Stationary configurations of the Standard Model potential: EW stability and Higgs inflation

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based on *arXiv: 1604.06046 - submitted to PRD*,
in collaboration with dr. Isabella Masina

University of Ferrara and INFN - Ferrara

28th Rencontres de Blois
BSM/DM parallel session

Blois, 1st June 2016

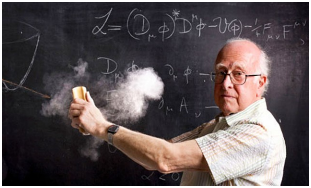
Outline

Outline

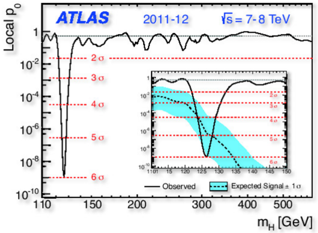
Scalar particle at LHC: very special universe!

Higgs boson mass

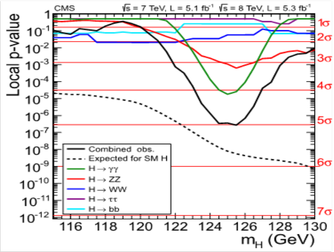
$$m_H = 125.09 \pm 0.21 \pm 0.11\,\text{GeV}$$



Now it's possible to study the SM potential up to the Planck scale



ATLAS, Phys.Lett.B 716(2012) 1-29



CMS, Phys.Lett.B 716(2012) 30

Stability and metastability

Higgs doublet (unitary gauge)

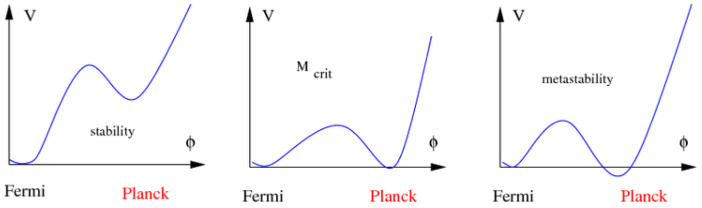
$$\mathcal{H}^{\mathcal{T}} = \begin{pmatrix} 0 & (\phi_H + v)/\sqrt{2} \end{pmatrix}$$

$$v = 1/(\sqrt{2}G_\mu)^{1/2} \simeq 246\, GeV$$

SM Higgs potential

$$V_0(\phi_H) = \frac{\lambda_H}{6}\left(|\mathcal{H}|^2 - \frac{v^2}{2}\right)^2$$

$$\sim \frac{\lambda_H}{24}\phi_H^4$$

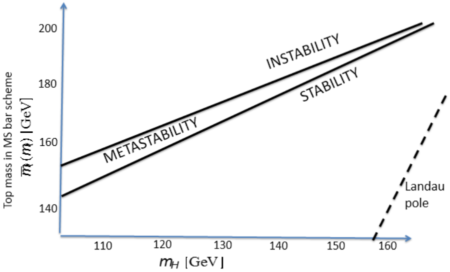


$\lambda\phi^4$, inflection point, plateau, ...

Degenerate vacua (Froggatt-Nielsen)

Tunneling rate $\tau > \tau_{universe}$

Vacuum stability diagram



Masina, Talk in Padua(2014)

$$\bar{m}_t^{\text{running}}(\mu) = y_t(\mu)\frac{v}{\sqrt{2}} =$$
$$= m_t^{pole}(1 + \delta_t(\mu))$$

Alekhin, Djouadi, Moch, Phys.Lett.B 716(2012) 214

- Extracted from the total cross-section $p\bar{p} \to t\bar{t} + X$
- Comparison rule: $m_t^{pole} \approx \bar{m}_t + 10\,GeV$

The two methods agree: Masina, PRD 87(2013) 053001

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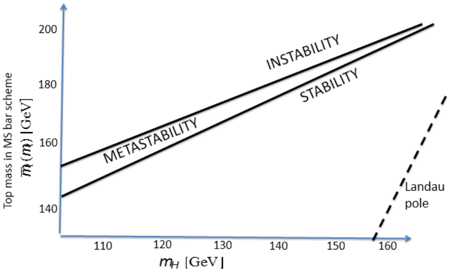

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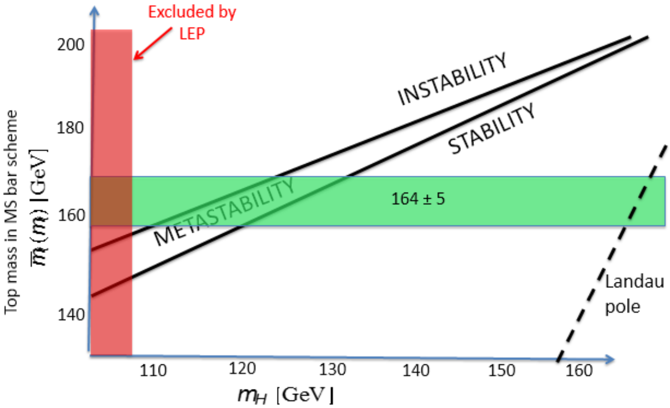
Experimental constraints (2000)

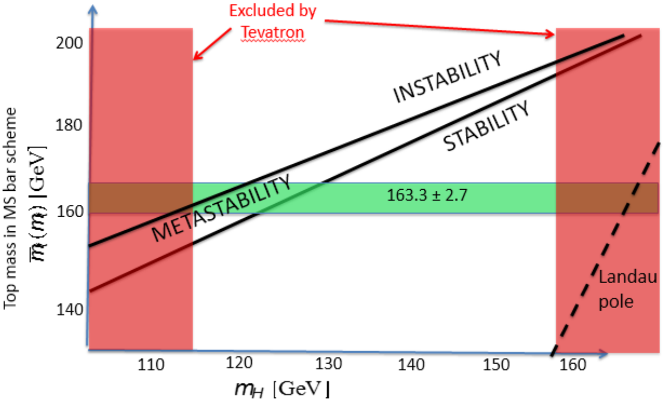
EW stability and inflection point

Giuseppe Iacobellis



Masina, Talk in Padua(2014)

Experimental constraints (2011)



Masina, Talk in Padua(2014)

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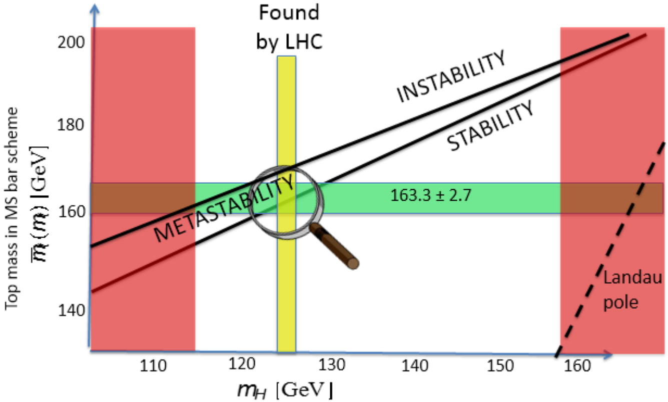

Masina, Talk in Padua(2014)

Is Nature trying to tell us something?

Outline

Calculation: some formulæ

Extrapolating the Higgs potential at high energies:
RGE evolution of the couplings (assuming "desert")

The effective potential must be independent of μ:

$$\left(\mu\frac{\partial}{\partial\mu}+\beta_i\frac{\partial}{\partial\lambda_i}-\gamma\frac{\partial}{\partial\phi}\right)V_{\text{eff}}=0\,,$$

$$\beta_i=\mu\frac{d\lambda_i}{d\mu}\,,\qquad \gamma=-\frac{\mu}{\phi}\frac{d\phi}{d\mu}$$

The formal solution of the RGE is

$$V_{\text{eff}}(\mu,\lambda_i,\phi)=V_{\text{eff}}(\mu(t),\lambda_i(t),\phi(t))\,,$$

$$\mu(t)=e^t\mu\,,\quad \phi(t)=e^{\Gamma(t)}\phi\,,\quad \Gamma(t)=-\int_0^t dt'\gamma(\lambda(t'))$$

t can be chosen in order to make $\phi(t)/\mu(t)\sim\mathcal{O}(1)$

Our choice: $\mu=m_t$

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Matching low-energy parameters at NNLO

Updating the results in literature:
Bednyakov et al., PRL 115(20) (2015) 201802

- Gauge couplings (g_3 in particular) matching is performed at m_Z and it is dominated by

$$\alpha_s^{(5)} = 0.1181 \pm 0.0013 \text{ (error doubled)}$$

- Uncertainty on λ is dominated by uncertainty on *Higgs mass* and *theoretical errors in the matching procedure* (scale variation and truncation)

$$\lambda(m_t) \simeq 0.7554 + 2.9 \times 10^{-3} \frac{m_H - m_H^{exp}}{\Delta m_H^{exp}} \pm 4.8 \times 10^{-3}$$

slight disagreement in literature (Degrassi et al., Buttazzo et al., Masina)

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Matching of Yukawa top coupling

Extrapolation of the $y_t(\mu)$ from the matching between the running top mass $\overline{m_t}(\mu)$ and the top pole mass m_t:

y_t matching

$$y_t(\mu)\frac{v}{\sqrt{2}} = \bar{m}_t(\mu) = m_t\left(1+\delta_t^W(\mu)+\delta_t^{QED}(\mu)+\delta_t^{QCD}(\mu)\right)$$

- *matching procedure theoretical uncertainty*
- *experimental top mass*: $m_t^{\text{exp}} = 173.34 \pm 0.74\, GeV$
  ATLAS, CDF, CMS, D0, arXiv: 1403.4427

(but what it is really measured is a MC parameter m_t^{MC}
$\rightarrow$ how much is the error committed in the identification?)

$$y_t(m_t) = 0.9359 + 4.4\times10^{-3}\frac{m_t - m_t^{\text{exp}}}{\Delta m_t^{\text{exp}}} \pm 1.4\times10^{-3}$$

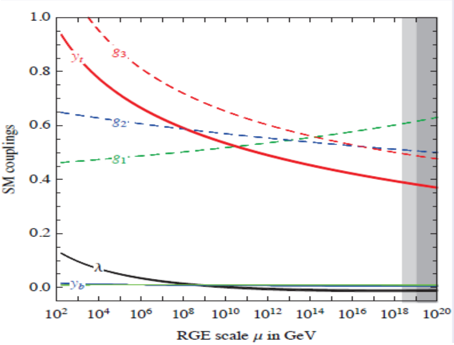
Running

(Mihaila, Salomon, Steinhauser; Chetyrkin, Zoller; Bednyakov, Pikelner, Velizhanin)

β-functions at NNLO (and 4-loop for strong coupling)

$$\frac{d\lambda_i(t)}{dt} = \kappa\beta^{(1)}_{\lambda_i} + \kappa^2\beta^{(2)}_{\lambda_i} + \kappa^3\beta^{(3)}_{\lambda_i}\,,$$

$$\kappa = 1/(16\pi^2)\,,\quad \lambda_i = \{g, g', g_3, y_t, \lambda, \gamma\}$$



We focus on the running of the Higgs quartic coupling λ

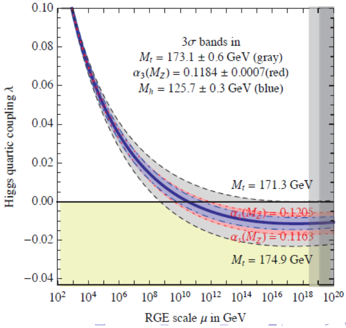
Degrassi et al., JHEP 08(2012) 098

Higgs quartic coupling: running of effective λ at NNLO

$$\frac{d\lambda}{d\ln\left(\frac{\mu}{m_t}\right)} \simeq \frac{1}{16\pi^2}\left[12\lambda^2 + 6\lambda y_t^2 - 3y_t^4 - \frac{3}{2}\lambda(3g'^2 + g^2) + \frac{3}{16}(2g'^4 + (g'^2 + g^2)^2)\right] + \lambda^{(2)}(\mu/m_t) + \lambda^{(3)}(\mu/m_t)$$

The running of λ is heavily dependent on the top *Yukawa* coupling (and α_s)

Stability or metastability?
Inflection point?



Buttazzo et al., JHEP 12(2013) 089

RGE-improved effective potential

$$V_{\text{eff}}(\phi) = V^{(0)}(\phi) + V^{(1)}(\phi) + V^{(2)}(\phi) \equiv \frac{1}{24}\lambda_{\text{eff}}(\mu)\phi^4$$

Coleman-Weinberg correction (Coleman, Weinberg, PRD 7(1973) 1888)

$$V^{(1)}(\phi) = \sum_{i=W^\pm,Z,t} \frac{n_i}{4(4\pi)^2} m_i(\phi)^4 \left[\ln \frac{m_i(\phi)^2}{\mu^2} - C_i\right]$$

Issue: gauge dependence?

2-loop correction
Ford, Jack, Jones, Nucl.Phys.B 387(1992) 373-390 and in more compact forms
Degrassi et al. JHEP 1208(2012) 098; Buttazzo et al. JHEP 12(2013) 089

$\lambda \to 0$: Higgs and Goldstone contributions are neglected

Issue: is this approximation theoretically justified?

Outline

Abuses of the CW radiative correction

- V_{eff} is gauge dependent: is it meaningful to extract physical quantities?
  For instance:
  $$\left.\frac{\partial^2 V_{\text{eff}}}{\partial \phi^2}\right|_{\phi_{\min}} \approx m_H^2 .$$

Solution!

Nielsen's identities and critical points

- Dangerous "hunting" imaginary part:

$$V^{(1)} \sim \ln \frac{m_i(\phi)^2}{\mu^2(t)}, \quad \text{but} \ldots \text{ some } m_i(\phi)^2 < 0\,!!$$

Solution!

$\hbar$-expansion (Andreassen, Frost, Schwartz, PRL 113(2014) 241801)

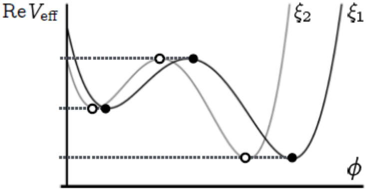
Gauge (in)dependence: Nielsen's identities

Nielsen, Nucl. Phys. B 101 (1975) 173-188

$$\xi \frac{\partial}{\partial \xi} V(\phi, \xi) = -C(\phi, \xi) \frac{\partial}{\partial \phi} V(\phi, \xi).$$

Variations w.r.t. gauge parameters are proportional to variations w.r.t. field.

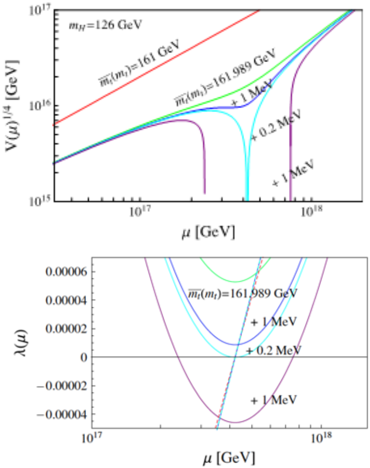
In other words, at critical points of V, the potential is gauge independent.



The top mass value at any stationary configuration m_t^s $(s = i, c)$ is a gauge-independent quantity

Outline

SM potential stationary configurations



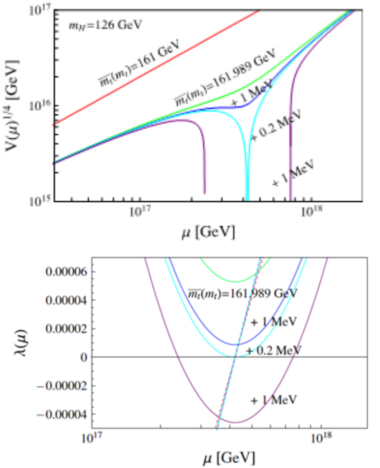
Masina, PRD 87(2013) 053001

As m_t increases, the potential is destabilized:

- stable
- stable with flex (*inflection point*) ⋆
- stability line (*degenerate vacua*) ⋆
- metastable
- deeper: unstable

$m_t^c \approx m_t^i$

SM potential stationary configurations



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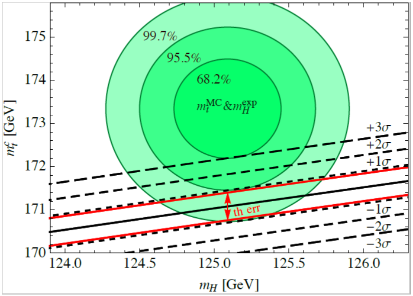
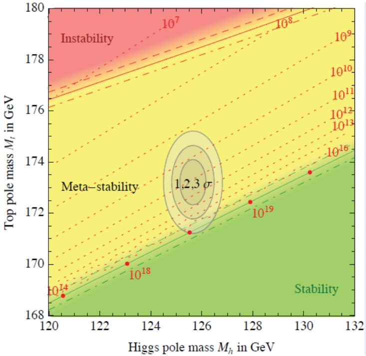
Stability results: degenerate vacua (1)



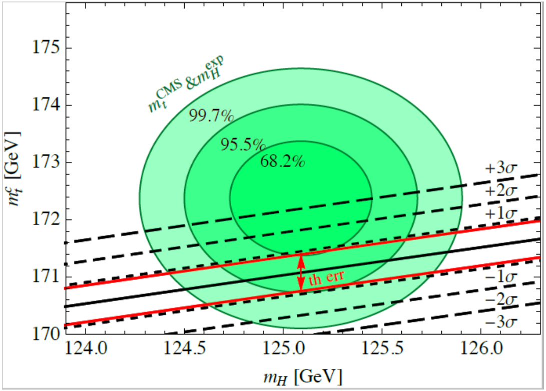


Only 1.5σ deviation from stability!

Buttazzo et al., JHEP 12(2013) 089

$$m_t^c = 171.08 \pm 0.37_{\alpha_s} \pm 0.12_{m_H} \pm 0.32_{\text{th}}\ GeV$$

Stability results: degenerate vacua (2)



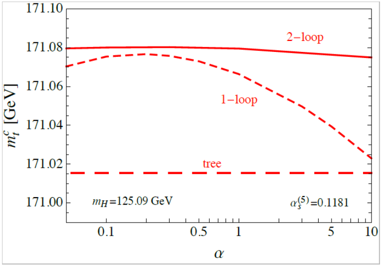
$$m_t^{\text{CMS}} = 172.38 \pm 0.66\, GeV$$

CMS, Rep. number: CMS-PAS-TOP-14-015(2014)

Stability would be excluded at less than 1σ

Degenerate vacua: theoretical uncertainties

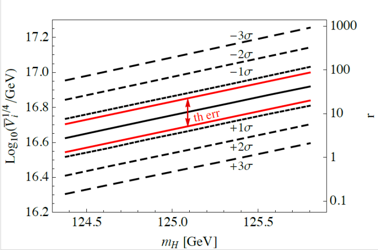
- NNLO matching (scale variation and truncation): $(\Delta\lambda)_{\rm th} \sim 1.6\sigma$ in m_H and $(\Delta y_t)_{\rm th} \sim 0.3\sigma$ in m_t $\rightarrow 0.32\,GeV$ in m_t^c (dominant one);
- Order of the β-functions in the RGE: error $\sim 10^{-5}$ (negligible);



- Truncation of the effective potential loop expansion at 2nd order: $\mu(t) = \alpha\phi(t)$

The higher the order, the less the dependence on α ($\sim \pm 5 \times 10^{-3}\,GeV$)

Inflection point configuration: results

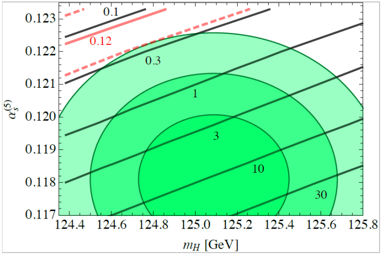


$$V = \frac{3\pi^2 A_s}{2} r$$

The potential spans one order of magnitude for decreasing α_s: dramatic variation of r

A tension of at least 3σ appears: all the false vacuum inflationary models seem to be ruled out

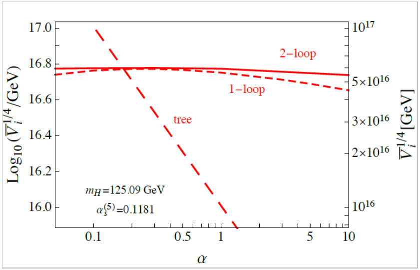
Exclusion should be less stronger than Ballesteros, Tamarit, JHEP 09(2015) 210



$$\log_{10} \bar{V}_i^{1/4} = 16.77 \pm 0.11_{\alpha_s} \pm 0.05_{m_H} \pm 0.08_{\mathrm{th}}$$

Inflection point: theoretical uncertainties

- NNLO matching (scale variation and truncation): $(\Delta\lambda)_{\text{th}} \sim 0.08\, GeV$ on $\bar{V}_i$ and $(\Delta y_t)_{\text{th}}$ has no significant impact on $\bar{V}_i \rightarrow 0.08\, GeV$ on $\bar{V}_i$;
- Order of the β-functions in the RGE: $\bar{V}_i^{1/4}$ changes at the per mille level (negligible);



- Now the dependence at tree-level is implicit, but significant (one order of magnitude). The 1-loop and 2-loop flatten the potential and make the uncertainty respectively of 20% and 5%

Outline

Conclusions

Most updated and detailed study of *gauge-independent observables* associated with m_t^c (which ensures stability) and m_t^i (SM Higgs potential at rising inflection point)

1. Stability of the SM is *compatible with present data at the level of* 1.5σ: it is still a viable possibility. *Higher precision measurements* of the top quark pole mass and α_s would be needed.
2. *False vacuum inflationary models* (Higgs scalar rolling down along an inflection point configuration) display a 3σ tension with the PLANCK bounds on the tensor-to-scalar ratio r.

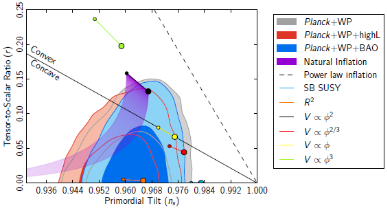
Thank you.

Outline

General inflationary parameters: slow-roll

1. scalar spectral index: $n_s = 1 - 6\epsilon + 2\eta$;
2. tensor-to-scalar ratio: $r = \mathcal{P}_t/\mathcal{P}_s \simeq 16\epsilon$;
3. number of e-folds:

$$N \equiv \ln\left(\frac{a_f}{a_i}\right) = \lambda_P^2 \int_{\phi_{end}}^{\phi_{CMB}} \frac{V}{V_\phi} d\phi \sim 50 \div 60$$



Planck collaboration, arXiv: 1303.5062

Inflationary scale

$$V_{infl} = 1.94 \times 10^{16}\, GeV \left(\frac{r}{0.12}\right)^{1/4}$$

Amplitude of scalar perturbations (slow-roll approx)

$$A_s \simeq \frac{V}{24\pi^2 \epsilon M_P^4}$$

Matching procedure (1)

- The matching for the gauge couplings is performed at the Z boson pole mass m_Z: the correction to the numerical values for the related $\overline{MS}$ observables (from PDG) is very small and can be neglected;
- The matching between $\lambda_h(\mu)$ with the Higgs pole mass m_h is given by:

$$\lambda_h(\mu) = \frac{1}{2}\frac{m_h^2}{v^2}\left(1 + \delta_h^{(1)}(\mu) + \delta_h^{(2)}(\mu) + \dots\right)$$

known at NLO: $\delta_h^{(1)}(\mu)$ is $\mathcal{O}(\alpha)$, while $\delta_h^{(2)}(\mu)$ is a Yukawa contribution and a QCD contribution $(\mathcal{O}(\alpha\alpha_3), \mathcal{O}(\alpha_s^4))$. “Theoretical” uncertainty is 0.7% at 2-loop:

$$\lambda_h(m_h) = 0.8065 + 0.0109(m_h[\text{GeV}] - 126) + \\ + 0.0015(m_t[\text{GeV}] - 172)^{+0.0002}_{-0.0060}$$

Matching procedure (2)

- Extrapolation of the $y_t(\mu)$ from the matching between the running top mass $\overline{m_t}(\mu)$ and the top pole mass m_t:

$$y_t(\mu)\frac{v}{\sqrt{2}} = \bar{m}_t(\mu) = m_t\left(1 + \delta_t^W(\mu) + \delta_t^{QED}(\mu) + \delta_t^{QCD}(\mu)\right)$$

known at NLO: $\delta_t^W(\mu) + \delta_t^{QED}(\mu)$ represent the EW contribution (at 2-loop), while $\delta_t^{QCD}(\mu)$ is the QCD (at 3-loop).
"Theoretical" uncertainty is related to the choice of μ, 2% at 2-loop:

$$y_t(m_t) = 0.933 + 0.006(m_t[\mathrm{GeV}] - 172)^{+0.017}_{-0.013}$$

SM two-loop effective potential[2]

Coleman-Weinberg correction[1]

$$V_{\rm eff}(\phi) = V^{(0)}(\phi) + V^{(1)}(\phi) + V^{(2)}(\phi) \equiv \frac{1}{4}\lambda_{\rm eff}(\mu)\phi^4,$$

$$V^{(1)}(\phi) = \sum_{i=W^\pm,Z,t} \frac{n_i}{4(4\pi)^2} m_i(\phi)^4 \left[\ln \frac{m_i(\phi)^2}{\mu^2(t)} - C_i\right],$$

$$C_{W^\pm} = C_Z = \frac{5}{6}, \quad C_t = \frac{3}{2}, \quad n_{W^\pm} = 6, \quad n_Z = 3, \quad n_t = -12$$

$$m_i(t)^2 = k_i \phi(t)^2, \qquad \mu(t) = m_Z e^t$$

$$\phi(t) = \xi(t)\phi_{cl}, \qquad \xi(t) \equiv e^{-\int_0^t \gamma(\tau)d\tau}$$

$$k_{W^\pm} = \frac{1}{4}g(t)^2, \quad k_Z = \frac{1}{4}\left[g(t)^2 + g'(t)^2\right], \quad k_t = \frac{1}{2}\phi(t)^2$$

[1] S. Coleman, E. Weinberg, Phys. Rev. D7, 1888 (1973).

[2] 't *Hooft-Landau* gauge and $\overline{MS}$ renormalization scheme.

Anomalous dimension

Dilatation in a scale-invariant QFT: $x \to \lambda x$,
each operator acquires a factor $\lambda^{-\Delta}$,
with Δ called *scaling dimension of the operator*

Free theories Δ_0 from dimensional analysis (classical one);

Interacting fields $\Delta = \Delta_0 + \gamma(g)$, where $\gamma(g)$ is the anomalous dimension[3]: the scale invariance is spoiled at quantum level
(or, in some cases, preserved approximatey over long distances).

Higgs field case

$$\Gamma(\mu) \equiv \int_{m_t}^{\mu} \gamma(\mu') d \ln \mu', \qquad \gamma(g) = -\frac{d \ln h}{d \ln \mu}$$

This quantity is independent by the cut-off of the theory but not by the gauge.

[3] It is generally expressed by power series in the couplings, with their running in energy.

$\hbar$ - expansion method (1)

(H. Patel, M. J. Ramsey-Musolf, JHEP 1107, 029 (2011) and also[5])

- $\hbar$ counts the number of loops, the effective potential is truncated to order $\hbar$ at NLO and $\hbar^2$ at NNLO, with a $\lambda \sim \hbar$ power counting[4].
- Effective potential will be a series in $\hbar$:

$$V_{\rm eff}(\phi) = V^{(0)}(\phi) + \hbar V^{(1)}(\phi) + \hbar^2 V^{(2)}(\phi) + \ldots \rightarrow$$
$$\phi_{\rm min} = \phi^{(0)} + \hbar\phi^{(1)} + \hbar^2\phi^{(2)} + \ldots,$$

where $\phi^{(0)}$ is the tree-level vev v and the others are the quantum corrections δv.

Inserting into the minimization condition $V'_{\rm eff}\Big|_{\phi_{\rm min}} = 0$:

$$V'_{\rm eff}(\phi_{\rm min}) = V'^{(0)}(\phi^{(0)} + \hbar\phi^{(1)} + \ldots) + V'^{(1)}(\ldots) + \ldots =$$
$$= V'^{(0)}(\phi^{(0)}) + \hbar[V'^{(1)}(\phi^{(0)}) + \phi^{(1)}V''^{(2)}(\phi^{(0)})] = 0$$

[4] Be careful to terms scaling like the inverse power of $\hbar$.

[5] A. Andreassen, W. Frost, D. Schwartz, arXiv: 1408.0292.

$\hbar$ - expansion method (2)

- Each power of $\hbar$ must satisfy the equality:

$$\mathcal{O}(1): \quad V'^{(0)} = 0 \quad \text{tree-level vev}$$

$$\mathcal{O}(\hbar): \quad \phi^{(1)} = -V''^{(0)}(\phi^{(0)})^{-1} V'^{(1)}(\phi^{(0)}) \quad \text{1-loop}$$

$$\mathcal{O}(\hbar^2): \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad \text{2-loop}$$

Vacuum energy

$$\varepsilon = V^{(0)}(\phi^{(0)}) + \hbar V^{(1)}(\phi^{(0)}) + \hbar^2 \left(V^{(2)}(\phi^{(0)}) - \frac{1}{2} \frac{V'^{(1)}(\phi^{(0)})^2}{V''^{(2)}(\phi^{(0)})} \right) + \ldots$$

ε depends only on extremal gauge-independent objects

- It can be applied also to *VEVs (δv), Masses, CW corrections, RG-improved vacua, ...*

Proof: gauge independence of m_t^s (1)

Absolute stability ($\phi_c > \phi_{ew}$)

$$\left.\frac{\partial V_{\text{eff}}}{\partial \phi}\right|_{\phi_{ew}, m_t^c} = \left.\frac{\partial V_{\text{eff}}}{\partial \phi}\right|_{\phi_c, m_t^c} = 0\,,\; V_{\text{eff}}(\phi_{ew}, m_t^c; \xi) = V_{\text{eff}}(\phi_c, m_t^c; \xi)$$

Inflection point ($\phi_i > \phi_{ew}$)

$$\left.\frac{\partial V_{\text{eff}}}{\partial \phi}\right|_{\phi_{ew}, m_t^i} = \left.\frac{\partial V_{\text{eff}}}{\partial \phi}\right|_{\phi_i, m_t^i} = 0\,,\quad \left.\frac{\partial^2 V_{\text{eff}}}{\partial \phi^2}\right|_{\phi_i, m_t^i} = 0$$

Due to Nielsen's identities

$$\left.\frac{\partial V_{\text{eff}}(\phi,\xi)}{\partial \phi}\right|_{\phi_s, m_t} = 0 \rightarrow \left.\frac{\partial V_{\text{eff}}(\phi,\xi)}{\partial \xi}\right|_{\phi_s, m_t} = 0$$

so $\quad V_{\text{eff}}(\phi_s, m_t^s; \xi) = V_{\text{eff}}(\phi_s^L, m_t^s; 0) \equiv \bar{V}_s$

Proof: gauge independence of m_t^s (2)

Inverting $V_{\text{eff}}(\phi_s, m_t^s; \xi) = V_{\text{eff}}(\phi_s^L, m_t^s; 0) \equiv \bar{V}_s$
would give a gauge-dependent field and top mass
$\phi_s = \phi_s(\xi)$ and $m_t^s = m_t^s(\xi)$
Applying a total derivative w.r.t. ξ

$$\left.\frac{\partial V_{\text{eff}}}{\partial \xi}\right|_{\phi_s, m_t^s} + \left.\frac{\partial V_{\text{eff}}}{\partial m_t}\right|_{\phi_s, m_t^s} \frac{\partial m_t^s}{\partial \xi} + \left.\frac{\partial V_{\text{eff}}}{\partial \phi}\right|_{\phi_s, m_t^s} \frac{\partial \phi_s}{\partial \xi} = 0$$

third and first term vanish because of stationary condition and Nielsen identity respectively.
Since in general $\left.\frac{\partial V_{\text{eff}}}{\partial m_t}\right|_{\phi_s, m_t^s} \neq 0$, we obtain that

$$\frac{\partial m_t^s}{\partial \xi} = 0$$

$\phi \to h$: Higgs inflation? (1)

EW stability and inflection point

Giuseppe Iacobellis

Who's the scalar field which drives inflation?

Minimal choice: the only scalar in SM, the Higgs field!

Main issue

The Higgs potential is not flat

$$V_0 = \lambda_h \left(\mathcal{H}^\dagger \mathcal{H} - \frac{v^2}{2} \right)^2$$

Electroweak (EW) scale: $v \simeq 246\, GeV$.

Higgs mass: $m_h \equiv \sqrt{2v^2\lambda_h} \simeq 125.1\, GeV$.

Extrapolation of the high-energy behaviour is needed!

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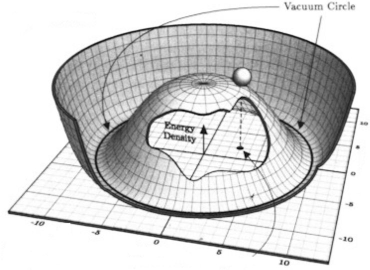
$\phi \to h$: Higgs inflation? (2)

Unitary gauge

$$\mathcal{H}^T = \begin{pmatrix} 0 & (h+v)/\sqrt{2} \end{pmatrix}$$



Guth, PRD 23(1981) 347

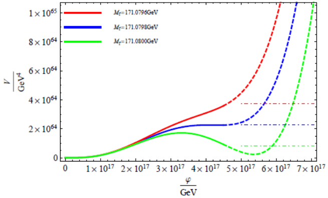
λ_h quartic coupling constant

For large field values

$$V_0 \sim \lambda_h h^4$$

Pure SM inflation

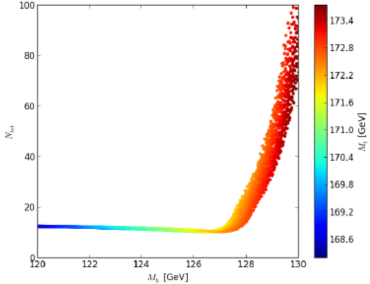
1. Sufficient e-folds N;
2. Correct A_s;
3. Power spectrum nearly scale invariant.



Y. Hamada et al., arXiv: 1308.6651

- For $m_h \simeq 126\,GeV \Rightarrow$ too low N_{tot}
- If N_{tot} correct, wrong A_s: no slow-roll?

Maybe the Higgs is not responsible of both inflation and scalar perturbations.



M. Fairbairn et al., arXiv: 1403.7483

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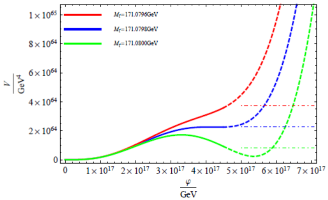
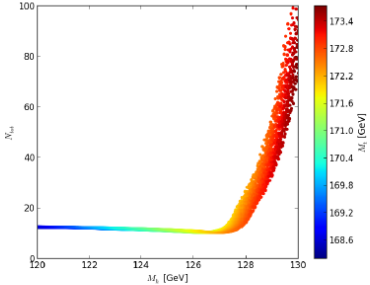


Y. Hamada et al., arXiv: 1308.6651



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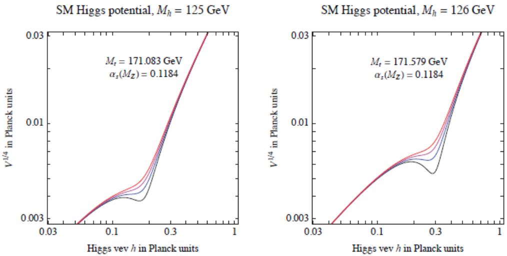
False vacuum inflation (1)[6]

- Tuning the top quark mass, it is possible to obtain a shallow local minimum at large field values (stability required);
- The Higgs boson sitting in this false vacuum would provide exponential inflation and then could tunnel to the EW one;
- The model needs another scalar responsible of scalar perturbations and a mechanism (tunnelling) for escaping from inflationary phase (graceful exit).



[6] I. Masina, A. Notari, arXiv: 1112.2659.

Real scalar singlet and right-handed neutrino: $U(1)_{B-L}$

Global Lagrangian

$$\mathcal{L} = \mathcal{L}_{\mathcal{SM}} + \mathcal{L}_{\mathcal{S}} + \mathcal{L}_{\mathcal{N}} \,,$$

$$\mathcal{L}_{\mathcal{S}} = -\frac{m_s^2}{2} s^2 - \frac{\lambda_{\phi s}}{2} \left|\mathcal{H}\right|^2 s^2 - \frac{\lambda_s}{24} s^4 + (\text{kinetic terms}) \,,$$

$$\mathcal{L}_{\mathcal{N}} = \left(\frac{M_N}{2} \bar{N}^c N + h_\nu \bar{L}_\alpha \mathcal{H} N + \text{c.c.} \right) + (\text{kinetic terms})$$

I-type seesaw mechanism

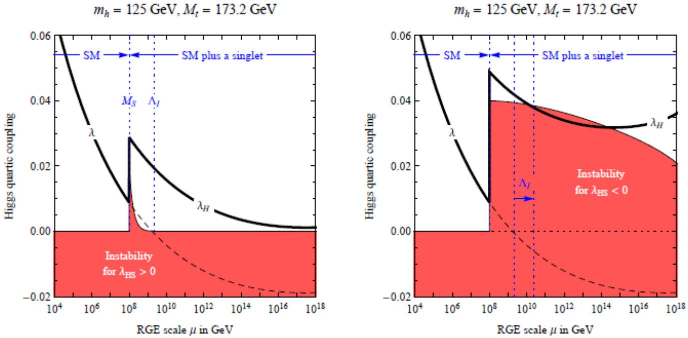
$$m_\nu = h_\nu \frac{v^2}{M_N}, \qquad M_N \gg v$$

Other generations can be generated by lighter right-handed neutrinos

- Z_2 symmetry
- $m_s <$ instability scale
- tree-level threshold effect:

$$\lambda = \lambda_\phi - \frac{\lambda_{\phi s}^2}{\lambda_s}$$

Extended running[7]



Under evaluation...

[7] J. Elias-Miró, J. R. Espinosa, G. F. Giudice, H. M. Lee, A. Strumia, JHEP 1206 (2012) 031.

Gravitational corrections

Near the cutoff of the theory, large Planckian effects are possibile: our *ignorance about the UV completion* of the theory could be parametrized through an effective field theory approach

$$V(\phi) = \frac{\lambda}{24}\phi^4 + \frac{\lambda_6}{6}\frac{\phi^6}{M_P^2} + \frac{\lambda_8}{8}\frac{\phi^8}{M_P^4} + \mathcal{O}\left(\frac{\phi^{10}}{M_P^6}\right)$$

The impact of gravitational effects is largely dependent on the free couplings towards stability or metastability

The effective theory expansion breaks down when $\phi \sim M_P$:

the use of an effective theory close to its cutoff might not be fully reliable