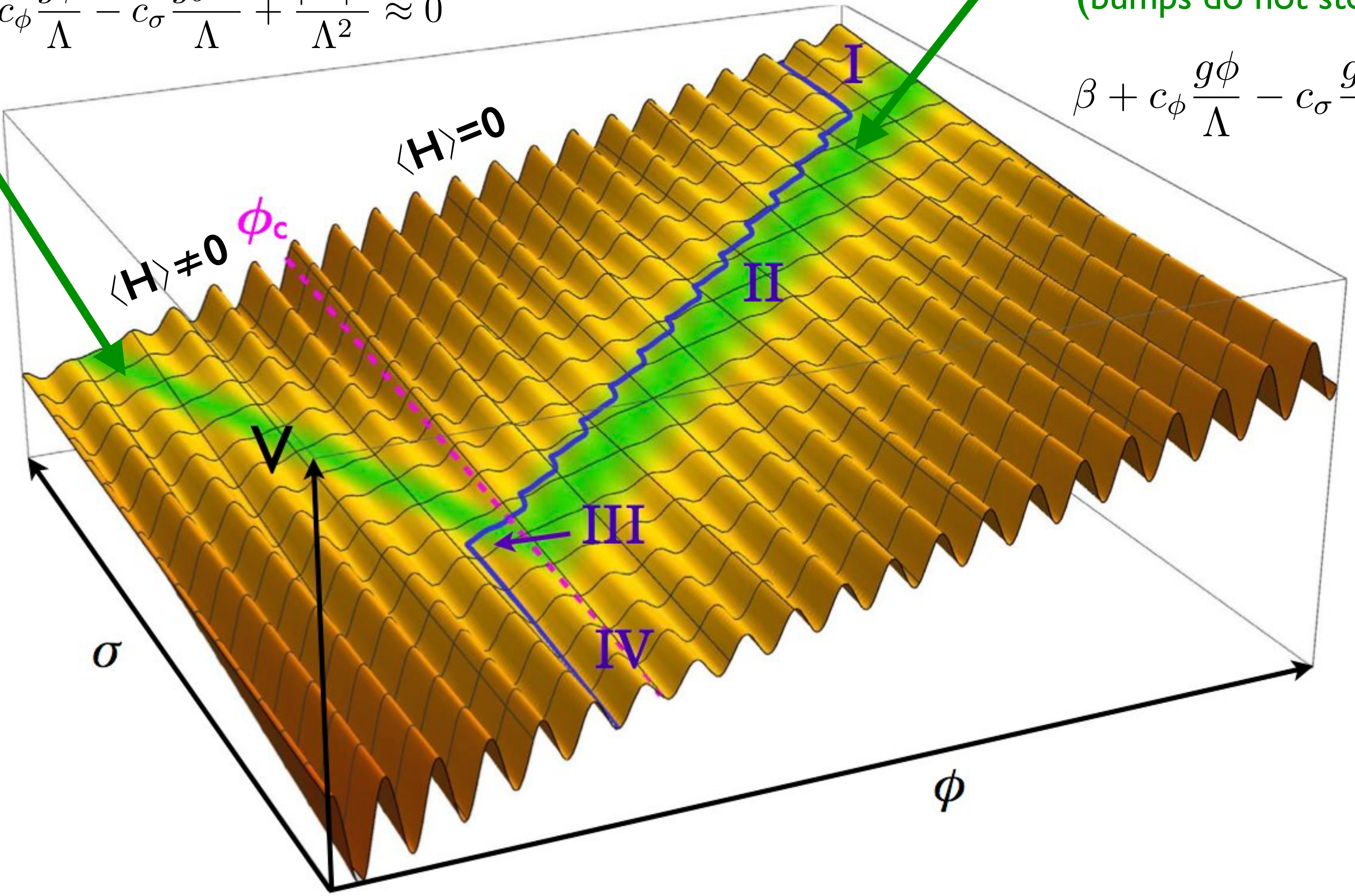
σ **scans the amplitude of the oscillating term**

generated by strong dynamics at scale Λ

ALPine Cosmology

$$V(\phi,\sigma,H)=\Lambda^4\left(\frac{g\phi}{\Lambda}+\frac{g_\sigma\sigma}{\Lambda}\right)+m^2(\phi)|H|^2+A(\phi,\sigma,H)\cos\left(\phi/f\right)$$

$$A(\phi,\sigma,H)\equiv\epsilon\Lambda^4\left(\beta+c_\phi\frac{g\phi}{\Lambda}-c_\sigma\frac{g_\sigma\,\sigma}{\Lambda}+\frac{|H|^2}{\Lambda^2}\right)$$



area where A≈0

$$\beta+c_\phi\frac{g\phi}{\Lambda}-c_\sigma\frac{g_\sigma\,\sigma}{\Lambda}+\frac{|H|^2}{\Lambda^2}\approx 0$$

area where A≈0

(bumps do not stop ϕ)

$$\beta+c_\phi\frac{g\phi}{\Lambda}-c_\sigma\frac{g_\sigma\,\sigma}{\Lambda}\approx 0$$

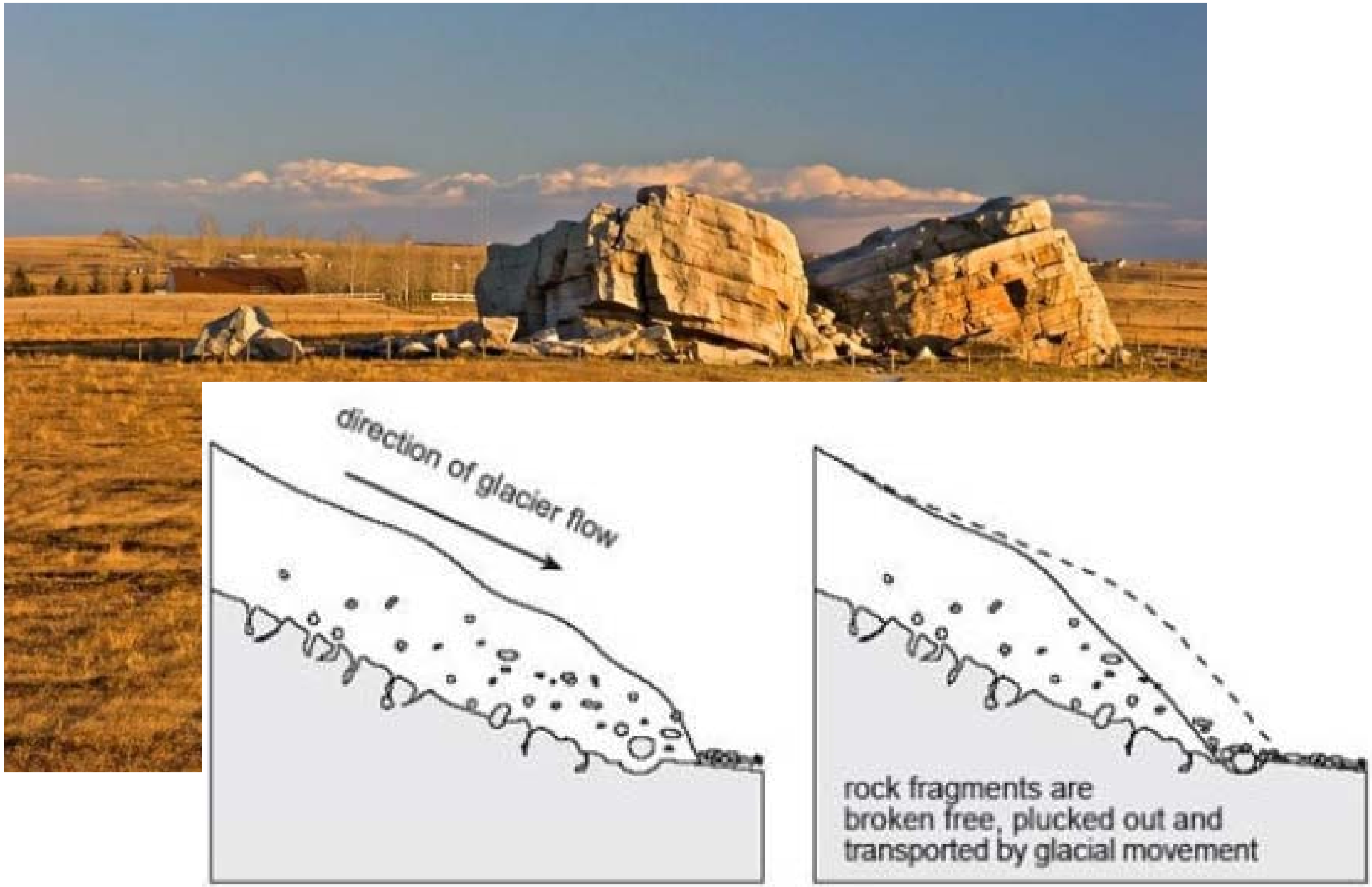
EW SCALE AS COSMOLOGICAL ERRATIC

[JR Espinosa]

okotoks glacial erratic,
Alberta, Canada

EW SCALE AS COSMOLOGICAL ERRATIC

[JR Espinosa]



Unnatural large rocks differing in composition from the typical surrounding ones as a result of a long geological history.

The apparently unnatural EW scale is the result of a long cosmological evolution of an axion-like particle.

Conditions on parameters:

- $\epsilon \lesssim v^2/\Lambda^2$ to avoid to be dominated by terms like $\epsilon^2\Lambda^4\cos^2(\phi/f)$
- $H_I^3 \lesssim g_\sigma \Lambda^3$ to avoid quantum wiggles spoiling classical rolling
- $g_\sigma \lesssim g$ to avoid $\boldsymbol{\phi}$ not tracking σ
- $\frac{\Lambda^2}{M_P} \lesssim H_I$ to avoid $\boldsymbol{\phi}$ & σ affect inflation

Minimization: $v^2 \simeq \frac{g\Lambda f}{\epsilon}$

$$\frac{\Lambda^3}{M_P^3} \lesssim \overbrace{g_\sigma \lesssim g}^{} \lesssim \frac{v^4}{f\Lambda^3} \longrightarrow \boxed{\Lambda \lesssim (v^4 M_P^3)^{1/7} \simeq 2 \times 10^9\,\mathrm{GeV}}$$

not yet fully solving the hierarchy problem
but pushing Λ beyond LHC & future colliders reach !

Phenomenological implications of this minimal model:

- Nothing at the LHC
- Only BSM below Λ :

Two light and very weakly coupled scalars:

$m_\phi \sim 10^{-20} - 10^{2}$ GeV

$m_\sigma \sim 10^{-45} - 10^{-2}$ GeV

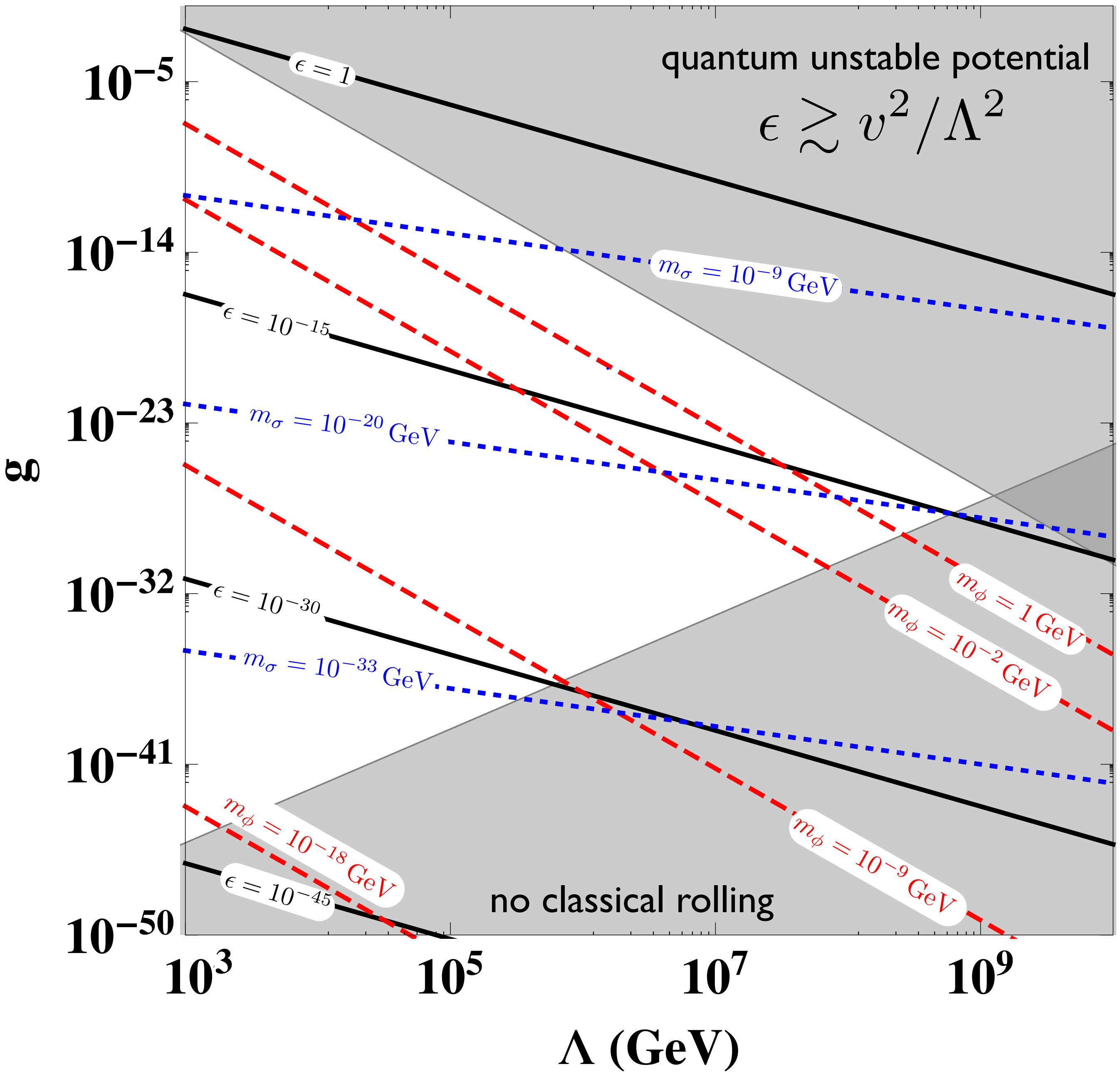
Couple to the SM through their mixing with the Higgs

benchmark values: $\Lambda \sim 10^{9}$ GeV ☛ $m_\phi \sim 100$ GeV
$\theta_{\phi h} \sim 10^{-21}$
$\phi\phi hh$-coupling $\sim 10^{-14}$
$m_\sigma \sim 10^{-18}$ GeV
$\theta_{\sigma h} \sim 10^{-50}$

- Experimental tests from cosmological overabundances, late decays, Big bang Nucleosynthesis, Gamma-rays, Cosmic Microwave Background ...

Phenomenological implications:

Taking $g_\sigma \sim 0.1g$ & $f \sim \Lambda$



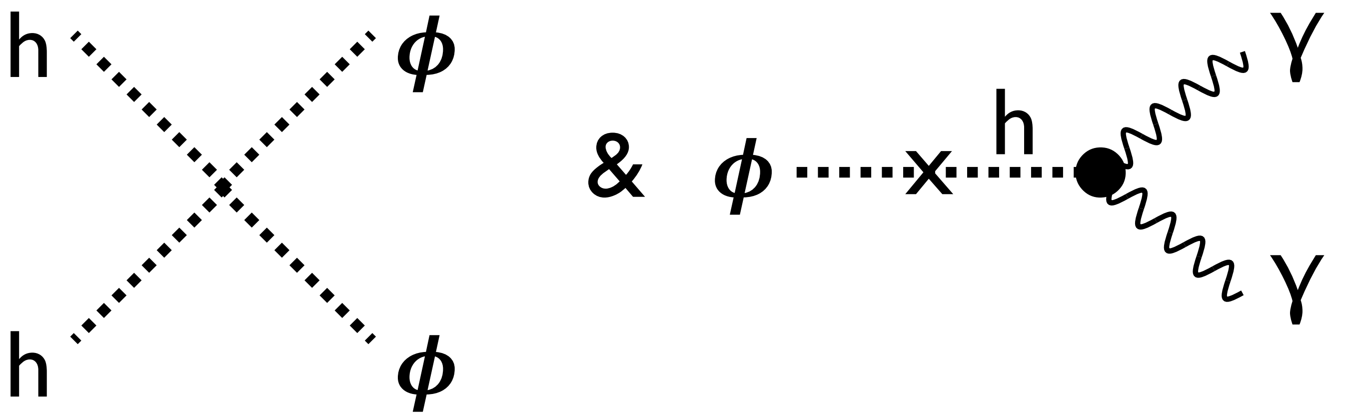
$$m_\phi^2 \sim \frac{\epsilon\Lambda^4}{f^2} \sim g\frac{\Lambda^5}{fv^2} \lesssim v^2$$

$$m_\sigma^2 \sim g_\sigma^2\Lambda^2 \ll m_\phi^2$$

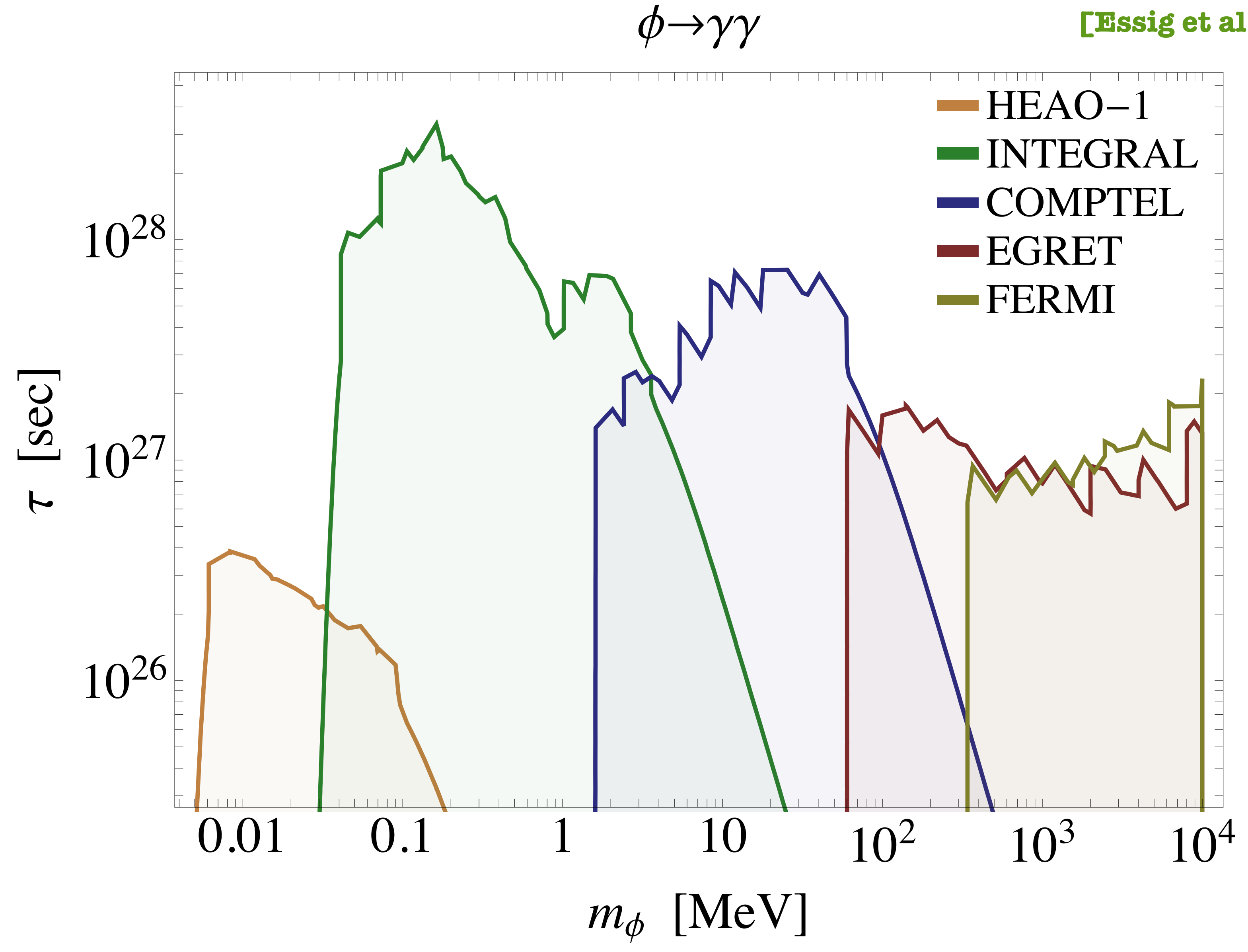
Physics of the slow-rollers:

σ **stable->Late classical oscillations-> cold dark matter**

ϕ **stable->Late decays**



Constraining Light and Long-Lived Dark Matter with gamma Ray observations



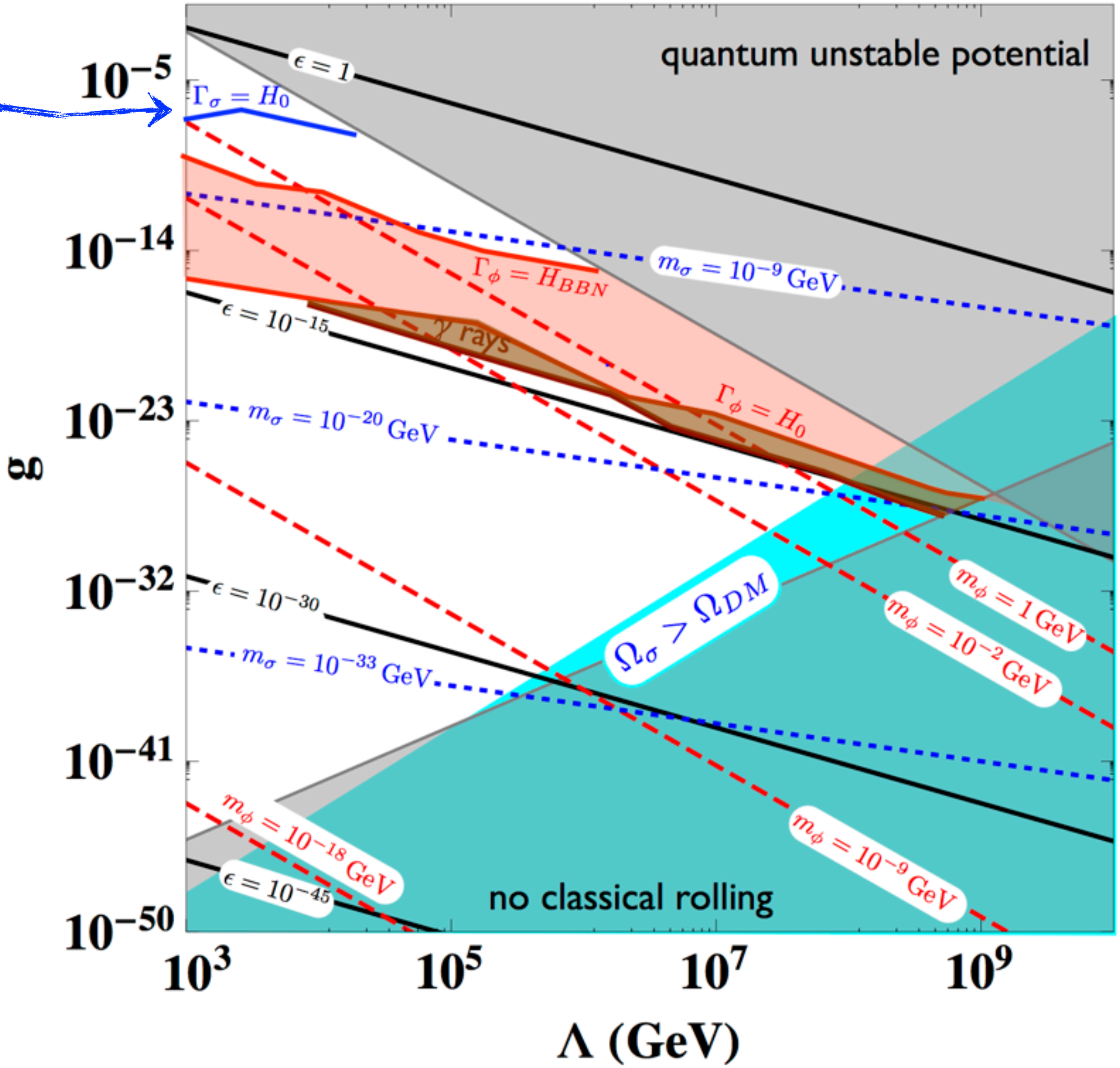
[Essig et al, 1309.4091]

Cosmological constraints

σ decays within the age of the Universe

ϕ decays after BBN

ϕ cosmologically stable



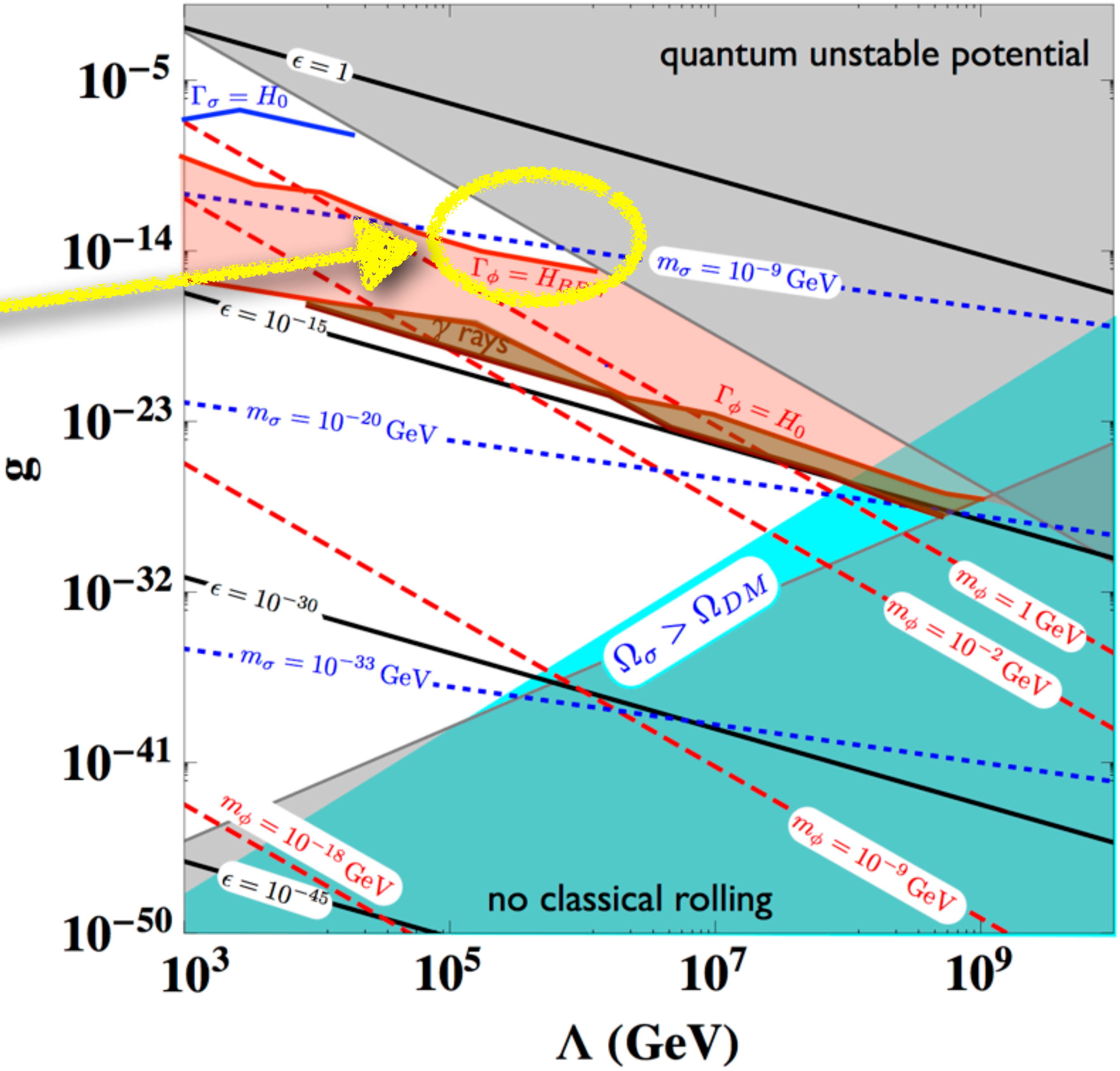
$$g = 10 g_\sigma$$

$$f = \Lambda$$

1506.09217

A minimal solution to the Little Hierarchy

`reasonable' region with moderately small coupling, moderately large field excursion, and a cut off scale @100-1000 TeV



$$g = 10 g_\sigma$$

$$f = \Lambda$$

1506.09217

The CHAIN mechanism

An existence proof of a model that generates a quantum stable large mass gap between the Higgs mass and the new physics threshold

Weak scale is not put by hand but generated dynamically

There are no light fermions to be found at the LHC

The only new physics scale:

$$\Lambda \sim \Lambda_c \gg v$$

Axions: ubiquitous in String Theory

Massless fields with axion-like properties generically arise in string theory compactifications

Their number is determined by the topology of the compact manifold
(non-equivalent embeddings of a closed two-surface into a reasonably complicated six-dimensional manifold.)

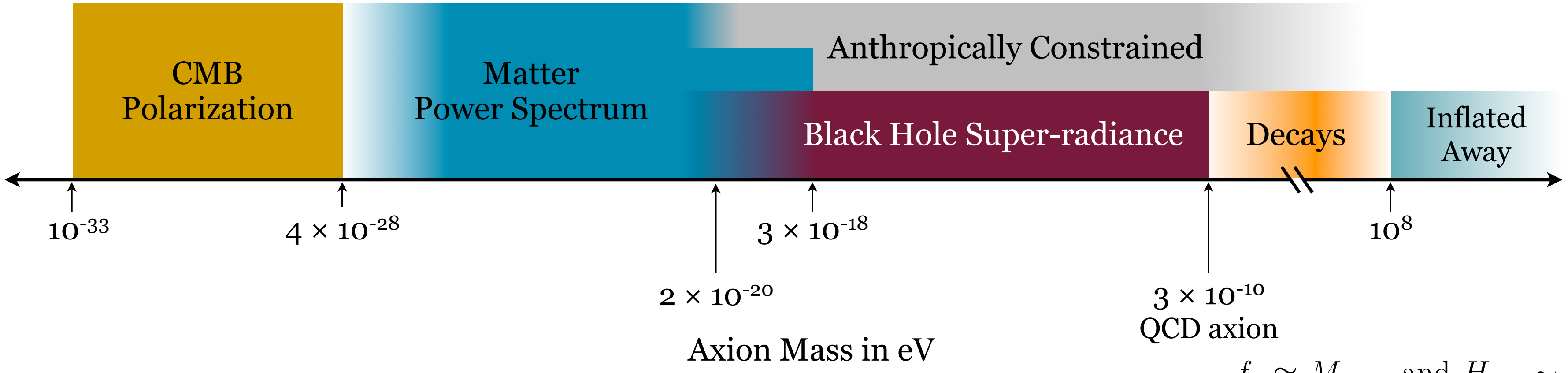
In most compactifications this number is of the order of several hundreds!

higher-dimensional gauge invariance -> shift symmetry in 4D!

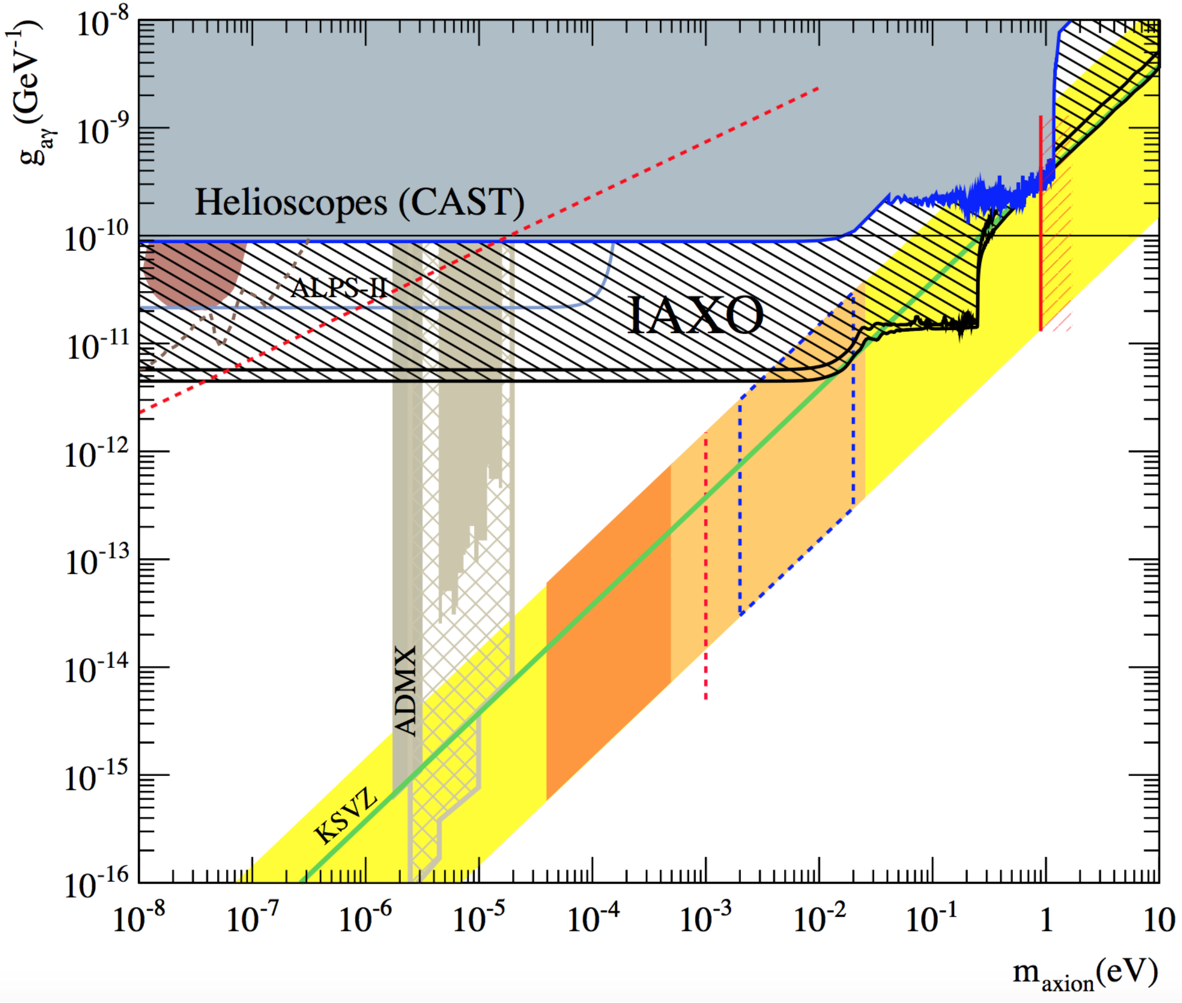
broken non-perturbatively by couplings to gauge fields -> generates a mass

-> provides many particles with the qualitative properties of the QCD axion.

["String Axiverse" 0905.4720]



$f_a \approx M_{GUT}$ and $H_{inf} \sim 10^8$ eV.


$g_{a\gamma}$ (GeV^{-1})
10^{-8}
10^{-9}
10^{-10}
10^{-11}
10^{-12}
10^{-13}
10^{-14}
10^{-15}
10^{-16}
Helioscopes (CAST)
ALPS-II
IAXO
ADMX
KSVZ
10^{-8}
10^{-7}
10^{-6}
10^{-5}
10^{-4}
10^{-3}
10^{-2}
10^{-1}
1
10
m_{axion}(eV)

Summary

- A new approach to the hierarchy problem based on intertwined cosmological history of Higgs and axion-like states. Connects Higgs physics with inflation & (DM) axions.

- An existence proof that technical naturalness does not require new physics at the weak scale

$$\Lambda < \left(v^4 M_P^3\right)^{1/7} = 3 \times 10^9 \, \mathrm{GeV}$$

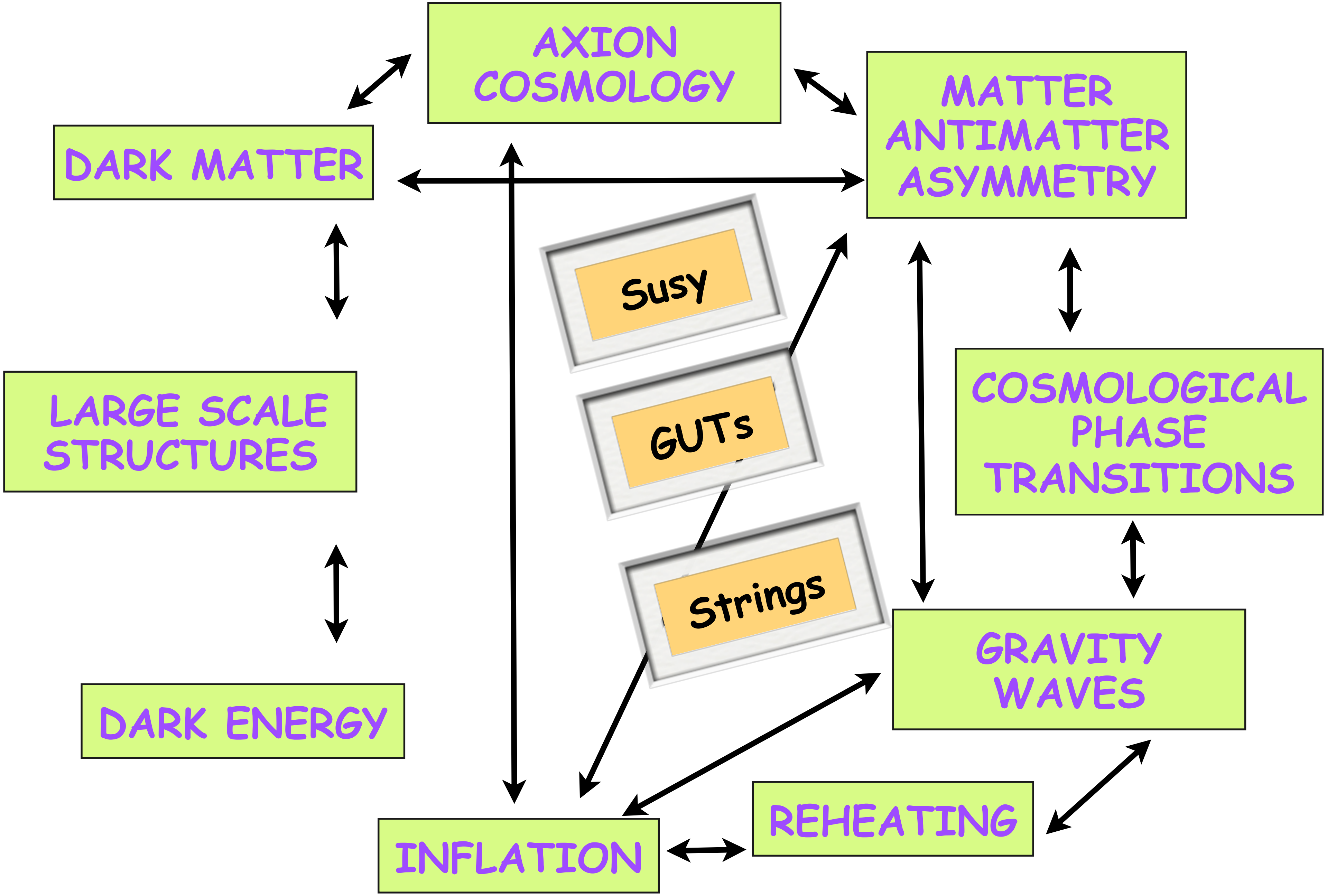
- Change of paradigm:

  no signature at the LHC , new physics are weakly coupled light states which couple to the Standard Model through their tiny mixing with the Higgs.

- Experimental tests from cosmological overabundances, late decays, Big Bang Nucleosynthesis, Gamma-rays, Cosmic Microwave Background...

Not a complete theory !

A new playground at the crossroads between particle phenomenology, cosmology, strings...



AXION COSMOLOGY

DARK MATTER

MATTER ANTIMATTER ASYMMETRY

Susy

GUTs

LARGE SCALE STRUCTURES

COSMOLOGICAL PHASE TRANSITIONS

Strings

GRAVITY WAVES

DARK ENERGY

REHEATING

INFLATION

Works in all these interface areas is needed to get a consistent picture

Open Questions

- Main challenge: Large (superplanckian) field excursions

  -> monodromy?

- Weak gravity conjecture

  Heidenreich, Reece, Rudelius '15
  Hebecker, Rompineve, Westphal '15

- UV completion?

  Choi, Im '15
  Kaplan, Rattazzi '15

- Inflation model building (at low scale)

- Signatures in low-energy experiments?

- Can other scales be relaxed too? SUSY breaking scale?

  Batell, Giudice, McCullough '15
  Evans, Gherghetta, Nagata, Thomas '16

  -> Use the relaxion mechanism to solve the Little Hierarchy and then SUSY takes over.

Supersymmetrize the SM + the QCD relaxion:

$$S = \frac{\underbrace{s}_{\text{srelaxion}} + i\underbrace{a}_{\text{relaxion}}}{\sqrt{2}} + \sqrt{2}\,\theta\underbrace{\tilde{a}}_{\text{relaxino (goldstino)}} + \theta^2\underbrace{F}_{\text{susy breaking}}$$

Batell, Giudice, McCullough '15

relaxion superfield is the SUSY breaking sector

scanning of Higgs mass through scanning of SUSY breaking scale

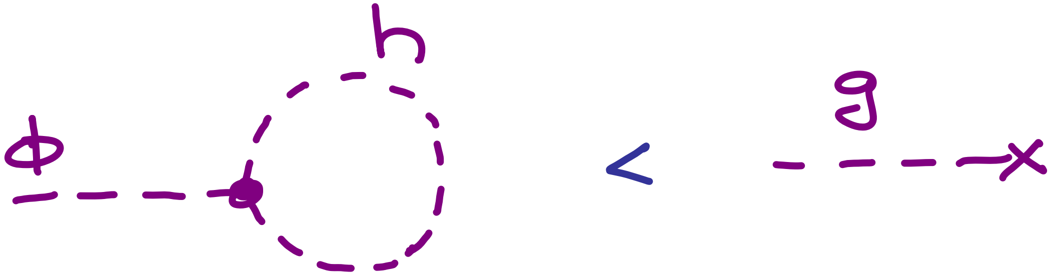
Mass spectrum		Phenomenology
$\tilde{q}, \tilde{\ell}, \tilde{H}, s$	$\sim$ 100 TeV	OK for m_H, flavor, dim-5 p-decay
$\tilde{g}, \tilde{W}, \tilde{B}$	$\sim$ 1 TeV	could be within LHC reach
SM	$\sim$ 100 GeV $\leftrightarrow$ MeV	
$\tilde{a}\,(\tilde{G})$	$\sim$ keV $\leftrightarrow$ GeV	LSP, DM for $T_{RH} \sim \tilde{m}$
a	$\sim 10^{-2} \leftrightarrow 10^{-5}$ eV	DM for $f \sim 10^{11\text{-}12}$ GeV (rel)axion couplings related to soft terms

"natural mini-split"

Annexes

Technical naturalness

V(H,Φ) is radiatively stable



$$g\Lambda \times \frac{\Lambda^2}{16\pi^2} < g\Lambda^3$$ ✓

Concerns about V(h,Φ) ?

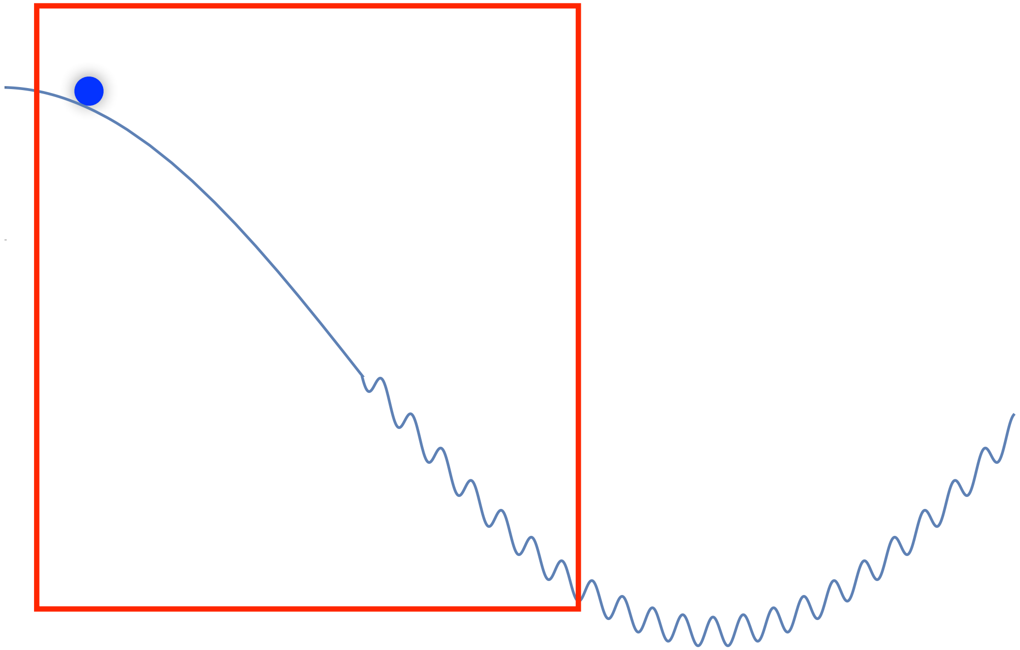
Relaxion potential may be obtained without breaking of shift symmetry but with hierarchy of decay constants, e.g. “clockwork axion”

Choi, Im’15

Kaplan, Rattazzi’15

Is this natural -> multiple axion models

$$V \sim A\cos(\frac{\phi}{f_{eff}}) + B\cos(\frac{\phi}{f_{eff}})h^2 + C(h)\cos(\frac{\phi}{f}), \qquad f_{eff} \sim e^{\zeta N} f \gg f$$



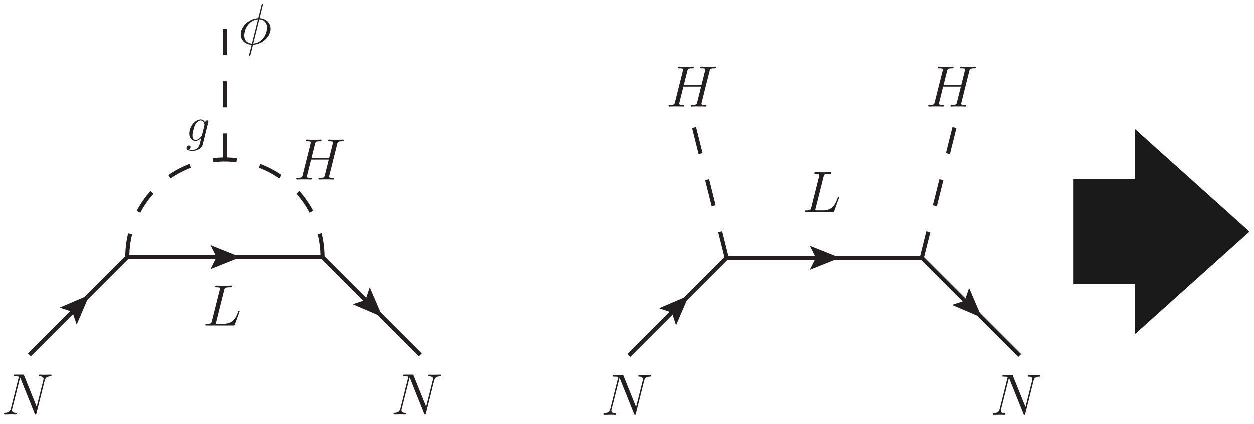
CHAIN UV Completion

New strong sector à la QCD with vector-like elementary quarks + axion-like field $\frac{\phi}{f} G'_{\mu\nu}\tilde{G}'^{\mu\nu}$.

L SU(2)$_L$ Dirac doublet
N SU(2)$_L$ Dirac singlet

$$\begin{aligned}\mathcal{L}_{\text{mass}} &= \Lambda \overline{L} L + \epsilon \Lambda \overline{N} N \\ \mathcal{L}_{\text{Yuk}} &= \sqrt{\epsilon}\, \overline{L} H N + h.c.\,. \\ \mathcal{L}_N &= \epsilon g \phi \overline{N} N + \epsilon g_\sigma \sigma \overline{N} N\end{aligned}$$

ε→0, additional chiral symmetry (broken by axial anomaly)



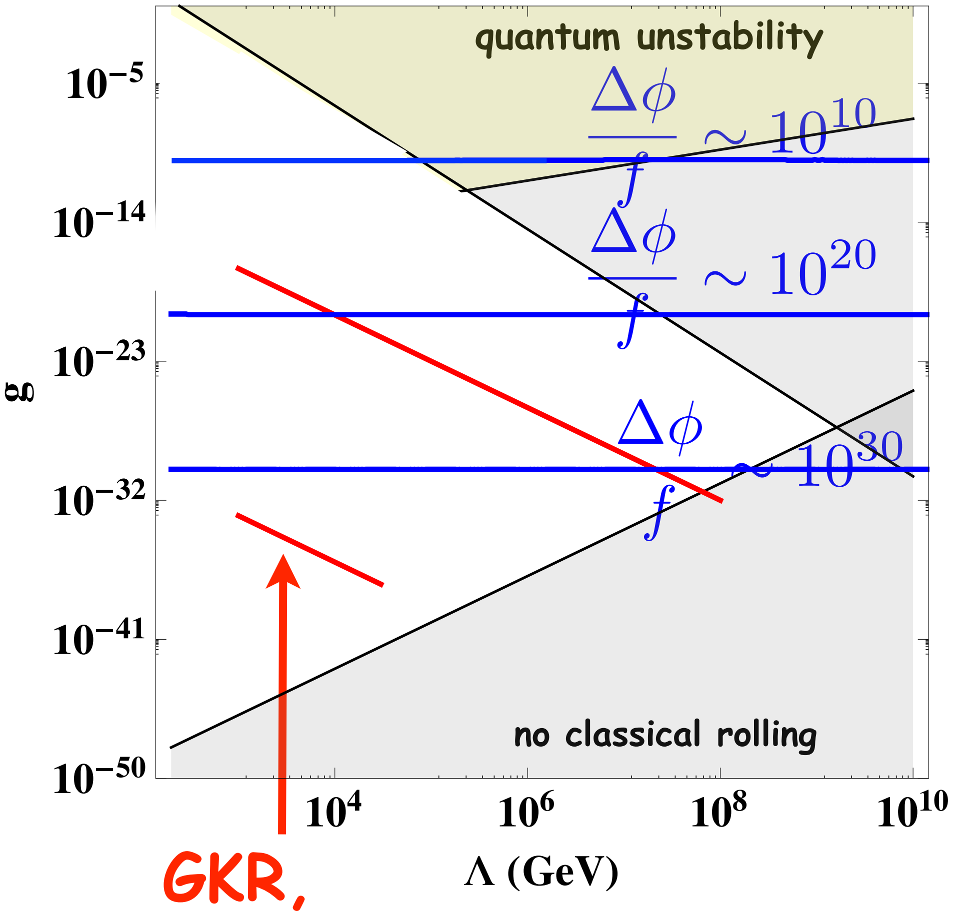
$$m_N \simeq \epsilon \left(\Lambda + g_\sigma \sigma + g\phi - \frac{|H|^2}{\Lambda} \right)$$

$$\boxed{\langle \bar{N} N \rangle \sim \Lambda^3 \Rightarrow V = \Lambda^3 m_N \cos\frac{\phi}{f}}$$

composite baryons and mesons @ Λ but no light meson since axial U(1) is anomalous

Comparison of relaxion models

CHAIN $f = \Lambda$

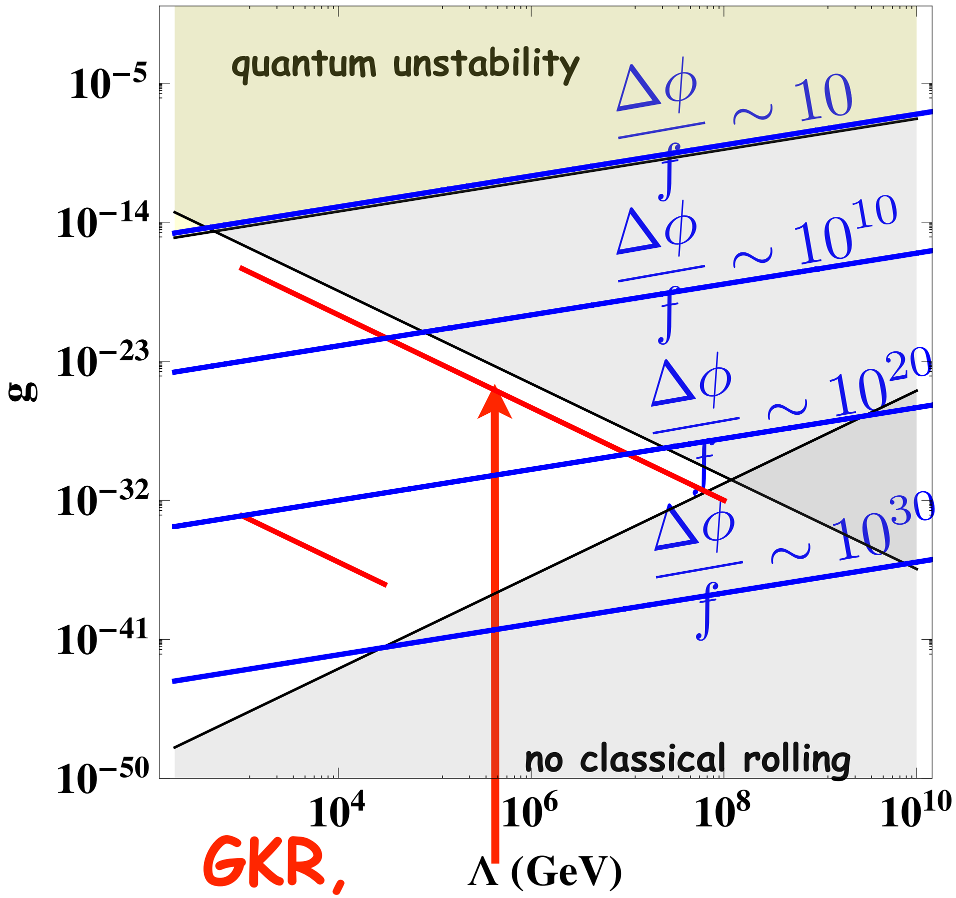


GKR,

QCD model

$\frac{\Delta\phi}{f} \sim 10^{20}$ $N_e \sim 10^{47}$

CHAIN $f = 10^{16}$ GeV



GKR,

non-QCD

model $\frac{\Delta\phi}{f} \sim 10^{8} - 10^{28}$

$N_e \sim 10^{15} - 10^{50}$

Comparison of relaxion models

	GKR 1	GKR 2	CHAIN with $f \sim M$
f	$f_{PQ} \sim 10^{10} - 10^{12}$ GeV	$\gtrsim M_{GUT} \sim 10^{16}$ GeV	$\gtrsim M$
g	$\frac{\Lambda^4_{QCD}\Theta_{QCD}}{(M^3 f_{PQ})} \lesssim 10^{-36}$	$\frac{\Lambda^4_{EW}}{(M^3 M_{GUT})} \sim 10^{-30} - 10^{-20}$	$\lesssim v^4/M^4 \sim 10^{-26} - 10^{-6}$
M_{max}	30 TeV	10^8 GeV	10^9 GeV
m_ϕ	$\frac{\Lambda^2_{QCD}}{f_{PQ}} \lesssim 10^{-11}$ GeV	$\frac{\Lambda^2_{EW}}{M_{GUT}} \lesssim 10^{-12}$GeV	$\sqrt{(gM^4/v^2)} \lesssim v$
$(\Delta\phi/f)$	$(\frac{M}{\Lambda_{QCD}})^4 \frac{1}{\Theta_{QCD}} \gtrsim 10^{30}$	$(M/\Lambda_{EW})^4 \sim 10^8 - 10^{24}$	$g^{-1} \sim 10^6 - 10^{26}$
$N_e\vert_{min}$	$\frac{M^8 f^2_{PQ}}{\Theta M^2_{Pl}\Lambda^8_{QCD}} \gtrsim 10^{47}$	$\frac{M^8 M^2_{GUT}}{M^2_{Pl}\Lambda^8_{EW}} \gtrsim 10^{12}$	$\frac{M^{10}}{v^8 M^2_{Pl}} \gtrsim \mathcal{O}(1)$

! Notation switched in this table . M is Λ !

Comparison of relaxion models