On-shell helicity methods for soft-collinear effective field theories

Angelo Raffaele Fazio

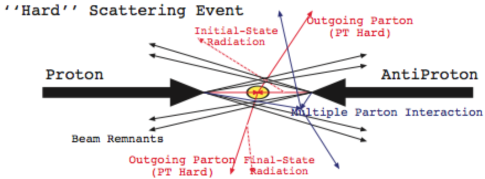
Departamento de Física, Universidad Nacional de Colombia, sede Bogotà

Outline

Introduction

Measurements look for hard leptons and jets to probe the hard interactions

- The underlying hard process is always accompanied by collinear initial state radiation, final state radiation and underlying soft radiation events.



- Almost never one measures a total cross section.
- Jet selection cuts can be sensitive to additional soft and collinear emissions (or any other type of exclusive cut).
  In perturbation theory this sensitivity to lower scales manifests itself via large logarithms needed to be resummed.

Perturbative Structure of jet Cross Section

The total cross section $\sigma_{\rm total}$, is divided into a 0-jet exclusive cross-section, $\sigma_0(p_T^{\rm cut})$, and the $(N \geq 1)$-jet inclusive cross section, $\sigma_{\geq 1}(p_T^{\rm cut})$,

$$\sigma_{\rm total} = \int_0^{p_T^{\rm cut}} dp \frac{d\sigma}{dp} + \int_{p_T^{\rm cut}}^{+\infty} dp \frac{d\sigma}{dp} \equiv \sigma_0(p_T^{\rm cut}) + \sigma_{\geq 1}(p_T^{\rm cut})$$

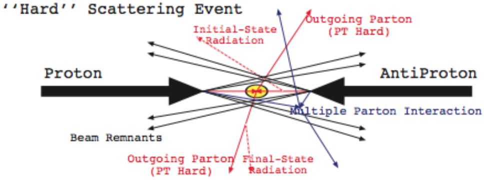
$$\frac{\sigma_{\rm total}}{\sigma_B} = 1 + \alpha_S + \alpha_S^2 + \dots$$

$$\sigma_{\geq 1}(p_T^{\rm cut}) = \alpha_S(1 + L + L^2) + \alpha_S^2(L^4 + L^3 + L^2 + L + 1) + \dots$$

where $L = \log\left(\frac{p_T^{\rm cut}}{Q}\right)$.

Soft Collinear Effective Field Theories (SCET) resummation

[Bauer, Fleming, Pirjol, Stewart; Rothstein, Beneke, Chapovsky, Diehl, Feldmann]
An effective theory of QCD with which we can study what goes on before and after the hard interaction
SOFT: low energy particles without a preferred direction



Collinear: energetic jets along incoming and outgoing directions.

- In SCET the soft-collinear limit of QCD is manifestly implemented at a Lagrangian and operator level using a systematic power expansion.
- SCET disentangle the relevant scales in jet production and resum the associated logarithms of ratios of these scales via renormalization group evolution.
- The QCD corrections appearing at the hard-interaction scale, i.e., away from any infrared singular limits, which contain the process-specific details, are incorporated via matching from QCD onto SCET.
- On the other hand, the infrared-sensitive QCD dynamics below the hard interaction scale that describes the collinear radiation within the jets and soft interactions between jets is contained in the effective theory.

Factorization of Exclusive Jet Cross Section

Contributions appear at different physical energy scales $\rightarrow$ Factorization

$$d\sigma = \text{hard interactions} \otimes \mathit{PDFs} \otimes \mathit{ISR} \otimes \mathit{FSR} \otimes \text{soft radiation}$$

$$d\sigma = H_N \times \left[(f_{a,b} \otimes I_{a,b}) \times \Pi_{j=1}^{N} J_j \right] \otimes S_N \qquad (1)$$

- SCET allow to derive factorized cross section
  - Each function has a precise definition in the effective field theory.
  - RG evolution between scales resums logarithms of ratios of scales.
- Beam, jets and soft functions
  - contain virtual and integrated real emission corrections in all IR-singular limits. They are IR finite.
  - Depend on the jet definition observable.
  - Only process dependence from parton type and number.

The Hard function

- Contains hard virtual corrections.
- Independent of jet definition/observable and precise form of factorization theorem.
- Contains explicit process dependence and hard kinematics.

Matching from QCD to SCET

$$L_{\rm eff} = L_{\rm SCET} + \sum_k C_k O_k$$

$$A_{\rm QCD} \stackrel{!}{=} A_{\rm SCET} = \sum_k iC_k < O_k >_{\rm SCET}$$

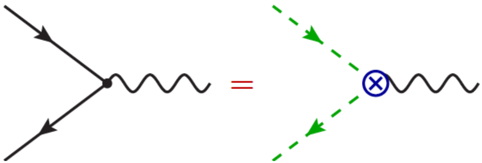
Wilson coefficients

- follow from matching amplitudes in full and effective theory;
- do not depend on IR regulator (but must be the same in QCD and SCET);
- do depend on renormalization scheme used for the SCET operators O_k (we will use dimensional regularization with MS);
- determine the Hard function

$$\sigma \sim |A|^2 = \sum_k |C_k|^2 | < O_k >_{SCET} |^2$$

Schematic matching at tree level

Consider the Drell-Yan simplest case:



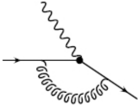
The matching condition is

$$< QCD|q\bar{q} >_Q = [C_2(\mu) < O_2|q\bar{q} >]_{\mu=Q} = C_2(Q)$$

if the chosen normalization operator is $< O_2|q\bar{q} >_{\mu=Q} = 1$ at tree level.

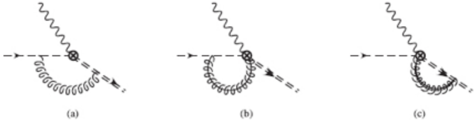
Schematic matching at one-loop

We compute matrix elements of the full QED-QCD current and the QED-SCET current to one-loop order in α_S. In QCD



$$< j^\mu >_{QCD} = \gamma^\mu \left[1 + \frac{\alpha_S C_F}{4\pi} \left(-\frac{2}{\varepsilon^2} - \frac{1}{\varepsilon} \left(2 \log \frac{\mu^2}{Q^2} + 3 \right) - \log^2 \frac{\mu^2}{Q^2} - 3 \log \frac{\mu^2}{Q^2} - 8 + \frac{\pi^2}{6} \right) \right] \quad (2)$$

In SCET the corresponding matrix element involves three nul diagrams



- The matching coefficient corresponds to the finite part of $< j^\mu >_{QCD}$, meaning up to one-loop

$$C_2(Q) = 1 + \frac{\alpha_S C_F}{4\pi}\left(-8 + \frac{\pi^2}{6}\right) \quad (3)$$

  to avoid troublesome logarithms the matching has been performed at the scale $\mu = Q$.
- The IR-divergent terms (cancelled by the UV divergent terms) are reproduced by the matrix elements in both theories.
- The SCET UV-divergences are the opposite of the IR-QCD divergences, from matching calculations we can simply read off the SCET counterterm and activate the RGE machine for the resummation.

Building an appropriate operator basis

The non trivial task is to construct a convenient operator basis

- Number of possible spin and color structures quickly proliferates when increasing the number of legs.
- Incorporate constraints from charge conjugation and parity invariance.
- Stay crossing symmetric.

Use the same helicity and color decompositions as for the amplitudes also to construct the operator basis in SCET.

- Yields an operator basis in SCET that is in one-to-one correspondence with the independent color-ordered helicity amplitudes.
- The SCET Wilson coefficients are directly equivalent to the IR-finite parts of the amplitudes

$$C_{(\pm,\pm,.....)} = A_{\text{finite}}(1^{\pm}, 2^{\pm}, ...)$$

Collinear Fields in SCET

SCET operators are built from collinear gauge invariant quark and gluon fields

$$O^{\mu}_{\bar{q}q}(x) = \bar{\chi}^{\alpha}_{n_1,\omega_1}(x)\gamma^{\mu}\chi^{\alpha}_{n_2,\omega_2}(x)$$
$$O^{\mu\nu}(x) = \mathcal{B}^{a\,\mu}_{n_1,\omega_1}(x)\mathcal{B}^{a\,\nu}_{n_2,\omega_2}(x)$$

Fields carry fixed large momentum indicated by their labels

$$\tilde{p}_i = \omega_i \frac{n_i}{2} \quad \text{with} \quad n_i = (1, \hat{n}_i), \quad \bar{n}_i = (1, -\hat{n}_i) \quad \hat{n}_i^2 = 1$$

Summing over operators really means integrating over field labels,

$$\sum_k C_k O_k \to \int d\tilde{p}_1 d\tilde{p}_2 C_{q\bar{q},\mu}(\tilde{p}_1, \tilde{p}_2) O^{\mu}(\tilde{p}_1, \tilde{p}_2)$$

Helicity fields

Use standard spinor representation for polarization vectors

$$\epsilon_+^\mu = \frac{< p + |\gamma^\mu| k + >}{\sqrt{2} < kp >} \quad \epsilon_-^\mu = -\frac{< p - |\gamma^\mu| k - >}{\sqrt{2}[kp]}$$

Define collinear gluon field and $q\bar{q}$-current of definite helicity

$$\mathcal{B}_{i,\pm}^a = -\epsilon_{\mp\mu}(n_i, \bar{n}_i)\mathcal{B}_{n_i,\omega_i}^{a\mu}$$

$$J_{ij\pm}^{\alpha\beta} = \mp\epsilon_\mp^\mu(\tilde{p}_i, \tilde{p}_j)\frac{< \bar{\chi}_{n_i,-\omega_i}^\alpha \pm |\gamma_\mu| \chi_{n_j,\omega_j}^\beta \pm >}{\sqrt{2} < \tilde{p}_j \mp |\tilde{p}_i \pm >}$$

with tree level Feynman rules

$$< g_\pm^a(p)|\mathcal{B}_{i\pm}^b|0 >= \delta^{ab}\tilde{\delta}(\tilde{p}_i - p) \quad < g_\mp^a(p)|\mathcal{B}_{i\pm}^b|0 >= 0$$

$$< q_\pm^{\alpha_1}\bar{q}_\mp^{\alpha_2}|J_{12\pm}^{\beta_1\beta_2}|0 >= \delta^{\alpha_1\beta_1}\delta^{\alpha_2\beta_2}\tilde{\delta}(\tilde{p}_1 - p_1)\tilde{\delta}(\tilde{p}_2 - p_2)$$

Helicity Operator Basis

Assemble helicity fields into helicity operators for each helicity configuration (S is a symmetry factor for identical particles)

$$O^{a_1 a_2 \cdots \alpha_{i-1}\alpha_i \cdots \alpha_{n-1}\alpha_n}(\tilde{p}_1, \tilde{p}_2, \cdots \tilde{p}_{i-1}, \tilde{p}_i \cdots \tilde{p}_{n-1}, \tilde{p}_n) =$$

$$S \mathcal{B}^{a_1}_{1\pm} \mathcal{B}^{a_2}_{2\pm} \cdots J^{\alpha_{i-1}\alpha_i}_{i-1,i\pm} \cdots J^{\alpha_{n-1}\alpha_n}_{n-1,n\pm}$$

$$\mathcal{L}_{eff} = \mathcal{L}_{SCET} + \sum_{hel} \int \Pi_{i=1}^{n} d\tilde{p}_i \, C^{a_1 \cdots \alpha_n}_{hel}(\tilde{p}_1, \cdots \tilde{p}_n) O^{a_1 \cdots \alpha_n}_{hel}(\tilde{p}_1, \cdots \tilde{p}_n)$$

A tree level scattering amplitude in SCET projects out single Wilson coefficient for a given helicity configuration

$$< g^{a_1}_{+}(p_1) g^{a_2}_{-}(p_2) \cdots q^{\alpha_{n-1}}_{-}(p_{n-1}) \bar{q}^{\alpha_n}_{+}(p_n) | \mathcal{L}_{\text{eff}} | 0 >^{\text{tree}}_{\text{SCET}} =$$

$$C^{a_1 a_2 \cdots \alpha_n}_{+-\cdots(\cdots -)}(p_1, p_2, \cdots p_n)$$

Color decomposition

Pick a complete basis of color-singlet structures to decompose coefficients

$$C^{a_1 a_2 \cdots \alpha_{n-1} \alpha_n}_{+-\cdots(\cdots-)} = \vec{T}^{\dagger a_1 a_2 \cdots \alpha_{n-1} \alpha_n} \cdot \vec{C}_{+-\cdots(\cdots-)}$$

for instance

$$\vec{T}^{\dagger \alpha\beta} = \delta^{\alpha\beta} \qquad \vec{T}^{\dagger ab} = \delta^{ab}$$

$$\vec{T}^{\dagger a\alpha\beta} = T^a{}_{\alpha\beta} \quad \vec{T}^{\dagger abc} = if^{abc}$$

$$\vec{T}^{\alpha\beta\gamma\delta} = (\delta_{\alpha\delta}\delta_{\gamma\beta}, \delta_{\alpha\beta}\delta_{\gamma\delta})$$

$$(\vec{T}^{ab\alpha\beta} = (T^a T^b)_{\alpha\beta}, (T^b T^a)_{\alpha\beta}, \mathrm{tr}[T^a T^b]\delta_{\alpha\beta})$$

Using the same basis as in the color decomposition of the amplitudes

$$A_{QCD}(g_+ g_- \cdots q_- \bar{q}_+) = \sum_k \vec{T}_k^{\dagger a_1 a_2 \cdots \alpha_{n-1} \alpha_n} A_k(1^+ 2^- \cdots n_{\bar{q}}^+)$$

MS-bar Wilson coefficients are equal to the color ordered amplitudes to all orders.

$$\vec{C}^k_{+-\cdots(\cdots-)}(p_1, p_2, \cdots p_{n-1}, p_n) = A^k_{\text{finite}}(1^+, 2^-, \cdots n_{\bar{q}}^+)$$

An example: $ggq\bar{q}H$

Operator basis (+ charge conjugated)

$$O^{ab\alpha\beta}_{++(+)} = \frac{1}{2}\mathcal{B}^a_{1+}\mathcal{B}^b_{2+}J^{\alpha\beta}_{34+}H$$

$$O^{ab\alpha\beta}_{+-(+)} = \mathcal{B}^a_{1+}\mathcal{B}^b_{2-}J^{\alpha\beta}_{34+}H$$

$$O^{ab\alpha\beta}_{--(+)} = \frac{1}{2}\mathcal{B}^a_{1-}\mathcal{B}^b_{2-}J^{\alpha\beta}_{34+}H$$

The color decomposition of QCD amplitude

$$A(g_{1\pm}, g_{2\pm}, q_{3+}, \bar{q}_{4-}, H) =$$

$$i\sum_{\sigma\in S_2}[T^{a_{\sigma(1)}}T^{a_{\sigma(2)}}]_{\alpha_3\alpha_4}A(\sigma(1^{\pm})), \sigma(2^{\pm}), 3^+_q, 4^-_{\bar{q}}5_H)$$

$$+i\mathrm{tr}[T^{a_1}T^{a_2}]\delta_{\alpha_3\alpha_4}B(1^{\pm}, 2^{\pm}, 3^+_q, 4^-_{\bar{q}}5_H)$$

The resulting Wilson coefficients are

$$\vec{C}_{\pm,\pm,(+)} = \begin{pmatrix} A_{\mathrm{finite}}(1^{\pm}, 2^{\pm}; 3^+_q, 4^-_{\bar{q}}; 5_H) \\ A_{\mathrm{finite}}(2^{\pm}, 1^{\pm}; 3^+_q, 4^-_{\bar{q}}; 5_H) \\ B_{\mathrm{finite}}(1^{\pm}, 2^{\pm}; 3^+_q, 4^-_{\bar{q}}; 5_H) \end{pmatrix}$$

Conclusions

- It is possible to construct a helicity operator basis in SCET such that the IR-finite parts of the color-ordered partial amplitudes are the hard matching coefficients.
- The different color structures in

$$\vec{O}^{\dagger}_{+\cdots(\cdots-)} = O^{a_1\cdots\alpha_N}_{+\cdots(\cdots-)} \vec{T}^{\dagger a_1\cdots\alpha_N}$$

  mixes under renormalization. Including the RGE running of the hard coefficients, the resummed cross section has the color structure

$$\sigma_N \sim \vec{C}^{\dagger} \cdot \hat{U}_H^{\dagger} \cdot \left[(B_a B_b \Pi_j J_j) \otimes \hat{S} \right] \cdot \hat{U}_H \cdot \vec{C}$$

  the hard evolution factors $\hat{U}_H$ as well as the soft function $\hat{S}$ are matrices in color space which depend on the kinematics.
- Individual color-ordered amplitudes are needed, not only their sum.
- Using resummation virtual amplitudes can be used to get physical cross sections without expensive integrations over real emission.